RISK EFFICIENCY IN THE INTERPRETATION OF AGRICULTURAL PRODUCTION RESEARCH

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Risk is often perceived by farmers as being more formidable in new technologies emanating from agricultural research than in more traditional practices. Consequently risk may tend to act as an impediment to adoption of improved practices as well as a general friction on efficient use of resources. Research and extension workers in agriculture may thus wish to identify technologies that are not only "improved" (more productive and profitable on the average) but are also less risky in that they would be preferred by "risk-averse" farmers. The extent to which this identification can proceed in the absence of knowledge of farmers' individual attitudes to risk is explored here through application of the concepts of stochastic dominance.

1 INTRODUCTION

Risk is now widely recognized as a key factor in nearly all farming activities and has especially come into prominence as an important factor in the adoption of new technology by farmers, especially those in traditional agricultures.

The well-developed analytical framework of Bernoullian decision theory exists for incorporating consideration of risk in planning but it is a personalized structure that emphasizes the individual's preferences for risk and his individual feelings of uncertainty and perception of risk [3, 10, 16]. Even such decision-theoretic methods have so far found very restricted practical application because of the difficulties and costs involved in eliciting farmers' subjective probabilities and encoding their preferences in utility functions. To date, preferences have probably been elicited for something less than 400 farmers. How, then, can anything useful be said about risky planning for the remaining millions of farmers?

An approach which holds some promise is the notion of ordering risky prospects according to stochastic dominance (SD) rules [14]. The present purpose is first (in section 2) to review these rules in a didactic manner that is hopefully easier to follow than the classical articles on the subject [15, 17, 18, 34, 42] and then to exploit the rules in several agricultural applications in section 3 in order to exemplify the promise.

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Most of the work on stochastic dominance [15, 18, 19, 29, 30, 31] has had as its rationalization the evident superiority of SD to orderings of uncertain quantities based only on moments, notably the mean and variance. This is not regarded here as a crucial advantage of SD—the prime purpose is to explore how far one can go in risk planning in the absence of specific assumptions about the algebraic form of farmers' preference functions.

The SD ordering rules prove computationally tedious in most applications, so an important part of the applications sections is the discussion of the development of several computer programs that greatly simplify SD procedures.

The empirical applications described below all relate to problems in the interpretation of agricultural research. It is believed that, whenever research is addressed to the development of new varieties and practices, etc. that are intended for adoption by "risk-averse" farmers, the principles of stochastic efficiency are pertinent and indeed offer an important method of filtering out inefficient technological packages (i.e. packages that would not be preferred and adopted by those averse to risk) so that they are not extended to the farming community.

2 MATHEMATICAL BACKGROUND OF SD ORDERING RULES

Definitions

Consider two probability density functions (PDF's) $f(x)$ and $g(x)$ for the random variable x which does not take values outside the range $[a, b]$, (i.e. outside $[a, b]$, $f(x)$ and $g(x)$ are everywhere zero). Assuming x to vary continuously over its range so that the PDF's are continuous, (less-than) cumulative distribution functions (CDF's) can be defined as:

(2.0.1) $F_1(R) = \int_a^R f(x)\,dx, \quad G_1(R) = \int_a^R g(x)\,dx,$ so that R varies continuously on the interval $[a, b]$.

The procedure of accumulating areas under $f(x)$ to define $F_1(R)$ can be applied to $F_1(R)$ to accumulate area under the CDF and thus define $F_2(R)$, i.e.,

(2.0.2) $F_2(R) = \int_a^R F_1(x)\,dx, \quad G_2(R) = \int_a^R G_2(x)\,dx.$

Analogously define

(2.0.3) $F_3(R) = \int_a^R F_2(x)\,dx, \quad G_3(R) = \int_a^R G_2(x)\,dx,$ so that in general,

(2.0.4) $F_n(R) = \int_a^R F_{n-1}(x)\,dx,$ for $n = 1, 2, 3, \ldots$

and for consistency, write $F_0(R) = f(x)$. These functions are illustrated in figure 1.

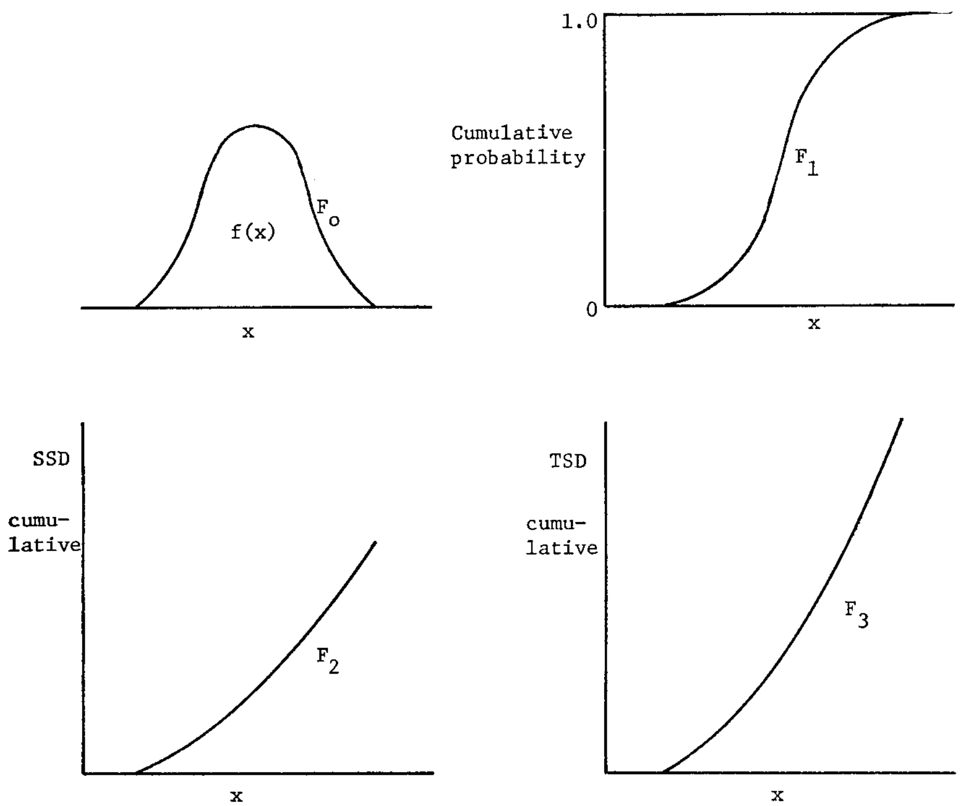

FIGURE 1: *Schematic probability and derived SD functions*

We shall be concerned with choice between alternatives described by a single uncertain quantity, x. A decision-maker's preferences for x are encoded in a utility function $U(x)$ which is defined for all x in $[a, b]$. Several increasingly restrictive assumptions shall be introduced concerning the preference function. These shall involve the first three derivatives of $U(x)$, of which the i-th is written $U_i(x)$, which are also defined for all x in $[a, b]$. Under risk, utility maximization implies maximizing expected utility.

2.1 First-Degree Stochastic Dominance (FSD)

Preference Assumption

This initial case presumes only that decision-makers prefer more to less of x. This implies that the function $U(x)$ is monotonically increasing between a and b or equivalently, $U_1(x) > 0$.

Ordering rule

The distribution $f(x)$ dominates $g(x)$ by first-degree stochastic dominance (FSD) if, and only if, $F_1(R) \leqslant G_1(R)$ for all R in $[a, b]$ with strict inequality for at least one value of R.

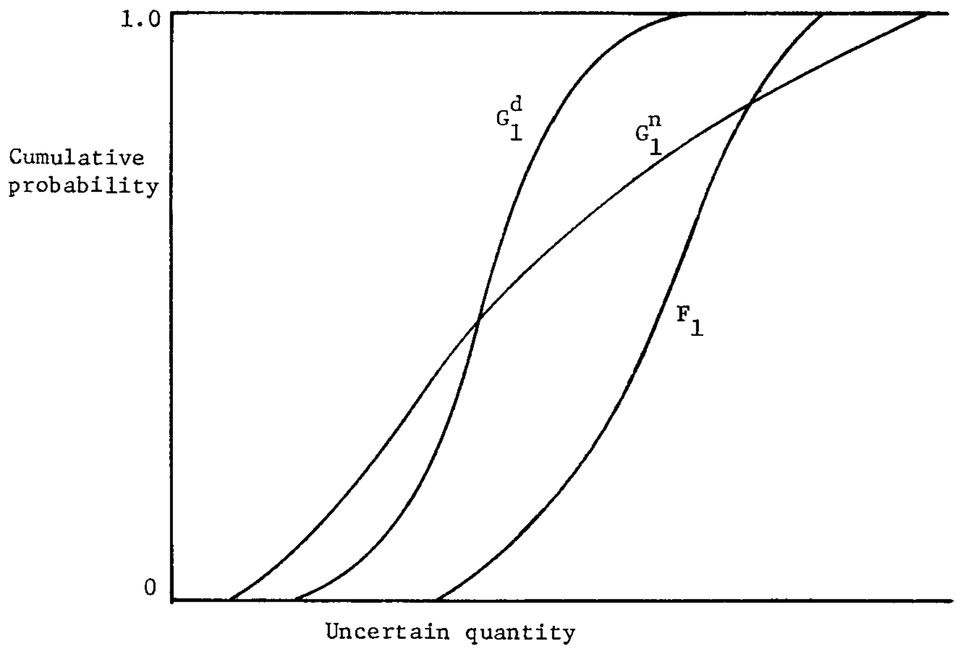

FIGURE 2: *Illustration of FSD* (F_1 *dominates* $G_1{}^d$ *but not* $G_1{}^n$)

Figure 2 depicts the FSD case graphically. For $f(x)$ to dominate $g(x)$, the F_1 (CDF) curve must nowhere lie to the left of the G_1 curve.

Theorem

If $F_1(R) \leqslant G_1(R)$ for all R in $[a, b]$, (i.e., if $f(x)$ dominates or is identical to $g(x)$ in the sense of FSD), then $f(x)$ is at least as preferred as $g(x)$ (i.e. the expected utility associated with $f(x)$, $\bar{u}_f$ is at least as large as the expected utility of $g(x)$, $\bar{u}_g$). That is, decision-makers with any $U(x)$ such that $U_1(x) > 0$ will prefer an FSD dominant distribution to one that is dominated.

Proof

It is required to prove that if $F_1(R) \leqslant G_1(R)$ (or $G_1(R) - F_1(R) \geqslant 0$) for all R in $[a, b]$, then $\bar{u}_f \geqslant \bar{u}_g$, or $\bar{u}_f - \bar{u}_g \geqslant 0$. An expected utility is defined by,

$$\bar{u}_f = \int_a^b U(x) f(x)\, dx.$$

Thus

$$\bar{u}_f - \bar{u}_g = \int_a^b U(x) f(x)\, dx - \int_a^b U(x)\, g(x)\, dx$$

and recalling that the derivative of a CDF is the corresponding PDF,

$$\begin{aligned} \bar{u}_f - \bar{u}_g &= \int_a^b U(x)\, (dF_1(x))\, dx - \int_a^b U(x)\, (dG_1(x))\, dx \\ &= \int_a^b U(x)\, [dF_1(x) - dG_1(x)]\, dx \end{aligned}$$

which is in a convenient form to integrate by parts,[1] thus

(2.1.1)

$$\bar{u}_f - \bar{u}_g = \Big[U(x)\,[F_1(x) - G_1(x)]\Big]_a^b - \int_a^b [F_1(x) - G_1(x)]\,U_1(x)\,dx.$$

Since a CDF ranges from zero at the lowest range ($x = a$) to unity at upper bound $x = b$, the first term in the above expression disappears since

$U(b)\,[1 - 1] - U(a)\,[0 - 0] = 0.$

Then

$$\text{(2.1.2)}\quad \bar{u}_f - \bar{u}_g = -\int_a^b U_1(x)\,[F_1(x) - G_1(x)]\,dx$$

$$= \int_a^b U_1(x)\,[G_1(x) - F_1(x)]\,dx$$

By assumption, $U_1(x) > 0$ and $[G_1(x) - F_1(x)] \geqslant 0$ for all possible values of x, so the integral in (2.1.2) consists of integrating the non-negative product of the positive marginal utility and non-negative differences in cumulative probabilities and accordingly is also non-negative, i.e. $\bar{u}_f - \bar{u}_g \geqslant 0$ which is what was required to be proved. When the theorem is modified by introducing strict inequality of $F_1(R)$ and $G_1(R)$ for at least one possible value of R, it can be proved that $\bar{u}_f > \bar{u}_g$, and it follows that FSD as defined implies strict preference for a dominant distribution.

Related results

A converse theorem can also be proved [15] but this is not done here since the converse result is not so important in the present context. If $f(x)$ is preferred to $g(x)$ according to all Bernoullian utility functions for which $U_1(x) > 0$, then $f(x)$ dominates $g(x)$ by FSD.

The FSD theorems hold also for the case of discrete distributions [15] where, say, x takes only a finite number of values x_i, $i = 1, n$ all on the interval $[a, b]$. A probability mass function can then be written as $f(x_i)$, and with the x_i arrayed in ascending order, a CDF defined as

(2.1.3) $F_1(R) = P(x \leqslant R) = \sum_{\text{all } x \leqslant R} f(x_i)$, which is a step function consisting of horizontal lines the leftmost heights of which are defined at the sample values of x_i as

$$\text{(2.1.4)}\quad F_1(x_r) = \sum_{i=1}^{r} f(x_i),\ r = 1, \ldots n.$$

The FSD ordering rule can then be stated as: $f(x_i)$ dominates $g(x_i)$ by FSD if, and only if, $F_1(x_i) \leqslant G_1(x_i)$ for all x_i's with strict inequality for at least one value.

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[1] If y and z are functions of x,

$$\int_a^b y\,(dz/dx)\,dx = [y\,z]_a^b - \int_a^b z\,(dy/dx)\,dx$$

2.2 Second-Degree Stochastic Dominance (SSD)

Preference assumptions

The second case adds the assumption that successive amounts of x have diminishing value to a decision-maker—e.g. the 1 000-th unit of income is not quite as valuable to its recipient as the 1-st or the 999-th. This is the assumption of diminishing marginal utility or of a concave preference function. Algebraically it amounts to assuming that, as well as $U_1(x) > 0$, the second derivative $U_2(x) < 0$, (i.e. $U(x)$ is concave with respect to x).

Behaviourally, individuals whose preferences accord with these assumptions are said to be *averse to risk*. The following rule, theorem and results also hold under the assumption of "non-preference" for risk ($U_2(x) \leqslant 0$) but this is not highlighted since the addition of risk indifference to the risk aversion assumption seems of trivial practical importance.

Ordering rule

The distribution $f(x)$ dominates $g(x)$ by second-degree stochastic dominance (SSD) if, and only if, $F_2(R) \leqslant G_2(R)$ for all possible R with strict inequality for at least one value of R.

Figure 3 depicts the SSD case graphically, where f is dominant if the F_2 curve lies nowhere to the left of the G_2 curve. Intuitive interpretation of this rule is not easy in terms of the F_2-type curves but is simplified by observing the corresponding CDF curves of figure 3. A necessary condition for f to be dominant in the sense of SSD is that the area labelled A is not less than the area labelled B [19].

Theorem

If $F_2(R) \leqslant G_2(R)$ for all possible R and $U_2(R) < 0$ for all R in $[a, b]$, then $f(x)$ is at least as preferred as $g(x)$.

