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Multiplicity of investment equilibria  
when pollution permits are not tradable

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## **Abstract**

We study a model in which the level of environmental regulation depends on abatement cost, which depend on aggregate levels of investment in abatement capital. Firms are non-strategic. When emissions quotas are not tradable, there are multiple competitive equilibria to the investment problem. Allowing trade in permits leads to a unique socially optimal equilibrium. For a given distribution of investment, allowing trade in permits has an ambiguous effect on the optimal level of regulation. Previous results on coordination games with non-atomic agents are applied to the problem of environmental regulation with endogenous investment in abatement capital.

# Multiplicity of investment equilibria when pollution permits are not tradable\*

Larry Karp<sup>†</sup>

May 10, 2006

## Abstract

We study a model in which the level of environmental regulation depends on abatement costs, which depend on aggregate levels of investment in abatement capital. Firms are non-strategic. When emissions quotas are not tradable, there are multiple competitive equilibria to the investment problem. Allowing trade in permits leads to a unique socially optimal equilibrium. For a given distribution of investment, allowing trade in permits has an ambiguous effect on the optimal level of regulation. Previous results on coordination games with non-atomic agents are applied to the problem of environmental regulation with endogenous investment in abatement capital.

*Keywords:* tradable permits, coordination games, multiple equilibria, global games

*JEL classification numbers* C79, L51, Q58

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# 1 Introduction

A non-strategic firm's belief about future environmental regulation affects its decision to install equipment that reduces abatement costs. Firms probably understand that future regulations might depend on industry-wide abatement costs. In a competitive setting, an individual firm is not able to affect the industry-wide abatement cost and therefore is unable to affect future regulation. Nevertheless, if the firm's investment decision depends on its beliefs about future environmental regulation, then its decision might also depend on its beliefs about aggregate industry investment. Firms in our setting are too small to affect the market outcome, so they are always non-strategic. However, following common usage we describe actions as “strategic substitutes or complements”, depending on the effect that *collective actions* have on a firm's payoff.

If the regulator uses quantity restrictions *without tradeable permits*, firms' investment decisions are strategic complements: it is more attractive for an individual firm to invest in pollution-reducing technology if it believes that a sizeable fraction of the industry is making this investment. Investment by many firms in a technology that reduces abatement costs makes future regulation more stringent, thus increasing the profitability of the investment. The fact that actions are strategic complements can create multiple equilibria. This possibility is transparent in the two-period binary action (invest or do not invest) model presented in the next section, but it can also arise when each firm's investment decision is a continuous variable.

In the binary action setting the two equilibria consist of all (homogeneous) firms or no firms making the investment. In general, neither outcome is socially optimal. Thus, optimal ex post regulation does not induce a socially optimal outcome, even in a competitive (non-strategic) setting. When all firms make the same decision, they all have the same abatement costs. Consequently, the lack of tradeable permits appears inconsequential. However, in a non-equilibrium subgame, in which not all firms make the same investment decision, there would be an incentive for trade. The inability for that trade to occur creates a distortion, causing the ex post optimal regulation to lead to suboptimal welfare.

When the regulator uses quantity restrictions *with tradeable permits* (equivalently, a pollution tax), it is still the case that a higher level of aggregate investment causes regulation to be more stringent. This relation tends to increase the value of the investment, as is the case without tradeable permits. However, when more firms invest in the pollution abatement technology, more firms want to sell permits and fewer firms want to buy them. Thus, although

higher investment reduces the aggregate supply of permits (chosen by the regulator), it always reduces the equilibrium price of permits, making investment less attractive. Therefore, when the regulator uses tradeable permits (or taxes), the investment decisions are strategic substitutes and there is always a unique equilibrium to the investment game.

Firms' ability to trade permits lowers aggregate abatement costs, conditional on an arbitrary distribution of investment. From this fact, it might seem that the equilibrium pollution standard would be stricter (fewer pollution permits) when firms are allowed to trade. However, the optimal regulation depends on a comparison of marginal pollution damages and marginal (not total) abatement costs. The ability to trade permits can make the optimal level of regulation either laxer or tighter, for a given distribution of investment.

Jaffe, Newell, and Stavins (2003) survey the literature on pollution control and endogenous investment. Many papers in this literature, including Biglaiser, Horowitz, and Quiggin (1995), Kennedy and Laplante (1999), Montero (2002), Fischer, Parry, and Pizer (2003), Moledina, Polasky, Coggins, and Costello (2003), Tarui and Polasky (2005) and Tarui and Polasky (2006) assume that firms behave strategically with respect to the regulator: firms believe that their investment decisions will affect future regulation. Several papers, including Milliman and Prince (1989), Requate (1998), and Karp and Zhang (2002) treat firms as non-strategic. None of these papers discuss multiple equilibria, so they do not consider the relation between the policy instrument and the existence of multiplicity. Multiple equilibria can arise regardless of whether firms are strategic. The possibility is particularly easy to identify when firms are non-strategic; this case is also useful because it helps to isolate the strategic incentive from the more general effect of forward-looking behavior.

Tradeable permits are a fairly recent innovation. Quantity restrictions without trade are still the norm. Therefore, the possibility of multiple equilibria and the lack of optimality in a competitive setting is significant. In our model, the regulator's decision is conditioned on industry costs. The equilibrium when the regulator uses *tradeable* permits (or taxes) equals the social optimum regardless of whether the regulator commits to a level of permits before investment occurs. The outcome is subgame perfect when the regulator moves after the firms invest. It is time consistent (but not subgame perfect) when the regulator moves before firms. If the regulator commits to a level of *nontradeable* permits before investment occurs, the outcome might be time consistent, but it is typically not subgame perfect. Trade in pollution permits creates efficiencies at both the abatement stage and the investment stage.

In some cases, regulation and investment coevolve, so a multiperiod alternating moves game probably provides a more realistic description than the two-period setting. However, as long as regulation responds to aggregate industry investment decisions and firms understand this, the effects that we discuss are present. If the regulator expects to learn about the severity of damages, there is a good reason to delay making the regulatory decisions. The depreciation of old capital may make it difficult for firms to delay their investment decisions. Although a two-stage model is unlikely to “get the timing right”, it nevertheless provides insight into the effect that state-contingent policymaking has on equilibrium outcomes.

Section 2 presents the model and establishes the results described above. Section 3 “imports” previous results on coordination games into the setting with investment and environmental regulation.

