ESTIMATING THE IMPORTANCE OF SHELF SPACE CONFIGURATION ON RETAILER'S PROFIT

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Maud Roucan-Kane, Bobby Martens,
and Paul V. Preckel

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Dept. of Agricultural Economics

Purdue University

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Maud Roucan-Kane
Dept. of Agricultural Economics, Purdue University
West Lafayette, Indiana 47907-2056
mroucan@purdue.edu

Bobby Martens
Dept. of Logistics and Supply Chain Management, Iowa State University
Ames, Iowa 50011
Martens1@iastate.edu

Paul Preckel
Dept. of Agricultural Economics, Purdue University
West Lafayette, Indiana 47907-2056
preckel@purdue.edu

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Abstract

Retail shelf space allocation remains a central issue in grocery retailing. A literature review produced many studies on retail shelf space allocation, but none which evaluated shelf space allocation using three major factors at once: space, vertical height, and price. In this study, shelf space allocation was modeled from the perspective of a retailer maximizing profit using space, vertical height, and price. Using benchmarking, the results show how shelf configuration affects consumer demand and retailer profit. Parameters for the model were based on experience-based intuition. Although the initial results are not valuable at this point, the method and results create a rationale and motivation to gather primary data. Once primary data is collected, this methodology has important applications. First, it develops an understanding of which parameters are important in determining optimal shelf space configuration. Second, a properly specified model would determine retailer's profit for specific shelf level configurations.

Keywords: shelf space allocation, retail, optimization, grocery, elasticity, GAMS, ketchup.

JEL Codes: C61, L81

Introduction

During the past three decades, the retail grocery industry has experienced steady industry wide consolidation, an exodus of traditional grocers, and the introduction of large grocery retailers (Wal-Mart, Target, and K-Mart). For traditional grocery retailers, the industry is extremely competitive. Therefore, properly using store shelf space to maximize profit is often mandatory for survival. One estimate is that 100,000 grocery products are available to retailers, but the typical supermarket only has shelf space for between 30,000 and 40,000 products (Santalla and Associates, 2006). Once the grocer decides which products to sell, space and location must be allocated to both categories and specific products within the categories.

Analysis of shelf space allocation has been the subject of numerous journal articles (Borin, Farris, and Freeland,1994; Urban, 1998; Bookbinder and Zarour, 2001; Martin-Herran et al.,2005). These studies evaluated shelf space allocation based on horizontal allocation of space (i.e., units of shelf allotted for products) or vertical positioning (i.e., lower shelf or higher shelf). However, most of the research focused on only allocation of space or vertical positioning, and none of the studies found in the literature modeled allocation of space, horizontal positioning, and price together. The purpose of this paper is therefore to add to the literature by creating a model that evaluates the effects of allocation of space, vertical positioning, and price on a retailer's goal of profit maximization. This paper will: 1) summarize previous works on the subject of shelf space allocation, 2) describe the math programming model being used to evaluate the retailer's profit based on different shelf space configuration, 3) present the data (provided by previous research and empirical evidence), and 3) discuss the results of the math programming model.

Previous Work

In 1981, Corstjens and Doyle wrote that "It is generally accepted that shelf space allocation is a central problem in retailing". In 1994, Borin, Freeland and Farris show that allocation is also important because consumers are not completely brand loyal; otherwise they would pick a product regardless of its in store-merchandising factors.[1] Early research worked to solve this central problem by allocating brands within category groups to scarce space (Hansen and Heinsbroek, 1979; Curhan, 1972, 1973). Closed loop solutions did not exist and most researchers approach the shelf space problem using heuristics which find an approximation to an optimal solution for tractability. For example, Malsagne, 1972 advocated giving more space to products that generate more sales. McKinsey, 1963 used the Direct Product Profitability (DPP) concept suggesting to allocate more space to products that generate more profits. These models simplified the complex problem of allocating scarce shelf space by ignoring space and cross elasticity effects on demand.

In 1981, Corstjens and Doyle looked at the optimization of shelf space for a chain of ice cream and candy stores with the demand rate being a function of shelf space allocated to the product. They applied a polynomial functional form of demand using own product space elasticities and inter-product cross elasticities of shelf space. They

[1] These include price, allocation of space, vertical positioning, and advertising.

also considered the supply side by taking into account inventory and handling costs. By modeling both shelf space and cross price elasticities with profit margins and costs, their model became the first to account for substitute and complement products. They used signomial geometric programming to solve the model. Their results indicate that retailers must give the highest share of their shelf space to high-growth products and remove the oldest ones unless they are complementary to products with high potential.