Proof

The proof follows a similar style to that used for FSD and the same notation is employed. In fact, proof is identical up to the stage of equation (2.1.1) at which time under FSD the residual term, $-\int_a^b U_1(x)\,[F_1(x) - G_1(x)]\,dx$, could be declared non-negative. Now it is no longer assumed that the $[F_1(x) - G_1(x)]$ is non-negative for all x—in fact the CDF's underlying SSD comparisons will, in general, intersect otherwise FSD would have indicated the dominated distributions as being unworthy of further attention by utility-maximizing decision makers. To proceed we need to disentangle the residual term of equation (2.1.1) by a further integration by parts. Recall that from the definition of $F_n(R)$ it follows by differentiating that $d[F_2(x)]/dx = F_1(x)$, so that

$$\int_a^b U_1(x)\,[F_1(x) - G_1(x)]\,dx = \int_a^b U_1(x)\,\{d[F_2(x) - G_2(x)]/dx\}\,dx$$

$$= U_1(x)\,[F_2(x) - G_2(x)] - \int_a^b [F_2(x) - G_2(x)]\,U_2(x)\,dx.$$

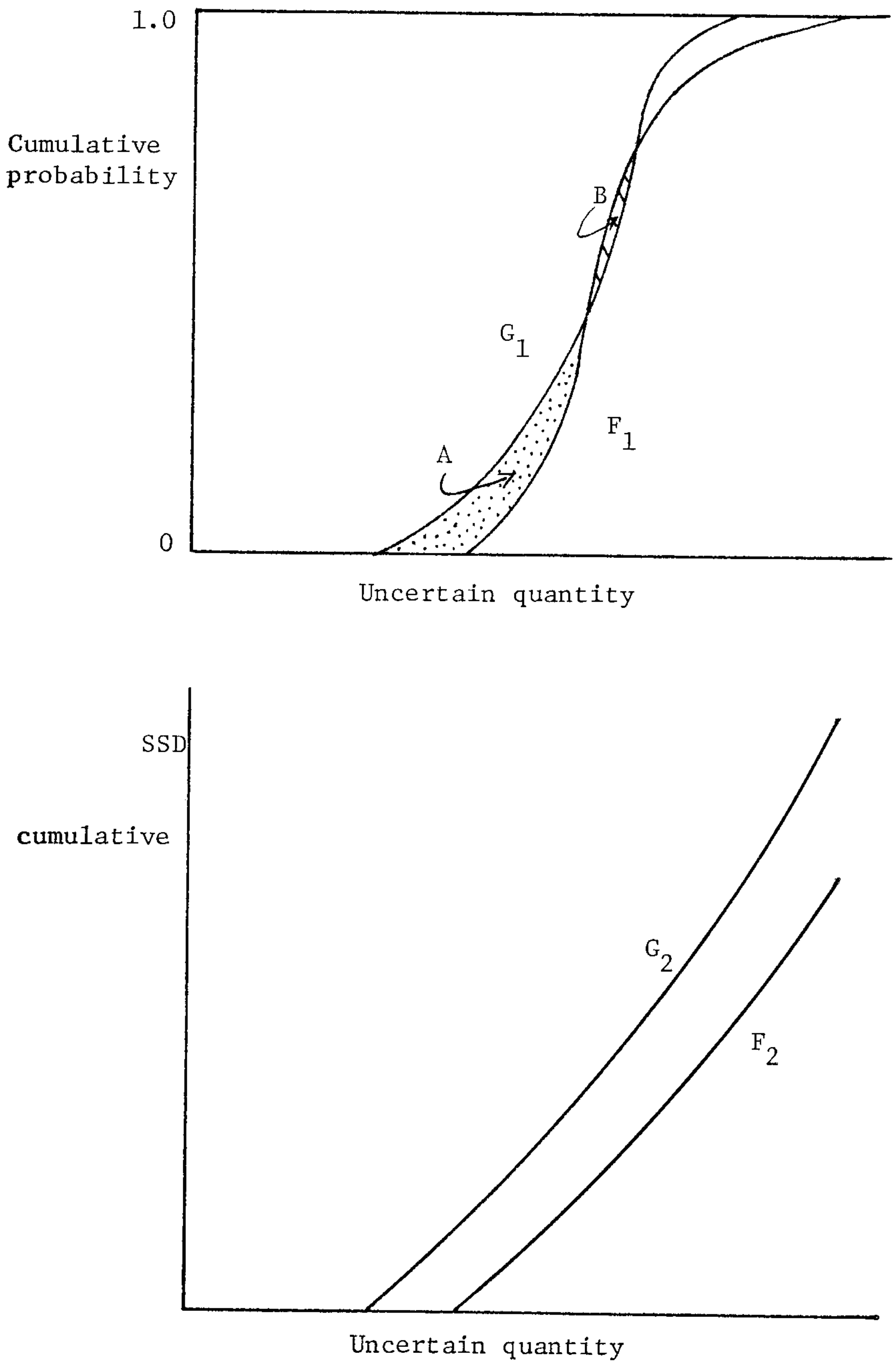

FIGURE 3: *Illustration of SSD where CDF's cross twice (area A > area B)*

This expanded expression can now be substituted in (2.1.2) to give

(2.2.1)

$$\bar{u}_f - \bar{u}_g = -\Big[U_1(x)\,[F_2(x) - G_2(x)]\Big]_a^b + \int_a^b U_2(x)\,[F_2(x) - G_2(x)]\,dx. \geqslant 0$$

because it has been assumed that $U_1(x) > 0$, $U_2(x) < 0$ and $[F_2(x) - G_2(x)] \leqslant 0$, including, of course, $[F_2(b) - G_2(b)] \leqslant 0$. Note that $F_2(a) = G_2(a) = 0$.

Related results

The converse theorem that: "if $\bar{u}_f > \bar{u}_g$ for all utility functions featuring $U_1(x) > 0$, $U_2(x) < 0$ for all x in $[a, b]$ then $f(x)$ dominates $g(x)$ by SSD" also can be proved. Hadar and Russell [15] offer a proof and the proof of the equivalent theorems of the case of discrete distributions. The operative discrete theorem can be stated by following the earlier notation, defining $\triangle x_i = x_{i+1} - x_i$, x_n as the highest value taken by x, and defining the analogue of $F_2(R)$ as

$$(2.2.2)\quad F_2(x_r) = \sum_{i=1}^{r} F_1(x_i)\,\triangle x_i,\ r = 1, \ldots n.$$

Then, if $F_2(x_r) \leqslant G_2(x_r)$ for all $r < n$, $U_1(x) > 0$ and $U_2(x) < 0$, then $f(x_i)$ is at least as preferred as $g(x_i)$.

SSD can usually order a larger set of risky prospects than FSD, which is as to be expected with the additional but still quite reasonable and defensible restriction on the preference function. Alternatively, SSD can be thought of as ordering some prospects that are not orderable under FSD. While the set of stochastically efficient alternatives under SSD will usually be smaller than under FSD, there is no guarantee that the set will be small, and to make further progress on narrowing down the efficient set it is necessary to make more restrictive assumptions about the nature of preferences. Of course, the limit to such activity is to define a particular preference function which will inexorably lead to the identification of a unique efficient (utility maximizing) prospect. There is, however, one more fairly reasonable assumption that can be introduced to narrow down the utility-nonspecific efficient set, namely a constraint on the third derivative.

2.3 Third-Degree Stochastic Dominance (TSD)

Preference assumptions

The third and final case considered is to add the assumption that the third derivative is positive, $U_3(x) > 0$, to the assumptions that $U_1(x) > 0$ and $U_2(x) < 0$.

This additional restriction is not so strong in intuitive appeal as the former but is likely to characterize the preferences of many decision-makers including peasant farmers. The restriction is implied by the requirement that decision-makers become decreasingly averse to risk as they become more wealthy [32], although it is a necessary but not a sufficient condition for decreasing risk aversion. The positive third derivative also usually (but ambiguously [20]) implies that owners of the utility functions prefer positive skewness in distributions of x to negative skewness (i.e. prefer long tails in the upper values of x).

Ordering rule

The distribution $f(x)$ dominates $g(x)$ by TSD if, and only if, $F_3(R) \leqslant G_3(R)$ for all R in $[a, b]$ with strict inequality for at least one value of R, and if $F_2(b) \leqslant G_2(b)$.

Theorem

If $F_3(x) \leqslant G_3(x)$ for all x in $[a, b]$, $F_2(b) \leqslant G_2(b)$, $U_1(x) > 0$, $U_2(x) < 0$ and $U_3(x) > 0$, then $f(x)$ is at least as preferred as $g(x)$.

Graphically, for f to dominate g, the F_3 curve must be nowhere to the left of the G_3 curve (*cf.* figure 11 p.162) and the top of the F_2 curve must not be to the left of the top of the G_2 curve. Integration by parts reveals that $F_2(b) \leqslant G_2(b)$ is equivalent to the requirement that the mean of f is not less than the mean of g and so is a necessary condition for TSD and SSD.

Proof

Proof proceeds as for the SSD case but this time continues from the position reached in equation (2.2.1). First take the term

$$\Big[-U_1(x)\,[F_2(x) - G_2(x)] \Big]_a^b$$

which is evaluated as

$$-U_1(b)\,[F_2(b) - G_2(b)] + 0,$$

and since $U_1(b) > 0$ by assumption and also $[F_2(b) - G_2(b)] \leqslant 0$ by assumption, the expression is non-negative.

To explore the second term recall that $d[F_3(x)]/dx = F_2(x)$, so that the second term of (2.2.1) can be rewritten and then integrated by parts as:

$$\int_a^b U_2(x)\,[F_2(x) - G_2(x)]\,dx$$

$$= \int_a^b U_2(x)\,\{d[F_3(x) - G_3(x)]/dx\}\,dx$$

$$= \Big[U_2(x)\,[F_3(x) - G_3(x)] \Big]_a^b - \int_a^b U_3(x)\,[F_3(x) - G_3(x)]\,dx$$

$$\geqslant 0.$$

The result is strictly non-negative because $U_2(x) < 0$, $[F_3(b) - G_3(b)] \leqslant 0$, $U_3(x) > 0$ and $[F_3(x) - G_3(x)] \leqslant 0$ for all possible x.

Substituting both these non-negative results for the two terms of equation (2.2.1) yields $\bar{u}_f - \bar{u}_g \geqslant 0$, and again an appeal to at least one strict inequality in the $F_3(x) \leqslant G_3(x)$ gives the strict TSD result.

Related results

As for earlier cases, a converse theorem exists and is proved by Whitmore [42] to whom the foregoing proof is also due. He does not prove a TSD theorem for discrete distributions but, by induction, the theorem can be stated after defining, in terms of earlier notation, the analogue of $F_3(R)$ as

$$(2.3.1) \quad F_3(x_r) = \sum_{i=1}^{r} F_2(x_i) \triangle x_i$$

Then, if $F_3(x_r) \leqslant G_3(x_r)$ for all $r < n$, $F_2(x_{n-1}) \leqslant G_2(x_{n-1})$, $U_1(x) > 0$, $U_2(x) < 0$ and $U_3(x) > 0$ for all x in $[a, b]$ then $f(x)$ is at least as preferred as $g(x)$.

2.4 Comments and Caveats

The literature contains numerous mentions of "weakness" and "strength" of criteria, orderings and results, which can be confusing. Results are said to be relatively strong if based on a strong criterion, i.e. one with relatively fewer restrictions and assumptions. Thus FSD is stronger than SSD which is stronger than TSD. Theoretically a criterion weaker than TSD can be based on a requirement for decreasing risk aversion, but this has not been worked out. Hammond [17] has approached such a criterion but at the cost of various additional assumptions about risk aversion and the relationships between probability distributions. It has also been suggested that further criteria be developed by confining attention to payoffs lower than, say, the median.[2]

TABLE 1

Summary of Assumptions and Rules for Stochastic Dominance Orderings

FSD	SSD	TSD	QSD
$U_1 > 0$	$U_1 > 0$	$U_1 > 0$	$U_1 > 0$
$F_1(x) \leqslant G_1(x)$	$U_2 < 0$	$U_2 < 0$	$U_2 < 0$
	$F_2(x) \leqslant G_2(x)$	$U_3 > 0$	$U_3 > 0$
		$F_3(x) \leqslant G_3(x)$	$U_4 < 0$
		$F_2(b) \leqslant G_2(b)$	$F_4(x) \leqslant G_4(x)$
			$F_2(b) \leqslant G_2(b)$
			$F_3(b) \leqslant G_3(b)$

[2] Robert T. Masson (personal communication). A similar suggestion is embodied in the idea of mean-semivariance (E-S) ordering/efficiency which Porter [30] demonstrates to be rather similar to SSD ordering and indeed proves that the E-S efficient set is a subset of the SSE set.

It will probably have occurred to the reader that the process taken here to three steps can be continued (e.g. to the fourth, "QSD") to yield progressively weaker ordering rules (i.e. rules that can order larger and larger sets of prospects) in the manner summarized in table 1.

However, the difficulty with rules such as fourth-degree stochastic dominance (QSD) and analogues of higher order is the lack of either theoretical or intuitive justification for the constraints on the derivatives of utility functions beyond the third.

Note that the presumption throughout of a finite range of the random variable x is not restrictive since the domain $[a, b]$ can be extended to embrace any interval. Proofs unfortunately become more complicated when the domain is infinite but the theorems still hold [15].

As always is the case with strong results, one pays a price for unrestrictive generality in a choice criterion. In all of the stochastic dominance criteria the "price" seems to be the important emphasis placed on the lower tails of the distributions compared. A review of the criteria reveals that a necessary condition for FSD, SSD and TSD is that the lower bound of a dominant distribution not be less than that of an unpreferred distribution. As an empirical matter this places inordinate emphasis on estimation of (lower) extreme values of uncertain quantities. As a choice-theoretic matter, one can easily envisage preferring one distribution to another in spite of the fact that the preferred has some small probability in its relatively leftwards lower tail.

However, when people talk of risk in farming it is usually the prospect of falling into the lower tails of probability distributions of yields, prices, profits or sustenance consumption that they have in mind. It thus seems appropriate to focus attention on these tails. Indeed this is the rationale for the emphasis on "safety-first" [38] and "safety-fixed" [8, 22] criteria in work related to this [4, 26, 36, 37]. The difficulties of working with these criteria are the theoretical implications of discontinuous preferences at the crucial or critical level and the empirical question of appropriate specification of critical levels and the probabilities with which they should be exceeded.

A second necessary condition for FSD, SSD and TSD that has not been highlighted in the above presentation is that the mean of a dominant distribution cannot be less than the mean of an unpreferred distribution. It might appear at first blush that these two necessary conditions (on means and lower bounds) provide an expedient first approach in computations for reviewing stochastic dominance. However, since they are necessary and not sufficient conditions, they can in general, only identify pairs of distributions that can *not* be separated through the SD criteria. General implementation of the SD rules requires identification (and then elimination) of distributions that are dominated in an SD sense and for this review process there is no alternative to comprehensive pairwise comparisons based on the rules elaborated above. The exception to this general result occurs in the case where the CDF's are known to intersect only once [17]—a case encountered in dealing with many theoretical continuous distributions, and introduced in section 3.1.3.

3 APPLICATIONS

The principles of stochastic dominance introduced and elaborated in section 2 are applicable to many decision problems in agriculture and other fields provided that (1) probability distributions can be usefully specified; and that, (2) computational problems inherent in the SD applications can be overcome. Probability specification is clearly of basic importance but because it is not the major focus of this study, discussion of its practice and problems is omitted here (*see* [3, 35, 40]). This section is primarily concerned with computational and practical aspects of examining alternative prospects for SD.

Risky decision problems can conveniently be categorized into two broad groups: (1) Those where only a finite number of risky prospects is to be compared, which hereafter are referred to as discrete actions (e.g. varieties); and, (2) those where the number of actions open to the decision-maker is logically (if not practically) infinite (e.g. fertilizer rates), hereafter called continuous actions. Application of SD is most straightforward in the case of discrete actions which is considered in section 3.1 through several examples of generally increasing analytical complexity. For each type of distribution (in sections 3.1.1 to 3.1.3) the introductory example is of a simplicity chosen to permit analysis without resort to electronic computation whereas subsequent examples generally demand such resort.

Analysis for continuous actions employs the procedures developed for discrete actions but raises some additional difficulties which are discussed and their solutions illustrated in section 3.2.

3.1 Discrete Actions

An SD review of several risky prospects involves pairwise comparisons among the prospects whilst progressively eliminating from further comparison those prospects (actions) that are revealed as being dominated at any degree, commencing with degree one (FSD). Conceptually, distributions of any type can be compared but the pragmatic simplification adopted here is to confine comparisons to those among distributions of the same category. Three categories are considered below, namely: (1) discrete distributions; (2) arbitrary continuous distributions (approximated as rectangular histograms); and, (3) theoretical continuous distributions.

3.1.1 Discrete Distributions

Analysis of stochastic efficiency (non-dominance) is simplest from a computational point of view in the case of discrete distributions. Against this simplicity must be balanced a recognition that the assumption that a random variable is (or the states of nature are) discrete is usually a rather simplistic interpretation of a probabilistic situation that is properly continuous. However, because of analysts' frequent (hitherto seemingly invariable [19, 29, 31]) resort to the assumption, the case is important and deserves careful attention.

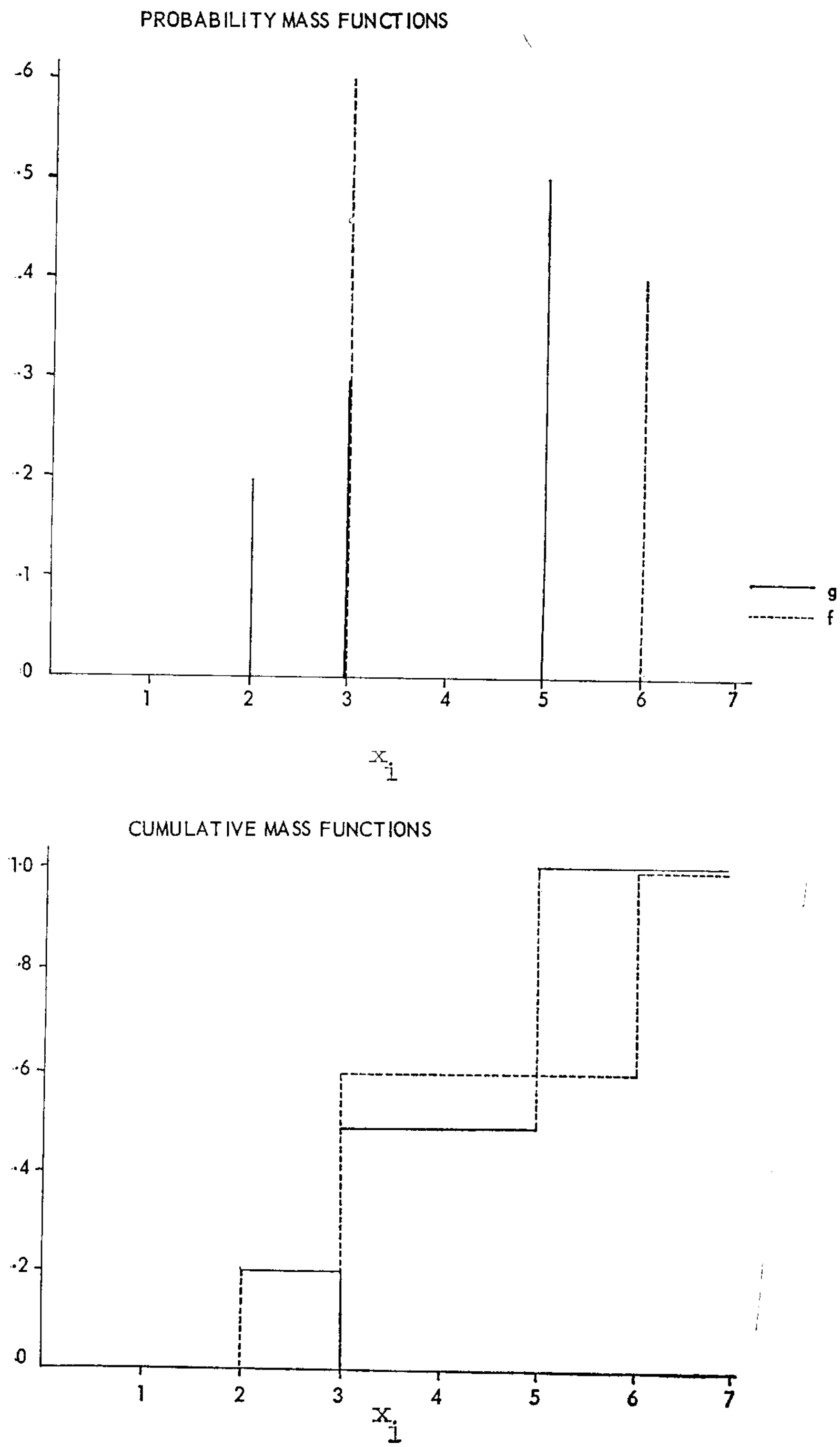

FIGURE 4: *Illustrative discrete probability functions*

The algebra of this case has already been spelled out in section 2 following the notation introduced in the ultimate subsection of section 2.1. For any two probability mass functions $f(x_i)$ and $g(x_i)$, compatibly defined in terms of an appropriate uncertain quantity x_i, analysis of stochastic efficiency proceeds straightforwardly by first listing all the combined (finite) values taken by x in ascending order such that if $i < j$ then $x_i < x_j$.[3] If two or more have the same numerical value each value is considered to be distinct and the rank allocated to ties is lowest for those ties associated with the distribution with the non-zero probability for the lowest value of x or, where this is also tied, with the potentially dominated distribution. With the x_i listed, the $f(x_i)$ and $g(x_i)$ can then be written out and the cumulative functions $F_1(x_i)$, $F_2(x_i)$ and $F_3(x_i)$ and those corresponding for $g(x_i)$ readily computed using the formulae in equations (2.1.3), (2.2.2) and (2.3.1), respectively and the definition of Δx_i defined in the preamble to (2.2.2).