## 2 The model and equilibria without and with trade

We use a two-period rational expectations equilibrium model. Prior to investment, all firms are identical. In the first period, each firm makes a binary decision: it does not invest in a new technology ( $K = 0$ ) or it does invest ( $K = 1$ ). If a firm is allowed to emit  $e$  units of pollution, its abatement cost is  $c(e, K)$ , a function that is decreasing and convex in  $e$  and decreasing in  $K$ , with  $c_{e,K} > 0$ . This inequality implies that the business-as-usual (BAU) level of emission, i.e. the level that satisfies  $c_e(e, K) = 0$ , is decreasing in  $K$ , and also that  $c_e(e, 1) - c_e(e, 0) > 0$ . The fraction of firms that invest is  $0 \leq \kappa \leq 1$ . The firm’s cost of investment is  $\phi$ . If  $0 < \kappa < 1$ , firms are heterogenous in the second stage, when the regulator decides on the level of pollution permits.

Each (non-atomic) firm is given an emissions allowance of  $e$ , independently of whether it invested. The mass of firms is normalized to 1, so aggregate emissions are  $e$ . The damage function is  $D(e)$ , an increasing convex function. If firms who did not invest emit at the rate  $e^0$  and firms that did invest emit at the rate  $e^1$ , total emissions are  $(1 - \kappa) e^0 + \kappa e^1 = e$  and social costs at the emissions stage are  $(1 - \kappa) c(e^0, 0) + \kappa c(e^1, 1) + D(e)$ .

### 2.1 No trade in permits

In the absence of trade (and given the assumption that the allocation of permits does not depend on whether firms invested), all firms emit at the same rate, so  $e^0 = e^1 = e$ . We use subscripts

to denote partial derivatives and superscripts to indicate the firm's investment decision. For example,  $c_e^i \equiv c_e(e^i, i)$ ,  $i = 0, 1$ . Given  $\kappa$ , minimization of social costs requires

$$\begin{aligned} (1 - \kappa) c_e^0 + \kappa c_e^1 + D' &= 0 \\ S &\equiv (1 - \kappa) c_{ee}^0 + \kappa c_{ee}^1 + D'' > 0. \end{aligned} \tag{1}$$

The first order condition, equation (1), defines the optimal  $e^*$  as a function of  $k$ . ("\*" denotes an equilibrium, or an optimal level.) The derivative is

$$\frac{de^*}{d\kappa} = \frac{c_e^0 - c_e^1}{S} < 0.$$

When more firms invest ( $\kappa$  is larger), marginal abatement costs are lower, so the equilibrium number of permits is lower.

In the investment stage, a firm's net benefit of investing (the costs when it does not invest minus the cost when it does invest) is

$$\Pi(\kappa) = c(e(\kappa), 0) - c(e(\kappa), 1) - \phi,$$

which implies

$$\Pi'(\kappa) = \frac{(c_e^1 - c_e^0)^2}{S} > 0. \tag{2}$$

The investment decisions are strategic complements.

The necessary and sufficient condition for multiple equilibria are

$$E \equiv c(e(0), 1) + \phi - c(e(0), 0) > 0 \tag{3}$$

$$H \equiv c(e(1), 0) - c(e(1), 1) - \phi > 0. \tag{4}$$

The net cost of adopting if no other firm adopts is  $E$ . Inequality (3) implies that a firm does not want to invest if it knows that no other firm will invest ( $\kappa = 0$ ); here the firm knows that the environmental standards will be lax. The net benefit of adopting if all other firms adopt is  $H$ . Inequality (4) implies that it pays a firm to invest if all other firms do so; here the firm knows that abatement standards will be strict.

If equations (3) and (4) hold there is an interior unstable equilibrium that satisfies  $\Pi(\kappa_u) = 0$ , where  $0 < \kappa_u < 1$ . At  $\kappa_u$  a firm is indifferent between investing and not investing. This equilibrium is unstable; for example, if slightly fewer than the equilibrium number of firms invest ( $\kappa < \kappa_u$ ), it becomes optimal for all other investors to change their decisions, and decide not to invest. In summary, we have



**Remark 1** *Inequalities (3) and (4) are necessary and sufficient for the existence of two stable boundary equilibria (all firms or no firms invest) and one unstable interior equilibrium. If either inequality fails, there exists a unique boundary equilibrium.*

If  $\phi$  is very small, it is always optimal to invest; it is never optimal to invest if  $\phi$  is very large. Multiplicity requires that  $\phi$  is neither very large nor very small.

## 2.2 Trade in permits

Trade in permits equates marginal costs of investors and non-investors, and the price of permits equals this marginal cost. Let  $e$  be each firm's endowment of permits, and  $e^t$  the equilibrium purchases of each non-abater. Since the mass of purchases equals  $(1 - \kappa) e^t$ , each of the  $\kappa$  low cost firms (the investors) must be willing to sell  $\frac{(1-\kappa)e^t}{\kappa}$ . The equilibrium conditions for quantity ( $e^t$ ) and price ( $p$ ) are

$$c_e(e + e^t, 0) = c_e\left(e - \frac{(1 - \kappa) e^t}{\kappa}, 1\right) \quad (5)$$

$$c_e(e + e^t, 0) = -p(e, \kappa). \quad (6)$$

Given  $\kappa$ , the planner's problem is to choose  $e$  to minimize

$$W(e; \kappa) = (1 - \kappa) c(e + e^t, 0) + \kappa c\left(e - \frac{(1 - \kappa) e^t}{\kappa}, 1\right) + D(e), \quad (7)$$

leading to the first order condition

$$(1 - \kappa) c_e(e + e^t, 0) + \kappa c_e\left(e - \frac{(1 - \kappa) e^t}{\kappa}, 1\right) + D'(e) = 0. \quad (8)$$

Because the market allocates permits optimally, the envelope result means that the first order condition does not involve  $\frac{de^t}{de}$ . We assume that the planner's problem is convex, so the second order condition holds:

$$S^t \equiv \frac{d^2 W}{de^2} = (1 - \kappa) c_{ee}^0 \left(1 + \frac{de^t}{de}\right) + \kappa c_{ee}^1 \left(1 - \frac{1-\kappa}{\kappa} \frac{de^t}{de}\right) + D'' > 0. \quad (9)$$

### 2.2.1 Comparison of pollution levels with and without trade, given $\kappa$

Here we compare the equilibrium levels of pollution permits with and without trade in permits, for a given  $\kappa$ . The two levels are equal at  $\kappa = 0$  or  $\kappa = 1$ , where there is no incentive to trade.

