

**AJAE appendix for ‘Land Market Imperfections and Agricultural Policy Impacts in the New EU Member States: A Partial Equilibrium Analysis’**

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7 February 2006

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

## A1. Proof of Proposition 1.

Part 1: To show:  $\frac{d\Pi^I}{ds} = 0$ ,  $\frac{d\Pi^C}{ds} = 0$  and  $\frac{d\Pi^L}{ds} > 0$  without transaction costs and perfect

competition. In this case, profits of landowners, IF, and CF, respectively, are  $\Pi^L = rA^T$ ,

$\Pi^I = pf^I(A^I) - (r-s)A^I$ , and  $\Pi^C = pf^C(A^C) - (r-s)A^C$ . Then we must show that:

$$(A1.1) \quad \frac{d\Pi^I}{ds} = -A^I \frac{dr}{ds} + A^I = 0$$

$$(A1.2) \quad \frac{d\Pi^C}{ds} = -A^C \frac{dr}{ds} + A^C = 0$$

$$(A1.3) \quad \frac{d\Pi^L}{ds} = r \frac{dA^T}{ds} + A^T \frac{dr}{ds} > 0$$

With  $\frac{dA^T}{ds} = 0$ , (A1.1 – A1.3) can only hold if  $\frac{dr}{ds} = 1$ . In equilibrium the following

conditions must be satisfied (with  $\frac{\partial f^I(A^I)}{\partial A^I} = f'_A$  and  $\frac{\partial f^C(A^C)}{\partial A^C} = f'_A$ ):

$$(A1.4) \quad pf'_A = r - s \quad \text{First order condition of a representative IF}$$

$$(A1.5) \quad pf'_A = r - s \quad \text{CF' first order condition}$$

$$(A1.6) \quad A^T = A^I + A^C \quad \text{Land equilibrium condition}$$

Total differentiating equations (A1.4 – A1.6) yields:

$$(A1.7) \quad pf''_{AA} dA^I = dr - ds$$

$$(A1.8) \quad pf''_{AA} dA^C = dr - ds$$

$$(A1.9) \quad dA^I + dA^C = 0$$

Using (A1.7–A1.9), it follows that:

$$(A1.10) \quad \frac{dr}{ds} = \frac{pf_{AA}^C + pf_{AA}^I}{pf_{AA}^C + pf_{AA}^I} = 1. \quad (\text{Q.E.D of part 1.})$$

Part 2: To show:  $\frac{d\Pi^I}{ds} = 0$ ,  $\frac{d\Pi^C}{ds} = 0$  and  $\frac{d\Pi^L}{ds} > 0$  with transaction costs and imperfect competition. Now IF profit is defined by equation  $\Pi^I = pf^I(A^I) - (r + t - s)A^I$ . For CF and landowners total profits are defined as in part 1. Then we must show that:

$$(A1.11) \quad \frac{d\Pi^I}{ds} = -A^I \frac{dr}{ds} - A^I t_A \frac{dA^I}{ds} + A^I = 0$$

$$(A1.12) \quad \frac{d\Pi^C}{ds} = pf_A^C \frac{dA^C}{ds} - (r - s) \frac{dA^C}{ds} - A^C \frac{dr}{ds} + A^C = 0$$

as well as (A1.3), where  $t(A^I)$  allows for increasing unit transaction costs

$$\left( \frac{\partial t}{\partial A^I} = t_A \geq 0 \right).$$

(A1.11) (A1.12) and (A1.3) hold if  $\frac{dA^I}{ds} = \frac{dA^C}{ds} = 0$  and  $\frac{dr}{ds} = 1$ . With imperfect

competition and transaction costs, condition (A1.6) must be satisfied, as well as:

$$(A1.13) \quad pf_A^I = r + t - s \quad \text{First order condition of a representative IF}$$

$$(A1.14) \quad pf_A^C = r - s + A^C \frac{\partial r}{\partial A^C} \quad \text{CF' first order condition}$$

From (1.13) and (A1.6)  $\frac{\partial r}{\partial A^C}$  can be obtained:

$$(A1.15) \quad \frac{\partial r}{\partial A^C} = -pf_{AA}^I + t_A$$

Total differentiating equations (A1.6) (1.13) and (A1.14) and using equation (A1.15)

(with  $\frac{\partial^2 t(A^I)}{\partial A^{I^2}} = t_{AA}$ ,  $\frac{\partial^3 f^I(A^I)}{\partial A^{I^3}} = f_{AAA}^I$ ) yields (A1.9), as well as:

$$(A1.16) \quad pf_{AA}^I dA^I = dr + t_A dA^I - ds$$

$$(A1.17) \quad (pf_{AA}^C + pf_{AA}^I - t_A) dA^C + (A^C pf_{AAA}^I - A^C t_{AA}) dA^I = dr - ds$$