In 1983, the same authors introduced a dynamic optimization model which allowed for product growth (Corstjens and Doyle, 1983). With this model, Corstjens and Doyle showed that maintaining long-run profitability requires that retailers sacrifice short-term profits by allocating some space to new, hi-growth products. This is necessary due to product's life cycles. As in the earlier model, the authors sought to maximize profit using polynomial functional forms.

In 1986, Zufryden produced another dynamic model that incorporated a general objective function accounting for space elasticity, cost of sales, and potential demand related marketing variables. The model was created to make optimal selection among a given set of products and allocate integer units of shelf space to such selected products in supermarkets. For simplicity, the model ignored the cross elasticity among different products and fixed the values of non-space marketing variables in the demand functions.

Bultez and Naert (1988) created a model similar to Corstjens and Doyle (1981) but used marginal analysis on a general theoretical formulation. They used an attraction model to represent the interaction between brands and applied it to space allocation for several products categories present in Belgian and Dutch supermarkets. They provide an allocation rule that gives priority to items whose displays are the most profitable.

Borin, Farris, and Freeland (1994) modeled shelf space using a constrained optimization problem with assortment and allocation of space as the choice variables. Their model was developed to aid retailers decide which products to stock and how much shelf space to allocate to each product. Their model accounts for actual product dimensions, delivery cycles, selling costs, stocking, storing, effects of substitute items, and strength of customer loyalty. They incorporate shelf space elasticities and cross-elasticities into their model and included stock-out demand. They used return on inventory as the objective and simulated annealing as the solution methodology. In a later article, they did a sensitivity analysis on the parameter estimates and found that the "parameters can vary by as much as 50 percent and still make application of the model useful" (Borin et al., 1994).

Drèze, Hoch, and Purk (1994) considered the effect of location of the shelf in the store, vertical position of the product on the shelves, number of facings for each product, item deletions, and assortment/categorization (by flavor and brand) on sales dollars and profits. They did not test the effect of price. They reported that optimizing the position of the product is more important than the number of facings given to the product.

Using a polynomial functional form, Urban (1998) created a model that integrates both inventory management and shelf space allocation approaches. This inventory model incorporates the effect of displayed inventory on demand, distinguishing between backroom and displayed inventory; and looks at multiple items. From the shelf space management standpoint, this model incorporates inventory costs and the effect of less-than-full shelves on demand. However, as the author pointed out, including price would have been beneficial.

Bookbinder and Zarour (2001) incorporated the concept of direct product profitability (DDP) into the framework developed by Corstjens and Doyle (1981). They used elasticities for own and cross shelf space and DDP to determine the optimal selling area which maximizes profit.

A more recent shelf space allocation study recognized that previous methods did not incorporate the manufacturer's actions. For example, a manufacturer can influence the shelf space decision through promotions or by paying shelf slotting fees. Martin-Herran et al. (2005) use an open-loop Stackelberg game theory models to evaluate how a manufacturer could interact with a retailer to influence shelf space allocation decisions.

In summary, previous research has evaluated shelf space allocation using only one to two demand parameters, which include position, space, and price. We extend the findings of the previous research by evaluating how allocation of space, vertical positioning, and price affect the retailers' profit.

<u>Model</u>

In this paper, we used the approach called benchmarking to parameterize demand. It is a fairly popular approach among math programmers. The idea with benchmarking is that one begins with a snapshot of consumer behavior – that is, prices, consumption quantities, and demand – and wants to choose parameter values that are consistent with the observed consumer behavior. More precisely, we choose values for the parameters of the utility function so that the first-order conditions for the utility maximization problem are exactly satisfied at the observed levels of prices and consumption.

