AJAE APPENDIX: TOURNAMENTS, FAIRNESS, AND RISK

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The Contracting Experiments in Detail

The experimental design of the contracting sessions builds upon BSW and Schotter and Weigelt (SW), and is nearly identical to those of Wu and Roe (WR), and Wu, et. al. Thus, we will not provide a detailed treatment of the underlying mathematics, and instead refer the reader to (WR), and Wu et. al. Here we will focus on providing the reader with only enough detail to conceptualize the experiments.

In each contracting session, subjects play 10 identical rounds of each contracting game. In each round, subjects begin by choosing a costly "decision number" (effort) from 0 to 100. Higher effort levels costs more and once a subject chooses an effort level, she looks up the cost of that effort level from a table distributed to her. She subsequently records this cost on a worksheet. An example of a cost table is shown on page 14. All subjects are informed of all experimental parameters, including opponents' cost tables. In fact, during asymmetric cost experiments, both the low and high cost tables are projected onto a screen during the experiments and each low cost subject knows that her pair member has the high cost table and vice versa. Only the identity of pair members is unknown. Next, a subject is randomly chosen to draw a "common shock" number from a bucket and all subjects add this to their decision numbers. Then each subject randomly draws a personal "idiosyncratic shock" number from another bucket and adds this number to her decision number and the common shock number. Both the idiosyncratic and common shocks are normally distributed (but may have different variances) and copies of the probability distributions for all shocks are given to subjects before the experiment and explained in detail. The sum of the decision number, common shock and

idiosyncratic shock yields a subject's performance for the round. The mathematical

expression of performance in a two-player tournament is:

(A1) $y_i = e_i + u_C + u_i$ $i = 1, 2$

where y_i is performance, e_i is effort, u_C is a common shock, and u_i is an idiosyncratic

shock that is independently and identically distributed across i. We also assume that

$u_C \sim N(0, \sigma_C^2)$, $u_i \sim N(0, \sigma^2)$, $Cov(u_C, u_i) = 0$, and $Cov(u_i, u_j) = 0$, $\forall\ i \neq j$. The way we

implement the random shock in the experiments is to approximate each shock using 300

pennies each marked with an outcome. The frequency for each outcome is determined by

approximating the number of outcomes out of 300 that occur under a normal distribution.

For experiment 1, for example, we calculate the probability function in Excel for a

normal distribution with mean zero and standard deviation 15.8, and multiply the

probability for each outcome by 300 and round to the nearest integer. We then place the

300 pennies in a bucket and have subjects draw a penny for each shock.

In T sessions, administrators compare outputs of pair members; the pair member

with more (less) output receives high payment, R (low payment r), where $R > r$. That is,

if $y_i > y_j$, then player i receives a high payment, R, and player j receives a low payment, r,

and vice versa (ties are broken by coin flip). All subjects were informed of the specific

values of R and r at the beginning of each session. Each subject only knows her payment

and not the difference in output. This is consistent with the way many comparative

performance contracts work where growers/workers are informed about their rankings

but are not provided detailed information about competitors' performance.

In F sessions, administrators compare output to some fixed standard, y^*, which

we set to "41." If performance exceeds 41, the subject receives high payment R. If not,

she receives the low payment, *r*. We avoided the obvious choice of $y^* = 37$ so that subjects are not provided a focal point to which they might naturally gravitate. Instead, we choose $y^* = 41$ and then adjust our payment spread to ensure that 37 is the optimal choice. To end the round, subjects subtract effort-costs, which were determined at the beginning of the round when effort was chosen, from revenue (*R* or *r*), and record the result (profits) in a worksheet. At the end of round 10, subjects calculate cumulative payoffs. A session lasts about 20 minutes.

We ran a total of seven experiments, with three of the experiments involving asymmetric cost subjects where half the subjects were randomly assigned "high" effort-cost tables and the other half "low" effort-cost tables. And then each high cost subject was paired with a low cost subject for the T-sessions. For the F-sessions, no such pairing was necessary because each subject competes only against the fixed threshold y^*. In the remaining four symmetric cost experiments, all subjects were assigned identical effort-cost tables. This table can also be expressed mathematically as:

(A2) $\quad c_i(e_i) = \dfrac{e_i^2}{10,000}$ $\qquad\qquad e_i = 1, 2, ..., 100$

which is identical to the cost function used by BSW. In the asymmetric cost experiments, the cost structure was modified slightly to get:

(A3) $\quad c_i(e_i) = \dfrac{\alpha_i e_i^2}{10,000}$

Heterogeneity is created by letting the parameter α_i vary across agents. For high cost subjects, we set $\alpha_i = 1.5$, and for low cost subjects, we set $\alpha_i = 1$, which is identical to (A2). Examples of cost tables for the asymmetric cost experiments are provided on pages 18 and 19.

In order to calculate expected payoffs under T and F for a given subject, we first have to define the probabilities of "winning" under T and the probability of exceeding the threshold $y*$ under F. Under T, this is Prob($y_i > y_j$) = Prob($u_i - u_j > e_j - e_i$) = $1 - F(e_j - e_i)$ where $u_i - u_j \sim N(0, 2\sigma^2)$ and $F(\bullet)$ is the normal CDF of $u_i - u_j$. Under F, the probability that agent i will exceed $y*$ is $\text{Pr}ob(y_i > y*)$ = Prob($u_C + u_i > y* - e_i$) = 1 - $G(y*-e_i)$, where $u_C + u_i \sim N(0, \sigma_C^2 + \sigma^2)$ and $G(\bullet)$ is the normal cumulative density function of $u_C + u_i$. To make $F(\bullet)$ and $G(\bullet)$ operational, we had to choose specific values for σ_C^2 and σ^2 which are listed in table 1 in the *AJAE* article. Now it is straightforward to calculate the per-round expected payoffs. The expected payoff in the symmetric T-session is:

$$(A4) \quad E(\pi_i^T) = r + \left[1 - F(e_j - e_i)\right]\left[R - r\right] - \frac{\alpha_i e_i^2}{10,000} \qquad i = 1,2$$

where $\alpha_i = 1$ for the low cost subject and $\alpha_i = 1.5$ for the high cost subject. For symmetric cost experiments, $\alpha_i = 1$ for all subjects. Finally, the expected per-round payoff under the F-contract is

$$(A5) \quad E(\pi_i^F) = r + \left[1 - G(y*-e_i)\right]\left[R - r\right] - \frac{\alpha_i e_i^2}{10,000} \qquad i = 1,2$$

where $\alpha_i = 1$ for all subjects in the symmetric cost experiments. In the asymmetric cost experiments, $\alpha_i = 1$ or $\alpha_i = 1.5$ depending on whether subject i is high or low cost.

We can calculate actual numeric values for expected payoffs by choosing specific numeric values for σ_C^2, σ^2, α_i, R and r. Moreover, choosing specific values of these parameters should also affect optimizing behavior of our subjects; that is, the optimal

effort they choose under each of the contracts. For example, under the T contract, the two-players paired together will choose effort levels that simultaneously satisfy their first order conditions from (A4) which are:

(A6) $\quad \dfrac{\partial E(\pi_1^T)}{\partial e_1} = f(e_2 - e_1)[R - r] - \dfrac{2\alpha_1 e_1}{10,000} = 0$

(A7) $\quad \dfrac{\partial E(\pi_2^T)}{\partial e_2} = f(e_2 - e_1)[R - r] - \dfrac{2\alpha_2 e_2}{10,000} = 0$

Solving (A6) and (A7) simultaneously will yield the Nash equilibrium effort levels for the two players. While the tournament was repeated over 10 rounds, the theory is based on a static model. Such repetition is common in experimental practice because subjects make complex decisions. Moreover, the only subgame perfect Nash equilibrium to a finitely repeated game is the Nash equilibrium decision level to the one-shot game. Thus, predictions concerning equilibrium play were independent of finite repetition (BSW). Note that the solutions will depend on the specific numeric values of $R,\ r, \alpha_i,\ \sigma_C^2$, and σ^2. The variances σ_C^2 and σ^2 will affect the first order conditions via the normal density function $f(e_2 - e_1)$. The specific numeric values of σ_C^2, σ^2, α_i, R and r for the seven experiments are listed in table 1. Optimal effort under F is determined by the first order condition of (A5), which is:

(A8) $\quad \dfrac{\partial E(\pi_i^F)}{\partial e_i} = g(y* - e_i)[R - r] - \dfrac{2\alpha_i e_i}{10,000} = 0$

Solving (A8) for each subject $i,$ would yield the optimal effort level under F.