Consider, for purposes of illustration, the example depicted in the mass and cumulative functions of figure 4. The steps of the previous paragraph are executed in table 2. Inspection of these hypothetical data indicates that the prospects cannot be differentiated on the FSD rule but, since $F_2(x_i) \leqslant G_2(x_i)$ for $i = 1, 4$, $f(x_i)$ dominates $g(x_i)$ in the SSD sense. It follows (as can be observed) that $f(x_i)$ is also dominant in the TSD sense.

TABLE 2

Illustrative SD Review for Discrete Distributions

xi	2	3	3	5	6	SD comparison
$f(x_i)$	0	0	·6	0	·4	
$g(x_i)$	·2	·3	0	·5	0	
$F_1(x_i)$	0	0	·6	·6	1·0	FSD
	$\wedge$	$\wedge$	$\vee$	$\wedge$	$\parallel$	
$G_1(x_i)$	·2	·5	·5	1·0	1·0	
Δx_i	1	0	2	1	..	
$F_2(x_i)$	0	0	1·2	1·8	..	SSD
	$\wedge$	$\wedge$	$\parallel$	$\wedge$		
$G_2(x_i)$	·2	·2	1·2	2·2	..	
$F_3(x_i)$	0	0	2·4	4·2	..	TSD
	$\wedge$	$\wedge$	$\wedge$	$\wedge$		
$G_3(x_i)$	·2	·2	2·6	4·8	..	

[3] By "compatibly" and "appropriate" is meant that x in each distribution should be comparable (e.g. returns net of the same cost categories) and should be a sensible argument of the (implicit) utility function (e.g. be some measure of profit).

With these introductory remarks, two empirical exemplifications are now considered. Example 1 treats the case of discrete distributions of unequal probability elements. In example 2 the distributions are assumed to have equal probability elements corresponding to the (assumed equal) relative sample frequencies. Thus, if there are N_f distinct observations on prospect f, then $f(x_i) = 1/N_f$, and in comparing two probability functions $f(x)$ and $g(x)$ there is a total of $N = N_f + N_g$ distinct observations and the relative frequencies of $g(x)$ are $g(x_i) = 1/N_g$ or $g(x_i) = 0$. That is, if the i-th of the merged observations belongs to prospect f, the $f(x_i) = 1/N_f$ and $g(x_i) = 0$. The assumptions of equal probability elements and equal numbers of observations also simplify the FSD review procedure as it is only necessary to show that, for f to dominate g, the ranked outcomes for f are never less than the corresponding ranked outcomes for g.

Example 1. *Control of a Maize Insect Pest*

Yield distributions of crops under different regimes of chemical controls are doubtless continuous. However, agronomists and farmers often think and talk about such distributions as if they were discrete (i.e. have only a finite, usually small number of states). By following this convention, an example is presented to illustrate the discrete SD analysis but, because of this simplification and others that are subsequently incorporated, the analysis must be classified as an extremely simplified didactic version.

The specific problem considered is the control of whorlworm (fall army worm) on maize at Poza Rica, Mexico. This pest is often a serious problem for farmers in this and many other areas. Quite satisfactory control measures have been evolved, even though they are not used as widely as would appear to be socially desirable.

Two treatments are compared here. The simplest is a "seed" treatment with an appropriate insecticide in wettable powder form which works out at about 120 pesos/ha for purchased ingredients. Since farm labour can readily perform this task during slack periods, labour costs are ignored. The second treatment consists of seed treatment as mentioned plus a foliar application early in the growth of the crop in the form of granules of another appropriate insecticide. The cost of the granules and labour is about 150 pesos/ha so that the total cost of the second, "both", treatment is about 270 pesos/ha. More sophisticated treatment strategies such as deciding on foliar applications on the basis of observed levels of insect infestation are presently ignored.

Three discrete states are assumed, namely "low", "medium" and "high" levels of infestation prevailing. "No" infestation was thought not to be a possibility. With these assumptions, the basic data assumed for this example are presented in table 3. The yields and probabilities are agronomists estimates of typical situations prevailing on farms in the area with appropriate fertilization. Valuing maize at 1·2 pesos/kg and subtracting the respective treatment costs converts the yields of table 3 to the gross margins of table 4.

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TABLE 3

Technical Data for the Insect Control Problem

State (infestation)	Probability	Treatment yields (kg/ha)		
		None	Seed	Both
Low	·15	4 500	4 600	4 700
Medium	·70	2 200	4 000	4 650
High	·15	600	2 500	4 600

TABLE 4

Economic Data for the Insect Control Problem

State	Probability	Returns (pesos/ha)		
		None	Seed	Both
Low	·15	5 400	5 400	5 370
Medium	·70	2 640	4 680	5 310
High	·15	720	2 880	5 250
	Expected returns[a] ..	2 106	4 086	5 328

[a] Returns weighted by respective probabilities. The act "Both" has the highest expected money value and accordingly would be preferred by farmers who are indifferent to risk.

With the data displayed as in table 4, one simple form of dominance is immediately apparent, namely "seed" is dominant over "none" since no matter which state occurs, the payoff from seed treatment exceeds or equals that from no treatment. This means that the act "none" need not be considered further in this analysis.

The decision problem is now reduced to exploring if the SD rules permit the identification of either "seed" or "both" treatments as being stochastically efficient. The analysis proceeds in table 5 by first ranking the discrete payoffs in the two acts to be considered and then defining the SD functions as previously illustrated. Since only the "both" treatment is efficient in the sense of SSD, analysis can cease at that point: but for completeness, the TSD functions are also presented in table 5. Naturally, "both" is also dominant over "seed" in the TSD sense. Thus, as one might anticipate when there is an effective and fairly cheap measure available for control of a serious pest, "risk-averse" farmers should adopt the safest treatment irrespective of their own particular attitudes to risk.

TABLE 5

SD Review for the Insect Control Problem

x_i		2 880	4 680	5 250	5 310	5 370	5 400
$F_0(x_i)$	Seed	·15	·70	..	..	..	·15
	Both	..	..	·15	·70	·15	..
$F_1(x_i)$	Seed	·15	·85	·85	·85	·85	1·0
	Both	0	0	·15	·85	1·0	1·0
$\triangle x_i$		1 800	570	60	60	30	..
$F_2(x_i)$	Seed	270	754·5	805·5	856·5	882	..
	Both[a]	0	0	9	60	90	..
$F_3(x_i)$	Seed	·486 m[b]	·916 m	·964 m	1·016 m	1·042 m	..
	Both	0	0	540	4·140	6·840	..

[a] The "Both" SSD function is less than the "Seed" function at each value of x so "Both" is second-degree stochastically efficient.

[b] Letter m denotes multiplication by 10^6.

Example 2. Selection of Wheat Varieties

Several methods have been used for identifying crop varieties that have wide environmental adaptability. The basic data for such work are usually obtained from nursery trials conducted in diverse environments, sometimes across many countries such as in the collaborative International Spring Wheat Yield Nursery (ISWYN) administered by the International Maize and Wheat Improvement Centre (CIMMYT) [6]. The identification methods used have ranged from comparisons of mean yields [6] to comparisons of statistics based on regressions of varietal yields on environmental indices [11, 12, 41].

Without presently indulging in a critical review, it seems clear that in the absence of specifically and carefully elaborated criteria, there can be no one perfect method of appraisal. The present example examines the question of adaptability from the new point of view of risk aversion and stochastic efficiency—a point of view believed to be relevant if the ultimate purpose of identifying widely adapted varieties is to make them available for adoption by farmers who generally are averse to risk. As Finlay and Wilkinson [12] have observed in an earlier day: "Plant breeders are inclined to ignore the results obtained in low-yielding environments (e.g. drought years), on the basis that the yields are too low and are therefore not very useful for sorting out the differences between selections. This is a serious error, because high-yielding selections under favourable conditions may show relatively greater failure under adverse conditions".

The notions of stochastic dominance and efficiency seem to provide a useful framework for posing the essentially empirical question of how different selections perform in diverse risky environments. This analysis is straightforward, as is demonstrated below, with some important provisos; most importantly, that it makes good sense to speak of a world (or large regional) probability distribution of wheat yields. Such a concept seems implicit and inherent in the conduct of world (or regional) yield nurseries and in the comparison of means (of probability distributions) from such nurseries. A second proviso, which necessarily almost amounts to a presumption, is that the selection of sites, co-operators, fields and growing and disease conditions is somehow representative of the relevant world (or regional) domain of production.

A third proviso is of importance for the logical application of the principles of stochastic dominance as outlined in section 2. This is that yield *per se*, provides a reasonable surrogate for the argument of the implicit utility function. This assumption which involves ignoring likely varying production costs is unavoidable in processing international nursery data since each trial is in general grown under differing regimes of irrigation (where practised), tillage, fertilizers and weed and pest control that are most difficult to cost.

Just whose utility function is implicit is not too obvious. It is *not* some omnipotent wheat producer since, with his global long-term vision, he would arguably be indifferent to risk on individual farms and accordingly focus only on mean yields. It is more like "Mr Average Farmer" hypothetically assuming responsibility for risk-bearing in the production of wheat on farms around the world.

To make the exemplification concrete, attention is now concentrated on the data from the Sixth ISWYN [6]. In this nursery, forty-nine varieties were compared in trials at sixty locations in thirty-seven countries during 1969–70. For each variety, each trial observation is regarded as a distinct component of the discrete sample probability function of that variety. The pairwise comparison of forty-nine discrete actions involves up to (49)(48)/2 = 1 176 FSD comparisons at each of up to (60)(2) = 120 values of the uncertain quantity yield. Such a computational burden can be faced with equanimity only with the aid of an electronic digital computer. To this end a program has been prepared to undertake analysis of stochastic efficiency in this case of discrete distributions from samples of equal size.[4] This program works on a fully defined yield matrix for varieties and states (sites) and provides listings of those varieties that are stochastically efficient of degrees 1, 2 and 3.

The results of applications to three (sub-) sets of the Sixth ISWYN yield data are summarized in table 6. The results can be appraised from two viewpoints; first the empirical identification of the numbered varieties into categories of stochastic efficiency—information that would seemingly be useful to plant breeders. Secondly, there is the methodological question of the relationship between stochastic efficiency and rank according to mean yield of each variety. Not surprisingly, there is a close relationship, especially between first-degree stochastic efficiency (FSE) and rank. The FSE set includes most of the top-ranked varieties, and perhaps the most useful aspect of this identification is in pinning-down a cut-off point of mean yields that is less than

[4] This, and subsequently mentioned programmes are written in FORTRAN IV and listings are available from the author on request.

arbitrary. It should also be emphasized that risk-efficient varieties in the second-degree stochastically efficient, (SSE) and third-degree stochastically efficient (TSE) sets are necessarily selected from the FSE set.

These three applications offer little scope for generalizations about the likely general composition of the SSE and TSE sets but it seems reasonable to risk one—namely that as the environmental scope of analysis becomes more restrictive (e.g. as in the irrigated and non-irrigated cases reported in table 6), the greater is the chance that only the very highly mean ranked varieties will be SSE or TSE. The implication of this shaky generalization is that, providing breeders confine environmental scope in some way for selecting "broadly" adapted varieties, then by focussing on mean yield they shall most probably be selecting varieties that are also stochastically efficient for "risk-averse" growers.

Finally, in assessing the proposed SD method of sorting out varieties, it would be useful to know how it matched up against the methods employing regressions of yields on environmental indices (inevitably equal to site mean yields). One shortcoming for such methods is that, in the absence of either sophisticated computer plotters or tolerant artistic/technical assistance, they are rather tedious to employ. Consequently, only the third case of table 6 is subjected to further such analysis in this comparison.

TABLE 6

Results of Stochastic Dominance Analysis of Yield Data from the Sixth ISWYN

Sites Number and type		Stochastically efficient varieties and their mean ranks* Degree 1									2†			3		
60 All sites ..	Var.	25	47	33	15	34	11	23	30	31	25	47	34	25	47	34
	Rank	1	2	3	4	5	6	7	8	9	1	2	5	1	2	5
	Var.	45	1	38	42	44	20	40	18	13	45	14	21	..	45	14
	Rank	10	11	12	13	14	15	16	17	18	10	27	34	..	10	27
	Var.	17	36	24	41	29	26	14	21	16	..	..	..	..	..	..
	Rank	19	21	22	23	24	26	27	34	35	..	..	..	..	..	..
28 Irrigated N. of 10° N. or S. of 10° S.	Var.	25	23	33	15	47	30	11	31	34	25	..	..	..	..	..
	Rank	1	2	3	4	5	6	7	8	10	1	..	..	..	..	..
	Var.	42	1	44	14	..	..	..	..	..	..	..	..	..	..	..
	Rank	12	13	15	22	..	..	..	..	..	..	..	..	..	..	..
22 Non-irrigated sites not affected by severe biological limitations and for which rainfall was reported.	Var.	15	47	25	1	33	34	30	23	44	15	47	..	..	15	..
	Rank	1	2	3	4	5	6	7	8	9	1	2	..	..	1	..
	Var.	31	40	42	11	13	41	22	38	45	..	..	..	..	..	..
	Rank	10	11	12	13	14	15	16	17	18	..	..	..	..	..	..
	Var.	17	26	24	36	18	43	21	..	..	..	..	..	..	..	..
	Rank	19	20	21	23	24	25	28	..	..	..	..	..	..	..	..

* Ranks of varietal means calculated within the specified sites.

† Risk-efficient varieties variously include: 14 Giza 155 (Egypt), 15 Siete Cerros 66 (Mexico), 25 LR-P4160³ (E) (Pakistan), 34 Tobari 66 (Mexico), 45 CIANO 5 (Mexico), 47 Sonalika (India).

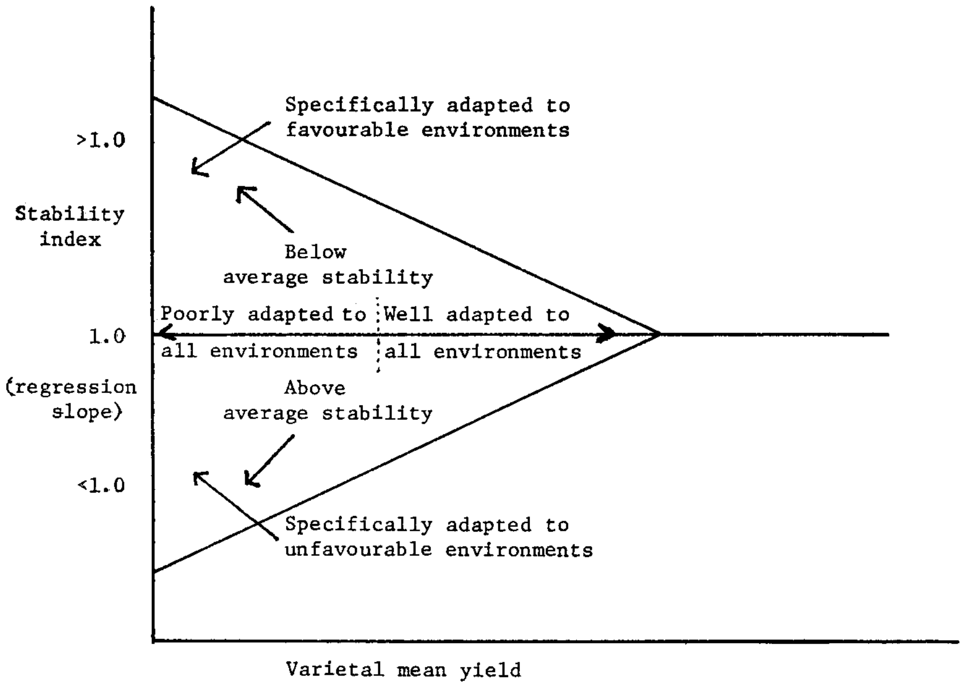

FIGURE 5: *Generalized interpretation of scatter diagrams of stability measure against varietal mean yield* [12]

Laing and Fischer [23] have conducted an analysis of the type discussed using only the regression model

$$(3.1.1)\quad Y_{ij} = a_i + b_i X_j$$

where Y_{ij} = yield of variety i at site j and X_j is mean yield at site j. The "stability" estimates, b_i, are then plotted against varietal means, V_i, to give a scatter diagram of the type elaborated by Finlay and Wilkinson [12] and to which they suggest the generalized interpretation sketched in figure 5. Such an interpretation strikes a strong intuitive note for categorizing a diverse range of genetic materials. However, the analysis hinges on describing stability in terms of the single statistic, namely the regression slope, and to me, questions arise as to the adequacy of such a simple regression model and the comparability of results based on alternative transformations of the data. For instance, Finlay and Wilkinson [12] employed a logarithmic transformation and fitted the model,

$$(3.1.2)\quad \log(Y_{ij}) = a + b_i \log(X_j).$$

Eberhart and Russell [11] and Laing and Fischer [23] rejected this model in favour of the linear model (3.1.1) on the grounds that equation (3.1.2) gives "too much weight" to low-yielding varieties and sites.