The model used in the study is:

$$\max\ profit = \sum_{choice=1}^{4}\left[\left(price_{choice} - \cos t_{choice}\right) * \frac{\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choice,a} + \alpha_{choice}\right)}{\sum_{choice=1}^{3}\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choice,a} + \alpha_{choice}\right)}\right]$$

$$\begin{aligned} \text{s.t.}\ & 0.5 \le \text{price}_{\text{i}} \le 5 \\ & \text{cost}_{\text{i}} = cstt \ge 0 \\ & \beta_{choice,a,choices} = cstt \\ & \alpha_{\text{choice}} = cstt \end{aligned}$$

Our assumptions in the research were the following:

1. The retailers' objective was to maximize profit of a single category, ketchup, ignoring slotting allowances.
2. The retailer sold three products within the ketchup category: a premium or national brand product, a store brand or private label, and a generic brand.[2] Generic products being quite inexistent in today's world, our generic represented a second national brand.
3. Consistent with prior research and the assumption of substitutability, a product's own space or position elasticity will lie between 0 and 1, and vice versa for the own and cross price elasticities.
4. We simplified the problem by ignoring the horizontal position and assumed that only the vertical dimension matters for consumers' demand. We restricted the number of vertical shelves to five.
5. Since we used "deviation from ideal height" as the proxy for product height, for simplicity, we had to assume that the bottom and top position would have the same effect on demand.
6. We assumed that to avoid stock-outs, the minimum number of facings is 12. For simplicity, we limited the space available to a length of 12 facings for each shelf and each shelf was filled up in each scenario.
7. For simplicity, we assumed that none of the products' packaging was more attractive than the other and all would be the same size. Thus, the main parameters that might affect demand was the vertical position of the product, number of facings and price.
8. For simplicity, we assumed that the products are storable i.e., non perishable and that there is no waste. Thus, all the products on the shelf can be sold.
9. For simplicity, we also assumed that there were not stockouts on the shelves.

Data

As a starting point for the benchmarking model, we used an initial shelf configuration. This configuration is based on how retailers usually allocate and organize their shelves (Figure 1).

Figure 1: Initial Configuration

	Product	# of Facings	Deviation from ideal height	Price
Shelf 1	**Store**	12	3	$0.78
Shelf 2	**Premium**	36	1	$1.57
Shelf 3 (Ideal Height)	**Premium**			
Shelf 4	**Premium**			
Shelf 5	**Generic**	12	3	$0.94

The deviation from ideal height was determined based on how far the shelf was from the ideal height (+1). For products stored on several shelves, the deviation was based on how far the closest shelf was from ideal height. The numeral 1 represents ideal

[2] Generic ketchup represents a fractional market share. In this study, the generic brand is representative of a secondary, lower quality brand.

shelf, 2 represents shelves immediately above or below the idea shelf (1), and 3 represents shelves two shelves above or below the idea shelf (1).

The prices are based on Wal-Mart prices as of May 10th, 2006 in West Lafayette, Indiana. The ketchup price of the brand Great Value was used to represent the price data for the store ketchup. The price of ketchup Heinz was used to represent the price of the premium ketchup. Finally, we used Hunt's for the generic/second national brand.

The elasticities associated with this configuration are hypothetical but partially based on literature (Bookbinder and Zarour, 2001; Hoch et al.,1995; Borin, Farris, Freeland, 1994) and experienced based intuition:

Figure 2: Own and Cross Elasticities

		Height/Facing/Price		
		Premium	Store	Generic
Sales	Premium.height deviation	0.05	-0.02[3]	-0.01
	Store.height deviation	-0.03	0.15	-0.14
	Generic.height deviation	-0.02	-0.12	0.125
	Premium.number of facings	0.1	-0.06[4]	-0.03
	Store.number of facings	-0.1	0.3	-0.2
	Generic.number of facings	-0.05	-0.15	0.25
	Premium.price	-0.8	0.75[5]	0.55
	Store.price	0.33	-1.2	1
	Generic.price	0.25	0.9	-1

Knowing the configuration and the elasticities, the model calculated the betas based on the following equation:

Elasticity[6]= $\frac{\partial proba}{\partial x_{a,choice}} * \frac{x_{a,choice}}{proba}$

(1)

[3] A 1% increase toward ideal position for the store brand would decrease sales of the premium brand by 0.02%.
[4] A 1% increase in space for the store brand would decrease sales of the premium brand by 0.06%.
[5] A 1% increase in the price of the store brand will increase sales of the premium brand by 0.75%.
[6] Note: "proba" refers to equations 2 and 3.