The only interesting situation is where  $0 < \kappa < 1$ , as we assume for the rest of this subsection. For any  $e, \kappa$ , total abatement costs are lower when permits are tradeable. This fact might appear to suggest that the equilibrium level of permits would be lower (i.e., abatement would be higher) under trade. This relation need not hold, because the optimal level of pollution depends on a comparison of marginal (not total) abatement costs, and abatement benefits.

The private marginal benefit of emissions equals the negative of private marginal costs. The social marginal benefit of emissions is  $G(e; \kappa, j) \equiv -((1 - \kappa)c_e^0 + \kappa c_e^1)$ ,  $j = \text{trade, no trade}$ . With trade, the two types of firms have the same marginal benefits,  $p$ , so  $G(e; \kappa, \text{trade}) = p(e, \kappa)$ . With and without trade,  $G(\cdot)$  is a convex combination, with weights equal to  $1 - \kappa, \kappa$ , of the marginal benefits of emissions for the two types of firms (those who did not invest and those who did).

Equations (1) and (8) have the same form, but they have different arguments. They are identical if  $e^t = 0$ . Figure 1 shows the graphs of marginal social benefits and marginal damages (the solid curves) in the absence of trade, for given  $\kappa$ . The equilibrium number of permits without trade is shown as  $e^*$ . The curves labelled  $A$  and  $B$  show possible graphs of marginal emissions benefits under trade, taken from equation (8). If the introduction of trade causes the industry marginal benefit curve to shift to  $A$ , trade reduces the equilibrium number of permits. If the introduction of trade causes the marginal industry cost curve to shift to  $B$ , trade increases the equilibrium number of permits.

Either of these scenarios can occur. Trade in permits shifts down the industry marginal cost curve (to a location such as  $A$  in Figure 1) if and only if the equilibrium price, under trade, is lower than the convex combination of marginal abatement costs without trade.

Figure 2 graphs the non-adopter's and the adopter's marginal benefit curves as a function of  $e^t$  (for fixed  $e$  and for  $\kappa = 0.5$ ) under two scenarios. These scenarios are shown by the curves labelled "low" and "high", corresponding to low and high marginal costs for the firm that invests. For simplicity, the figure illustrates a case where the intercept of the adopter's marginal cost, point  $b$ , is the same in the high and the low scenario. The non-adopter's intercept of marginal cost is at point  $a$ , so the social marginal benefit without trade is  $\frac{a+b}{2}$  (for this particular value of  $e$  and for  $\kappa = 0.5$ ). If the investor's marginal benefit curve is "low", the equilibrium price is lower than  $\frac{a+b}{2}$ . This possibility corresponds to curve  $A$  in Figure 1. Here the volume of trade is high, and the price is low when permits are tradeable. In this case, trade reduces the equilibrium level of  $e$ , for given  $\kappa$ . Trade has the opposite effect in the "high" scenario. There

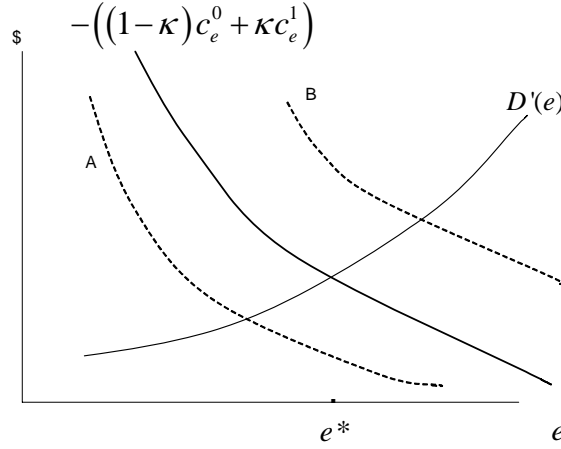


Figure 1: Comparison of emissions level for given  $0 < \kappa < 1$ , without trade (solid curve) and with trade (curves labelled  $A$  and  $B$ ).

the volume of trade is low and the price is high; trade increases the equilibrium level of  $e$  for given  $\kappa$ . A simple argument (see Appendix for details) establishes

**Remark 2** Assume that  $0 < \kappa < 1$ . A sufficient condition for trade in permits to decrease the equilibrium supply of permits (i.e. to lead to stronger environmental regulation) is

$$\Delta(\kappa, e, s) \equiv c_{ee}(e + s, 0) - c_{ee}\left(e - \frac{(1 - \kappa)s}{\kappa}, 1\right) > 0 \quad (10)$$

for all  $e, \kappa, s$ . A sufficient condition for trade to lead to weaker environmental regulation is for inequality (10) to be reversed for all  $e, \kappa, s$ .

In general, there is no reason to suppose that inequality (10) holds. For example, suppose that we can approximate costs using the function  $c(e, K) = f(K) - \left(a(K) - \frac{b(K)}{2}e\right)e$ ;  $f(K)$  is a fixed cost of abatement,  $a(K)$  is the intercept of the marginal benefit of emissions, and  $b(K) > 0$  is the slope of marginal benefits. This approximation only makes sense for  $0 \leq e \leq \frac{a(K)}{b(K)}$  (the BAU level of emissions). An increase in  $K$  reduces BAU emissions if and only if

$$\frac{a'(K)}{a(K)} < \frac{b'(K)}{b(K)}. \quad (11)$$

For this linear-quadratic example, inequality (10) is satisfied if and only if  $b(0) - b(1) > 0$ , i.e.  $b'(K) < 0$ . Inequalities (10) and (11) are mutually consistent, so it is possible that allowing trade in emissions permits would lead to tightening environmental regulations. However,

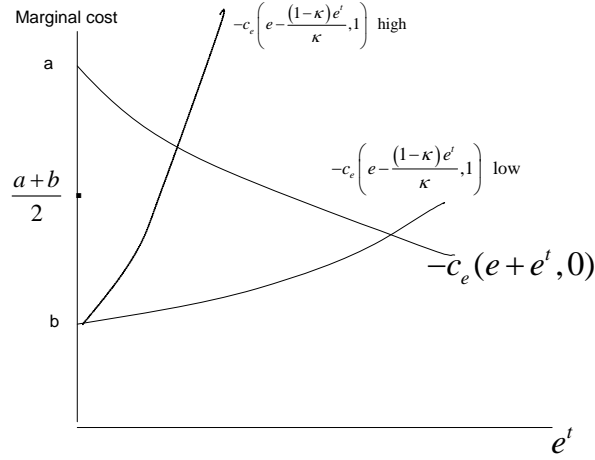


Figure 2: Marginal benefit of emissions trade and marginal cost under high and low scenarios, fixed  $e$  and  $\kappa = 0.5$ .

investment can increase the slope of marginal benefits, so that  $b'(K) > 0$ . In this case, trade would lead to weaker environmental standards (given  $k$ ).