All first order conditions, (A6) and (A7), and (A8) were solved in Maple software program which yields solutions listed in column (1) of table 2 in the *AJAE* article. The Maple software programs are available upon request from the authors, but examples are

provided later in this document. For readers interested in more detail, we refer them to (WR), and Wu, et. al.

Contracting Under Inequity Aversion

Fehr and Schmidt (FS) argue that, while many people may adhere to standard assumptions and care only about material payoffs, a non-negligible fraction also care about relative payoffs. Inequity aversion means people will forgo some payment to increase the equity of payouts across subjects. Consider a simple version of FS's model:

(B1) $U_i(x) = x_i - \delta_i \max[x_j - x_i, 0] - \beta_i \max[x_i - x_j, 0]$ for $i \neq j$

where x_i is agent *i's* payoffs, and the index *j* denotes a reference agent. The second term in (B1) captures utility loss from *disadvantaged* inequity, while the third term captures *advantageous* inequity. FS assume $\delta_i > \beta_i$ or that agent *i's* utility loss is greater when agent *i* has the lower payoff. This utility structure implies that, *given* some absolute monetary payoff, agent *i's* utility is highest when payoffs are equal, and that people are more averse to disadvantaged inequity than advantaged inequity.

Grund and Sliwka used FS's insights to develop a model of tournaments where agents exhibit inequity aversion.   In a two-player tournament, if player *i* "wins", she will have an utility of $v_i^w = R - \beta_i[R - r] - \frac{\alpha_i e_i^2}{k}$ whereas if she "loses" her utility is

$v_i^L = r - \delta_i[R - r] - \frac{\alpha_i e_i^2}{k}$. Expected utility for agent *i* is then:

(B2)

$$E\left[U_{IE}^T\right] = r + \left[1 - F(e_j - e_i)\right][R - r] - \left[1 - F(e_j - e_i)\right]\beta_i[R - r] - F(e_j - e_i)\delta_i[R - r] - \frac{\alpha_i e_i^2}{10,000}$$

Comparing (B2) to (A4), one can see inequity aversion causes an expected utility loss in

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the amount $\left[1-F(e_j-e_i)\right]\beta_i\left[R-r\right]+F(e_j-e_i)\delta_i\left[R-r\right]$. If we assume that agents are

not averse to advantaged inequity, which is consistent with our econometric results, then

this loss reduces to just $F(e_j-e_i)\delta_i\left[R-r\right]$ because $\beta_i=0$.

Moving to F contracts, note that there are four scenarios that might be relevant.

Both agents can earn the high payment, which is not possible under T, resulting in no

inequity. Agent i can earn the high payment while agent j earns the low payment

resulting in advantaged inequity for agent i, and vice versa. Finally, both agents can earn

the low payment, which, again, is not possible under T, resulting in no inequity.

Mathematically, these four scenarios are expressed respectively as, $v_i^1=R-\dfrac{\alpha_i e_i^2}{k}$,

$v_i^2=R-\beta_i(R-r)-\dfrac{\alpha_i e_i^2}{k}$, $v_i^3=r-\delta_i(R-r)-\dfrac{\alpha_i e_i^2}{k}$, and $v_i^4=r-\dfrac{\alpha_i e_i^2}{k}$. Hence, we have:

(B3) $E\left[U_{IE}^F\right]=r+\left[1-G(y*-e_i)\right]\left[R-r\right]-\left[1-G(y*-e_i)\right]G(y*-e_j)\beta_i\left[R-r\right]$

$$-\left[1-G(y*-e_j)\right]G(y*-e_i)\delta_i(R-r)-\frac{\alpha_i e_i^2}{k}$$

The expected loss from inequity is thus $\left[1-G(y*-e_i)\right]G(y*-e_j)\beta_i\left[R-r\right]$

$+\left[1-G(y*-e_j)\right]G(y*-e_i)\delta_i(R-r)$. If $\beta_i=0$ so that the agent is not averse to

advantaged inequity, then this loss reduces to $\left[1-G(y*-e_j)\right]G(y*-e_i)\delta_i(R-r)$. Note

that the loss from disadvantaged inequity is not as severe under F as it is under T due to

the fact inequity will occur less frequently under F.

We now turn to an analysis of how incentives are affected by inequity. This also

provides an outline of how predicted effort levels under inequity aversion (table 2 in

AJAE article) were calculated. Using B2, we can calculate the first order conditions for

both players in a two-player tournament as:

(B4) $\quad \dfrac{\partial E\left[U_{IE}^{T}\right]}{\partial e_{1}} = f(e_{2}-e_{1})[R-r][1+\delta_{1}-\beta_{1}] - \dfrac{2\alpha_{1}e_{1}}{k} = 0$

(B5) $\quad \dfrac{\partial E\left[U_{IE}^{T}\right]}{\partial e_{1}} = f(e_{2}-e_{1})[R-r][1+\delta_{2}-\beta_{2}] - \dfrac{2\alpha_{2}e_{2}}{k} = 0$

Solving (B4) and (B5) simultaneously will yield the Nash equilibrium effort levels. The

main difference between the standard first-order conditions, (A6) and (A7) and the

inequity averse first-order conditions is the presence of the term $[1+\delta_{i}-\beta_{i}]$, which, by

the assumption that subjects dislike disadvantaged inequity more than advantaged

inequity, is greater than 1. Hence, effort will be greater for inequity averse agents. We

use Maple software to generate numeric solutions using the same parameter values for *R,*

r, α_{i}, σ_{C}^{2}, and σ^{2} as we used for the non-inequity averse simulations. We also had to

choose specific values for the parameters β_{i} and δ_{i}. We chose $\beta_{i} = 0$ (subjects not

averse to advantaged inequity, which is consistent with our econometric results) and

$\delta_{i} = 0.15$ (mild aversion to disadvantaged inequity). The results of these simulations are

contained in the parentheses in column 2 of table 2 in the *AJAE* article.

Equation (B3) can be used to generate first order conditions for the F-contract:

(B6) $\quad g(y^{*}-e_{i})(R-r)\left[1+(1-G(y^{*}-e_{j}))\delta_{i} - G(y^{*}-e_{j})\beta_{j}\right] - \dfrac{\alpha_{i}e_{i}^{2}}{k} = 0.$

Although disadvantaged inequity still affects effort under F, the impact is scaled down by

the probability that another subject will receive a different payoff, so that the impact of

inequity is less under F than T. Numerical solutions for optimal effort are also

calculated using Maple and all software programs are available from the authors,

although examples are provided later in this document.

Example of Experimental Instructions for Symmetric Cost Tournaments

Introduction

This is an experiment about decision making. The instructions are simple, and if you follow them carefully and make good decisions, you could earn a considerable amount of money, which will be paid in cash to you at the end of today's session.

Specific Instructions

As you read these instructions you will be in a room with a number of other subjects. One of these subjects has been chosen to be paired with you by a random drawing of subject numbers conducted before you arrived. This subject will be called your pair member. The identity of your pair member will never be revealed to you and your pair member will never know your identity.

In the experiment you will perform a simple task. In each round of the experimental game you will choose a number between 0 and 100 – this is called your 'Decision Number'. Associated with each Decision Number is decision cost, which is listed in Column B of Table 1. Note that the higher the Decision Number you choose, the higher is the associated decision cost. Your pair member has an identical table.

At the beginning of each round of the experimental game you and your pair member will each select a Decision Number separately. Write your number in Column 1 of Sheet 1. Also, record the decision cost associated with your decision number in Column 6 of Sheet 1.