It seems to me that any such choice should be strongly influenced by the data and pertinent extraneous information. In particular, if all varieties yielded zero at a site, then mean site yield would necessarily be zero. This information can be encapsulated in (3.1.1) by insisting that the regression intercept is zero which is equivalent to "forcing the regression through the origin" as in the model

(3.1.3) $Y_{ij} = b_i X_j$.

Since equation (3.1.2) only applies to strictly positive values of Y_{ij} and X_j (i.e. does not accommodate zero yields), this extraneous information cannot be applied in equation (3.1.2) and indeed, on these grounds alone this model seems seriously questionable.

A question related to the somewhat arbitrary selection of data transformations is the empirical adequacy of simple linear models. This question was presently approached by fitting quadratic and cubic variants of (3.1.1) and (3.1.3). The results suggested that this was not too important a problem as only three of forty-nine fits of (3.1.1) and nine of forty-nine fits of (3.1.3) indicated significant non-linearity.

This discussion of possible alternative models and their incumbent problems has been discursive but necessary in leading up to discussion of the relationship with stochastic efficiency. The reason is simply that the pattern obtained by plotting b_i against V_i varies according to the model used, and the related results for the last case of stochastic efficiency reported in table 6 must be interpreted accordingly. The diverse results are most easily interpreted, albeit crudely, in the graphical summaries depicted in figure 6.

Detailed comparisons of the regression statistics (especially among comparably computed estimates of standard errors for the equations) suggests that models (3.1.1) and (3.1.3) are about equally good and both are quite superior to the log model (3.1.2) for these wheat data. For the log model, about all that can be said is that stochastically efficient varieties are those with high means (as already noted in table 6) and the risk-efficient selections are those with highest yields and general adaptation.

The results for models (3.1.1) and (3.1.3) are more interesting and rather surprising. This analyst's *a priori* anticipation was that, with the emphasis on lower-probability tails in the SD criteria, the stochastically efficient sets would be located below the unit slope (stability) line, i.e. emphasizing specific adaptation to unfavourable environments. Such, however, is not the case as these results place the stochastically-efficient varieties mainly in the region of below-average stability and good general adaptation. Again, of course, the risk-efficient varieties are those with highest mean yields and accordingly lie to the right-hand side of the sketches.[5] In fact, the pattern of results obtained for (the author's preferred) model (3.1.3) indicates a very close correspondence between (this version of) the regression approach and the analysis of stochastic efficiency but probably little generality can be attached to this correspondence.

[5] This appears to contrast with the observations of Purvis [33] in Tunisia.

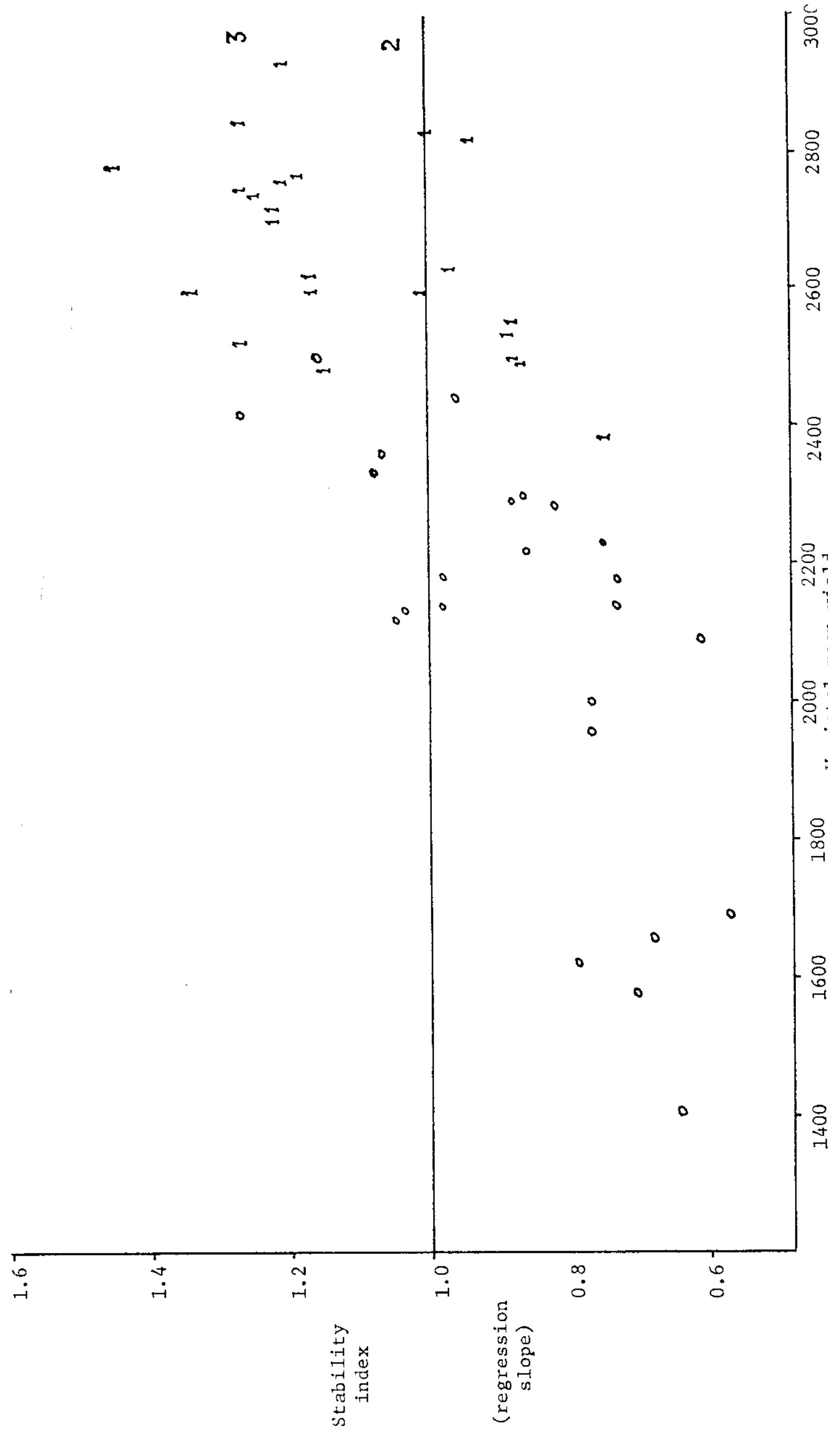

FIGURE 6 (a): *Degree of stochastic efficiency, and adaptation versus varietal mean yield—equation* (3.1.1)

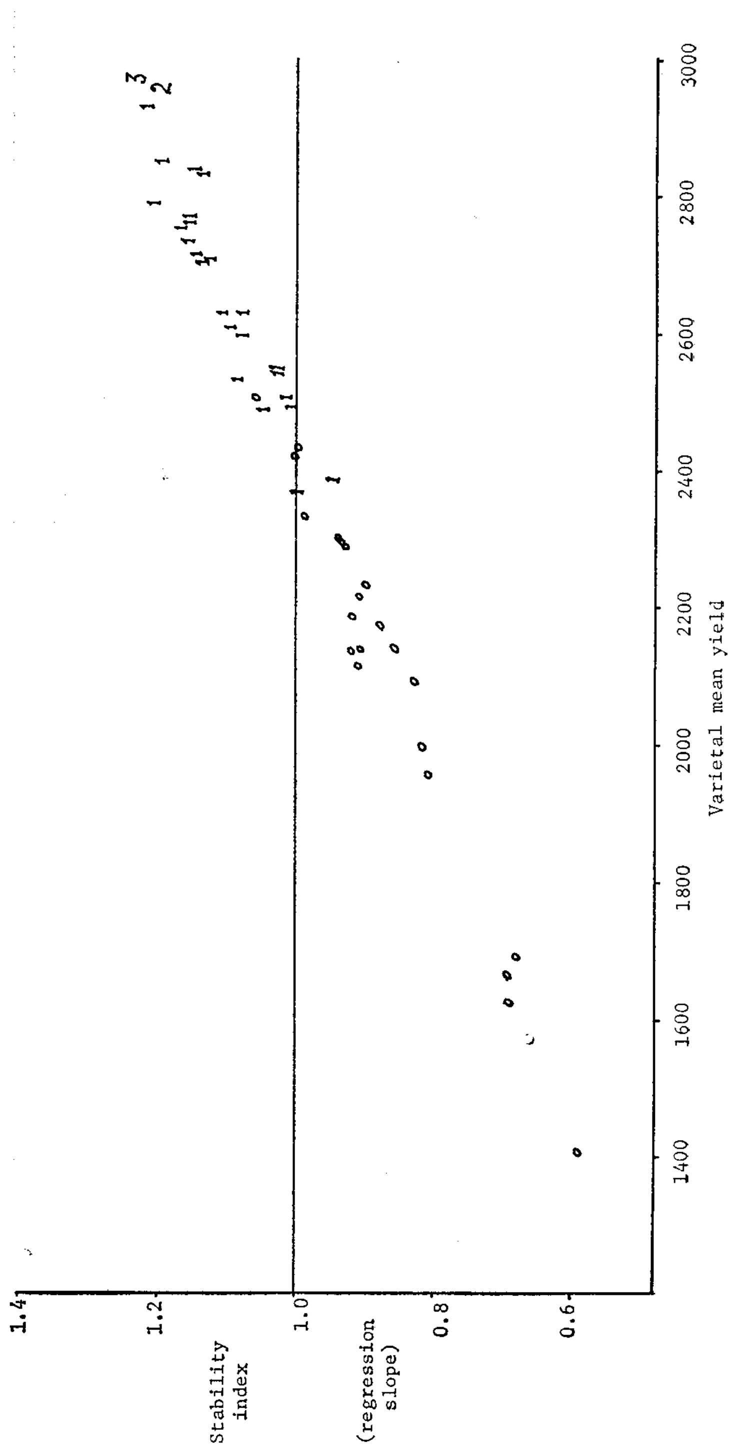

FIGURE 6 (b): ***Degree** of stochastic efficiency, and adaptation versus varietal mean yield—equation* (3.1.3)

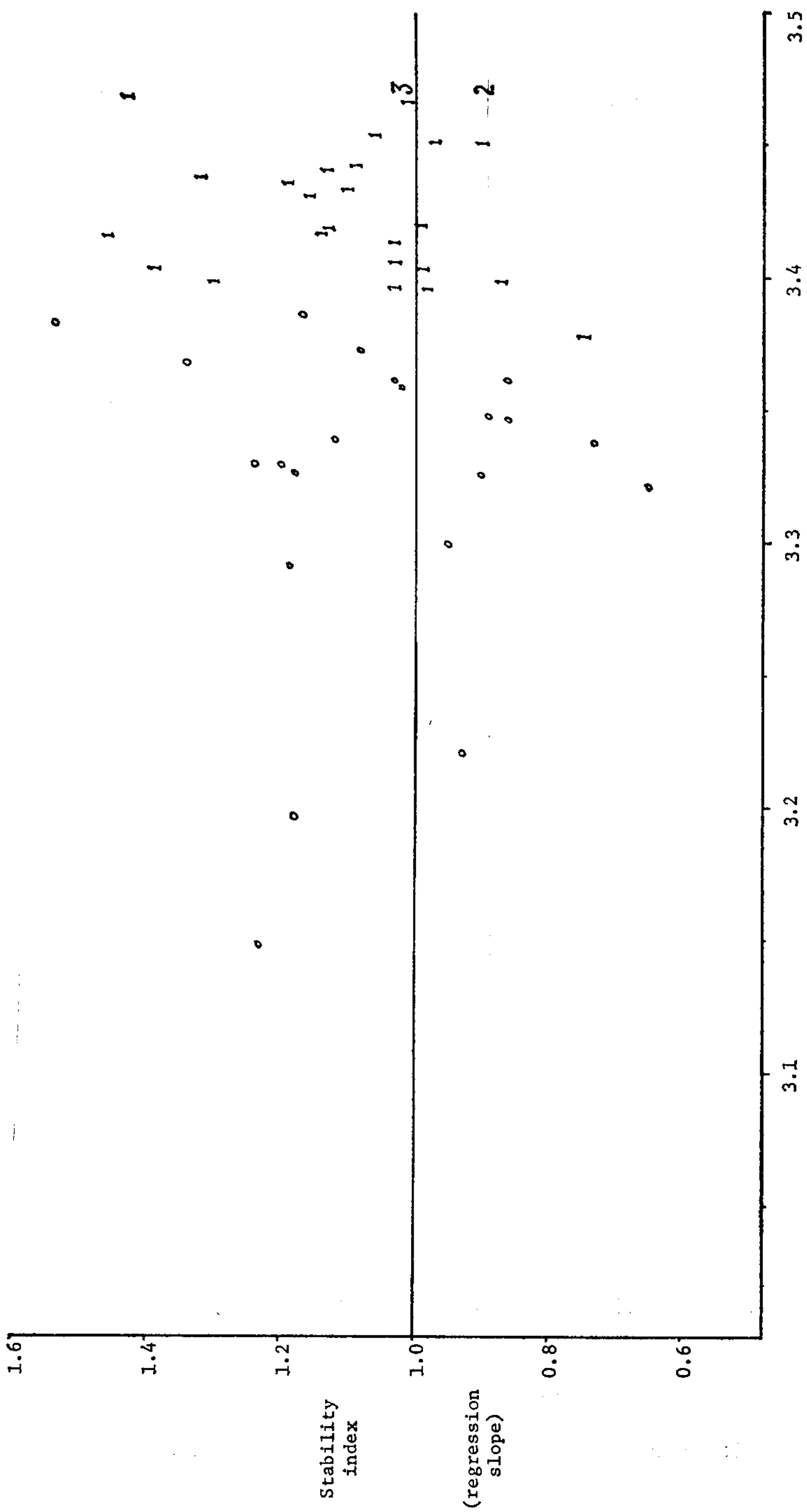

FIGURE 6 (c): *Degree of stochastic efficiency, and adaptation versus varietal mean yield—equation* (3.1.2)

To summarize, the regression approach was originally developed by Finlay and Wilkinson [12] to sort out very diverse materials in early screening trials conducted in diverse environments and it still seems useful for this purpose. The analysis employing notions of stochastic efficiency also seems to provide useful information but is probably applicable at a much later stage of screening materials that are relatively more similar in their adaptation and yield characteristics.

3.1.2 ARBITRARY CONTINUOUS DISTRIBUTIONS

The probability distributions that emerge from analyses of data or from judgements are often asymmetric or otherwise irregular so that a search for a convenient theoretical distribution that fits adequately may be tedious, futile or simply too costly. One pragmatic alternative is simply to describe a graphical CDF by a smooth hand-sketched curve and in turn approximate this curve by a number of segments of simple algebraic form.

The simplest form consists of linear segments and for several reasons this is the alternative adopted here. A linear-segmented CDF corresponds to a rectangular histogram form of PDF and, if sufficient segments (rectangles) are taken, this is bound to be an adequate approximation. Analogously, a multiquadratic-segmented CDF corresponds to a trapezahedral "histogram" form of PDF and if the segments are constrained to have continuous first derivatives at the join points, the PDF would have an "angular mountain" shape.[6] Unfortunately, the additional constraints imposed by the necessity for a CDF to have strictly positive slope (i.e. be monotonically increasing but not beyond the perpendicular) make this next-most-logical alternative rather impracticable.

Apart from conceptual simplicity, the advantage of specifying a CDF in terms of linear segments is the relative simplicity afforded to the integrations required to specify the SSD and TSD functions (see equations (2.0.2) and (2.0.3)) and to the solution of simultaneous equations required in SD comparisons implemented on a computer.

There is an infinity of ways of arranging linear approximation of an arbitrary CDF. To narrow down the possibilities a little, the alternative adopted here is to assume that each of a total number of NS segments spans an equal interval of cumulative probability ($DP = 1/NS$). The algebra of this case is now briefly reviewed.

Figure 7 depicts a linear segmented CDF for the i-th action. It consists of NS ($=6$) segments and is described completely by the coordinates $H_{i,k}$, $k = 1, \ldots, NC$, $NC = NS + 1$.[7] The insistence

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[6] A CDF consisting of two quadratic segments corresponds to the triangular PDF.

[7] The F_i and G_i notation of section 2 is inadequate here because generally more than two (in fact NA) actions are to be considered. Notational equivalences are as follows: $H_{i,k}$ denotes the particular values of R at the endpoints of the segments of $F_1(R)$ of section 2, $H_{j,k}$ now denotes the values of R at the endpoints of the segments of $G_1(R)$ of section 2. The values of the SSD ($F_2(R)$, $G_2(R)$) functions at the corresponding endpoints are stored in matrix S and the corresponding TSD ($F_3(R)$, $G_3(R)$) functional values in matrix T.