$$
= \left(\frac{\beta_{i,a,choice} * \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right) * \sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)}{\left(\sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)\right)^2} + \frac{\exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a.choice} + \alpha_{choice}\right) * \beta_{i,a,choice} * \sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)}{\left(\sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)\right)^2} \right)
$$

$$
* \left(\frac{x_{a,choice} * \sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)}{\exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)} \right)
$$

$$
= \left(\frac{\beta_{i,a,choice} * \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right) + \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right) * \beta_{i,a,choice}}{\left(\sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)\right)} \right)
$$

$$
* \left(\frac{x_{a,choice} * \sum_{choice=1}^{3} \exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)}{\exp\left(\sum_{a=1}^{3} \beta_{i,a,choice} x_{a,choice} + \alpha_{choice}\right)} \right)
$$

$$
= 2\beta_{i,a,choice} x_{a,choice}
$$

$$
\Leftrightarrow \beta_{\mathrm{i,a,choice}} = \frac{e_{i,a,choice}}{2x_{a,choice}}
$$

$\beta_{'nothing','price',i}$ satisfies the FOC:

$$
\mathrm{FOC_i} : \mathrm{proba_i} +
$$

$$
\sum_{choice} \left(\begin{array}{l} \left(x_{'price',choice} - \cos t_{choice}\right) * \begin{pmatrix} beta_{choice,price,i} \text{ if ord(choice)} < 4, \\ beta_{'nothing','price',i} \text{ if ord(choice)} = 4 \end{pmatrix} * proba_{choice} \\ - proba(choice) * \sum_{\mathrm{choice}} \left(\begin{pmatrix} beta_{choice,price,i} \text{ if ord(choice)} < 4, \\ beta_{'nothing','price',i} \text{ if ord(choice)} = 4 \end{pmatrix} * proba_{choice} \right) \end{array} \right) = 0
$$

with *choice* being purchase options for the customer (premium, store, generic or nothing). *i* being the products (premium, store, and generic). *a* represents the attributes (deviation from ideal height, number of facings, and price). x represents the values for each attribute of each product.

Based on the literature and hands-on experience, we also assumed that for each customer walking down the aisle of ketchup there is a 25% chance that he/she will not buy anything, 37.5% probability that the customer will choose the premium product, 15% probability for the store product, and 22.5% probability for the generic product (Fraser, 2004; Ross, 2004; Mollenkamp, 2004; Wysocki, 2005). Based on these assumptions and equations 2 and 3 for the probability, we determine the alphas (with $\alpha_{generic}$ being the base i.e., $\alpha_{generic} = 0$) (see equation 3).

Probability consumer chooses product 'choice'

$$(2) = \frac{\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choices,a} + \alpha_{choice}\right)}{\sum_{choice=1}^{3}\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choices,a} + \alpha_{choice}\right)}$$

with *'choice'* being the choices available to the customer (premium, store, generic, and nothing) and *'choices'* being an alias of choice

$$\frac{\text{proba (choose product 1)}}{\text{proba(choose product 2)}} = \exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{1,a,choices}x_{choices,a} - \sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{2,a,choices}x_{choices,a} + \alpha_1 - \alpha_2\right)$$

assuming $\alpha_2 = \alpha_{generic} = 0$

and we can calculate α_1 as being the alpha for the premium, the store product, or 'nothing' with :

$$\Leftrightarrow \alpha_1 = \ln\left(\frac{\text{proba (choose product 1)}}{\text{proba(choose product 2)}}\right) - \sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{1,a,choices}x_{choices,a} + \sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{2,a,choices}x_{choices,a}$$

$$\Leftrightarrow \alpha_{choice}\ \text{(if choice < 3)} = \ln\left(\frac{\text{proba}_{\text{choice}}}{\text{proba}_{\text{'generic'}}}\right) - \sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choices,a}$$

$$+ \sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choices,a}$$

Now the benchmarking model consisted of using the betas we calculated to determine how we should price the products to maximize profit for each of the eighteen configurations (presented in Appendix A). Using the data we had for the alphas, the costs, and the betas, we determined the optimal price for each product by maximizing the following profit function:

$$profit = \sum_{choice=1}^{4}\left[\left(price_{choice} - \cos t_{choice}\right) * proba_i\right]$$

$$= \sum_{choice=1}^{4}\left[\left(price_{choice} - \cos t_{choice}\right) * \frac{\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choice,a} + \alpha_{choice}\right)}{\sum_{choice=1}^{3}\exp\left(\sum_{choices=1}^{4}\sum_{a=1}^{3}\beta_{choice,a,choices}x_{choice,a} + \alpha_{choice}\right)}\right]$$

