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AJAE appendix for “Risk, Wealth and Sectoral Choice in Rural Credit Markets”

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Note: The material contained herein is supplementary to the article named in the title and published in the *American Journal of Agricultural Economics* (AJAE)

Proof of Proposition 1

Assume $P > 3A$, where P and A are the coefficients of absolute prudence and risk aversion respectively. Consider two wealth levels $W^0 > W^1$. The analysis of the problem in the formal sector implies that, for any given level of monitoring the certainty equivalent of the optimal contract is increasing in wealth, so that $CE(W^1, m^*(W^0)) > CE(W^0, m^*(W^0))$. By definition, $m^*(W^1)$ is the optimal monitoring level for W^1 , therefore $CE(W^1, m^*(W^1)) > CE(W^1, m^*(W^0))$. Combining these two inequalities gives: $CE(W^1, m^*(W^1)) > CE(W^0, m^*(W^0))$, so that the certainty equivalent of the optimal informal contract is increasing in wealth. A similar proof holds when $P < 3A$.

Proof of Proposition 2

Restrict attention to CRRA and assume $P > 3A$. Consider the agent with wealth \hat{W}_{FI} who is indifferent between the best available contracts in the two sectors. We will show here that $V'_I(\hat{W}_{FI}) < V'_F(\hat{W}_{FI})$. Since $V_F(W)$ and $V_I(W)$ are continuous, increasing functions of wealth, the strict inequality implies that agents with wealth greater than \hat{W}_{FI} would prefer the informal contract while those with wealth less than \hat{W}_{FI} would prefer the formal contract. Using the envelope theorem and the first order conditions from the optimization problem in each sector, it is straightforward to show:

$$V'_F(W) = \frac{u'(W + R_b^{F*})u'(W + R_g^{F*})}{p^H u'(W + R_b^{F*}) + (1 - p^H)u'(W + R_g^{F*})} \text{ and } V'_I(W) = \frac{u'(W + R_b^{I*})u'(W + R_g^{I*})}{p^H u'(W + R_b^{I*}) + (1 - p^H)u'(W + R_g^{I*})}.$$

The agent with wealth \hat{W}_{FI} is indifferent between loan sectors so that

$Eu(\hat{W}_{FI} + R_j^{F*}) = Eu(\hat{W}_{FI} + R_j^{I*})$. Furthermore, we know that $E(R_j^{F*}) > E(R_j^{I*})$ and the

incentive compatibility requires that $R_g^* > R_b^*$ in both sectors. The optimal informal

contract must then have a smaller spread in returns across states: $R_b^{F*} < R_b^{I*}$ and

$R_g^{F*} > R_g^{I*}$. Consequently $u(\hat{W}_{FI} + R_b^{F*}) < u(\hat{W}_{FI} + R_b^{I*})$ and

$u(\hat{W}_{FI} + R_g^{F*}) > u(\hat{W}_{FI} + R_g^{I*})$. Now consider the following two lotteries, L^I and L^F . L^I

yields payoffs $(u(\hat{W}_{FI} + R_g^{F*}), u(\hat{W}_{FI} + R_b^{F*}))$ with accompanying probabilities

$(p^H, 1 - p^H)$ and L^F yields payoffs $(u(\hat{W}_{FI} + R_g^{I*}), u(\hat{W}_{FI} + R_b^{I*}))$ with the same

probabilities. L^F is a mean preserving spread of L^I so that any risk averse agent would strictly prefer L^I to L^F . It is easy to show that if $P > 3A$, any agent with utility function

$\frac{1}{u'}$ is more risk averse than an agent with utility function u . Thus there exists a strictly

concave and increasing function Φ such that $\frac{1}{u'} = \Phi(u)$. Since Φ is increasing and

concave, it can be thought of as the utility function of a risk averse agent. This agent

would prefer L^I to L^F so that:
$$\frac{p^H}{u'(\hat{W}_{FI} + R_g^{I*})} + \frac{1 - p^H}{u'(\hat{W}_{FI} + R_b^{I*})} > \frac{p^H}{u'(\hat{W}_{FI} + R_g^{F*})} + \frac{1 - p^H}{u'(\hat{W}_{FI} + R_b^{F*})},$$

inverting both sides of this inequality yields:

$$V'_I(\hat{W}_{FI}) = \frac{u'(W + R_b^{I*})u'(W + R_g^{I*})}{p^H u'(W + R_b^{I*}) + (1 - p^H)u'(W + R_g^{I*})} < \frac{u'(W + R_b^{F*})u'(W + R_g^{F*})}{p^H u'(W + R_b^{F*}) + (1 - p^H)u'(W + R_g^{F*})} = V'_F(\hat{W}_{FI})$$

Model with multiple farm size

This discussion shows that when we extend the basic model to allow for continuous farm size, $\rho < 0.5$ is sufficient but not necessary for risk rationing of the land poor in the

formal sector where ρ is the coefficient of relative risk aversion. Consider an agent who is indifferent between the reservation activity and her optimal informal contract so that:

$V_F(W, T) = U(W + T\gamma, H, 0)$. The impact of a marginal change in her land endowment on

activity choice depends on the relative size of $\frac{\partial V_F(W, T)}{\partial T}$ and $\gamma u'(W + T\gamma)$. Using the

envelope theorem and the first order conditions from the formal sector optimization

problem, it is straightforward to show that

$$\frac{\partial V_F(W, T)}{\partial T} = ((1 - p^H)R_b^* + p^H R_g^*) \frac{u'(W + TR_b^*)u'(W + TR_g^*)}{p^H u'(W + TR_b^*) + (1 - p^H)u'(W + TR_g^*)}. \text{ As discussed in the paper,}$$

$P < 3A$ implies that risk rationing is biased against the relatively rich, or equivalently:

$$P < 3A \Leftrightarrow \frac{u'(W + TR_b^*)u'(W + TR_g^*)}{p^H u'(W + TR_b^*) + (1 - p^H)u'(W + TR_g^*)} < u'(W + T\gamma). \text{ Furthermore, the expected return}$$

per-hectare from the risky contract must be higher than the certain reservation income

per-hectare (i.e., the rental rate) for a risk averse agent to be indifferent between the two

activities so that: $(1 - p^H)R_b^* + p^H R_g^* > \gamma$. Consequently, if the difference in expected returns

from farming and renting out is large enough, then even under $\rho > 0.5$, we can obtain

$$\frac{\partial V_F}{\partial T}(W, T) > \gamma u'(W + T\gamma), \text{ so that the land poor are risk rationed while the land rich}$$

undertake the project with a formal loan. Without additional assumption, we can only say

that the threshold degree of relative risk aversion implying the intuitive result of the risk

rationing of the land poor is greater than 0.5. Furthermore – holding other parameters

constant – a larger difference between the rental rate and the expected return from the

contract increases this threshold.