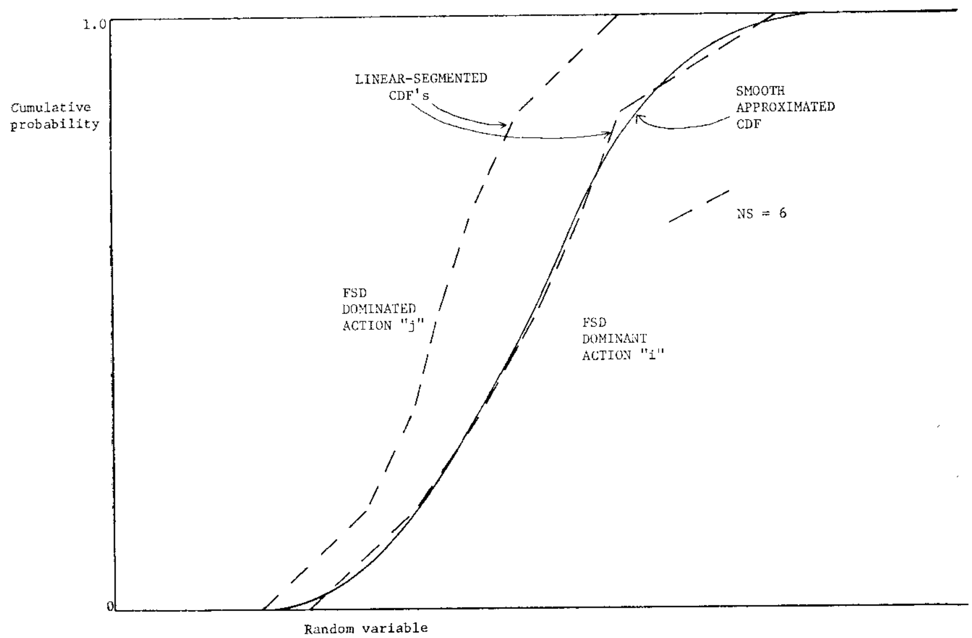

FIGURE 7: *Illustration of FSD with linear-segmented CDF's*

on equal probability segments makes FSD review simpler when CDF's with equal numbers of segments are compared. Consider the j-th action described by $H_{j,k}$, $k = 1, \ldots, NC$. Corresponding to the FSD ordering rule of section 2.1, action i dominates action j if, and only if, $H_{i,k} \geqslant H_{j,k}$, for all $k = 1, \ldots, NC$ with strict inequality for at least one value of k.

SSD comparison involves functions defined as cumulative areas under the linear-segmented CDF's. Such areas are readily computed at the endpoints of the linear segments from the areas of the component triangles and rectangles. These are stored in matrix S for which the i-th row (for the i-th action) is defined by the first element $S_{i,1} = 0$ and the k-th element.

(3.1.1) $S_{i,k} = S_{i,k-1} + DP\,(H_{i,k} - H_{i,k-1})\,(k - 1{\cdot}5),\ k = 2, \ldots, NC.$

SSD comparison cannot be based merely on comparing the functions at the segmental endpoints and interpolation is required. Comparison by machine calculation (as opposed to graphical methods) is facilitated by using two guidelines established in graphical comparisons, namely (a) that a necessary condition for the i-th action to dominate the j-th is that $H_{i,1} \geqslant H_{j,1}$ and (b) if the SSD functions intersect below the NC-th endpoint of the initially lesser function, it will most probably be at a joinpoint on that function (i.e. joinpoint of the convex quadratic segments). SSD review can thus proceed initially as follows: (a) Identify the rightmost (lesser) SSD function at its lower bound by the greater lowest limit of the H coordinates and label it ik and the potentially dominated function as jk. (b) Find for each joinpoint $k = 2, \ldots, NC$ on the $H_{ik,k}$ function the segment (iz-th) of the

$H_{jk,iz}$ function vertically above or below $H_{ik,k}$. (c) Compare the SSD function evaluated at $H_{ik,k}$, i.e. $S_{ik,k}$, with the interpolated value of the jk-th SSD function at the same value of $H_{ik,k}$, namely

$$(3.1.2)\quad SCF = S_{jk,iz} + \cdot 5\ DP^*RI^2/(H_{jk,iz+1} - H_{jk,iz}) + DP^*RI^*(iz - 1),$$

where the marginal region of integration $RI = H_{ik,k} - H_{jk,iz}$. It will be recalled that function comparisons are required over the total range of variables so that in general, one of the SSD functions must be extrapolated up to the level of its underlying variable corresponding to the larger of the NC-th coordinates of the distributions being compared.

When intersection of SSD functions (and consequently non-SSD) is not detected in the fast and direct manner, a more tedious and thorough review needs to be made. This is done by searching for intersection of the SSD functions in all the intervals of the merged and ranked H joinpoints. Generally (i.e. excepting for the linearly extrapolated upper portions), intersection is found by solving (a quadratic expression) for the equality of two quadratic segments and checking if any real solution lies within the respective interval of interest. If one does, then of course intersection is indicated.

TSD comparison involves exactly parallel steps to those outlined for SSD with a marginal increase in algebraic complexity occasioned by integration of the scalloped quadratic-segmented SSD functions yielding steeper cubic-segmented TSD functions. The joinpoints of these are stored in matrix T and defined by $T_{i,1} = 0$ and for the k-th element by

$$(3.1.3)\quad T_{i,k} = T_{i,k-1} + \cdot 5\ DP^*DI^{2*}(k - 2 + 1/3) + DI^*S_{i,k-1},\ k = 2, \ldots, NC,$$

where $DI = H_{i,k} - H_{i,k-1}$. After the initial check on the upper values of the SSD functions (noted in the ordering rule of section 2.3), analysis for TSD first follows a similar pattern of interpolation at the joinpoints (and extrapolation at the endpoints) as noted for SSD. The intermediate interpolation expression corresponding to (3.1.2) is, with RI as there defined,

$$(3.1.4)\quad TCF = T_{jk,iz} + (1/6)^*DP^*RI^3/(H_{jk,iz+1} - H_{jk,iz}) + \cdot 5\ DP^*RI^{2*}(iz - 1) + RI^*S_{jk,iz}.$$

This advocated review procedure is only approximate for TSD because the final careful check for intersection at other than ik joinpoints is omitted on grounds of the relative difficulty of solving for possible intersections of pairs of cubic equations. However, experience gleaned from graphical analysis suggests that this will result in very few, if any, errors (i.e. few false declarations of TSD or non-TSD) but this is clearly one point for possible methodological improvement.

The examples presented below are based on the assumptions that not only do the linear segments span equal intervals of probability but also that, in any comparison, each distribution shall have the same number of segments (NS). These assumptions result in some simplification of computation and programming but they could, if desired, be relaxed without a great deal of effort. Example 3 (with rather crude distributional representation based on $NS = 4$) is presented to illustrate graphically the nature of the computations discussed above. Choice of this example in which no SD of any order is encountered is deliberate as it is only in such an example that the entire sequence of comparisons should be performed. No generality should be attached to the empirical aspects of this example which is atypical of the maize technological situation in Mexico. Example 4 is both more realistic and more complex. A computer programme was used to conduct the analysis which was based on a fairly accurate distributional representation employing $NS = 20$ linear segments.[8]

Example 3. *Adoption Decision Concerning a Package of New Maize Technology*

The example chosen to illustrate review of SD based on distributions described arbitrarily by linear-segmented CDF's, concerns a hypothetical choice between a presently-employed maize technology and a "new" technology based on improved varieties and more intensive use of fertilizer, seed and irrigation. The "new" technology here is that recommended in the *Program de Altos Rendimientos* in the Chapingo area of Mexico. All data used here come from a study of O'Mara [27] and the particular subjective yield distributions are those elicited in his farmer interview case 49. The use made here of these data is simplistic in several ways including the simplification of couching the adoption question in a "yes/no" or "all or nothing" context where, in reality, partial adoption (i.e. on part of the maize area of a farm) is clearly an important possibility. However, this simplification permits us to ignore the dependence (included in O'Mara's work) between yield distributions under the two technologies.

The data are a farmer's subjective estimates of several fractiles of grain yield distributions under each technology and are presented in table 7.[9] These data can be presented graphically in the form of a histogram as in figure 8. The bimodal character of these histograms makes them atypical of yield distributions of field crops and a more appropriate analysis would first submit these to a smoothing process and probably result in unimodiality. However, these distributions will serve well in the present exemplification and accordingly are not altered.

TABLE 7

Yield Fractiles (t/ha) of Maize Under Alternative Technologies

Fractile	0	·25	·5	·75	1·0
A "present" technology	·75	1·25	1·5	2·5	3·0
B "new" technology	1·0	2·0	3·5	4·75	6·0

[8] *See* footnote 4.

[9] A "point b fractile", $f_{.b}$, is that value of a random variable x such that $P(x \leqslant f_{.b}) = .b$.

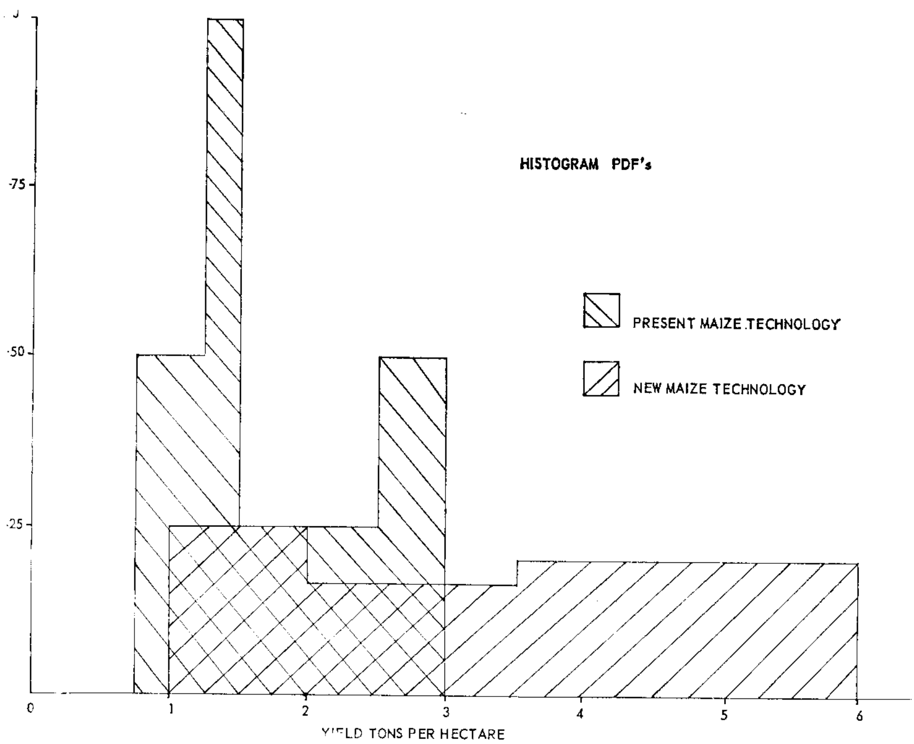

FIGURE 8: *PDF's for yields under two maize technologies*

Economic analysis of such data first involves bringing them to a common basis with due account of the costs and returns in each technology. Following O'Mara [28, pp. 97, 100, 268ff], this is done by computing returns per hectare as: (grain yield times government guaranteed grain price (940 pesos (1970) per tonne) less (variable costs excluding harvest costs, land rent and fixed labour costs). This simplification is based on the assumption that (a) returns from fodder; and, (b) costs of harvesting grain and fodder ((a) and (b) both vary with grain yield) are equal. The variable costs budgeted for technologies A and B were 866 and 1 770 pesos/ha, respectively. The fractile data for yields were then linearly transformed to fractiles for net returns and are reported in table 8.

TABLE 8

Returns Fractiles (pesos/ha) for Two Maize Technologies

Fractile	0	·25	·5	·75	1·0
A	−161	309	544	1 484	1 954
B	−830	110	1 520	2 695	3 870

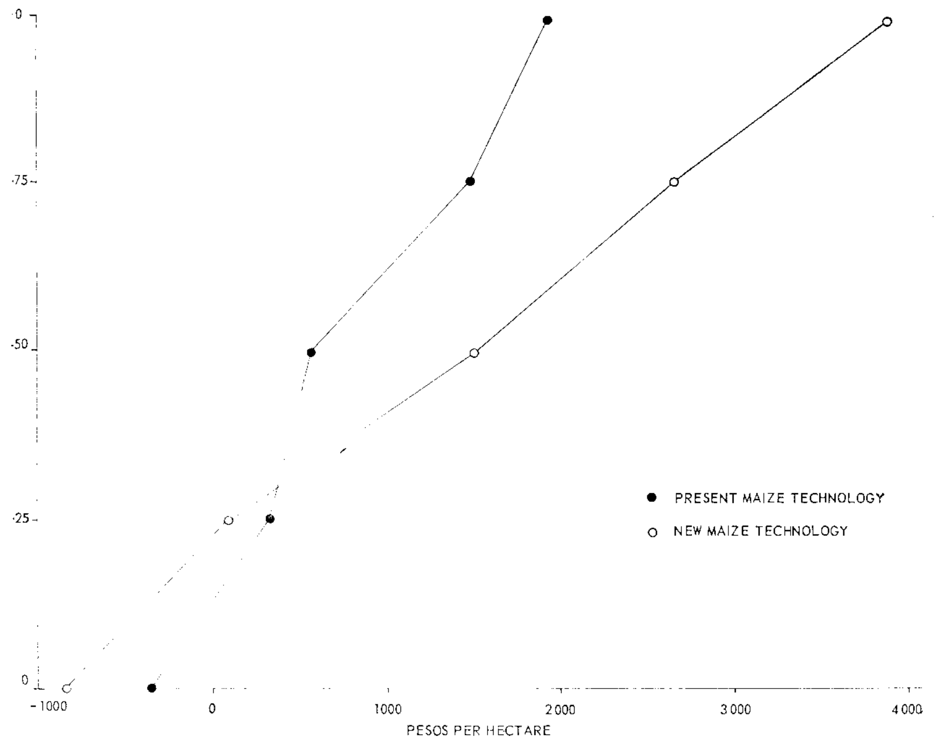

FIGURE 9: *CDF's (FSD functions) for profits under two maize technologies*

In terms of the symbols introduced in this section, the data of table 8 constitute matrix H with $NS = 4$, $NC = 5$ and, because $-161 > -830$, coordinates for technology A will be referenced as ik (and B as jk). As is most convenient for SD review, these return fractiles are charted in figure 9 and (corresponding with the rectangular PDF histograms) are linked by linear segments to constitute two arbitrary CDF's. The intersection of these functions means that neither technology is dominated (or dominant) in the sense of FSD.

Further analysis proceeds first by computing the SSD functions at the join- and end-points of the basic FSD functions as elaborated in equation (3.1.1). These points are tabulated in the $(2 \times NC)$ matrix S, and presented in table 9.

TABLE 9

SSD Function Ordinates at Segment Ends (Two Maize Technologies)

A	0	58·8	146·9	734·4	1 145·6
B	0	117·5	646·3	1 380·6	2 408·8

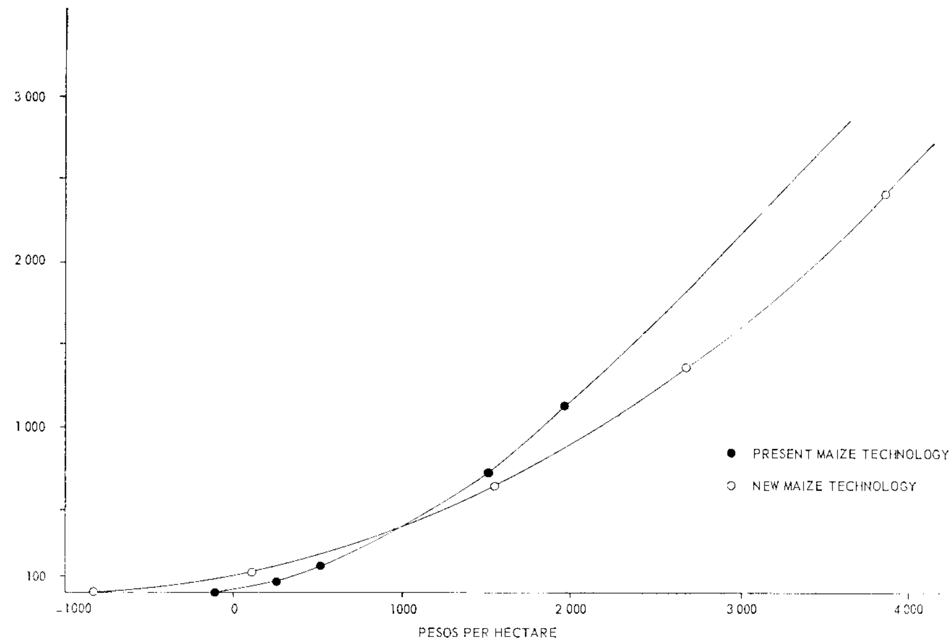

FIGURE 10: *SSD functions for profits under two maize technologies*

The elements of matrix H and matrix S together specify the coordinates of the segment endpoints of the SSD functions and these are plotted in figure 10. Values of the jk SSD function in the vertical direction of the ik abscissas are computed directly from equation (3.1.2). More generally, however, equation (3.1.2) provides the means for calculating the ordinate of any quadratic segment up to the 1·0 fractile of the respective function. In this case, since the jk function has the larger 1·0 fractile, the ik function must be extrapolated. This is a linear extrapolation (slope unity) since the area under the CDF when it is unity is simply the region of integration itself. Again it is apparent that the functions intersect so that A does not dominate B in the sense of SSD.