[Equation 4]

The costs for each product were kept constant among the different configurations and were based on the prices used in the initial configuration. The cost represented 95% of the price ($1.4915) for the premium ketchup, 90% of the price ($0.846) for the generic/second national branded ketchup, 80% of the price ($0.624) for the store product (Ross, 2004; Radhakrishnan, K. 2002; Ward et al.). The probabilities of buying each product were calculated using equation 2.

Empirical Results

Using experience based intuition, data from Wal Mart for prices, and data from the literature for the elasticities (Bookbinder and Zarour, 2001; Hoch et al.,1995; Borin, Farris, Freeland, 1994), the probabilities (Fraser, 2004; Ross, 2004; Mollenkamp, 2004; Wysocki, 2005), and the margins (Ross, 2004; Radhakrishnan, K. 2002; Ward et al.) we developed a benchmarking model using the software GAMS (refer to Appendix B for the code).

Eighteen configurations (presented in Appendix A) were tested. The optimal prices maximizing retailer's profit were determined for each of the three products and for each configuration (see Figure 5). The model also gave us the opportunity to compare the profit among the eighteen configurations and determine which configuration yielded the best profit (see Figure 5). Profit was calculated on a per unit basis, which explains the small numbers.

Figure 3: Optimal Price and Profit for Each Configuration

Configuration #	Premium	Store	Generic	Profit
Initial	1.57	0.78	0.94	73,987.50
2	1.52	0.92	0.93	82,404.38
3	1.49	0.78	1.08	78,767.08
4	1.54	0.86	0.94	77,228.42
5	1.49	0.86	1.02	78,634.70
6	1.52	0.79	1.02	75,526.65
7	1.54	0.9	0.92	81,793.61
8	1.51	0.75	1.06	78,142.84
9	1.57	0.77	0.94	73,829.77
10	1.52	0.82	0.99	76,714.28
11	1.54	0.75	1.01	75,202.95

12	1.56	0.83	0.93	76,313.07
13	1.5	0.77	1.06	77,898.86
14	1.58	0.79	0.93	74,236.45
15	1.54	0.91	0.9	82,652.38
16	1.54	0.78	0.99	74,897.02
17	1.56	0.85	0.9	78,059.23
18	1.53	0.84	0.98	77,226.26

The model also reported the probability that the consumer buys a product when he walks down the aisle of ketchup. The probabilities were reported for each configuration and Figure 6 also reports the probability of buying nothing. The results from Figure 5 and 6 and additional results are available in Appendix C.

Figure 4:Probabilities for Each Configuration

	Probabilities			
Configuration #	**Premium**	**Store**	**Generic**	**Nothing**
Initial	0.375	0.15	0.225	0.25
2	0.373643	0.18029	0.216102	0.229965
3	0.367216	0.142784	0.252172	0.237828
4	0.372368	0.164758	0.221072	0.241802
5	0.366307	0.16131	0.23422	0.238163
6	0.368982	0.146813	0.238414	0.245791
7	0.375799	0.178265	0.214539	0.231397
8	0.369803	0.139117	0.251675	0.239405
9	0.375513	0.147519	0.226544	0.250424
10	0.370315	0.155052	0.2317	0.242933
11	0.372547	0.140787	0.239921	0.246745
12	0.374843	0.160168	0.220851	0.244139
13	0.368949	0.141351	0.249742	0.239958
14	0.376079	0.151829	0.22681	0.249411
15	0.377288	0.180639	0.212499	0.229575
16	0.37174	0.14707	0.233684	0.247506
17	0.377404	0.167322	0.215243	0.240031
18	0.370903	0.159739	0.227632	0.241726

The initial configuration yielded the second lowest profit. Configurations 15, 2, and 7 yielded the highest profit; and these three configurations had 36 facings of the store product at the ideal height. Configurations 9, 1, and 14 yielded the lowest profit, and there three configurations had 36 fad 36 facings of the premium product at the ideal height. These results suggested that having the storec product at the ideal shelf may maximize profit, while having the premium product at the ideal shelf may minimize profit. These results would explain why retailers have had a grown interest in promoting and offering store/private-label brands.