When all subjects have selected their decision numbers, an experimenter will have one subject choose a penny from a bucket with a large number of pennies in it. Each penny in the bucket has a number written on it and the set of all possible numbers range from – 35 to + 35. The sheet "Distribution of the Random Number Draw" contains both the frequency (number of pennies for each specific number) and the probability of drawing a particular number. You will note that more pennies feature numbers closer to zero and the fewer pennies feature numbers close to –35 and +35. In other words, there is a higher probability of drawing numbers closer to zero than numbers far from zero. The penny chosen will be called the 'Group Random Draw Number'. Everyone in the room will enter this number in Column 2.

Then the experimenters will bring buckets around to each of you. You will draw a penny from the bucket and the number on this penny will be called your 'Individual Random Draw Number'. Record your Individual Random Draw Number in column 3 of Sheet 1 and then return the penny to the bucket.

Calculation of Payments

The amount of money you earn in each round will be determined as follows. You will add your Decision Number (column 1) to the Group Random Draw Number (column 2) and to your Individual Random Draw Number (column 3) – write this total in Column 4 of Sheet 1. Your pair member will do the same. The experimenter will also record this information after you receive your Individual Random Draw Number.

Since all subjects have worked in privacy, the experimenter will then compare the totals of you and your pair member. If your total in Column 4 is greater than your pair member, you receive the high payment of $0.81; if your point total is smaller than your pair member, you receive the low payment of $0.40. Whether you receive $0.81 or $0.40 depends only on whether your point total is greater than or less than the point total of your pair member. It does not depend on how much bigger or smaller it is. If there is a tie in total points, the experimenter will flip a coin to determine who gets the high payment.

The experimenter will announce whether you have received a high or low payment. Circle the appropriate payment in Column 5 and subtract the decision cost associated with your decision number, which is in Column 6. Record this difference in Column 7. The amount in Column 7 is your earnings in dollars for the round unless this is a practice round. If this is a paying round, this amount will be added to your running total, which is tabulated in Column 8. Your running total at the end of the 10th paying round is then carried forward to the next experiment.

Before we get started, make sure that you write your chair number on "Sheet 1".

Are there any questions?

Review of Instructions

1. Beginning of Round Announced

2. Choose Decision Number   → Record in Column 1

3. Locate associated Decision Cost from Table 1   → Record in Column 6

4. One Subject Draws Group Random Number   → Record in Column 2

5. Each subject draw Individual Random Number → Record in Column 3

6. Add Numbers in Columns 1, 2 and 3   → Record in Column 4

7. If your sum is:

    a. Higher than your 'pair member'   → Circle $0.81 as your payment

    b. Lower than your 'pair member'   → Circle $0.40 as your payment

8. Subtract your 'Decision Cost' from your payment → Record in Column 7

9. If this is a paying round then   → Update running total (Col. 8)

Example of Experimental Instructions for Symmetric Cost Fixed Standard Contract

This experiment is identical to Experiment A in all aspects except the following.

In Experiment A you received the high payment if the sum of your Decision Number, the Group Random Draw Number and your Individual Random Draw Number was greater than your pair member's sum. If your sum was lower than your pair member, you would receive the low payment.

In this Experiment, you will receive a high payment of $0.85 if the sum of your Decision Number, the Group Random Draw Number and your Individual Random Draw Number is greater than or equal to 41. If this sum is less than 41, you will receive a low payment of $0.43. Whether you receive $0.85 or $0.43 as your payment depends only on whether your point total is greater than or equal to 41 – it does not depend on how much bigger or smaller.

All instructions for recording your Decision Number, Decision Cost, Group Random Number, Individual Random Number and payment amount and all instructions for calculating your per round earnings are the same as before.

You will resume tabulating your running total after the one practice round. Please remember to carry forward your net running total from the bottom of Sheet 1 to the top of Column 7 on Sheet 2 so that you can correctly tabulate your running total for this experiment. That is, your running total builds upon your net earnings from the previous experiment and will be carried forward to the next experiment.

Are there any questions?

Review of Instructions

10. Beginning of Round Announced

11. Choose Decision Number → Record in Column 1

12. Locate associated Decision Cost from Table 1 → Record in Column 6

13. One subject draws Group Random Number → Record in Column 2

14. Each subject draw Individual Random Number → Record in Column 3

15. Add numbers in Columns 1, 2 and 3 → Record in Column 4

16. If your sum is:

    a. Greater than or equal to 41 → Circle $0.85 as your payment

    b. Less than 41 → Circle $0.43 as your payment

17. Subtract your 'Decision Cost' from your payment → Record in Column 7

18. If this is a paying round then → Update running total (Col. 8)

Auction Instructions

In the second half of today's session you will have the opportunity to earn more money by participating in two more experiments identical to the two experiments played in the first half of today's session only without the initial, non-paying practice rounds. That is, the rules and the number of paying rounds for the experiments played in the second half will be exactly like those played in the first half.

For each experiment, however, only 10 of you will be allowed to participate. Which 10 of you will participate in each experiment will be decided as follows.

For Experiment A you will fill out a Experiment A bid card. On this card you will place your chair number and the maximum number of dollars you would be willing to pay from your experimental earnings today in order to participate. You will then fill out a similar card for Experiment B. The total amount of your bids for Experiment A and Experiment B combined cannot exceed the running total of dollars you have earned so far in the experiment.

We will collect the Experiment A and Experiment B bid cards from all participants and rank them from highest to lowest for each experiment. The top 10 bids for each experiment will be allowed to play in that additional experiment.

Each participant that gains entry into an additional experiment will have his/her running dollar total decreased by the amount of the 10th place bid for that experiment. Note: if your bid is higher than the 10th place bid, you will pay less than the amount you bid. In other words, you will gain no advantage by bidding less than your true value for entry to the additional experiment, since it is unlikely you would have to pay the full amount you bid.

The two people with the lowest bids for each experiment will not be allowed to play in the additional experimental session and will not have any dollars deducted from their running total. They must sit quietly as the additional experiment is played.

Review of Instructions

19. Write maximum amount you are willing to pay to play an additional round of Experiment A on the Experiment A Bid Card

20. Write maximum amount you are willing to pay to play an additional round of Experiment B on the Experiment B Bid Card

21. Verify the sum of bids for Experiment A and B are not greater than your net running total.

22. The top 10 bidders for each experiment will play in an additional experiment.

23. Your net running totals will be reduced by the amount of the 10th place bid if you were one of the top 10 bidders for that experiment.

24. Your net running total will not be reduced if you are not in the top 10 bidders, but you can't play in the additional experiment.

Effort-Cost Table Used by Subjects During the Symmetric Cost Experiments.

Table 1. Decision Numbers and Associated Point Deductions

Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost
0	0.0000	25	0.0625	50	0.2500	75	0.5625
1	0.0001	26	0.0676	51	0.2601	76	0.5776
2	0.0004	27	0.0729	52	0.2704	77	0.5929
3	0.0009	28	0.0784	53	0.2809	78	0.6084
4	0.0016	29	0.0841	54	0.2916	79	0.6241
5	0.0025	30	0.0900	55	0.3025	80	0.6400
6	0.0036	31	0.0961	56	0.3136	81	0.6561
7	0.0049	32	0.1024	57	0.3249	82	0.6724
8	0.0064	33	0.1089	58	0.3364	83	0.6889
9	0.0081	34	0.1156	59	0.3481	84	0.7056
10	0.0100	35	0.1225	60	0.3600	85	0.7225
11	0.0121	36	0.1296	61	0.3721	86	0.7396
12	0.0144	37	0.1369	62	0.3844	87	0.7569
13	0.0169	38	0.1444	63	0.3969	88	0.7744
14	0.0196	39	0.1521	64	0.4096	89	0.7921
15	0.0225	40	0.1600	65	0.4225	90	0.8100
16	0.0256	41	0.1681	66	0.4356	91	0.8281
17	0.0289	42	0.1764	67	0.4489	92	0.8464
18	0.0324	43	0.1849	68	0.4624	93	0.8649
19	0.0361	44	0.1936	69	0.4761	94	0.8836
20	0.0400	45	0.2025	70	0.4900	95	0.9025
21	0.0441	46	0.2116	71	0.5041	96	0.9216
22	0.0484	47	0.2209	72	0.5184	97	0.9409
23	0.0529	48	0.2304	73	0.5329	98	0.9604
24	0.0576	49	0.2401	74	0.5476	99	0.9801
						100	1.0000

Note. This table allows subjects to calculate the cost of each effort level. For example, if a subject chooses an effort (decision) level of "60", it would cost her $0.36 or 36 cents.