The final check for SD resorts to TSD and the first step is to compute the TSD functions at the endpoints of the segments using equation (3.1.3). These TSD endpoints are reported in table 10 and intermediate

TABLE 10

TSD Function Ordinates at Segment Ends (Two Maize Technologies)

A	0	9 204	32 215	427 994	865 192
B	0	36 817	533 842	1 695 867	3 893 361

G 65389—3¶

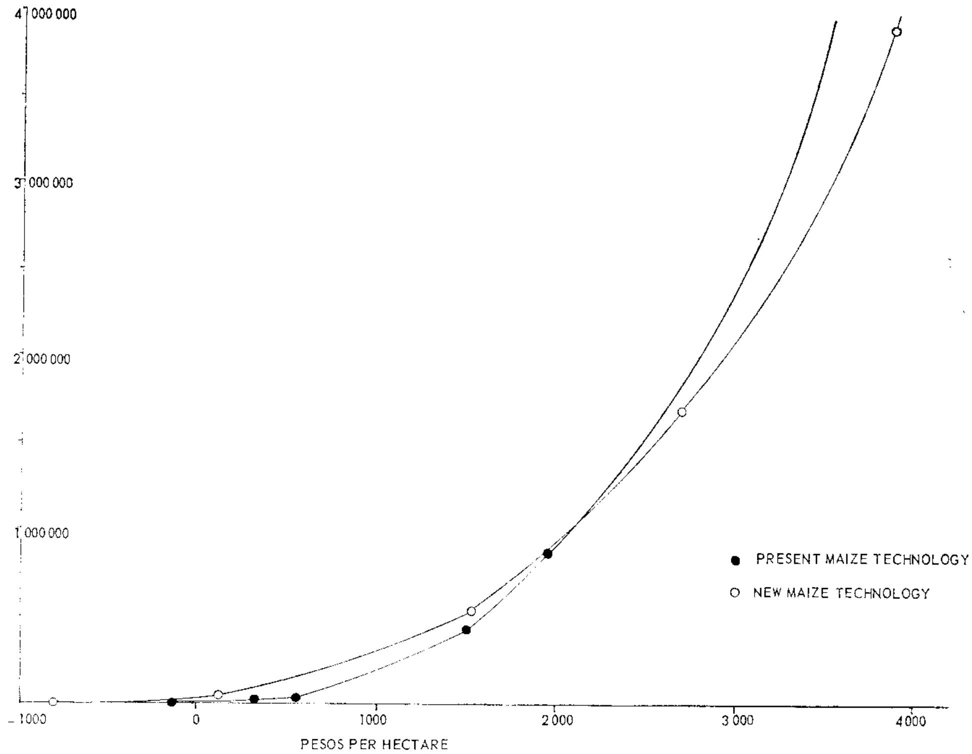

FIGURE 11: *TSD functions for profits under two maize technologies*

values are computed using equation (3.1.4) or some analogous variant. Those functions are charted in figure 11 and again it is clear that intersection occurs so that neither technology is dominant in the sense of TSD.

In turn, this means that, given the farmer's subjective probabilities and the arbitrary representation accorded them here, it is not possible to say which he would prefer without knowing something more of his specific attitudes to risk beyond the assumptions involved in the SSD and TSD ordering rules. The virtue of this example is to make explicit the pertinent SD review procedures which, even from this small exemplification, will be sensed to be computationally burdensome where many (say 20) linear segments are used to approximate a CDF and (especially) where many distributions are to be reviewed. Such an example is explored in example 4.

Example 4. *Discrete Rates of N and P on Wheat*

In this example no attempt is made to treat fertilizer rates as the continuous variables that (up to limits of machinery calibration) they are. The example builds on material described more fully elsewhere [1]. The starting point here is to use the thirty-six probability distributions of unirrigated wheat yield estimated for each of the design points of a 6 × 6 complete factorial. The treatments are for *N* approx. 0, 22·5, 44·9, 67·4, 89·8, 112·2 or 0, 22, 45, 67, 90, 112 kg/ha and for *P* approx. 0, 9, 18, 27, 36, 45 kg/ha.

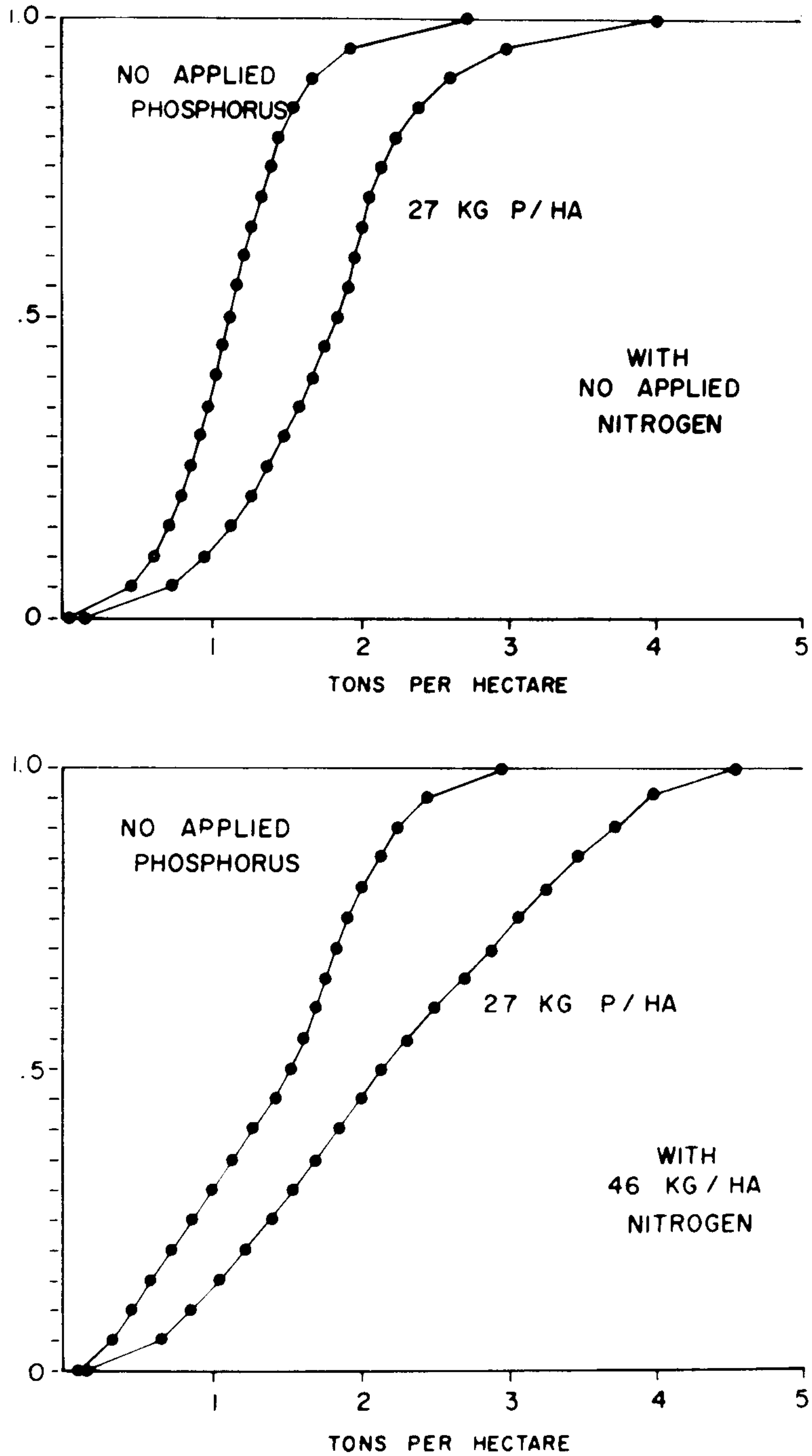

FIGURE 12: *Some examples of linear-segmented CDF's for yields of unirrigated wheat under different fertilizer treatments*

The estimation of the distributions, which reflect between-year variability, was based on sparse data [1, 2, 40] and some examples of these are shown in figure 12 in the linear-segmented CDF form in which all are subsequently described in this exemplification. Each distribution is described by twenty linear segments of equal height. The yield distributions were transformed to return distributions by the linear expression $R_{ijk} = p_y Y_{ijk} - p_n N_i - p_p P_j$, where R denotes return, Y denotes yield, N denotes nitrogen applied and P denotes elemental phosphorus applied, all per unit area, p_y, p_n and p_p are the respective unit prices of Y, N and P and the subscripts ijk denote respectively the i-th level of N, the j-th level of P and the k-th fractile. Prices assumed are as previously reported [1]. Note that fixed costs are not included in this expression as they have no influence on the determination of stochastic efficiency.

The outlined procedure for reviewing stochastic dominance with arbitrary continuous distributions was applied to these thirty-six discrete actions through the implemented computer programme. The results are most easily overviewed in table 11 with rows and columns defined by rates of N and P and the elements by the degree of stochastic efficiency, where zero denotes dominated in the sense of FSD. FSE combinations of fertilizers are indicated in table 11 by an integer $\geqslant 1$. FSE combinations become candidates for SSD review and those that are not dominated in this sense (the SSE set) would be indicated by an integer $\geqslant 2$ and in turn become candidates for TSD review. Those not dominated in the sense of TSD are TSE and are indicated by the integer 3. In this case the SSE and TSE sets are identical so that no "2" entries appear in table 11. The SSE and TSE combinations shall be referred to as "risk-efficient".

TABLE 11

Degree of Stochastic Efficiency of Specified Fertilizer Combinations

		P (kg/ha)					
		0	9	18	27	36	45
N (kg/ha)	0	0	1	3	3	3	3
	22	0	0	0	3	3	1
	45	0	0	1	3[a]	1	1
	67	0	0	0	1	1	1
	90	0	0	0	1	1	1
	112	0	0	0	0	1	1

[a] The discrete combination with greatest mean return.

These results indicate a fairly consistent pattern wherein, in this case which related to crop response on a red-brown earth in eastern Australia, a necessary condition for stochastic efficiency of any order is a reasonable dose of phosphorus. Nitrogen is indicated as being a rather risky proposal, since most of the risk-efficient combinations involve zero levels of N and the highest risk-efficient rate of N is 45 kg/ha in this non-irrigated situation.

TABLE 12

Median[a] returns for the Risk-Efficient Rates

N (kg/ha)	P (kg/ha)	Return[b] ($/ha)
0	18	63·7
0	27	63·5
0	36	61·3
0	45	51·1
22	27	65·2
22	36	62·2
45	27	64·5

[a] The probability of achieving less than (or more than) the median return is 0·5.
[b] Gross return less fertilizer cost.

Within the risk-efficient set, choice of fertilizer rate properly depends upon the risk preferences of individual farmers. However, it should be observed that in terms of average returns there is little difference between several of these rates.

Given the consistent pattern of risk-efficient rates, it seems reasonable to interpolate within the set. Making such an interpolation suggests that all the risk-optimal rates computed after resorting to specific assumptions about risk preference functions (assumptions and rates reported in [1]) fall in the efficient range. This, of course, should not be surprising, but the specific risk-optimal rates were computed by an approximate procedure that used only the first two moments of the yield distribution. In this particular example, as in others from the field of portfolio analysis [29, 31], it does turn out that the risk-efficient set corresponds very closely with the mean-variance efficient set [2].[10]

3.1.3 THEORETICAL CONTINUOUS DISTRIBUTIONS

Man's unending search for simplification has led him, in the development of statistical theory and practice, to devote most attention to a relatively small number of families of so-called theoretical probability distributions. These are "theoretical" in two senses—first because some represent theoretical deductions through probability theory from assumptions about sampling procedures; and secondly because, given their concise mathematical and parametric structures, it is possible to deduce many results about the probabilistic behaviour of the described random variables.

[10] A risky prospect is mean-variance efficient if it is not dominated by another in this sense. Distribution f dominates g in the mean-variance sense if $\text{mean}_f \geqslant \text{mean}_g$, and $\text{variance}_f \leqslant \text{variance}_g$, with at least one of the strict inequalities holding.

Many of these distributions are potentially important in empirical work in agricultural research in the sense that they often provide a useful approximation for describing observed (and judgmental) probabilistic phenomena. For instance, the Beta distribution is one of sufficient flexibility to describe adequately a diversity of uncertain yield and price distributions. However, the theoretical distribution that has received the greatest exploitation in agriculture is the normal distribution—in spite of the mounting evidence [7] suggesting that the normal is often a poor approximation for crop-yield and price distributions.

The two theoretical distributions selected for discussion here are the Beta and the normal, for the mentioned reasons of flexibility/relevance and popularity/familiarity, respectively. Several difficulties arise in employing such distributions in reviews of stochastic dominance. The foremost practical difficulty lies in defining the successive SD cumulative functions. Even the first step of defining the CDF of standard normal and Beta distributions is of such difficulty that these are presented as tables in statistics books. Precise computation of the required successive functions is considerably more difficult (not least because it must be done for particular parameter settings and not just for the standard (0,1) distributions) and for all intents and purposes in our context is practically impossible.

However, this difficulty can be circumnavigated in two ways, both of which are presently employed. First, the CDF of the theoretical distribution can be approximated by a series of linear segments, wherein the procedures of section 3.1.2 are directly applicable or secondly, modified concepts of dominance can be used. An alternative concept of dominance due to Hammond [17] is very useful for theoretical distributions and is discussed first before the approximate procedures.

Hammond's "Corollary 3.3" may be restated as: If the CDF's of $f(x)$ and $g(x)$ cross not more than once, if the mean of f is not less than the mean of g and if f is less prone to low outcomes than g, then f will be at least as preferred as g by all risk averse ($U_1(x) > 0$, $U_2(x) < 0$) decision-makers. A distribution is less prone to low outcomes than another if either its lower finite bound is greater or, in the case of equal (finite or infinite) lower bounds, if its (left-hand tail) CDF lies to the right of the other before (in the case of single intersection) it lies to the left. If f and g are normal, f will be less prone to low outcomes than g if its variance is lesser.

This concept of (second-degree) stochastic dominance is based on the assumption that CDF's do not cross more than once. This assumption does indeed hold for some probability families (e.g. normal, standardized Beta (0–1 range), and some others [17]) provided that comparisons are confined to distributions from the same family but will generally not hold for comparisons of distributions from different families (e.g. a normal compared with a Beta distribution). For the normal distribution, it means that SSD analysis is equivalent to mean-variance analysis (*see* footnote 10). Unfortunately, generalized Beta distributions (lower range limit a, upper range limit b and shape parameters c and d [27]) may entail more than one CDF intersection and so for these distributions, analysis of risk efficiency cannot be confined to comparisons of the lower range parameters, a, and the means, $(ad + bc)/(c + d)$.

Example 5. Rice Production Packages

The procedures for normal distributions are now introduced in a small example due to Roumasset [37]. It concerns four alternative packages of rice technology under irrigated conditions in the Philippines (Bicol region). Techniques M_1, M_2 and M_3, involve progressively more intensive use of fertilizer, other chemicals and labour on improved variety IR-5 while technique T involves traditional practices applied to local varieties. Mean profits are budgeted in Roumasset's Table 3-2 [37, p. 54] and are 960, 1 055, 1 135 and 510 pesos/ha, respectively. The introduced assumption of normality, and the estimation of standard deviations are much more arbitrary being designed to equilibrate the profit distributions at two standard deviations below each mean. The standard deviations suggested are, respectively, 320, 400, 480 and 128 pesos/ha.

Since all four distributions are from the same family, the modified procedure can be employed and as the family is normal, this devolves to a mean-variance (equivalently, mean-standard deviation) analysis. However, comparison among these distributions reveals that for no pair does the condition hold that simultaneously a mean is greater and a standard deviation lesser. Thus all four distributions belong to the SSE set.

One of the difficulties associated with the assumption of normality is that the range is necessarily $\pm$ infinity. Clearly this is not a very realistic assumption for most empirical phenomena in agriculture and indeed this is one reason for favouring the Beta distribution above. Presumably, however, the frequent resort to the normal distribution must imply that it does fit empirical distributions tolerably well over much of their relevant domains. Thus perhaps in risk analysis we should "go along" with the assumption of normality, but somehow ignore the extreme tails. This is the rationale behind the following suggestion for a modified definition of FSD for normal distributions.

Consider two normal distributions with means μ_i, μ_j and standard deviations σ_i, σ_j, respectively. Assume that $\mu_i > \mu_j$ and that $\sigma_i \neq \sigma_j$ so that the CDF's will intersect once at x^*, which lies z^* standard deviations from each mean and at a cumulative probability P^*. The important case is where $\sigma_i > \sigma_j$, $z^* < 0$ and $P^* < \cdot 5$ but for completeness we look also to the case of upper-tail intersections where $\sigma_i < \sigma_j$ and $P^* > \cdot 5$. For the lower-tail intersection, $z^* = (\mu_i - x^*)/\sigma_i = (\mu_j - x^*)/\sigma_j$ from which it follows that $C^* = 1/z^* = (\sigma_i - \sigma_j)/(\mu_i - \mu_j)$ and analogously, for intersections in both tails, $C^* = | \sigma_i - \sigma_j | /(\mu_i - \mu_j)$. Now, if intersections in the extreme tails of normal distributions can in some pragmatic sense be ignored, then pragmatic FSD orderings can be based on calculated values of C^*. The procedure is as follows: (1) define a critical left-tail probability, P^{**}, below which CDF crossovers can be ignored, (2) from tables of the cumulative standard normal distribution, look up z^{**} and find $C^{**} = 1/z^{**}$ (or consult table 13 below), (3) compute C^* and (4) if $\mu_i > \mu_j$ and $C^* \leqslant C^{**}$, distribution i pragmatically dominates j in the sense of approximate FSD.

TABLE 13

Critical Values for Approximate FSD Testing for Normal Distributions

P**	C**
·0001	·267
·001	·308
·01	·429
·05	·606
·1	·772

While choice of any critical probability is necessarily arbitrary, the suggestion offered here is that a value of ·01 seems a reasonable guideline, bearing in mind the typical adequacy of the normal assumption in empirical applications. This is the critical probability used in applying this approximate FSD procedure to the alternative rice technology data. The calculations are reported in table 14.

TABLE 14

Approximate FSD Analysis of Four Normal Distributions

Rice production techniques	Data	Ratio
M_1 M_2	| (320–400) | / (960–1 055)	·842
M_1 M_3	| (320–480) | / (960–1 135)	·914
M_1 T	| (320–128) | / (960–510)	·426
M_2 M_3	| (400–480) | / (1 055–1 135)	1·000
M_2 T	| (400–128) | / (1 055–510)	·499
M_3 T	| (480–128) | / (1 135–510)	·563

According to the criteria developed, and the critical levels reported, only one technique, T, is dominated in the sense of approximate FSD, for with P^{**} = ·01 and C^{**} = ·429, since 320 > 128 and ·426 < ·429, then M_1 dominates T. Similarly, if the critical probability is raised to ·05, T is also dominated by M_2 and M_3. However, for critical probabilities up to ·1, all the three improved-variety techniques are stochastically efficient of first degree.