The price for each product did not vary much from one configuration to another. Compared to the initial configuration, the three configurations that yielded the highest profit had lower prices for the premium and generic products and higher prices for the store product. This result suggested that probably retailers should lower their price (and therefore margins) on premium and generic products, but could increase the prices on the store products, while still selling more as the probability results suggest. Compared to the initial configuration, the premium product was the one that proportionally faced the lowest variation in price, while the generic and store products faced about the same variation. This could be explained by the fact that we assumed that the premium product had the largest market share but the lowest margin; so increasing prices won't have much effect on the retailer's margin. The generic product had a larger market share than the store product but a lower margin which may explain why they face about the same variation in price. Increasing the store product's price will mainly have an effect on the retailer's margin; while increasing the generic product's price will mainly have an effect on total profit.

Conclusion and Future Research

As stated at the beginning of the paper, the main objective of this study was to show how one could create a model taking into account the three main parameters affecting demand and retailers' profit: allocation of space, vertical positioning, and price. The objective was reached. Several conclusions can be drawn from this exercise. However, the conclusions stated below may be taken with extreme cautions as we used secondary data and hands-on experience. The profits did not vary hugely from one configuration to another suggesting that depending on the volume sales of a store and its space limits, there may not always be a need or justification for the search of the optimal configuration. However, this research shows evidence of better profits when the best configuration (more facings and position at ideal height) is chosen for the store product. The results also highlight that the worst profits are attained when the premium product is given the best configuration. These last results could maybe be mitigated if we had taken into account the slotting fees.

This research has several limitations. First, the elasticities we used are not completely accurate, we did not test our initial configuration, but instead used secondary data. Second, only one category of product was studied with only three "brands" of products being studied in that category. Finally, we did not take into account an important aspect of the retail industry: slotting allowances. It would be easy to introduce slotting allowances in the model in the calculation of the margins/costs per product.

For lack of time and data, we used secondary data and experiences based intuition, which made our results not totally reliable but the methodology is still relevant. Consequently, this research with more time, more fundings, and partnership with retailers, could be taken a step further by gathering the data needed for the model. We could also look at other configurations (two products on the same shelf for example), more products and other categories of products. For example, in this model, we used a staple product; it would be interesting to look at products such as ice cream or chocolate that probably have different elasticities. Studying perishable product such as milk would also be of interest. We could also look at the interaction by studying several categories at once and using elasticities with substitute and complementary products. We could also take into account marketing strategies (promotion, attractiveness of the product's packaging, size, and so on). We could also create a dynamic model and include the effect of replenishment, stockouts and supply availability restrictions from the backroom.

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Appendix A: The 18 Configurations

		Product	Deviation from Ideal Height	# of Facings
Initial Configuration	**Shelf 1**	**Store**	3	12
	Shelf 2	**Premium**	1	36
	Shelf 3 (Ideal Height)	**Premium**		
	Shelf 4	**Premium**		
	Shelf 5	**Generic**	3	12
Configuration 2	**Shelf 1**	**Store**	1	36
	Shelf 2	**Store**		
	Shelf 3 (Ideal Height)	**Store**		
	Shelf 4	**Premium**	2	12
	Shelf 5	**Generic**	3	12
Configuration 3	**Shelf 1**	**Store**	3	12
	Shelf 2	**Premium**	2	12
	Shelf 3 (Ideal Height)	**Generic**	1	36
	Shelf 4	**Generic**		
	Shelf 5	**Generic**		
Configuration 4	**Shelf 1**	**Store**	2	24
	Shelf 2	**Store**		
	Shelf 3 (Ideal Height)	**Premium**	1	24
	Shelf 4	**Premium**		
	Shelf 5	**Generic**	3	12
Configuration 5	**Shelf 1**	**Store**	2	24
	Shelf 2	**Store**		
	Shelf 3 (Ideal Height)	**Premium**	1	12
	Shelf 4	**Generic**	2	24
	Shelf 5	**Generic**		