Investment decreases marginal abatement costs, but when  $b'(K) > 0$  investment increases the slope of the marginal benefit of an additional unit of emission. This situation corresponds to the “high” scenario in Figure 2, where the equilibrium permit price is higher than the convex combination of marginal abatement costs without trade, for a given level of emissions.

### 2.2.2 The equilibrium value of $\kappa$ under tradeable permits

In order to determine the equilibrium value(s) of  $\kappa$  we need to determine the effect of  $\kappa$  on a firm’s payoff. We totally differentiate equation (8), using the second order condition  $S^t > 0$ . The result is

$$\frac{de}{d\kappa} = -\frac{c_{ee}^0 c_{ee}^1}{\kappa S^t} \left( \frac{e^t}{\kappa c_{ee}^0 + (1 - \kappa) c_{ee}^1} \right) < 0. \quad (12)$$

(The appendix shows intermediate steps for the derivations of several equations.) Just as is the case without trade in permits, an increase in the number of adopters (larger  $k$ ) causes the regulator to use stricter environmental standards (smaller  $e$ ).

A larger value of  $\kappa$  has an ambiguous effect on the purchases per non-adopter,  $e^t$ . Totally

differentiating equation (5), the equilibrium condition for quantity traded, implies

$$\frac{de^t}{d\kappa} = \frac{\frac{\Delta}{s^t} \left( \frac{c_{ee}^0}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) + \frac{1}{\kappa}}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} c_{ee}^1 e^t. \quad (13)$$

This equation shows that a sufficient condition for the purchases per non-adopter to increase with the number of adopters is  $\Delta(\kappa, e, e^t) > 0$ . (From Remark 2, this inequality also implies that trade in permits leads to tighter environmental regulations, given  $\kappa$ .) An increase in  $\kappa$  causes the equilibrium price of permits to fall:

$$\frac{dp}{d\kappa} = -\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \frac{D''}{\kappa} < 0. \quad (14)$$

The benefit of investing (equal to the cost savings) when trade in permits is allowed is

$$\begin{aligned} \Pi^t(\kappa) \equiv & (c(e + e^t, 0) + p e^t) - (c(e - \frac{1-\kappa}{\kappa} e^t, 1) + \phi - p \frac{1-\kappa}{\kappa} e^t) = \\ & c(e + e^t, 0) - c(e - \frac{1-\kappa}{\kappa} e^t, 1) - \phi + p \frac{1-\kappa}{\kappa} e^t. \end{aligned} \quad (15)$$

Using the equilibrium conditions (5) and (6) we can write the derivative of the benefit of adoption as

$$\frac{d\Pi^t}{d\kappa} = \frac{e^t}{\kappa} \frac{dp}{dk} < 0. \quad (16)$$

Equation (16) implies

**Remark 3** *When permits are tradeable, investment is a strategic substitute; there always exists a unique equilibrium. The equilibrium involves the fraction  $0 < \kappa < 1$  of firms investing if and only if there is a solution to the equation  $\Pi^t(\kappa) = 0$  for  $0 < \kappa < 1$ . If no solution exists, the equilibrium is on the boundary. The solution exists if and only if*

$$\Pi^t(0) > 0 > \Pi^t(1). \quad (17)$$

We now find necessary and sufficient conditions to insure that equation (17) holds. The optimal level of  $e$  is continuous in  $\kappa$ , and as we noted above,  $e(\kappa)$  is the same with and without trade for  $\kappa = 0$  and  $\kappa = 1$ . We cannot mechanically use equation (15) to evaluate the benefit of adoption at  $\kappa = 0$  because of the terms  $\frac{1-\kappa}{\kappa} e^t$  and  $\frac{1}{\kappa} e^t$ . As  $\kappa \rightarrow 0$ , each of the adopters buys an infinitesimal amount. The clearest way to proceed is to treat the two boundaries ( $\kappa = 0$  and  $\kappa = 1$ ) symmetrically, recognizing that the benefit of adoption depends on *individual* levels of

trade.<sup>1</sup> For example, if almost all firms adopt ( $k \approx 1$ ), the aggregate volume of transactions is approximately 0. Each of the large mass of adopting firms sells an infinitesimal amount, and has almost no benefit from trade. However, each of the small mass of non-adopters can buy a substantial amount, and obtain substantial benefits from trade. This logic enables us to approximate the benefit of non-adoption in the neighborhood of  $\kappa = 0$  and  $\kappa = 1$ .

Consider  $\kappa \approx 1$ , where almost all firms adopt the new technology, so the mass of buyers is small. Denote the socially optimal level of permits when  $\kappa = 1$  as  $e^{*1} = e(1)$ . A firm that wants to buy permits can do so at (approximately) the price  $p(1) = -c_e(e^{*1}, 1)$ ; each adopter sells an infinitesimal quantity and each non-adopter buys the amount  $e^t$  that satisfies  $c_e(e^{*1} + e^t, 0) = c_e(e^{*1}, 1)$ . Thus, in the neighborhood of  $\kappa = 1$ , we have

$$\begin{aligned} \Pi^t(\kappa) &\approx (c(e^{*1} + e^t, 0) + pe^t) - (c(e^{*1} - \frac{1-\kappa}{\kappa}e^t, 1) + \phi - p\frac{1-\kappa}{\kappa}e^t) = \\ &\quad (c(e^{*1} + e^t, 0) + pe^t) - (c(e^{*1}, 1) + \phi) = \\ &\quad H - B \end{aligned} \tag{18}$$

where

$$B \equiv c(e^{*1}, 0) - c(e^{*1} + e^t, 0) - pe^t \text{ and } H \equiv c(e^{*1}, 0) - c(e^{*1}, 1) - \phi$$

The term denoted  $B$  is the consumer surplus that the non-adopters obtain by buying permits from the adopters. The term denoted  $H$  (previously defined in equation (4)) is net benefit of adoption when all other firms adopt and trade is not permitted. In view of the continuity of all functions,  $\Pi^t(1) = H - B$ .