Using (A1.9), (A1.16) and (A1.17), it follows that:

$$(A1.18) \quad \frac{dA^C}{ds} = \frac{dA^I}{ds} = \frac{1-1}{-pf_{AA}^C - 2pf_{AA}^I + 2t_A + A^C(pf_{AAA}^I - t_{AA})} = 0$$

$$(A1.19) \quad \frac{dr}{ds} = \frac{-pf_{AA}^C - 2pf_{AA}^I + 2t_A + A^C(pf_{AAA}^I - t_{AA})}{-pf_{AA}^C - 2pf_{AA}^I + 2t_A + A^C(pf_{AAA}^I - t_{AA})} = 1. \quad (\text{Q.E.D.})$$

## A2. Proof of proposition 2

To show:  $\frac{d\Pi^I}{ds} < 0$ ,  $\frac{d\Pi^C}{ds} > 0$  and  $\frac{d\Pi^L}{ds} > 0$ , if  $s^I = \mathbf{a}s$  and  $0 < \mathbf{a} < 1$ . With perfect

competition and no transaction costs,<sup>1</sup> (A1.5) and (A1.6) must be satisfied, as well as:

$$(A2.1) \quad pf_A^I = r - \mathbf{a}s$$

Total differentiating (A1.5), (A1.6) and (A2.1) yields (A1.8) and (A1.9) as well as:

$$(A2.2) \quad pf_{AA}^I dA^I = dr - \mathbf{a}ds$$

Combining (A1.8), (A1.9) and (A2.2) it follows that:

$$(A2.3) \quad \frac{dA^I}{ds} = \frac{1 - \mathbf{a}}{pf_{AA}^C + pf_{AA}^I} < 0$$

The denominator is negative with  $f_{AA}^C < 0$  and  $f_{AA}^I < 0$ , implying a decline of IF land use. The effect of unequal subsidies on land rent is:

$$(A2.4) \quad \frac{dr}{ds} = \frac{pf_{AA}^I + \mathbf{a}pf_{AA}^C}{pf_{AA}^C + pf_{AA}^I} \quad \text{and } 0 < \frac{dr}{ds} < 1.$$

Using these results it follows that:

$$(A2.5) \quad \frac{d\Pi^I}{ds} = \frac{-A^I pf_{AA}^I (1-\mathbf{a})}{pf_{AA}^C + pf_{AA}^I} < 0$$

$$(A2.6) \quad \frac{d\Pi^C}{ds} = \frac{A^C pf_{AA}^C (1-\mathbf{a})}{pf_{AA}^C + pf_{AA}^I} > 0$$

$$(A2.7) \quad \frac{d\Pi^L}{ds} = \frac{A^T (pf_{AA}^I + \mathbf{a}pf_{AA}^C)}{pf_{AA}^C + pf_{AA}^I} > 0$$

Landowners and CF gain while IF loose with unequal subsidies. (Q.E.D.)

### A3. Proof of proposition 4

The proof is similar to the proof of proposition 1. When new entrants are eligible for SFP, the IF and CF marginal conditions with transaction costs and imperfect competition are given by equations (11) and (10) respectively, equivalent to equations (A1.13) and (A1.14). Thus the effect with new entrants eligible for SFP payments is equivalent to the effect of area payments. (Proof for perfect competition is analogous.)

### A4. Proof of proposition 5

**Part a:** Step 1: To show:  $\frac{d(Q^T/A^T)}{dt} < 0$ , where  $Q^T = f^I(A^I) + f^C(A^C)$  is total output,

and  $\frac{Q^T}{A^T}$  is land productivity. Hence, using (A1.9), we need to show:

$$(A4.1) \quad \frac{d(Q^T/A^T)}{dt} = \frac{(f_A^I - f_A^C) dA^I}{A^T dt} < 0$$

With transaction costs (assuming fixed per unit  $t$ ), perfect competition, and without subsidies condition (A1.6) must be satisfied, as well as:

$$(A4.2) \quad pf_A^I = r + t \quad \text{and} \quad pf_A^C = r.$$

Total differentiating equations (A1.6) and (A4.2) yields (A1.9), as well as:

$$(A4.3) \quad pf_{AA}^I dA^I - dr = dt \quad \text{and} \quad pf_{AA}^C dA^C - dr = 0.$$

Using (A1.9) and (A4.3) it follows that  $\frac{dA^I}{dt} = \frac{1}{pf_{AA}^C + pf_{AA}^I} < 0$ . Equations (A4.2) imply

that in equilibrium (at point  $A_{I2}^*$  in figure 5)  $f_A^I > f_A^C$  with  $t > 0$  (with  $t_2 > 0$  in figure 5).