The second (previously-noted) general approach to analyzing stochastic efficiency among theoretical distributions involves approximating the theoretical CDF with linear segments and proceeding as in section 3.1.2. This notion is presently introduced by reference to the normal distribution, and is exemplified in section 3.2.2 below.

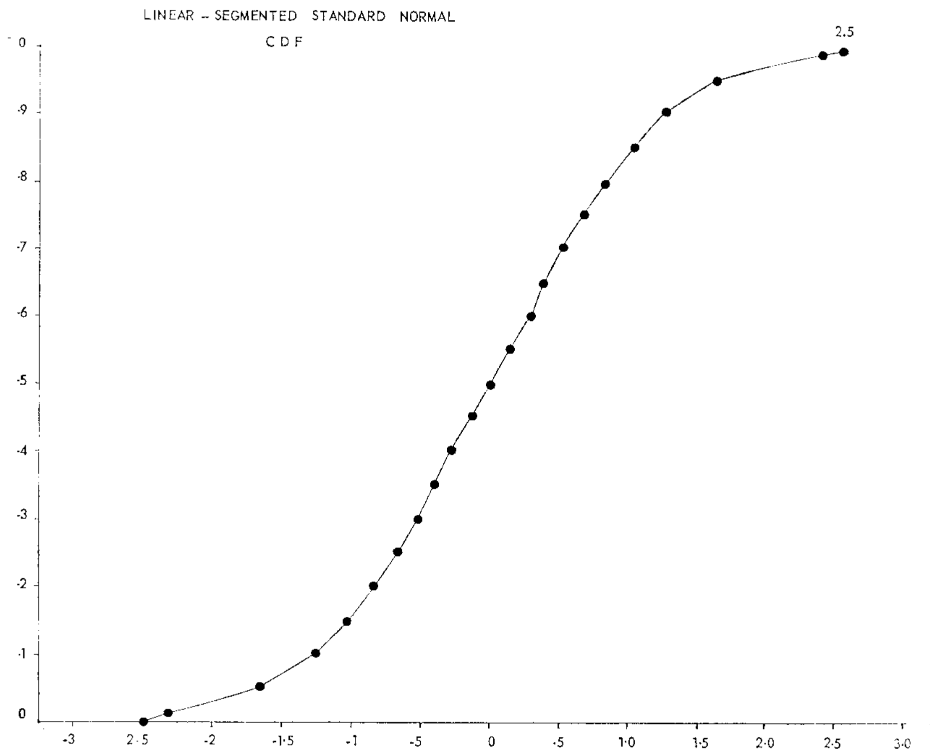

FIGURE 13: *A linear-segmented approximation to the standard normal CDF*

Figure 13 depicts a standard normal CDF represented by 20 linear segments each spanning ·05 cumulative probability. The inner 19 ordinates come directly from a tabulation of the percentiles of the standard normal distribution [27]. The extreme values (the 0·0 and 1·0 fractiles) are further arbitrary approximations (for $\pm\infty$) of $\pm$ 2·5, which are the approximate intercepts with the 0 and 1 probability lines formed by extrapolating the extreme linear segments drawn through the coordinates of the (5 and 1) and (95 and 99) percentiles. The coordinates for linear approximations for any normal distribution could then be found simply by multiplying each of these standard ordinates by the respective standard deviation and adding the respective mean. Thereafter, SD analysis could proceed as for the case of arbitrary continuous distributions.

3.2 CONTINUOUS ACTIONS

The foregoing theoretical, methodological and exemplary sections have illustrated that analysis of stochastic dominance is intrinsically a discrete affair involving as it does pairwise comparisons of the SD functions. It appears to be impossible to make the procedures continuous, say, analogously to the typical response analysis procedures. This means that applications of the SD principles to a problem that is inherently continuous must involve making the problem discrete in such a way that the essence of the original problem is not lost.

A crude illustration of this has already been provided by example 4, the results of which seemed to enable some degree of interpretation of a continuous nature. There the approach taken was to interpolate among the results to draw conclusions about the intervening continuum. The alternative approach is to follow a more familiar analytical route and interpolate among the data. For example, if normal distributions have been fitted to response data at several levels of a continuous variable, an analysis might reasonably postulate smooth functional relationships between the continuous variable and the distributional parameters. Then analysis for stochastic efficiency could proceed on the basis of (discrete) predicted or interpolated distributions at levels of the continuous variable other than those at which the observations were available. The interpolation procedure could presumably be carried to any desired intensity to allow effectively a continuous analysis. Such is broadly the approach adopted in this section.

Only two distributional cases are considered as it is speculated that analysts would very seldom wish to specify a continuous-action decision problem in terms of discrete distributions. Interpolation of linear-segmented arbitrary continuous distributions is considered in section 3.2.1. As the interpolated distributions have the same linear-segmented character, analysis then proceeds as elaborated in section 3.1.2. Continuous-action decision problems involving continuous theoretical distributions are investigated in section 3.2.2 employing the procedures introduced in section 3.1.3.

3.2.1 ARBITRARY CONTINUOUS DISTRIBUTIONS

As just noted, the only issue novel to this section is how to interpolate among related linear-segmented distributions. There are several possibilities all in keeping with the arbitrary nature of the distributions and all hinging on the underlying continuous decision variable(s) as a basis for interpolation. The most arbitrary is simply a linear interpolation between respective fractiles leading directly to (proper) intermediate CDF's. While this has the advantage of simplicity it suffers from an overarbitrariness in that interpolators usually like to undertake non-linear interpolation on a non-linear albeit approximate basis. Most simply, this involves using low-order polynomial functions such as the quadratic and cubic. Depending on the number of data, these can be fitted directly or with least-squares or other regression analysis.

The most serious difficulty with such non-linear interpolation is that the derived CDF's may not be proper (i.e. may imply negative probabilities) and this may require some adjustment and correction. Such problems are best discussed empirically through example 6.

Example 6. *Rate of N on rice*

This example is chosen for three main reasons: (a) there is a single continuous decision variable, namely rate of nitrogen; (b) resulting from a good deal of analytical effort, probability distributions for rice yield have been specified and reported at several levels of nitrogen;

and, (c) the data have been subjected to alternative analysis intended to give due attention to the risk component. The selected example is developed from one of the sets of experimental results on rice reported by Barker, Cordova and Roumasset [4, 37], namely for IR8 grown at the Maligaya research station (Philippines) during the wet seasons 1966–71.

Their fitted yield distributions were presented in PDF form. For the purpose of this analysis, these are required in CDF form and, because of their varied asymmetric shapes, the linear-segmented approximation seems useful and appropriate. This interpretation of the distributions is presented graphically in figure 14.

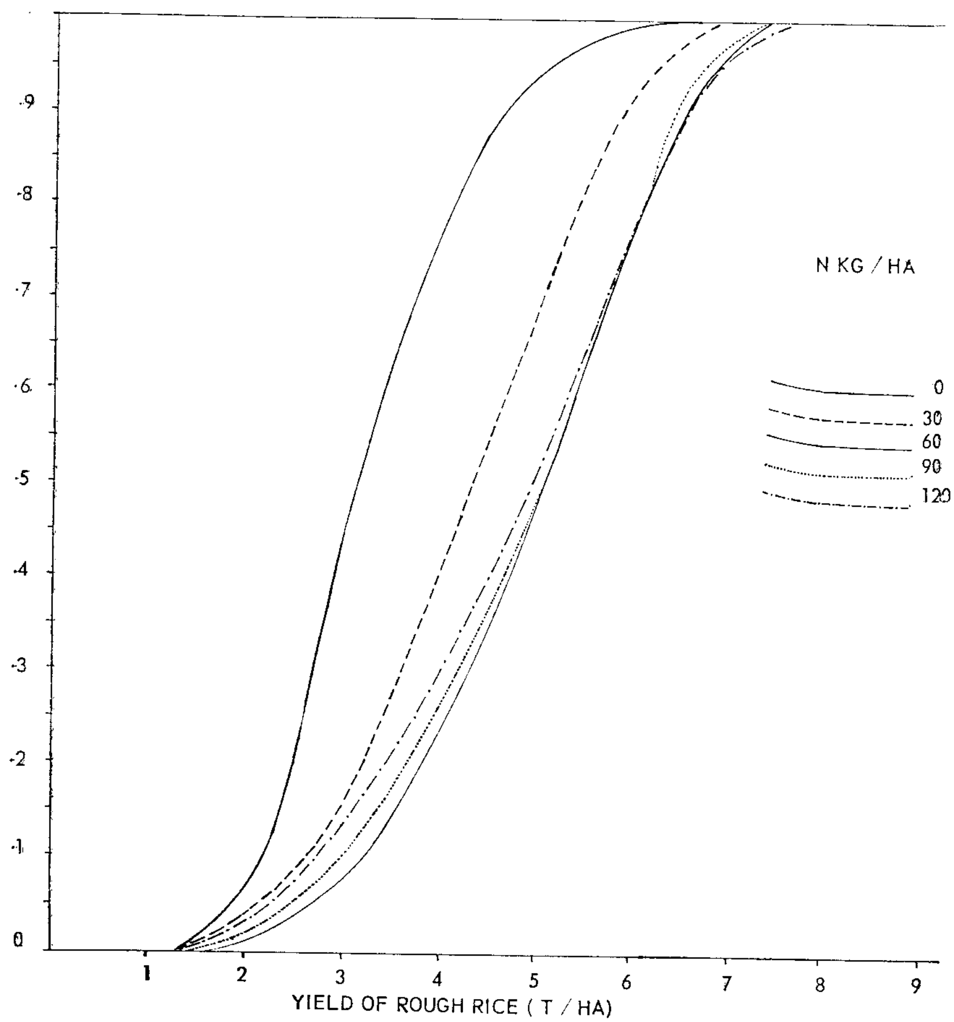

FIGURE 14: *CDF's for Rice Yield under five rates of nitrogen fertilizer (fitted by Roumasset* [37] *using the Pearson moment method)*

Analysis of stochastic efficiency again must proceed first by transforming the yield data to an appropriate argument of the implicit utility. Gross margin again seems appropriate here in the fertilizer decision as fixed costs play no role in influencing stochastic efficiency when they are constant among compared prospects. The only prospect for which this presumption may not be valid is the "zero *N*" action but this possibility is presently ignored.

Gross margins are computed in the manner described for example 4, for all the coordinates of the linear-segmented CDF's assuming ·5 pesos/kg of rough rice and two prices for nitrogen fertilizer, a prevailing acquisition price of 1·5 pesos/kg and an inflated price that allows for cost of credit, opportunity costs, etc., namely 3 pesos/kg.

Three schemes were examined for interpolating distributions for rates between the five experimental rates of 0, 30, 60, 90 and 120 kg/ha. The simplest (and worst) was simply to interpolate linearly between the coordinate fractiles but naturally this leads to kinked relationships between fractile estimates (like median response) and rate of *N*. Clearly such kinked linear segments make for a poor representation of the response to *N* and a curvilinear representation seems more intuitively acceptable—an intuition borne out by graphical doodling.

Quadratic and cubic functions were fitted to each of the twenty-one sets of fractiles for the five rates as a simple-minded attempt at non-linear interpolation and both these attempts seemed to work fairly well. The anticipated problem of interpolated CDF's being improper because of inconsistent predictions of fractiles from the regressions did not arise. There is, however, no guarantee that such interpolations will be proper and the possibility must always be examined. The intensity of interpolation is an arbitrary and judgmental matter which seems properly influenced by considering the precision readily obtainable in applying fertilizers. The compromise adopted here was to consider intervals of 3 kg/ha over the experimental range of 0 to 120 kg/ha. This amounts to comparing forty-one discrete actions along the continuum of fertilizer rates. Of course, more intense interpolation over all or part of the range would be readily possible but does not seem strongly justified on practical grounds.

Results are summarized in table 15 in the left-hand side. For each price, the first comparison is only among the actual experimental rates (i.e. no interpolation). "Same" (indicated by two stops) records that the stochastically efficient set is identical to that reported for the lower degree of stochastic efficiency appearing to the left. In overview, these results may be viewed as disappointing in that, especially with the non-linear interpolations which are believed to be the most accurate representation of reality, the risk-efficient range of rates of *N* is quite wide. (That is, more precise tailoring of *N* recommendations properly depends on individuals' attitudes to risk). However, these results also imply that use of fertilizer *N* is not as risky as some people may have presumed—at least in the case of rice production—a conclusion presented strongly by Roumasset [36] on the basis of an alternative analytical framework of "safety-first" criteria.

TABLE 15

Stochastically Efficient Rates of N on Irrigated Rice

	Probability specification					
	Pearson moment method [37]			Sparse data rule [1]		
High N price interpolation	FSE	SSE	TSE	FSE	SSE	TSE
None ..	0, 60	..[c]	..	0, 30, 60, 90, 120	30, 60	..
Linear ..	0, 3, 6, 60[a]	..	..	0–120	30–60	..
Quadratic ..	0–84[b]	0–72	..	0–120	18–72	..
Cubic ..	0–69	0–63	..	0–120	21–72	..
Low N price interpolation	FSE	SSE	TSE	FSE	SSE	TSE
None ..	60, 120	60	..	0, 30, 60, 90, 120	30, 60, 90	..
Linear ..	60, 117, 120	60	..	0–120	30–90	..
Quadratic ..	0–96	0–78	..	0–120	30–87	33–87
Cubic ..	0–78, 108–120	0–72	0–69	0–120	30–87	..

[a] All rates are in kg/ha and, where interpolated, are at intervals of 3 kg/ha over the experimental range 0(30)120.

[b] A dash separating two rates indicates all the examined intervening rates as well as the listed extremes. Discrete rates are separated by commas.

[c] Two stops indicate that this set is identical to the set of next lowest degree of stochastic efficiency.

On the right-hand side of table 15 are presented some analogous results of an alternative means of specifying the probability distributions. Roumasset's [37] specification was based on frequency histograms of experimental *observations* with a subsequent smoothing process based on the notion of a smooth bell-shaped Pearson Type I frequency function. The alternative estimational process for which the results are now discussed is based on taking *predictions* from yearly response functions and processing and smoothing according to the procedures suggested by Anderson [1, 40]. The resulting distributions were generally rather similar to those presented in figure 14, so it comes as no surprise that the results of analysis of stochastic efficiency are somewhat similar. Generally, however, the range of risk-efficient rates is somewhat narrower under this alternative specification of probabilities—although still wider and higher than this analyst's *a priori* anticipation.

3.2.2 THEORETICAL CONTINUOUS DISTRIBUTIONS

Theoretical distributions offer considerable convenience for analysis of continuous-action decisions because of their economical parametric structure. The central idea is that the few parameters of a convenient distribution can be simply related to the continuous decision variables.

The distributions selected here for discussion and exemplification are again the Beta and the normal distributions (in examples 7 and 8 respectively).

Example 7. Continuous rates of N and P on wheat

Example 7 illustrates the use of the Beta distribution and in this instance the suggested procedure of approximating fitted Beta distributions by linear-segmented CDF's for purpose of analysis of stochastic efficiency is adopted. For convenient access to empirical material, the data of example 4 are again employed.

Interpolation of distributions is accomplished in two stages: (a) relating sufficient parameters of the thirty-six estimated distributions to the decision variables N and P; and (b) fitting (interpolated) distributions via the parameters predicted for any specified combination of fertilizer nutrients. The selected equations are reported below: those for the mean response, $E(y)$ and the variance of response, $V(y)$, are metric versions of those reported elsewhere [1], while those for the lower bound of yield, $A(y)$ and the upper bound, $B(y)$ are new to this section. Response, y, and N and P are in kg/ha and numbers in parentheses are respective standard errors of the regression coefficients.

$$(3.2.1)\quad E(y) = 1170 + 9{\cdot}16N + 42{\cdot}4P - {\cdot}0765N^2 - {\cdot}695P^2 + {\cdot}146NP$$
$$\bar{R}^2 = {\cdot}994 \quad (19) \quad ({\cdot}51) \quad (1{\cdot}28) \quad ({\cdot}0040) \quad ({\cdot}025) \quad ({\cdot}009)$$

$$(3.2.2)\quad V(y) = 164200 + 10700N + 26500P - 88{\cdot}6N^2 - 716P^2 + 1320NP$$
$$\bar{R}^2 = {\cdot}943 \quad (67070) \quad (1840) \quad (4590) \quad (14{\cdot}5) \quad (90{\cdot}8) \quad (31)$$

$$(3.2.3)\quad A(y) = 106 - 4{\cdot}33N - {\cdot}76P + {\cdot}040N^2 + {\cdot}357P^2$$
$$\bar{R}^2 = {\cdot}800 \quad (62) \quad (1{\cdot}94) \quad (4{\cdot}8) \quad ({\cdot}016) \quad ({\cdot}104)$$

$$(3.2.4)\quad B(y) = 2{,}840 + 10{\cdot}0N + 63{\cdot}5P - {\cdot}137N^2 - 1{\cdot}50P^2 + {\cdot}831NP$$
$$\bar{R}^2 = {\cdot}938 \quad (162) \quad (4{\cdot}45) \quad (11{\cdot}1) \quad ({\cdot}035) \quad ({\cdot}220) \quad ({\cdot}075)$$

For a given combination of N and P within the experimental range, these equations predict with a fairly high degree of accuracy the mean and variance of response and the upper bounds of response. The lower bound equation (3.2.3) is unfortunately not so precise reflecting the less consistent pattern of the zero fractile with respect to N and P. These features of yield distributions are readily transformed (as elaborated in example 4, and using the same prices) into the corresponding mean, variance and bounds of the relevant net revenue distribution, which for the moment shall be denoted respectively as m, v, a and b. To fit a Beta distribution by the moment method, which presently seems convenient, it is first necessary to compute the mean and variance of the corresponding standard Beta distribution (range 0 to 1), denoted by m^* and v^* respectively. Hence, $m^* = (m - a)/(b - a)$ and $v^* = v/(b - a)^2$ from which the shape parameters [27] can be found directly as $c + d = [m^*(1{\cdot}0 - m^*)/v^*] - 1{\cdot}0$ and $c = (c + d)m^*$.