We now consider the other extreme, where almost no firms adopt, i.e. in the neighborhood of  $\kappa = 0$ . Denote the socially optimal level of permits when  $\kappa = 0$  as  $e^{*0} = e(0)$ . When there are very few sellers ( $\kappa \approx 0$ ) each buyer purchases an infinitesimal amount, so the price is determined by buyer's marginal cost. Thus,  $p(0) = -c_e(e^{*0}, 0)$  and an adopter is willing to sell  $s$  permits, a level that satisfies  $-c_e(e^{*1} - s, 1) = p(0)$ . In the neighborhood of  $\kappa = 0$  the benefit of adopting is

$$\begin{aligned} \Pi^t(\kappa) &\approx (c(e^{*0} + e^t, 0) + pe^t) - (c(e^{*0} - \frac{1-\kappa}{\kappa}e^t, 1) + \phi - p\frac{1-\kappa}{\kappa}e^t) = \\ &\quad c(e^{*0}, 0) - (c(e^{*0} - s, 1) + \phi - ps) = \\ &\quad F - E, \end{aligned} \tag{19}$$

---

<sup>1</sup>It may seem that the two boundaries,  $\kappa = 0$  and  $\kappa = 1$  are asymmetric, since we *can* mechanically use equation (15) to study the neighborhood of  $\kappa = 1$ , but not to study the neighborhood of  $\kappa = 0$ . This asymmetry is not real; for example, it is “reversed” if we write the payoff as a function of the measure of firms who do not invest,  $k = 1 - \kappa$ .

where

$$F \equiv ps + c(e^{*0}, 1) - c(e^{*0} - s, 1) \quad \text{and} \quad E \equiv c(e^{*0}, 1) + \phi - c(e^{*0}, 0).$$

The term  $F$  is the producer surplus that each adopter receives, when almost all firms do not adopt. The term  $E$  (previously defined in equation (3)) is the loss due to adoption when no other firms adopt. when trade is not permitted.

As a consequence of Remark 3 and equations (18) and (19) we have

**Remark 4** *With tradeable permits there is an interior (unique) equilibrium if and only if*

$$F - E > 0 > H - B. \quad (20)$$

There may be multiple (boundary) equilibria without trade, but a unique interior equilibrium with trade. This possibility requires that the private benefits that arise from trade, measured by the consumer and producer surplus ( $B$  and  $F$ ) are sufficiently large.

The following linear quadratic example shows that for some parameterizations there are two boundary equilibria without trade (equations (3) and (4) are both satisfied) and a unique interior equilibrium with trade (equation (17) is satisfied). We define  $\chi = c(0, 0) - c(0, 1) - \phi$ , the benefit of adopting the technology if emissions are restricted to 0. In order for adoption to be optimal in some circumstances, it must be the case that  $\chi > 0$ , as we hereafter assume.

**Example 1** *Let  $c(e, K) = f(K) - \left(a(K) - \frac{b(K)}{2}e\right)e$  with  $b(0) = b(1) \equiv b$  and  $D(e) = \frac{be^2}{2}$ . Here we have  $\chi = f(0) - f(1) - \phi$ . For this example,*

$$E = \frac{1}{2}a\frac{a-A}{b} - \chi, \quad H = \chi - \frac{1}{2}A\frac{a-A}{b}, \quad B = F = \frac{1}{2}\frac{(a-A)^2}{b}.$$

*Inequalities (3), (4) and (17) are all satisfied if and only if  $\frac{1}{2}a\frac{a-A}{b} > \chi > \frac{1}{2}A\frac{a-A}{b}$ . When trade in permits is not allowed, there are multiple boundary equilibria under circumstances where there would be a unique interior equilibrium with trade. These circumstances require that the cost of adoption is neither very large nor very small.*

### 2.2.3 A graphical treatment

Figure 3 shows the optimal levels of emissions if all firms or no firms adopt,  $e^1$  and  $e^0$  respectively. (The linearity and equal slopes in the figure are of no consequence for this graphical

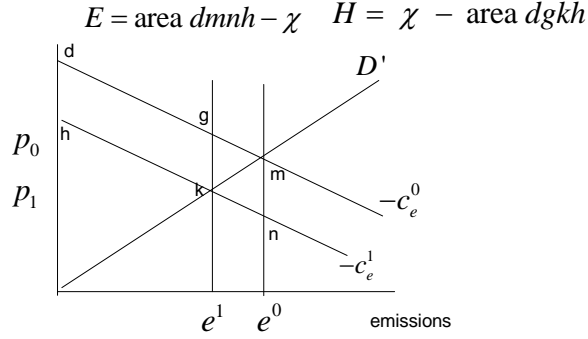


Figure 3: No trade in permits. Benefit of mimicking non-adopters =  $B$ ; Benefit of mimicking adopters =  $H$ .

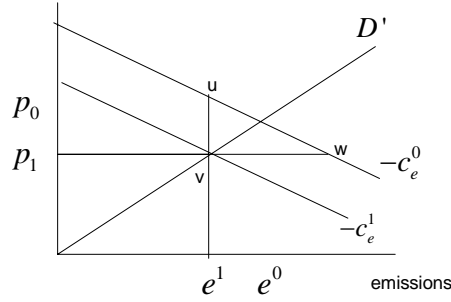


Figure 4: Permits are tradable,  $\kappa \approx 1$  Purchases per non-adopter =  $vw$ . Consumer surplus is  $B = \text{area } uvw$

analysis.) When permits are not tradeable, the advantage of not adopting if no other firms adopt is  $E = \text{area } dmnh - \chi$ ; the advantage of adopting when all other firms adopt is  $H = \chi - \text{area } dgkh$ .

When almost all firms adopt, any adopter is willing to sell a unit of permits for the price  $p_1$ . Figure 4 shows the level of purchases per non-adopter when almost all firms adopt ( $vw$ ) and the corresponding level of consumer surplus (area  $uvw$ ). When almost no firms adopt, any non-adopter is willing to buy a unit of permits at the price  $p_0$ . Figure 5 shows the level of sales per adopter when almost no firms adopt ( $xy$ ) and the corresponding level of producer surplus ( $xyz$ ).