		Product	Deviation from Ideal Height	# of Facings
Configuration 6	Shelf 1	Store	3	12
	Shelf 2	Premium	1	24
	Shelf 3 (Ideal Height)	Premium		
	Shelf 4	Generic	2	24
	Shelf 5	Generic		
Configuration 7	Shelf 1	Generic	3	12
	Shelf 2	Store	1	36
	Shelf 3 (Ideal Height)	Store		
	Shelf 4	Store		
	Shelf 5	Premium	3	12
Configuration 8	Shelf 1	Generic	1	36
	Shelf 2	Generic		
	Shelf 3 (Ideal Height)	Generic		
	Shelf 4	Store	2	12
	Shelf 5	Premium	3	12
Configuration 9	Shelf 1	Generic	3	12
	Shelf 2	Store	2	12
	Shelf 3 (Ideal Height)	Premium	1	36
	Shelf 4	Premium		
	Shelf 5	Premium		
Configuration 10	Shelf 1	Generic	2	24
	Shelf 2	Generic		
	Shelf 3 (Ideal Height)	Store	1	24
	Shelf 4	Store		
	Shelf 5	Premium	3	12

		Product	Deviation from Ideal Height	# of Facings
Configuration 11	Shelf 1	Generic	2	24
	Shelf 2	Generic		
	Shelf 3 (Ideal Height)	Store	1	12
	Shelf 4	Premium	2	24
	Shelf 5	Premium		
Configuration 12	Shelf 1	Generic	3	12
	Shelf 2	Store	1	24
	Shelf 3 (Ideal Height)	Store		
	Shelf 4	Premium	2	24
	Shelf 5	Premium		
Configuration 13	Shelf 1	Premium	3	12
	Shelf 2	Generic	1	36
	Shelf 3 (Ideal Height)	Generic		
	Shelf 4	Generic		
	Shelf 5	Store	3	12
Configuration 14	Shelf 1	Premium	1	36
	Shelf 2	Premium		
	Shelf 3 (Ideal Height)	Premium		
	Shelf 4	Generic	2	12
	Shelf 5	Store	3	12
Configuration 15	Shelf 1	Premium	3	12
	Shelf 2	Generic	2	12
	Shelf 3 (Ideal Height)	Store	1	36
	Shelf 4	Store		
	Shelf 5	Store		

		Product	Deviation from Ideal Height	# of Facings
Configuration 16	Shelf 1	Premium	2	24
	Shelf 2	Premium		
	Shelf 3 (Ideal Height)	Generic	1	24
	Shelf 4	Generic		
	Shelf 5	Store	3	12
Configuration 17	Shelf 1	Premium	2	24
	Shelf 2	Premium		
	Shelf 3 (Ideal Height)	Generic	1	12
	Shelf 4	Store	2	24
	Shelf 5	Store		
Configuration 18	Shelf 1	Premium	3	12
	Shelf 2	Generic	1	24
	Shelf 3 (Ideal Height)	Generic		
	Shelf 4	Store	2	24
	Shelf 5	Store		