Example of Instructions for the Asymmetric Cost Tournament

Introduction

This is an experiment about decision making. The instructions are simple, and if you follow them carefully and make good decisions, you could earn a considerable amount of money, which will be paid in cash to you at the end of today's session.

Specific Instructions

As you read these instructions you will be in a room with a number of other subjects. One of these subjects has been chosen to be paired with you by a random drawing of subject numbers conducted before you arrived. This subject will be called your pair member. The identity of your pair member will never be revealed to you and your pair member will never know your identity.

In the experiment you will perform a simple task. In each round of the experimental game you will choose a number between 0 and 100 – this is called your 'Decision Number'. Associated with each Decision Number is decision cost, which is listed in Column B of Table 1. Note that the higher the Decision Number you choose, the higher is the associated decision cost. Also, for each decision number, costs are lower in Table 1A and higher in Table 1B. Whether you have been assigned Table 1A or Table 1B depends on the seat you were assigned. Those that arrived early were randomly assigned to either seat 1, 3, 9, or 2, 4, 10 and will have Table 1A. If you are in seats 5, 7, 11, or 6, 8, 12 you will have Table 1B. If you have Table 1A, then your pair member will have Table 1B and vice versa.

At the beginning of each round of the experimental game you and your pair member will each select a Decision Number separately. Write your number in Column 1 of Sheet 1. Also, record the decision cost associated with your decision number in Column 6 of Sheet 1.

When all subjects have selected their decision numbers, an experimenter will have one subject choose a penny from a bucket with a large number of pennies in it. Each penny in the bucket has a number written on it and the set of all possible numbers range from – 35 to + 35. The sheet "Distribution of the Random Number Draw" contains the frequency (number of pennies for each specific number). You will note that more pennies feature numbers closer to zero and the fewer pennies feature numbers close to – 35 and +35. In other words, there is a higher probability of drawing numbers closer to zero than numbers far from zero. The penny chosen will be called the 'Group Random Draw Number'. Everyone in the room will enter this number in Column 2.

Then the experimenters will bring buckets around to each of you. You will draw a penny from the bucket and the number on this penny will be called your 'Individual Random Draw Number'. Record your Individual Random Draw Number in column 3 of Sheet 1 and then return the penny to the bucket.

Calculation of Payments

The amount of money you earn in each round will be determined as follows. You will add your Decision Number (column 1) to the Group Random Draw Number (column 2) and to your Individual Random Draw Number (column 3) – write this total in Column 4 of Sheet 1. Your pair member will do the same. The experimenter will also record this information after you receive your Individual Random Draw Number.

Since all subjects have worked in privacy, the experimenter will then compare the totals of you and your pair member. If your total in Column 4 is greater than your pair member, you receive the high payment of $0.95; if your point total is smaller than your pair member, you receive the low payment of $0.33. Whether you receive $0.95 or $0.33 depends only on whether your point total is greater than or less than the point total of your pair member. It does not depend on how much bigger or smaller it is. If there is a tie in total points, the Table 1B pair member gets the high payment.

The experimenter will announce whether you have received a high or low payment. Circle the appropriate payment in Column 5 and subtract the decision cost associated with your decision number, which is in Column 6. Record this difference in Column 7. The amount in Column 7 is your earnings in dollars for the round unless this is a practice round. If this is a paying round, this amount will be added to your running total, which is tabulated in Column 8. Your running total at the end of the 10th paying round is then carried forward to the next experiment.

Before we get started, make sure that you write your chair number on "Sheet 1".

You may also take a minute to look at your pair member's decision cost sheet. Once you have looked at it, please pass it to one of the experimenters and work strictly off of your own decision cost sheet.

Review of Instructions

26. Beginning of Round Announced

27. Choose Decision Number   → Record in Column 1

28. Locate associated Decision Cost from Table 1   → Record in Column 6

29. One Subject Draws Group Random Number   → Record in Column 2

30. Each subject draw Individual Random Number   → Record in Column 3

31. Add Numbers in Columns 1, 2 and 3   → Record in Column 4

32. If your sum is:

     a. Higher than your 'pair member'   → Circle $0.95 as your payment

     b. Lower than your 'pair member'   → Circle $0.33 as your payment

33. Subtract your 'Decision Cost' from your payment   → Record in Column 7

34. If this is a paying round then   → Update running total (Col. 8)

Example of Fixed Standard Contract Instructions ("Experiment B")
Example of Instructions for the Asymmetric Cost Fixed Performance Session

This experiment is identical to Experiment A in all aspects except the following.

In Experiment A you received the high payment if the sum of your Decision Number, the Group Random Draw Number and your Individual Random Draw Number was greater than your pair member's sum. If your sum was lower than your pair member, you would receive the low payment.

In this Experiment, you will receive a high payment of $0.95 if the sum of your Decision Number, the Group Random Draw Number and your Individual Random Draw Number is greater than or equal to 41. If this sum is less than 41, you will receive a low payment of $0.40. Whether you receive $0.95 or $0.40 as your payment depends only on whether your point total is greater than or equal to 41 – it does not depend on how much bigger or smaller.

All instructions for recording your Decision Number, Decision Cost, Group Random Number, Individual Random Number and payment amount and all instructions for calculating your per round earnings are the same as before.

You will resume tabulating your running total after the one practice round. Please remember to carry forward your net running total from the bottom of Sheet 1 to the top of Column 7 on Sheet 2 so that you can correctly tabulate your running total for this experiment. That is, your running total builds upon your net earnings from the previous experiment and will be carried forward to the next experiment.

Are there any questions?

Review of Instructions

35. Beginning of Round Announced

36. Choose Decision Number → Record in Column 1

37. Locate associated Decision Cost from Table 1 → Record in Column 6

38. One subject draws Group Random Number → Record in Column 2

39. Each subject draw Individual Random Number → Record in Column 3

40. Add numbers in Columns 1, 2 and 3 → Record in Column 4

41. If your sum is:

    a. Greater than or equal to 41 → Circle $0.95 as your payment

    b. Less than 41 → Circle $0.40 as your payment

42. Subtract your 'Decision Cost' from your payment → Record in Column 7

43. If this is a paying round then → Update running total (Col. 8)

Effort-Cost Table for the Low Cost Subjects.

Table 1A. Decision Numbers and Associated Point Deductions (Seats 1, 2, 3, 4, 9, 10)

Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost
0	0.0000	25	0.0625	50	0.2500	75	0.5625
1	0.0001	26	0.0676	51	0.2601	76	0.5776
2	0.0004	27	0.0729	52	0.2704	77	0.5929
3	0.0009	28	0.0784	53	0.2809	78	0.6084
4	0.0016	29	0.0841	54	0.2916	79	0.6241
5	0.0025	30	0.0900	55	0.3025	80	0.6400
6	0.0036	31	0.0961	56	0.3136	81	0.6561
7	0.0049	32	0.1024	57	0.3249	82	0.6724
8	0.0064	33	0.1089	58	0.3364	83	0.6889
9	0.0081	34	0.1156	59	0.3481	84	0.7056
10	0.0100	35	0.1225	60	0.3600	85	0.7225
11	0.0121	36	0.1296	61	0.3721	86	0.7396
12	0.0144	37	0.1369	62	0.3844	87	0.7569
13	0.0169	38	0.1444	63	0.3969	88	0.7744
14	0.0196	39	0.1521	64	0.4096	89	0.7921
15	0.0225	40	0.1600	65	0.4225	90	0.8100
16	0.0256	41	0.1681	66	0.4356	91	0.8281
17	0.0289	42	0.1764	67	0.4489	92	0.8464
18	0.0324	43	0.1849	68	0.4624	93	0.8649
19	0.0361	44	0.1936	69	0.4761	94	0.8836
20	0.0400	45	0.2025	70	0.4900	95	0.9025
21	0.0441	46	0.2116	71	0.5041	96	0.9216
22	0.0484	47	0.2209	72	0.5184	97	0.9409
23	0.0529	48	0.2304	73	0.5329	98	0.9604
24	0.0576	49	0.2401	74	0.5476	99	0.9801
						100	1.0000

Effort-Cost Table for the High Cost Subjects.