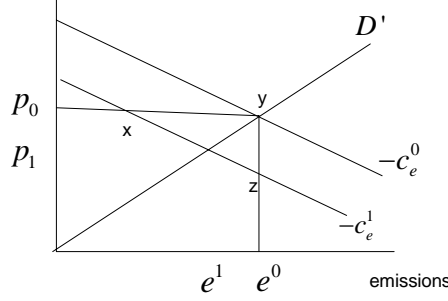


Figure 5: Permits are tradable;  $\kappa \approx 0$ . Sales per adopter =  $xy$ , producer surplus per adopter is

$$F = \text{area } xyz$$

In summary, there are two boundary equilibria in the absence of trade if  $\text{area } dmnh > \chi > \text{area } dgkh$ . There is a unique interior equilibrium with tradeable permits if  $\text{area } dmnh - \text{area } xyz < \chi < \text{area } dgkh + \text{area } uvw < 0$ .

### 2.3 Social optimality and the timing of actions

(i) When permits are tradeable, the outcome produces the social optimum regardless of whether the planner announces the quota before or after firms decide on investment. (ii) When permits are not tradeable, the outcome may depend on whether the planner announces the quota before or after firms decide on investment; in general the outcome is not socially optimal.

In order to establish the first claim, note that the social planner who can choose  $\kappa$  and  $e$ , allowing trade, wants to minimize environmental damages plus abatement and investment costs,  $W(e; \kappa) + \kappa\phi$ . (See equation (7).) The first order condition for this problem is

$$\frac{dW(e; \kappa)}{d\kappa} + \phi = -\Pi^t(\kappa) = 0. \quad (21)$$

The first order condition for an interior value of  $\kappa$  is identical to the condition for an interior competitive equilibrium. The second order condition for an interior equilibrium,  $\frac{d\Pi^t(\kappa)}{d\kappa} < 0$ , is identical to the condition that an interior competitive equilibrium is stable. The announcement  $e^*(\kappa^*)$  is time consistent, but it is not subgame perfect: if a positive measure of firms deviate from equilibrium, the planner wants to change the level of quotas.

To establish the second claim, consider the constrained social optimum, where there is no trade in permits. Suppose that this optimum requires an interior solution,  $0 < \kappa^* < 1$ ,

with  $e = e^*(\kappa^*)$ . The assumption that the solution is interior implies (using the first order condition to the planner's choice of  $k$ ) that  $\Pi(\kappa^*, e^*(\kappa^*)) = c(e^*, 0) - c(e^*, 1) - \phi = 0$ . In view of inequality (2)  $\kappa^*, e^*$  is unstable when the regulator chooses quotas conditional on the distribution of investment. A regulator who chooses quotas prior to the investment decision can announce  $e^*(\kappa^*)$ . At this value of the quota, firms are indifferent between investing and not investing, so any value  $\kappa \in [0, 1]$  is an equilibrium value to the investment game. Clearly, the announcement  $e^*(\kappa^*)$  is not subgame perfect, and for most outcomes it is not even time-consistent.

If the social optimum (when trade is allowed) involves a boundary equilibrium, it is easy to show that the competitive equilibrium reproduces the socially optimal outcome regardless of whether permits are tradeable. Not surprisingly, allowing trade in permits is significant only if the optimal outcome involves heterogeneous firms.

### 3 Investment as a coordination game

The last fifteen years has seen an explosion of literature on coordination games, but there have been few applications in environmental economics of the results from this literature. Here we sketch some of the insights from coordination games that are applicable to the problem of regulating emissions when investment is endogenous.

Our starting point is the observation that restricting emissions, without trade in permits, is a common policy instrument; in some cases it is reasonable to think that the regulator adjusts pollution targets in response to changing abatement costs. Therefore, it is at least worth considering the possibility that non-strategic firms' investment problem is a coordination game, with multiple equilibria. The multiplicity of equilibria has obvious implications for policy and welfare analysis. We discuss some of the possible resolutions of the question of multiplicity.

**Model 1 (Adapted from Matsuyama (1991).)** Suppose that each firm's capital wears out and must be replaced at random times; alternatively, a firm fails at a random time and is replaced by a new firm that chooses whether to incur the fixed cost to install the technology that reduces abatement costs. The environmental standard adjusts according to the fraction of firms that have installed the technology. If the failure rate is high, so that firms replace their capital at short intervals (or old firms are replaced by new firms at short intervals), the

model approximates a repeated version of our model in Section 2.1, and there are two boundary equilibria. However, if the failure rate is low, there is a unique equilibrium. In this case, firms that replace their capital have a dominant strategy; their optimal action depends on the fraction of firms that currently have low abatement costs. With a low failure rate, the equilibrium depends on market fundamentals, not beliefs about future actions. (Herrendorf, Valentinyi, and Waldman (2000) show that if firms are sufficiently heterogeneous before investment, there is a unique equilibrium to this model.)

**Model 2 (Adapted from Frankel and Pauzner (2000).)** Consider a variation of Model 1, in which the damage of emissions (equivalently, the regulator’s beliefs about the damage of emissions) changes randomly and exogenously; future damage parameters depend on the current damage parameter. In this case, the optimal level of emissions depends on the current fraction of firms that have the efficient technology, and also on (the regulator’s current belief about) the damage parameter. Suppose also that there are “dominance regions”: if the damage parameter exceeds a critical threshold, the regulator chooses such strict regulation that investment is optimal regardless of the level of  $\kappa$ ; if the damage parameter is lower than a (different) critical threshold, the regulator chooses such lax regulation that it is not optimal to make the investment, regardless of the level of  $\kappa$ . Using an argument of iterated deletion of dominated strategies, the equilibrium is again unique. Here the optimal investment decision depends on  $\kappa$  and the current value of the damage parameter, but not on beliefs about future investment decisions.

**Model 3 (Adapted from Morris and Shin (1998).)** Consider the two period model in Section 2.1, but suppose that the damage parameter,  $\delta$ , is unknown at the investment stage. Moreover, there are dominance regions; if the realization of  $\delta$  falls in a dominance region, the second period regulation is either so weak or so stringent that the optimal investment decision – had firms been able to predict the regulation – is independent of  $\kappa$ . Before making the investment decision firm  $i$  observes a *private* signal  $x_i$  that is correlated with  $\delta$ . In some examples, as we assume here, the signal is distributed uniformly,  $x_i \sim U(\delta - \epsilon, \delta + \epsilon)$ . For small  $\epsilon$ , a firm almost knows the truth about  $\delta$ , it almost knows what signals other firms received, it almost knows what other firms know about other firms, and so on. However, these higher order beliefs become less precise, as the order of the belief increases, so that there remains substantial “strategic uncertainty” (i.e. uncertainty about what other agents will do). For  $\epsilon = 0$

there is common knowledge and multiple equilibria, but for  $\epsilon$  arbitrarily small, there is “almost common knowledge” and a unique equilibrium. The unique equilibrium involves threshold strategies: each firm’s decision depends on whether its signal is greater or less than a threshold level.