Appendix B: GAMS Code

```
Option limrow=0, limcol=0;
option decimals=6;

*part I, find beta;
set
choice choices /premium, store, generic, nothing/
i(choice) product /premium, store, generic/
a attributes /height, facing, price/;


alias (i,ii), (choice,choices);

Parameter
totdem demand for all products
proba(choice) probability of consumer choosing product p
beta(choice,a,choices) demand parameters;


Table x(a, choice)  Initial configuration

          Premium   Store   Generic   Nothing
Height       1        3        3        0
Facing      36       12       12        0
Price     1.57      .78      .94        0
;


Table e(i,a,choices) elasticities
                  Premium   Store   Generic
Premium. Height     0.05    -0.02    -0.01
Store.Height       -0.03     0.15    -0.14
Generic.Height     -0.02    -0.12     0.125
Premium.Facing      0.1     -0.06    -0.03
Store.Facing       -0.1      0.3     -0.2
Generic.Facing     -0.05    -0.15     0.25
Premium.Price      -0.8      0.75     0.55
Store.Price         0.33    -1.2      1
Generic.Price       0.25     0.9     -1
;

beta(i,a,choices)= e(i,a,choices)/(2*x(a,i));


Parameter cost(choice) costs for each product /Premium 1.4915, Store .624,
```

```
Generic .846/;

Parameter proba(choice) probability product i will be bought
/premium .375, store .15, generic .225, nothing .25/;


alias (choice,choicess);
variable betav(i) beta('nothing''price'ii);
Equation
FOC(i) FOC to find beta nothing;
FOC(i).. proba(i)+sum(choice,(x('price',choice)-cost(choice))*
((beta(choice,'price',i)$(ord(choice) lt card(choice))
+ betav(i)$(ord(choice) eq card(choice)))
*proba(choice)- proba(choice)*sum(choicess,
(beta(choicess,'price',i)$(ord(choicess) lt card(choicess))
+ betav(i)$(ord(choicess) eq card(choicess)))
*proba(choicess)))) =e= 0;

Model FOC1 /FOC/;
Solve FOC1 using CNS;

beta('nothing','price',i) = betav.l(i) ;

display beta, x;

Parameter
alpha(choice) bias the bigger the alpha the highest the proba of buying that product;

alpha('generic')=0;
alpha(choice)$(ord(choice) ne 3)=log(proba(choice)/proba('generic'))-
sum((a,choices),beta(choice,a,choices)*x(a,choices))
+sum((a,choices),beta('generic',a,choices)*x(a,choices));

set
config /1*18/;

Parameter
int(config,choice) part of the profit function;
int('1',choice) = sum(choices,sum(a,
beta(choice,a,choices)*x(a,choices)))+alpha(choice);


Display proba, alpha;

parameter
profit1 profit value;
```

```
profit1 = sum(i,(x('Price',i)-cost(i))*proba(i));

Display profit1;


Table x2(config,a, choice)  attributes for the configurations

Height.Premium  Height.Store  Height.Generic  Facing.Premium  Facing.Store
Facing.Generic
1    1    3    3    36    12    12
2    2    1    3    12    36    12
3    2    3    1    12    12    36
4    1    2    3    24    24    12
5    1    2    2    12    24    24
6    1    3    2    24    12    24
7    3    1    3    12    36    12
8    3    2    1    12    12    36
9    1    2    3    36    12    12
10   3    1    2    12    24    24
11   2    1    2    24    12    24
12   2    1    3    24    24    12
13   3    3    1    12    12    36
14   1    3    2    36    12    12
15   3    1    2    12    36    12
16   2    3    1    24    12    24
17   2    2    1    24    24    12
18   3    2    1    12    24    24
;

alias (config, configg)  ;

Parameter price2(choice) price for each product  /premium 1.57, store .78, generic .94/;
Variable
r2 profit value
p2(choice) price to be optimized;
p2.lo(choice) = 0.01 ;

Scalar oconf;

int(config,choice) = sum(choices,sum(a$(ord(a)lt card(a)),
beta(choice,a,choices)*x2(config,a,choices)))+alpha(choice);

alias (choice,choicess);
Equations
profit2 profit constraint 2 ;
p2.l(choice) = price2(choice) + uniform(-.1,.1) ;
```

```
p2.fx('nothing') = 0 ;
profit2.. r2 =e= 1000000*sum(choice,(p2(choice)-cost(choice))*
(exp(sum(config$(ord(config) eq oconf), int(config,choice))
+sum(choices,beta(choice,'price',choices)*p2(choices)))/
sum(choicess,exp(sum(config$(ord(config) eq oconf), int(config,choicess))
+sum(choices,beta(choicess,'price',choices)*p2(choices))))));

Model SS2  /profit2/;

Parameter
profit3(configg) revenue generated by each configuration
price3(configg,i) price for each product in each configuration
proba2(configg,choices) probabilities of buying a product
x3(configg,a,i) configurations;

loop(configg,
oconf = ord(configg);
Solve SS2 using nlp maximizing r2;
profit3 (configg) = r2.l;
price3(configg,i) = p2.l(i);
x3(configg,a,i) = x2(configg, a, i);
proba2(configg,choice)= exp(int(configg,choice)
+sum(choices,beta(choice,'price',choices)*p2.l(choices)))/
sum(choicess,exp(int(configg,choicess)
+sum(choices,beta(choicess,'price',choices)*p2.l(choices))));
);

Display beta, x, alpha, price3, x3, profit1, profit3, proba, proba2;
```