Table 1B. Decision Numbers and Associated Point Deductions (5, 6, 7, 8, 11, 12)

Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost	Column A: Decision Number	Column B: Decision Cost
0	0.00000	25	0.09375	50	0.37500	75	0.84375
1	0.00015	26	0.10140	51	0.39015	76	0.86640
2	0.00060	27	0.10935	52	0.40560	77	0.88935
3	0.00135	28	0.11760	53	0.42135	78	0.91260
4	0.00240	29	0.12615	54	0.43740	79	0.93615
5	0.00375	30	0.13500	55	0.45375	80	0.96000
6	0.00540	31	0.14415	56	0.47040	81	0.98415
7	0.00735	32	0.15360	57	0.48735	82	1.00860
8	0.00960	33	0.16335	58	0.50460	83	1.03335
9	0.01215	34	0.17340	59	0.52215	84	1.05840
10	0.01500	35	0.18375	60	0.54000	85	1.08375
11	0.01815	36	0.19440	61	0.55815	86	1.10940
12	0.02160	37	0.20535	62	0.57660	87	1.13535
13	0.02535	38	0.21660	63	0.59535	88	1.16160
14	0.02940	39	0.22815	64	0.61440	89	1.18815
15	0.03375	40	0.24000	65	0.63375	90	1.21500
16	0.03840	41	0.25215	66	0.65340	91	1.24215
17	0.04335	42	0.26460	67	0.67335	92	1.26960
18	0.04860	43	0.27735	68	0.69360	93	1.29735
19	0.05415	44	0.29040	69	0.71415	94	1.32540
20	0.06000	45	0.30375	70	0.73500	95	1.35375
21	0.06615	46	0.31740	71	0.75615	96	1.38240
22	0.07260	47	0.33135	72	0.77760	97	1.41135
23	0.07935	48	0.34560	73	0.79935	98	1.44060
24	0.08640	49	0.36015	74	0.82140	99	1.47015
						100	1.50000

Maple Code and Output Example: Risk Neutral F Contract Simulations[1]

```
> #note: this is the Maple program for the fixed
performance contract under symmetric costs, risk neutrality
and no inequality aversion.  This is essentially the
benchmark case.  Note: it is straightforward to get the
asymmetric cost simulations using this file.  Just do the
following: (1) change "alpha" from 1 to 1.5, (2) change
payment parameters.  That is, let m=.40 and "spread" be
.55, and (3) once you have the new optimal effort levels,
be sure to plug them into the equations that simulate
expected profits, expected utility, variance of profits
etc.  These equations are near the bottom of this program.
If these substitutions are not made, results can be
misleading.
> restart;


> sigmac:=0;sigma :=500;#sigmac is common shock variance,
sigma is idiosyncratic variance, although for fixed
performance contracts, it is irrelevant so long as total
variance is equal to 500.
> alpha:=1;  #for symmetric costs, set this equal to "1".
For asymmetric, set this equal to "1.5"
> m:=.43;spread:=.42; M:=m+spread;  #m is "r" in the paper.
M is "R".  Note M>m. For the fixed performance contract,
these payoffs change across symmetric and asymmetric cost
experiments.  See paper.
>
```

$$sigmac := 0$$

$$\sigma := 500$$

$$\alpha := 1$$

$$m := 0.43$$

$$spread := 0.42$$

$$M := 0.85$$

```
>
```

```
> n1:=(1/((2*3.14*(sigma+sigmac))^(1/2)))*exp((-
1/2)*(((u)^2)/(sigma+sigmac))); #This is the normal
probability density function for agent 1.
```

$$n1 := 0.01784576526 \, \mathbf{e}^{\left(-\frac{1}{1000} u^2 \right)}$$

```
>
> n2:=(1/((2*3.14*(sigma+sigmac))^(1/2)))*exp((-
1/2)*(((u2)^2)/(sigma+sigmac))); #This is the normal
probability density function for agent 2.
```

$$n2 := 0.01784576526 \, \mathbf{e}^{\left(-\frac{1}{1000} u2^2 \right)}$$

```
>
>  UM:=M-alpha*(e1^2)/10000; #This agent 1's "utility"
under high performance. Note utility=profit under risk
neutrality.
```

$$UM := 0.85 - \frac{1}{10000} \, e1^2$$

```
> UMa:=M-alpha*(e1^2)/10000;#This is agent 1's utility
under high performance.
```

$$UMa := 0.85 - \frac{1}{10000} \, e1^2$$

```
> Um:=m-alpha*(e1^2)/10000;#This is agent 1's utility under
low performance.
```

$$Um := 0.43 - \frac{1}{10000} \, e1^2$$

```
> Umd:=m-alpha*(e1^2)/10000;#This is agent 1's utility
under low performance.
```

$$Umd := 0.43 - \frac{1}{10000} \, e1^2$$

```
> cdf1:=int(n1,u=-infinity...(41-e1));#This is agent 1's
cumulative distribution function.
```

$$cdf1 := 0.5001267877 - 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e1)$$

```
> UM2:=M-(e2^2)/10000;# Repeat above for agent 2 (next 5
lines).
```

$$UM2 := 0.85 - \frac{1}{10000} \, e2^2$$

```
> UM2a:=M-(e2^2)/10000;
```

$$UM2a := 0.85 - \frac{1}{10000}\, e2^2$$

```
> Um2:=m-(e2^2)/10000;
```

$$Um2 := 0.43 - \frac{1}{10000}\, e2^2$$

```
> Um2d:=m-(e2^2)/10000;
```

$$Um2d := 0.43 - \frac{1}{10000}\, e2^2$$

```
> cdf2:=int(n2,u2=-infinity...(41-e2));
```

$$cdf2 := 0.5001267877 - 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2)$$

```
>
> EU1:=(1-cdf1)*(1-cdf2)*UM+(1-cdf1)*cdf2*UMa+cdf1*(1-
cdf2)*Umd+cdf1*cdf2*Um;   #Agent 1's expected utility
(profit).
```

$EU1 := (0.4998732123 + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e1))\, (0.4998732123$

$\qquad + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2))\left(0.85 - \frac{1}{10000}\, e1^2 \right) + (0.4998732123$

$\qquad + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e1))\, (0.5001267877$

$\qquad - 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2))\left(0.85 - \frac{1}{10000}\, e1^2 \right) + (0.5001267877$

$\qquad - 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e1))\, (0.4998732123$

$\qquad + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2))\left(0.43 - \frac{1}{10000}\, e1^2 \right) + (0.5001267877$

$\qquad - 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e1))\, (0.5001267877$

$\qquad - 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2))\left(0.43 - \frac{1}{10000}\, e1^2 \right)$

```
> DEU1:=diff(EU1,e1);#Derivative of expected utility with
respect to e1.
```

$DEU1 := \dfrac{1}{\sqrt{\pi}}\left(0.03163079536\, \mathbf{e}^{(-(-1.296533841 + 0.03162277660\, e1)^2)}\, (0.4998732123 \right.$

$\qquad + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e2))\left(0.85 - \frac{1}{10000}\, e1^2 \right)\Bigg) - \dfrac{1}{5000}\, (0.4998732123$