If the Beta distribution could be readily integrated to the CDF and successive SD functions, the analysis of stochastic efficiency could now proceed in a precise and direct manner. However, because such integrations are not feasible, the linear-segmented CDF approximation is suggested as an adequate and expedient approach to completing the analysis. Unfortunately, a difficulty with using the rather flexible Beta distribution is that tabulations of the percentiles are only readily available for integer values of c and $c + d$, and for fractiles ·05, ·10, . . . , ·95 [24]. The latter restriction conveniently fits the use of 20 linear CDF segments of equal probability span but, for non-integer values of c and d, the fractiles, f_s must be interpolated (linearly seems adequate) from the fractiles of the four embracing standard Beta distributions with interger shape parameters. This done, fractiles for the respective fitted distribution are then found through the transformation $[f_s\,(b - a) + a]$.

This procedure is demonstrated by reviewing stochastic efficiency first among 40 factorial combinations of N and P involving $N = 0, 20, \ldots, 80$ kg/ha and $P = 0, 5, \ldots, 35$ kg/ha. The results are summarized in table 16.

TABLE 16

Stochastically Efficient[a] *Fertilizers of a Relatively Coarse Grid*

(based on interpolated Beta distributions)

		P(kg/ha)				
		0(5)15[b]	20	25	30	35
N(kg/ha)	0	0	1	3	3	3
	20	0	0	3	3	3
	40	0	0	1	1	1
	60	0	0	0	1	1
	80	0	0	0	1	1

[a] Degree of stochastic efficiency is indicated as described for table 11.

[b] The shorthand $x(y)z$ means all rates $x, x + y, x + 2y, \ldots, z$.

The results shown in table 16 reveal that, given all the assumptions made, the combinations of fertilizers that are risk-efficient are in the region of low rates of N and high rates of P. Should more precise information be required, this can be obtained by interpolation on a grid finer than that used for table 16, in the determined region of interest. The results of such a more intensive and more localized analysis are reported in table 17 wherein the risk-efficient fertilizer policy is described in greater detail. Such a procedure could be carried out to any desired intensity of interpolation to approach a continuous analysis but, with due regard to the accuracy of the whole specification and procedure, could clearly be taken to nonsensical limits. Hopefully, such limits have not been too far transcended here.

Example 8. *Continuous rates of N and P on wheat (continued)*

In the limited number of empirical studies of risky response analysis [1, 2, 4, 9, 36] it has been assumed that risk is confined to the physical yield of the process and that all prices, including that of the product

TABLE 17

Stochastically Efficient Fertilizers of a Relatively Fine Grid

		20	25	30	35	40
N(kg/ha)	0	1	3	3	3	3
	5	0	0	1	3	3
	10	0	0	1	1	1
	15	0	3	3	3	1
	20	0	3	3	3	3
	25	0	3	3	1	3
	30	0	3	3	1	1
	35	0	3	3	1	1

are known with certainty. This assumption, which has also been made in the foregoing examples, has been rationalized on empirical grounds—firstly because of yields usually being relatively much more variable than prices and, relatedly, the frequent existence of price stabilization schemes for key crops such as cereals.

However, even where such price support or stabilization schemes are operative, it is not uncommon for farmers to experience variation (and consequently to face risk) in product prices and so it is useful to explore the possible impact of such risk on input decisions.

The only apposite theoretical work is a brief mention by Magnusson [25, section 5.11] in his analysis of risky decision-making in terms of mean and variance of profits. He concludes that the crucial influence in this case is the covariance between price per unit of output and marginal physical product. This covariance is neither easily conceptualized nor readily estimated.

A covariance that is more readily conceptualized and is probably easier to estimate is that between price, p_y, and yield, y. The sign and magnitude of the correlation between price and yield will depend on the degree of climatic homogeneity in the producing areas of each crop. One might argue that there is unlikely to be any stochastic dependence between the volume an *individual* farmer produces and the price *he* receives, in which case the correlation would be zero. Alternatively, if the great majority of producers in a country tends to experience exceptionally good or exceptionally bad conditions during the same season, then we might argue convincingly for a negative correlation between price and yield for each individual. Irrespective of correlation, it seems reasonable to assume that the marginal distribution for price will not be influenced by the decisions of individual farmers.

There are three broad methods of approaching the analysis of the joint distribution of price and yield: (a) To obtain data on both prices and yields over time and thus estimate directly marginal distributions of revenue and relate the parameters of these distributions to the decision variables. (b) To attempt to estimate the joint distribution for price and yield and incorporate the influence of the decision variables on the joint distribution. (c) To specify the marginal distributions of price and yield in such a way that the dependence in their joint distribution can be readily parameterized in a sensitivity analysis.

None of these methods is entirely satisfactory. Method (a) suffers from likely obsolescence of the historical price data especially. (New varieties may also make the yield data obsolete.) The estimational task in method (b) seems beyond the present data and analytical resources. Method (c) circumnavigates the task of specifying empirical price variability and is chosen as the method to be illustrated. With historical data available that seem pertinent to present decisions, method (a), or some variant of it, appears to offer the most generally practicable alternative.

In general, the distribution of a product ($p_y Y$) will not be of the same family as those random variables entering the product (p_y and Y). However, formulae for the mean and variance of the product (in terms of the means, variances and correlation) are available [5, 13] and do not depend on the distributional forms. Analysis of stochastic efficiency requires specification of the distribution and the arbitrary assumption made here for illustrative purposes is that the distribution of the revenue, $p_y Y$, is normal. There do not seem to be any guidelines available as to the conditions under which such an assumption would be reasonable excepting for the work of Hayya and Ferrara [21] which indicates how closely the normal may approximate the non-normal distribution of $p_y Y$ when each of p_y and Y is normally distributed.

Again to economize on the introduction of new empirical material, the example considered is the wheat-*N*-*P* fertilizer problem of example 7. As the analysis is to be couched in terms of only mean and variance, it is necessary to recall only equations (3.2.1) and (3.2.2) which describe the mean and variance of response of wheat to nitrogen and phosphorus. All price assumptions are as were previously introduced in example 4 (namely, p_n = \$.22/kg, p_p = \$.308/kg) excepting that whereas the price of output was assumed certain (p_y = \$.0385/kg), the mean of the price distribution is now given this value ($E(p_y)$ = \$.0385/kg). The price variance and the correlation with output are arbitrarily varied to explore the sensitivity of risk-efficient rate with respect to these parameters.

An analysis was made with the extreme possibilities that the correlation, r, is -1, 0 or $+1$, and with three alternative levels of variability, namely that the standard deviation of price, σ_p in \$/kg is 0, .00367 or .00733.

Under the assumption of normally distributed revenues, $R = p_y Y - p_n N - p_p P$, the revenue distributions are completely specified by their means and variances which are computed from the standard formulae as:

$$E(R) = E(p_y)\, E(Y) + r\sigma_p\sigma_y - p_n N - p_p P,$$

$$V(R) = V(P)\, V(Y) + E(p_y)^2\, V(Y) + E(Y)^2\, V(p_y) + r^2\, [rV(p_y)\, V(Y) + 2E(p_y)\, E(Y)\sigma_p\sigma_y],$$

where

$$V(p_y) = \sigma_p{}^2 \text{ and } \sigma_y = +V(y)^{.5}.$$

Analysis of stochastic efficiency must again be made in terms of discrete rates of N and P, according to a desired intensity of interpolation as indicated in example 7. In the present example we choose to employ neither the approximate FSD for normal distributions nor the method of approximating normal CDF's with linear segments. Instead we identify only a set of fertilizer combinations efficient in the sense of SSD which, as shown in section 3.1.3, is identical with the mean-variance efficient set.[11] For each of the parameterizations of σ_p and r, the grid of 144 combinations of fertilizers examined consists of a complete factorial arrangement of $N = 0, 11{\cdot}2, 22{\cdot}4, \ldots, 123{\cdot}2$ kg/ha (i.e. approx. $N = 0, 11, 22, 34, 45, 56, 67, 78, 90, 101, 112, 123$ kg/ha) and (approx.) $P = 0, 4{\cdot}5, 9, \ldots, 49{\cdot}5$. The relatively fine grid was selected to provide a reasonable chance of detecting sensitivity that would be of practical consequence.

TABLE 18

Risk-Efficient[a] Fertilizer Combinations with Risky Yield and Price

		P (kg/ha)							
		0(4·5)18	22·5	27	31·5	36	49·5	45	49·5
N (kg/ha)	0	0	0	2	2	2	2	2	2
	11	0	0	2	2	2	0	0	0
	22	0	0	2	2	0	0	0	0
	34	0	2*[b]	2	2	0	0	0	0
	45	0	0	2*	2	0	0	0	0
	56	0	0	0	2	0	0	0	0
	67(11)123	0	0	0	0	0	0	0	0

[a] The zero elements of the table denote "not in the mean-variance efficient set" and integer "2" values indicate members of the mean-variance or SSD efficient set.

[b] Asterisked elements are additional members of the SSE set under alternative parameter settings discussed in the text.

The results are summarized in table 18 and consist of two sets of risk-efficient combinations (broadly similar to that displayed in table 11). Combinations identified by a "2" without an asterisk are those that are SSE under the "riskless price" assumption, $\sigma_p = 0$. This was identical with the SSE set for $\sigma_p = {\cdot}00367$ and $r = 0$. For all the other parameterizations explored, namely $\sigma_p = {\cdot}00367$, $r = \pm 1$, and $\sigma_p = {\cdot}00733$, $r = -1, 0, +1$, the risk-efficient set consisted of the previously identified set and the two additional combinations identified by "2*". Clearly the risk-efficient set is not very sensitive to assumptions about uncertainty in output price—at least in this particular example based on the normal distribution.[12] A very tentative conclusion from these

[11] With normal distributions, it seems intuitively likely that the SSE and TSE set are identical.

[12] Restricting adjustments of the parameters to $r = -{\cdot}5, 0, +{\cdot}5$ and $\sigma_p = {\cdot}00184$, ${\cdot}00367$ caused no changes from the SSE set for $\sigma_p = 0$.

few results is that the slight effect of varying correlation from zero is seemingly independent of the sign of correlation coefficient and is equivalent in its impact to an increase in the variance of price. More generally, these results tempt one to speculate that analysts who have simply ignored price variability in response analysis have committed no grave sin.

4 CONCLUSIONS

While well-known procedures are at hand for formulating recommendations destined for farmers concerned solely with (average) profits, such is not the case for farmers concerned with risks as well as with average profits. The author believes that farmers are generally influenced in their decisions by aspects of profit distributions other than the averages, in a way that can be described as risk aversion[13]. This is not to say that farmers don't take risks—they must do so continually—but rather that they can take into account variability of profits as well as the average. A second, widely-agreed belief is that farming generally is risky and new technology in particular is perceived as risky. This study then is predicated on the notions that (a) research, extension and new technology will be more effective and successful if proper account is taken of risk and (b) it is impossible to account individually for the attitudes to risk of the millions of farmers who feed the world.

A key question, and one to which the answer has not been obvious, is "Can proper account of risk be taken in research and extension?" It is the author's hope that this study has contributed to an affirmative answer. The principles of stochastic dominance permit orderings of risky prospects that are as complete as is theoretically possible without knowing more (and much more) about the attitudes to risk of millions of farmers than will ever be possible. The question we must then ask is whether it is feasible to exploit these principles in interpreting practical research and extension programmes.

An affirmative answer hinges on two considerations: (1) the ability to specify probability distributions in data situations that are often sketchy or of questionable applicability; and, (2) the ability to undertake the numerical task involved in applying the principles. The methods developed and presented herein suggest that these considerations now present no serious obstacle to the exploitation of stochastic dominance in practical research and extension in agriculture. This conclusion is a necessary but not sufficient condition for the validity of the following implications.

5 IMPLICATIONS

Increasingly over recent years, lip service has been paid to the notion that risk is an important aspect of agricultural technology. While this recognition is valuable in itself, a machinery that deals analytically with risk in the absence of knowledge of farmers individual attitudes to risk has not hitherto been exploited in agriculture. Following the conclusion

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[13] More precisely, for farmers in general, the certainty equivalent of a risky prospect is believed to be less than the corresponding subjective expected money value.

that a satisfactory machinery is now available, what are the implications for agriculture generally? Speculation on this question can be made under several distinct headings.

5.1 For Agricultural Research

(a) Rather than focus only on estimation of treatment means, whole probability distributions should be explored and estimated to complement conventional "average-oriented" research, if risk-averse consumers are to be well served.

(b) This implies that "risk-oriented" research will be generally more demanding and more expensive than "average-oriented" research. Seemingly this is the price one pays for work that is potentially relevant in this context.

(c) More particularly, appraisal of stochastic efficiency in the absence of knowledge of individuals risk attitudes demands pinning down the lower tails of probability distributions. This estimational task suggests that agricultural innovations need to be evaluated and reported under bad as well as typical or average environmental conditions that potential adopters face (e.g. with respect to moisture stress, disease exposure, nutrient suboptimization, etc.).

(d) To the extent that identifiable groups of potentially adopting farmers face different "worst" conditions (if not also different "average" or "good" conditions), technological packages that are stochastically efficient may differ between groups. This implies that risk-oriented research should deliberately span an appropriate range of both environments and environmental conditions, which will usually mean replication over space and time.

(e) In the short run especially, there seems to be much unexploited scope for formal documentation of research agronomists considerable experience of and largely unpublished knowledge of the tails, especially the left, of relevant probability distributions.

5.2 For Agricultural Extension

(a) Extension of technological advice in risky agricultures will probably be more effective if due recognition is given to the impact of risk and the importance of technologically induced risk.

(b) Such extension will be simplified by dealing with farmers grouped according to the worst environmental conditions faced and its success will be enhanced by promoting practices that are tailored to be stochastically efficient (at least of degree two) for the identified groups.

(c) When an extension effort is mounted on such a scale and intensity that it is possible for extensionists to elicit and account for individual farmers attitudes to risk and perceived probability distributions (risks), then the analysis of stochastic efficiency as elaborated herein becomes redundant. As of 1974, such a situation has apparently not been attained anywhere and nowhere seems to be an imminent prospect.

(d) In judging extension efforts, recognition that a recommendation efficient in terms of average profit may not be risk-efficient should temper appraisal of programmes.

(e) Probabilistically based educational programmes can provide farmers with valuable information on what situations they face under various seasonal conditions and thereby appropriately modify their perceptions of risk inherent in various technologies.

5.3 For Training in Research and Extension

(a) Perceptive practitioners of the arts of agricultural research and extension inevitably develop a keen intuition for the importance of risk in most agricultural production. However, their formal training has usually done little or nothing to equip them with an analytic apparatus with which to deal directly with this aspect of their work. Clearly, educational programmes should do more to sensitize intending practitioners to the impact of risk in farming, and consequently in research and extension. Particular attention needs to be drawn to the fact that espoused experimental methodology is addressed to estimating only average effects, responses, etc. and accordingly is only directly applicable to risk-indifferent farmer-users.

5.4 For Agricultural Policy

(a) Formulators of agricultural policy will generally suffer fewer "surprises" in programme results if their economic models of farmer behaviour include an adequate recognition of farming risks and farmers attitudes to them.

(b) The main pertinent policy instruments in dealing with risk have been crop insurance schemes and minimal price supports for agricultural products. Our discussion of the stochastic dominance orderings places a new slant on such schemes. By focussing on values in the lower tails of distributions of yields and prices rather than on values near the averages, relatively low premiums may still exert significant adjustments to actions of farmers.

(c) More specifically, for example, a crop "insurance" scheme which effectively truncates yield distributions below a cross-over point in the lower tails of two varietal distributions makes the variety that yields higher on average stochastically dominant in the sense of FSD. Ensuant adoption by farmers of the now FSE new variety would doubtless, *ceteris paribus*, be in the national interest. Typically, a recommended technological practice will dominate (FSD) traditional practices if it can be "insured" to the extent that under really poor eventualities, farmers are not disadvantaged by adoption.

(d) Again, effective truncation of the lower tails of market price distributions through minimal (but low) price and income supports may effectively "take the risk out of" a programme under governmental sponsorship.[14]

5.5 For Methodological Research

The tone of the foregoing implications may have conveyed the impression of a complacent or self-satisfied attitude to the methodology developed herein. Any such impression should be immediately corrected by reiterating some perceived methodological shortcomings. However, it is salutary to recall first that the SD ordering rules do represent the theoretical limit to risk analysis in the absence of either knowledge of individuals attitudes to risk or probably unjustified and indefensible presumptions about these.

(a) Analysis of stochastic efficiency can only be as good as the specification of the respective probability distributions. There will always be scope for methodological improvement in such specification which is an intrinsically risky procedure itself. If the "ideal" is accurate representation of farmers' perceptions, we need to recognize that information gained only from experiments on experiment stations or small plots may be of very distant pertinence.

(b) Present understanding of the risks perceived and experienced in farming generally and in adoption of new technology particularly, is insufficient. Careful and imaginative farm-level research is called for in a cross-section of countries of diverse cultural traditions.

5.6 For the Economics of Research

(a) Applied agricultural research can be judged as potentially successful and economical when it leads to new farming practices that are stochastically efficient. If the new practices are also stochastically dominant (minimally of degree three and most desirably of degree one) then the chance of the research being positively beneficial is correspondingly greater. Of course, the cost of the research should enter the economic evaluation.

(b) Conversely if, after a research programme has been completed and extended to the farming community, farmers prior (competing) practices fall in the risk-efficient set then returns from the research must be highly uncertain and may well be negative.

(c) From a methodological point of view, research planning in the context of stochastic efficiency appears to be intrinsically difficult in the sense that only retrospective analysis of efficiency is in any way straightforward. However, if research *is* directed towards the development of stochastically efficient new technologies, research planners necessarily must aim to identify technologies that are not only more profitable on average but are also less prone to low outcomes under unfavourable conditions.

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[14] See [39] for a discussion of stochastic dominance in taxation policy.

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