**Model 4 (Adapted from Karp and Paul (2007).)**<sup>2</sup> Consider a variation of Model 1 above, but suppose that the environmental regulation adjusts with a lag. For example, if the conditional steady state environmental regulation (the optimal level of permits for a fixed value of  $k$ ) is  $e(\kappa)$ , and the actual level at time  $t$  is  $e_t$ , then the level adjusts according to  $\frac{de}{dt} = \alpha (e(\kappa_t) - e_t)$ ; here  $\alpha$  is a speed of adjustment parameter. As  $\alpha \rightarrow \infty$  adjustment becomes instantaneous, and we are back in Model 1. With instantaneous adjustment of  $e$ , we noted that multiplicity is likely to occur when firm turnover is rapid, because in this case we are close to a repeated version of the static model. However, if firm turnover is rapid but the regulation does not adjust instantaneously ( $\alpha$  is finite) multiplicity of equilibria is very unlikely. The explanation is that firms are able to choose their investment to track the changing level of regulation, so that they have a dominant strategy.

## 4 Conclusion

When a regulator chooses the level of non-tradeable emissions conditioned on the industry marginal cost, the firms’ investment decisions are typically strategic complements. In this case, firms play a coordination game (with non-atomic agents) at the investment stage, typically leading to multiple equilibria. The introduction of a market for tradeable permits fundamentally alters the investment game. With tradeable permits, the equilibrium is unique; it equals the first best outcome, regardless of whether the regulator announces the level of permits before or after firms invest.

The market for permits reduces the cost of achieving any target level of abatement, whenever firms are heterogenous. However, for a fixed distribution of investment, introducing a market for permits has an ambiguous effect on the equilibrium level of regulation.

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<sup>2</sup>This paper is based on an extension of Krugman (1991) which is similar – but much easier to work with – than Matsuyama’s model. Key results in the paper rely on numerical examples. Karp and Paul (2005) contains a simple two period version of this model that leads to analytic results.

Using previous results from coordination games, we sketched several modifications of the model that would eliminate multiplicity (without trade). For example, different firms might make their investment decision at different times; firms might be heterogenous prior to investment; the optimal level of regulation might adjust slowly, tracking changing beliefs about a damage parameter; firms might receive private signals about a payoff-relevant state, leading to strategic uncertainty.

## References

- BIGLAISER, G., J. HOROWITZ, AND J. QUIGGIN (1995): "Dynamic pollution regulation," *Journal of Regulatory Economics*, 8, 33–44.
- FISCHER, C., I. PARRY, AND W. PIZER (2003): "Instrument choice for environmental protection when technological innovation is endogenous," *Journal of Environmental Economics and Management*, 45, 523–45.
- FRANKEL, D., AND A. PAUZNER (2000): "Resolving Indeterminacy in Dynamic Settings: the Role of Shocks," *Quarterly Journal of Economics*, CXV, 285–305.
- HERRENDORF, B., A. VALENTINYI, AND R. WALDMAN (2000): "Ruling out Multiplicity and Indeterminacy: the Role of Heterogeneity," *Review of Economic Studies*, 67(2), 295–307.
- JAFFE, A., R. NEWELL, AND R. STAVINS (2003): "Technological change and the environment," in *Handbook of Environmental Economics, volume 1*, ed. by K.-G. Maler, and J. Vincent, pp. 461–516. North-Holland, Amstredam.
- KARP, L., AND T. PAUL (2005): "Intersectoral migration costs and multiple equilibria," Unpublished working paper; <http://are.Berkeley.EDU/karp/>.
- (2007): "Indeterminacy with Environmental and Labor Dynamics," Forthcoming, Structural Change and Economic Dynamics.
- KARP, L., AND J. ZHANG (2002): "Controlling a Stock Pollutant with Endogenous Abatement Capital and Asymmetric Information," Giannini Foundation of Agricultural Economics, Working Paper No. 928, University of California, Berkeley; <http://are.Berkeley.EDU/karp/>.
- KENNEDY, P., AND B. LAPLANTE (1999): "Environmental policy and time consistency: emission taxes and emission trading," in *Environmental Regulation and Market Power*, ed. by E. Petrakis, E. Sartzetakis, and A. Xepapadeas, pp. 116–144. Edward Elgar, Cheltenham.
- KRUGMAN, P. (1991): "History versus Expectations," *Quarterly Journal of Economics*, (106), 651–67.
- MATSUYAMA, K. (1991): "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium," *Quarterly Journal of Economics*, 106, 587–597.

- MILLIMAN, S., AND R. PRINCE (1989): “Firm incentives to promote technological change in pollution control,” *Journal of Environmental Economics and Management*, 17, 247–65.
- MOLEDINA, A., S. POLASKY, J. COGGINS, AND C. COSTELLO (2003): “Dynamic Environmental Policy with Strategic Firms: prices vs. quantities,” *Journal of Environmental Economics and Management*, 45.
- MONTERO, J. (2002): “Permits, standards, and technology innovation,” *Journal of Environmental Economics and Management*, 44.
- MORRIS, S., AND H. SHIN (1998): “Unique Equilibrium in a Model of Self-Fulfilling Expectation,” *American Economic Review*, 88, 587–597.
- REQUATE, T. (1998): “Incentives to innovate under emissions taxes and tradeable permits,” *European Journal of Political Economy*, 14, 139–65.
- TARUI, N., AND S. POLASKY (2005): “Environmental regulation with technology adoption, learning and strategic behavior,” *Journal of Environmental Economics and Management*, 50, 447–67.
- (2006): “Environmental regulation in a dynamic model with uncertainty and investment,” Presented at NBER conference.