$\qquad + 0.5001267877\, \text{erf}(-1.296533841 + 0.03162277660\, e1))\, (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 + \dfrac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536$

$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \dfrac{1}{10000} \, e1^2 \right) \Bigg) - \dfrac{1}{5000} \, (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 - \dfrac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536$

$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \dfrac{1}{10000} \, e1^2 \right) \Bigg) - \dfrac{1}{5000} \, (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 - \dfrac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536$

$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \dfrac{1}{10000} \, e1^2 \right) \Bigg) - \dfrac{1}{5000} \, (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1$

```
> EU2:=(1-cdf1)*(1-cdf2)*UM2+cdf1*(1-cdf2)*UM2a+(1-
cdf1)*cdf2*Um2d+cdf1*cdf2*Um2;#Agent 2's expected utility.
```

$EU2 := (0.4998732123 + 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \dfrac{1}{10000} \, e2^2 \right) + (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \dfrac{1}{10000} \, e2^2 \right) + (0.4998732123$

$+ \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$

$- \; 0.5001267877 \; \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \dfrac{1}{10000} \, e2^2 \right) + (0.5001267877$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \frac{1}{10000} \, e2^2 \right)$$

> `DEU2:=diff(EU2,e2);#Derivative of agent 2's expected utility with respect to e2.`

$$DEU2 := \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \right.$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 + \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.5001267877 \right.$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 \right.$$

$$\left. - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$(0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \right.$$

$$(0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.5001267877 \right.$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 \right.$$

$$\left. - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$(0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2$$

> `focEU1:=DEU1=0;focEU2:=DEU2=0;#First order conditions`

$$focEU1 := \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.4998732123 \right.$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \frac{1}{10000} \, e1^2 \right) \right) - \frac{1}{5000} \, (0.4998732123$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 + \frac{1}{\sqrt{\pi}} \left(0.03163079536 \right.$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \frac{1}{10000} \, e1^2 \right) \right) - \frac{1}{5000} \, (0.4998732123$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \right.$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.4998732123$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \frac{1}{10000} \, e1^2 \right) \right) - \frac{1}{5000} \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \right.$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \frac{1}{10000} \, e1^2 \right) \right) - \frac{1}{5000} \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- \, 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e1 = 0$$

$$focEU2 := \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \right.$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 - \frac{1}{10000} \, e2^2 \right) \bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 + \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.5001267877 \right.$$

$$- \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 \right.$$

$$\left. - \frac{1}{10000} \, e2^2 \right) \bigg) - \frac{1}{5000} \, (0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$(0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \right.$$

$$(0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$\mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 - \frac{1}{10000} \, e2^2 \right) \bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.5001267877 \right.$$

$$- \; 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 \right.$$

$$\left. - \frac{1}{10000} \, e2^2 \right) \bigg) - \frac{1}{5000} \, (0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

$$(0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) \, e2 = 0$$

```
> fsolve({focEU1,focEU2},{e1=0..100,e2=0..100});#solving
first order conditions for optimal efforts.
```
$$\{e2 = 36.83010387, \; e1 = 36.83010387\}$$

```
> DDEU1:=diff(DEU1,e1); DDEU2:=diff(DEU2,e2);#second order
derivative of expected utility
```

$$DDEU1 := \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.08200000002 - 0.002000000000 \, e1) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \right.$$

$$(0.4998732123 + 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \frac{1}{10000} \, e1^2 \right) \Bigg) - \frac{1}{5000}$$

$$(0.4998732123 + 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) + \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.08200000002 \right.$$

$$- 0.002000000000 \, e1) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.85 - \frac{1}{10000} \, e1^2 \right) \Bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.08200000002 \right.$$

$$- 0.002000000000 \, e1) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.4998732123$$

$$+ 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \frac{1}{10000} \, e1^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) - \frac{1}{\sqrt{\pi}} \left(0.03163079536 \, (0.08200000002 \right.$$

$$- 0.002000000000 \, e1) \, \mathbf{e}^{(-(-1.296533841 + 0.03162277660 \, e1)^2)} \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2)) \left(0.43 - \frac{1}{10000} \, e1^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \mathrm{erf}(-1.296533841 + 0.03162277660 \, e2))$$

$$DDEU2 := \frac{1}{\sqrt{\pi}} \left(\; 0.03163079536 \, (0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \right.$$

$$(0.08200000002 - 0.002000000000 \, e2) \; e^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 - \frac{1}{10000} \, e2^2 \right) \Bigg) -$$

$$\frac{1}{5000} \, (0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) + \frac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536 \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.08200000002 - 0.002000000000 \, e2)$$

$$e^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.85 - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) - \frac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536 \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.08200000002 - 0.002000000000 \, e2)$$

$$e^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.4998732123$$

$$+ 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2)) - \frac{1}{\sqrt{\pi}} \Bigg(\; 0.03163079536 \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.08200000002 - 0.002000000000 \, e2)$$

$$e^{(-(-1.296533841 + 0.03162277660 \, e2)^2)} \left(0.43 - \frac{1}{10000} \, e2^2 \right) \Bigg) - \frac{1}{5000} \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.5001267877$$

$$- 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e2))$$

```
> 
soc1:=eval(DDEU1,{e1=36.83010387,e2=36.83010387});soc2:=eva
l(DDEU2,{e1=36.83010387,e2=36.83010387});#evaluating second
order conditions to ensure maximum.  Note: make sure you
plug in the right numbers for effort level when you change
other parameters.  For example, changing variance and
payments will affect optimal effort and these new optimal
```

efforts will have to be plugged into this and the next few formulas below.

$$soc1 := \frac{0.0001088837600}{\sqrt{\pi}} - 0.0002000000000$$

$$soc2 := \frac{0.0001088837600}{\sqrt{\pi}} - 0.0002000000000$$

> winprob1:=eval((1-cdf1),e1=36.83010387);winprob2:=eval((1-cdf2),e2=36.83010387);#probabilities of achieving the high payment M for both agents at optimal effort levels.

$$winprob1 := 0.4258872942$$

$$winprob2 := 0.4258872942$$

>
Eutility1:=eval(EU1,{e1=36.83010387,e2=36.83010387});#Indir ect utility

$$Eutility1 := 0.4732270085$$

>
Eutility2:=eval(EU2,{e1=36.83010387,e2=36.83010387});#Indir ect utility

$$Eutility2 := 0.4732270085$$

>
> eprofit:=(1-cdf1)*(M)+cdf1*(m)-(e1^2)/10000;#same as indirect utility because this is the risk neutral case. For the risk averse and inequality aversion cases, separating expected profit from expected utility is more meaningful.

$$eprofit := 0.6399467492 + 0.2100532508 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1) - \frac{1}{10000} \, e1^2$$

> vprofit:=cdf1*(1-cdf1)*(spread)^2;#variance of expected profit.

$$vprofit := 0.1764 \, (0.5001267877 - 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1)) \, (0.4998732123 + 0.5001267877 \, \text{erf}(-1.296533841 + 0.03162277660 \, e1))$$

> eval(eprofit,{e1=36.83010387,e2=36.83010387}); #expected profit at optimal effort

$$0.4732270085$$

> eval(vprofit,{e1=36.83010387,e2=36.83010387});#variance of profit evaluated at optimal effort.