Hence  $f_A^I - f_A^C > 0$ . With  $\frac{dA^I}{dt} < 0$  and  $f_A^I - f_A^C > 0$ , it follows that  $\frac{d(Q^T/A^T)}{dt} < 0$ .

Step 2: To show:  $\left. \frac{d(Q^T/A^T)}{dt} \right|_{s=0} = \left. \frac{d(Q^T/A^T)}{dt} \right|_{s>0}$ . (A4.1) implies that this will be the case

if  $\left. \frac{dA^I}{dt} \right|_{s=0} = \left. \frac{dA^I}{dt} \right|_{s>0}$  and  $(f_A^I - f_A^C)|_{s=0} = (f_A^I - f_A^C)|_{s>0}$ . From proposition 1 it follows that

subsidies do not change land allocation. Hence  $\left. \frac{dA^I}{dt} \right|_{s=0} = \left. \frac{dA^I}{dt} \right|_{s>0}$ . At the initial

equilibrium ( $A_{I2}^*$  in figure 5), the marginal land productivity of IF and CF are not affected

by  $s$ :  $(f_A^I - f_A^C)|_{s=0} = (f_A^I - f_A^C)|_{s>0}$ . Combining these results, it follows that

$$\left. \frac{d(Q^T/A^T)}{dt} \right|_{s=0} = \left. \frac{d(Q^T/A^T)}{dt} \right|_{s>0} < 0. \quad (\text{Q.E.D. of part a.})$$

**Part b:** Assume  $s = e > 0$ . Since the SFP effects are not continuous, we analyze part b with discrete changes in  $t$ . From (A4.2) and (A4.3) it follows that for all  $A^I < A^T - A^*$  (where  $A^*$  is the CF equilibrium land renting with  $t = 0$ ) it holds (a) that  $f_A^I > f_A^C$ , (b) that

$\frac{\Delta A^I}{\Delta t} < 0$ , which implies that  $\frac{\Delta(Q^T/A^T)}{\Delta t} < 0$ , and (c) that  $\left. \frac{\Delta A^I}{\Delta t} \right|_s < \left. \frac{\Delta A^I}{\Delta t} \right|_e$ , which implies

that  $\left. \frac{\Delta(Q^T/A^T)}{\Delta t} \right|_s < \left. \frac{\Delta(Q^T/A^T)}{\Delta t} \right|_e$ . (This is bounded by  $|Dt| = t$ .) What is then left to show

is:  $\left. \Delta A^I \right|_s > \left. \Delta A^I \right|_e$  for  $|Dt| = t$ .

Case 1:  $e > |Dt| = t$

In equilibrium (at  $A_2^*$  in figure 5) for  $DA^I > 0$  the marginal land revenue for the IF remains smaller than the marginal land revenue of the CF:  $pf_A^I - t < pf_A^C + e$  (since IF do not get SFP for  $DA^I$  because it is above the eligible area). Proposition 3 implies that the difference is equal to  $e$ :  $(pf_A^I - t) - (pf_A^C + e) = e$ . The reverse holds for  $DA^I < 0$ :  $pf_A^I - t + e > pf_A^C$ , where  $(pf_A^I - t + e) - pf_A^C = e$ . Because  $e > |Dt|$  this implies that  $pf_A^I - (t - |\Delta t|) < pf_A^C + e$ . Hence, there will be no change in land allocation:

$$\left. \Delta A^I \right|_s > \left. \Delta A^I \right|_{e > |\Delta t|} = 0.$$

Case 2:  $e < |Dt| = t$

The SFP( $e$ ) equilibrium is determined by conditions (A1.6), and by  $pf_A^I = r + (t - |\Delta t|)$  and  $pf_A^C = r - e$ . The area payments equilibrium is determined by conditions (A1.5), (A1.6) and by  $pf_A^I = r + (t - |\Delta t|) - s$ . Comparing these conditions

implies that for each  $|Dt| = t$  it must be that in equilibrium  $f_A^I|_s < f_A^I|_e$ , and hence that

$$\Delta A^I|_s > \Delta A^I|_{e < |Dt|}. \text{ (Q.E.D. of part b.)}$$

**Part c:** This follows directly from the combination of part a and proposition 4. (Q.E.D.)



Footnotes

<sup>1</sup> The proof with transaction costs and imperfect competition is in Ciaian and Swinnen (2005).

## **References**

Ciaian, P., and J.F.M. Swinnen. 2005. "Land Market Imperfections and Agricultural Policy Impacts in the New EU Member States: A Partial Equilibrium Analysis."

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