## Appendix: Details of derivations

**Proof of Remark 2** We define

$$\tilde{G}(e, \kappa, s) \equiv - \left( (1 - \kappa) c_e(e + s, 0) + \kappa c_e \left( e - \frac{(1 - \kappa) s}{\kappa}, 1 \right) \right),$$

the social marginal benefit of emissions given  $e, \kappa$  and permit purchases of level  $s$  (per non-investing firm). Using this definition,  $\tilde{G}(e, \kappa, s) = G(e, \kappa, \text{trade})$  for  $s = e^t(e, \kappa)$ , and  $\tilde{G}(e, \kappa, s) = G(e, \kappa, \text{no trade})$  for  $s = 0$ . Thus,

$$G(e; \kappa, \text{no trade}) - G(e; \kappa, \text{trade}) = \tilde{G}(e, \kappa, 0) - \tilde{G}(e, \kappa, e^t) = \quad (22)$$

$$- \int_0^{e^t} \frac{\partial \tilde{G}(e, \kappa, s)}{\partial s} ds = (1 - \kappa) \int_0^{e^t} \left( c_{ee}(e + s, 0) - c_{ee} \left( e - \frac{(1 - \kappa)s}{\kappa}, 1 \right) \right) ds.$$

If the integrand in equation (22) is positive (respectively, negative), then trade causes the social marginal benefit of emissions curve in Figure 1 to shift down to curve  $A$  (respectively, shift up to curve  $B$ ) leading to a reduction (respectively, increase) in emissions.

**Derivation of equation (12) (the effect of  $k$  on the equilibrium level of emissions with trade):** We begin by showing how  $\kappa$  affects the volume of trade for given  $e$ . Differentiating equation (5) with respect to  $e^t$  and  $\kappa$ , holding  $e$  fixed, implies<sup>3</sup>

$$\frac{\partial e^t}{\partial \kappa} = \frac{c_{ee}^1}{\kappa c_{ee}^0 + (1 - \kappa) c_{ee}^1} \left( \frac{e^t}{\kappa} \right) > 0. \quad (23)$$

If there are more adopters (larger  $\kappa$ ) then each non-adopter buys more permits, holding fixed the aggregate supply of permits,  $e$ . Hereafter we use the definition of  $\Delta = \Delta(k, e, e^t)$  from equation (10), i.e. we set  $s = e^t$ . Differentiating the planner's first order condition, equation (8) implies

$$\begin{aligned} \frac{de}{d\kappa} &= \frac{- \left[ (1 - \kappa) \Delta \frac{\partial e^t}{\partial \kappa} - (c_e^0 - c_e^1) + \frac{e^t}{\kappa} c_{ee}^1 \right]}{S^t} \\ &= \frac{- \left[ (1 - \kappa) \Delta \frac{\partial e^t}{\partial \kappa} + \frac{e^t}{\kappa} c_{ee}^1 \right]}{S^t} \end{aligned}$$

The second equality uses equation (5). Using equation (23) to eliminate  $\frac{\partial e^t}{\partial \kappa}$  and simplifying produces equation (12).

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<sup>3</sup>Recall the meaning of superscripts. These indicate that the function is evaluated at arguments corresponding to the type of firm (non-investor or investor). For example  $c_{ee}^1 = c_{ee} \left( e - \frac{(1 - \kappa)e^t}{\kappa}, 1 \right)$ .



### Derivation of equation (13) (the effect of $k$ on equilibrium purchases per non-adopter)

We begin by totally differentiating equation (5), again setting  $s = e^t$  and using the definition of  $\Delta(k, e, e^t)$  from equation (10).

$$\begin{aligned} \frac{de^t}{d\kappa} &= \frac{-\left(\Delta \frac{de}{d\kappa} - c_{ee}^1 \frac{e^t}{\kappa^2}\right)}{c_{ee}^0 + c_{ee}^1 \frac{1-\kappa}{\kappa}} \\ &= \frac{\left(\Delta \frac{c_{ee}^0 c_{ee}^1}{\kappa s^t} \left(\frac{e^t}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1}\right) + c_{ee}^1 \frac{e^t}{\kappa^2}\right)}{c_{ee}^0 + c_{ee}^1 \frac{1-\kappa}{\kappa}} \end{aligned}$$

The second equality uses equation (12). We obtain equation (13) from simplification.

**The effect of investment on the equilibrium price of permits** We begin with an intermediate result. Differentiating equation (5) (holding  $\kappa$  fixed) implies

$$\frac{de^t}{de} = -\frac{\kappa \Delta}{\kappa c_{ee}^0 + (1-\kappa) c_{ee}^1}.$$

Substituting this result into the expression for  $c_{ee}^0 - S^t$  yields

$$\begin{aligned} c_{ee}^0 - S^t &= \Delta \left( \kappa - (1-\kappa) \frac{de^t}{de} \right) - D'' = \\ &\Delta \left( \kappa + (1-\kappa) \frac{\kappa \Delta}{\kappa c_{ee}^0 + (1-\kappa) c_{ee}^1} \right) - D'' \end{aligned} \tag{24}$$

With slight abuse of notation, we write  $p = p(\kappa) = p(e(\kappa), \kappa)$ . Totally differentiating equation (6) and using equations (12), (13), and (24) implies

$$\begin{aligned}
\frac{dp}{d\kappa} &= -c_{ee}^0 \left( \frac{de}{d\kappa} + \frac{de^t}{d\kappa} \right) = \\
&-c_{ee}^0 \left( -\frac{c_{ee}^0 c_{ee}^1}{\kappa s^t} \left( \frac{e^t}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) + \frac{\left( \left( \frac{\Delta}{s^t} \left( \frac{c_{ee}^0}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) + \frac{1}{\kappa} \right) \right)}{(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} c_{ee}^1 e^t \right) = \\
&\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \left( \frac{c_{ee}^0}{\kappa} - \left( \Delta \left( \frac{c_{ee}^0}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) + \frac{s^t}{\kappa} \right) \right) = \\
&\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \left( \frac{1}{\kappa} \left[ \Delta \left( \kappa + (1-\kappa) \frac{\kappa \Delta}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) - D'' \right] - \Delta \left( \frac{c_{ee}^0}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) \right) = \quad (25) \\
&\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \left( \Delta \left( 1 + (1-\kappa) \frac{\Delta}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} - \frac{c_{ee}^0}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) - \frac{D''}{\kappa} \right) = \\
&\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \left( \Delta \left( 1 - \frac{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1}{\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1} \right) - \frac{D''}{\kappa} \right) = \\
&-\frac{c_{ee}^0 c_{ee}^1 e^t}{S^t(\kappa c_{ee}^0 + (1-\kappa)c_{ee}^1)} \frac{D''}{\kappa} < 0
\end{aligned}$$