Maple Code and Output Example: Risk Averse-Inequality Averse T Simulations

```
> #This is a tournament simulation where the common shock
variance is 250.   Agents are risk averse and inequality
averse. Costs are symmetric. Note: it is straightforward to
get the asymmetric cost simulations using this file.  Just
do the following: (1) change "alpha" from 1 to 1.5, (2)
change payment parameters.  That is, let m=.40 and "spread"
be .41, and (3) once you have the new equilibrium effort
levels, be sure to plug them into the equations that
simulate expected profits, expected utility, variance of
profits etc.  These equations are near the bottom of this
program.  If these substitutions are not made, results can
be misleading.
> #Also note that it is straightforward to generate the
other tournament simulations with different common shock
variances.  For example, if you wanted to generate the
experiment where the common shock variance is 350, do the
following: (1) Change "Sigma" from 250 to 150 (note that
sigma is the idiosyncratic shock variance and the sum of
the variances need to be 500), (2) Change payments so that
m =.45 and spread=.32, and (3) once you have the new
equilibrium effort levels, be sure to plug them into the
equations that simulate expected utility, expected profits,
variance of profits etc.  These equations are near the
bottom of this program.  If these substitutions are not
made, results can be misleading.  Experiments for common
shock variance equaling 0 and 50 by following a similar
sequence of steps (make sure payments m and M are adjusted
accordingly.  See paper for specific values).
>
> restart;
> sigma :=250;alpha:=1;evalf(sigma^(0.5));#sigma is
idiosyncratic variance, common shock variance is filtered
away by the tournament.
```

$$\sigma := 250$$

$$\alpha := 1$$

15.81138830

```
> m:=.40;spread:=.41; M:=m+spread; #m is "r" in the paper.
M is "R".  Note M>m.   See paper.
```

$$m := 0.40$$

$$spread := 0.41$$

$$M := 0.81$$

> **w:=1; #this is initial wealth. Because our utility function exponentiates payments, we had to ensure that total payments exceeds 1. We did this by giving the agent an initial wealth of 1 which guarantees that total payment under each state of nature exceeds 1. While we set w = 1, it can be arbitrarily set at any value greater than 1. We conducted sensivity analysis by setting w = 20 to make sure that our qualitative predictions did not change. This was important to check because our utility funciton exhibits decreasing absolute risk aversion. We found that no qualitative results changed. You can verify this be changing w = 20 and running the program.**

$$w := 1$$

$$\sigma := 250$$

$$\alpha := 1$$

$$15.81138830$$

> **n1:=(1/((2*3.14*sigma*(2))^(1/2)))*exp((-1/2)*(((u1)^2)/(sigma*(2))));#This is the normal probability density of the agents.**

$$n1 := 0.001128665296 \sqrt{250} \; \mathbf{e}^{\left(- \frac{1}{1000} u1^2 \right)}$$

> **A:=0.731;beta:=0.029;theta:=5; #These are the parameters for the utility function.**

$$A := 0.731$$

$$\beta := 0.029$$

$$\theta := 5$$

> **adv:=0; disadv:=.15; #these are the inequality aversion parameters.**

$$adv := 0$$

$$disadv := 0.15$$

> **spread1:=theta-exp(-beta*(w+M)^A)-(theta-exp(-beta*(w+m)^A));#This is "inequality" in utility terms.**

$$spread1 := 0.0073529735$$

```
> UM1:=(theta-exp(-beta*(w+M-alpha*(e1^2)/10000)^A))-
adv*spread1;#Agent 1's utility when he "wins".
```

$$UM1 := 5. - e^{\left(-0.029\left(1.81 - \frac{1}{10000}\,e1^2\right)^{0.731}\right)}$$

```
> Um1:=(theta-exp(-beta*(w+m-alpha*(e1^2)/10000)^A))-
disadv*spread1;#Agent 1's utility when he "loses".
```

$$Um1 := 4.998897054 - e^{\left(-0.029\left(1.40 - \frac{1}{10000}\,e1^2\right)^{0.731}\right)}$$

```
> UM2:=(theta-exp(-beta*(w+M-(e2^2)/10000)^A))-
adv*spread1;#Agent 2's utility when he "wins"
```

$$UM2 := 5. - e^{\left(-0.029\left(1.81 - \frac{1}{10000}\,e2^2\right)^{0.731}\right)}$$

```
> Um2:=(theta-exp(-beta*(w+m-(e2^2)/10000)^A))-
disadv*spread1;#Agent 2's utility when he "loses".
```

$$Um2 := 4.998897054 - e^{\left(-0.029\left(1.40 - \frac{1}{10000}\,e2^2\right)^{0.731}\right)}$$

```
> cdf:=int(n1,u1=-infinity...(e2-e1));# the cumulative
distribution function.
```

$$cdf := 0.5001267876 + 0.5001267876\,\mathrm{erf}(0.03162277660\,e2 - 0.03162277660\,e1)$$

```
> EU1:=(1-cdf)*UM1+cdf*Um1; EU2:=cdf*UM2+(1-
cdf)*Um2;#Expected utilities for the two agents.
```

$$EU1 := (0.4998732124 - 0.5001267876\,\mathrm{erf}(0.03162277660\,e2 - 0.03162277660\,e1))\left(5. \right.$$

$$\left. - e^{\left(-0.029\left(1.81 - \frac{1}{10000}\,e1^2\right)^{0.731}\right)}\right) + (0.5001267876$$

$$+ 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \left(4.998897054 \right.$$

$$\left. - \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right)$$

$$EU2 := (0.5001267876 + 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \left(5. \right.$$

$$\left. - \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right) + (0.4998732124$$

$$- 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \left(4.998897054 \right.$$

$$\left. - \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right)$$

```
> DEU1:=diff(EU1,e1); DEU2:=diff(EU2,e2);#First order
derivatives of expected utilities.
```

$$DEU1 := \frac{0.03163079535 \, \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \left(5. - \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right)}{\sqrt{\pi}} -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.269}} \left(0.000004239800000 \, (0.4998732124 \right.$$

$$\left. - 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\sqrt{\pi}}\left(0.03163079535\, \mathbf{e}^{\left(-(0.03162277660\, e2 - 0.03162277660\, e1)^2\right)}\left(4.998897054 \right.\right.$$

$$\left.\left. - \mathbf{e}^{\left(-0.029\left(1.40 - \frac{1}{10000}\, e1^2\right)^{0.731}\right)}\right)\right) - \frac{1}{\left(1.40 - \frac{1}{10000}\, e1^2\right)^{0.269}}\left(0.000004239800000 \right.$$

$$(0.5001267876 + 0.5001267876\, \mathrm{erf}(0.03162277660\, e2 - 0.03162277660\, e1))\, e1$$

$$\left. \mathbf{e}^{\left(-0.029\left(1.40 - \frac{1}{10000}\, e1^2\right)^{0.731}\right)}\right)$$

$$DEU2 := \frac{0.03163079535\, \mathbf{e}^{\left(-(0.03162277660\, e2 - 0.03162277660\, e1)^2\right)}\left(5. - \mathbf{e}^{\left(-0.029\left(1.81 - \frac{1}{10000}\, e2^2\right)^{0.731}\right)}\right)}{\sqrt{\pi}} -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000}\, e2^2\right)^{0.269}}\left(0.000004239800000\, (0.5001267876 \right.$$

$$\left. + 0.5001267876\, \mathrm{erf}(0.03162277660\, e2 - 0.03162277660\, e1))\, e2\, \mathbf{e}^{\left(-0.029\left(1.81 - \frac{1}{10000}\, e2^2\right)^{0.731}\right)}\right) -$$

$$\frac{1}{\sqrt{\pi}}\left(0.03163079535\, \mathbf{e}^{\left(-(0.03162277660\, e2 - 0.03162277660\, e1)^2\right)}\left(4.998897054 \right.\right.$$

$$\left.\left. - \mathbf{e}^{\left(-0.029\left(1.40 - \frac{1}{10000}\, e2^2\right)^{0.731}\right)}\right)\right) - \frac{1}{\left(1.40 - \frac{1}{10000}\, e2^2\right)^{0.269}}\left(0.000004239800000 \right.$$

$$(0.4998732124 - 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e2$$

$$\mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \Bigg)$$

> **foc1:=DEU1=0; foc2:=DEU2=0;#First order conditions for both agents.**

$$foc1 := \frac{0.03163079535 \, \mathbf{e}^{\left(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2 \right)} \left(5. - \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right)}{\sqrt{\pi}} \; -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.269}} \Bigg(0.000004239800000 \, (0.4998732124$$

$$- 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1 \; \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \Bigg) \; -$$

$$\frac{1}{\sqrt{\pi}} \Bigg(0.03163079535 \, \mathbf{e}^{\left(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2 \right)} \Bigg(4.998897054$$

$$- \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \Bigg) \Bigg) \; - \; \frac{1}{\left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.269}} \Bigg(0.000004239800000$$

$$(0.5001267876 + 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1$$

$$\mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \Bigg) = 0$$

$$foc2 := \frac{0.03163079535 \, e^{\left(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2\right)} \left(5. - e^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2\right)^{0.731}\right)}\right)}{\sqrt{\pi}} -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e2^2\right)^{0.269}} \Bigg(0.000004239800000 \, (0.5001267876$$

$$+ 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e2 \, e^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2\right)^{0.731}\right)} \Bigg) -$$

$$\frac{1}{\sqrt{\pi}} \Bigg(0.03163079535 \, e^{\left(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2\right)} \Bigg(4.998897054$$

$$- e^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e2^2\right)^{0.731}\right)} \Bigg) \Bigg) - \frac{1}{\left(1.40 - \frac{1}{10000} \, e2^2\right)^{0.269}} \Bigg(0.000004239800000$$

$$(0.4998732124 - 0.5001267876 \, \text{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e2$$

$$e^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e2^2\right)^{0.731}\right)} \Bigg) = 0$$

```
> fsolve({foc1,foc2},{e1=0..100,e2=0..100});#solving first
order conditions for equilibrium efforts.
```
$$\{e1 = 41.77893461, \, e2 = 41.77996031\}$$

```
> DDEU1:=diff(DEU1,e1); DDEU2:=diff(DEU2,e2); #Second order
derivatives.
```

$$DDEU1 := \frac{1}{\sqrt{\pi}} \left(0.03163079535 \, (0.002000000000 \, e2 - 0.002000000000 \, e1) \right.$$

$$\left. \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \left(5. - \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) \right)$$

$$- \frac{2.682164922 \, 10^{-7} \, \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \, e1 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)}}{\sqrt{\pi} \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.269}} -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e1^2 \right)^{1.269}} \left(2.281012400 \, 10^{-10} \, (0.4998732124 \right.$$

$$\left. - 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1^2 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.269}} \left(0.000004239800000 \, (0.4998732124 \right.$$

$$\left. - 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.538}} \left(1.797590404 \, 10^{-11} \, (0.4998732124 \right.$$

$$\left. - 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1^2 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\sqrt{\pi}} \left(0.03163079535 \, (0.002000000000 \, e2 - 0.002000000000 \, e1) \right.$$

$$\left. \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \left(4.998897054 - \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) \right)$$

$$+ \frac{2.682164922 \, 10^{-7} \, \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \, e1 \, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)}}{\sqrt{\pi} \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.269}} -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000} \, e1^2 \right)^{1.269}} \left(2.281012400 \, 10^{-10} \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1^2 \, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.269}} \left(0.000004239800000 \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.538}} \left(1.797590404 \, 10^{-11} \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e1^2 \, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000} \, e1^2 \right)^{0.731} \right)} \right)$$

$$DDEU2 := \frac{1}{\sqrt{\pi}} \left(0.03163079535 \, (-0.002000000000 \, e2 + 0.002000000000 \, e1) \right.$$

$$\left. \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \left(5. - \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right) \right)$$

$$- \frac{2.682164922 \, 10^{-7} \, \mathbf{e}^{(-(0.03162277660 \, e2 - 0.03162277660 \, e1)^2)} \, e2 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)}}{\sqrt{\pi} \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.269}} -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e2^2 \right)^{1.269}} \left(2.281012400 \, 10^{-10} \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e2^2 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.269}} \left(0.000004239800000 \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.538}} \left(1.797590404 \, 10^{-11} \, (0.5001267876 \right.$$

$$\left. + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) \, e2^2 \, \mathbf{e}^{\left(-0.029 \left(1.81 - \frac{1}{10000} \, e2^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\sqrt{\pi}} \left(0.03163079535\, (-0.002000000000\, e2 + 0.002000000000\, e1) \right.$$

$$\left. \mathbf{e}^{\left(-(0.03162277660\, e2 - 0.03162277660\, e1)^2\right)} \left(4.998897054 - \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.731} \right)} \right) \right)$$

$$+\, \frac{2.682164922\ 10^{-7}\, \mathbf{e}^{\left(-(0.03162277660\, e2 - 0.03162277660\, e1)^2\right)}\, e2\, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.731} \right)}}{\sqrt{\pi} \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.269}}\, -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000}\, e2^2 \right)^{1.269}} \left(2.281012400\ 10^{-10}\, (0.4998732124 \right.$$

$$\left. -\, 0.5001267876\, \mathrm{erf}(0.03162277660\, e2 - 0.03162277660\, e1))\, e2^2\, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.269}} \left(0.000004239800000\, (0.4998732124 \right.$$

$$\left. -\, 0.5001267876\, \mathrm{erf}(0.03162277660\, e2 - 0.03162277660\, e1))\, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.731} \right)} \right) -$$

$$\frac{1}{\left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.538}} \left(1.797590404\ 10^{-11}\, (0.4998732124 \right.$$

$$\left. -\, 0.5001267876\, \mathrm{erf}(0.03162277660\, e2 - 0.03162277660\, e1))\, e2^2\, \mathbf{e}^{\left(-0.029 \left(1.40 - \frac{1}{10000}\, e2^2 \right)^{0.731} \right)} \right)$$

```
> soc1:=eval(DDEU1,{e1=41.77893461,e2=41.77996031});#Agent
1's second order condition evaluated at equilibrium efforts
```

$$soc1 := \frac{8.41573050\ 10^{-7}}{\sqrt{\pi}} - 0.000003998452331$$

```
> soc2:=eval(DDEU2,{e1=41.77893461,e2=41.77996031});#Agent
2's second order condition evaluated at equilibrium
efforts.
```

$$soc2 := \frac{8.40469084\ 10^{-7}}{\sqrt{\pi}} - 0.000003998354336$$

```
> winprob1:=eval((1-
cdf),{e1=41.77893461,e2=41.77996031});#Agent 1's
probability of winning the tournament
>
winprob2:=eval(cdf,{e1=41.77893461,e2=41.77996031});#Agent
2's probability of winning the tournament
```

$$winprob1 := 0.4998549083$$

$$winprob2 := 0.5001450917$$

```
>
> Eprofits1:=(1-cdf)*M+cdf*m-
alpha*(e1^2)/10000;eval(Eprofits1,{e1=41.77893461,e2=41.779
96031});#expected profit for agent 1
```

$$Eprofits1 := 0.6049480170 - 0.2050519830\ \text{erf}(0.03162277660\ e2 - 0.03162277660\ e1) - \frac{1}{10000}\ e1^2$$

$$0.4303925746$$

```
> Eprofits2:=cdf*M+(1-cdf)*m-
(e2^2)/10000;eval(Eprofits2,{e1=41.77893461,e2=41.77996031}
);#expected profit for agent 2
```

$$Eprofits2 := 0.6050519830 + 0.2050519830\ \text{erf}(0.03162277660\ e2 - 0.03162277660\ e1) - \frac{1}{10000}\ e2^2$$

$$0.4305029793$$

```
>
>
EUtility1:=eval(EU1,{e1=41.77893461,e2=41.77996031});#indir
ect expected utility for agent 1
```

$$EUtility1 := 4.036339882$$

```
>
EUtility2:=eval(EU2,{e1=41.77893461,e2=41.77996031});#indir
ect expected utility for agent 2
```
$$EUtility2 := 4.036342251$$

```
> vprofit:=cdf*(1-cdf)*(spread)^2;#variance of profit
```
$vprofit := 0.1681 (0.5001267876 + 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1)) (0.4998732124$

$\qquad - 0.5001267876 \, \mathrm{erf}(0.03162277660 \, e2 - 0.03162277660 \, e1))$

```
>
> eval(vprofit,{e1=41.77893461,e2=41.77996031});#variance
of profit evaluated at the equilibrium efforts.
```
$$0.04202499645$$

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