The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

No endorsement of AgEcon Search or its fundraising activities by the author(s) of the following work or their employer(s) is intended or implied.

AJAE appendix for 'Risk rationing and wealth effects in credit markets: Theory and implications for agriculture development'

Stephen R. Boucher
Agricultural and Resource Economics
UC-Davis
boucher@primal.ucdavis.edu

Michael R. Carter
Agricultural and Applied Economics
UW-Madison
mrcarter@wisc.edu

Catherine Guirkinger
Center for Research on Economic Development (CRED)
Department of Economics
University of Namur
catherine.guirkinger@fundp.ac.be

August 29, 2007

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).

AJAE appendix for 'Risk Rationing and Wealth Effects in Credit Markets: Theory and Implications for Agricultural Development'

Proof of Proposition 1

The proof consists of three steps. First, we describe $W^*(T)$ and show that it is the minimum collateral requirement necessary for a non-empty feasible set so that quantity rationing is biased against the financially poor. Second, we show that increases in productive (land) wealth have the same qualitative effect – namely quantity rationing, if it occurs, will affect the productive wealth poor. Finally, we show that equation 7 identified in Proposition 1 is necessary and sufficient for the existence of the marginally quantity rationed agent, $W^*(T)$, within the relevant range of financial wealth.

Define the following payoffs to the agent in the good state:

$$\text{(1a)} \qquad s_g^{min}(W;T) \quad : \quad [u(W+(s_g^{min}+p_T)T)-u(0)](\phi^H-\phi^L)=d(H)-d(L)$$

$$\text{(1b)} \qquad s_g^{max}(W) \quad : \quad \phi^H(x_g-s_g^{max})+(1-\phi^H)(x_b+W/T+p_T)=rk$$

s_g^{min} is minimum incentive compatible payoff in the good state when the agent posts her full wealth as collateral (i.e., when $s_b=-W/T-p_T$). Similarly, s_g^{max} is the payoff to the agent in the good state such that the lender just breaks even when the agent again posts maximum collateral. Holding s_b at $-W/T-p_T$, any contract with $s_g<s_g^{min}(W;T)$ would violate the ICC; while any contract with $s_g>s_g^{max}(W;T)$ would violate the LPC. Thus feasible, full collateral contracts require $s_g^{min}(W;T)\le s_g\le s_g^{max}(W;T)$. Now let $W^*(T)$ be the financial wealth level such that the LPC, ICC, and the agent's wealth constraint all bind and consider a marginal increase in W. From equation 1a, $\frac{\partial s_g^{min}}{\partial W}=-1/T<0$ and from equation 1b, $\frac{\partial s_g^{\max}}{\partial W}=\frac{1-\phi^H}{\phi^H}\frac{1}{T}>0$, so that if $W^*(T)$ exists, then any agent with productive wealth T and financial wealth $W<W^*(T)$ will have an

empty feasible contract set and will be quantity rationed. Agents with $W > W^*(T)$ will have access to some contracts and will not be quantity rationed.

Now return to our marginally quantity rationed agent, $W = W^*(T)$ and consider an increase in productive wealth, T. Again, using equations 1a and 1b we have: $\frac{\partial s_g^{min}}{\partial T} = -(s_g^{min} + p_T)/T < 0$ and $\frac{\partial s_g^{\max}}{\partial T} = -\left(\frac{1-\phi^H}{\phi^H}\right)\frac{W}{T^2} < 0$. Since both the lower and upper bounds of the success payoff decrease, an increase in productive wealth will imply a non-empty feasible contract iff $\left|\frac{\partial s_g^{min}}{\partial T}\right| > \left|\frac{\partial s_g^{\max}}{\partial T}\right|$ or, equivalently, $s_g^{min} > \left(\frac{1-\phi^H}{\phi^H}\right)\frac{W}{T} - p_T$. Next, rewrite equation 1b as:

$$s_g^{max}(W^*(T);T)) = \frac{\overline{x} - rk}{\phi^H} + \frac{1-\phi^H}{\phi^H}\left(\frac{W^*}{T} + p_T\right). \tag{2}$$

Since, by definition, the agent with $W^*(T)$ has a single contract available, we know that $s_g^{min}(W^*(T);T) = s_g^{max}(W^*(T);T)$ so that:

$$s_g^{min}(W^*(T);T) = s_g^{max}(W^*(T);T) > \left(\frac{1-\phi^H}{\phi^H}\right)\frac{W^*}{T} - p_T$$

which is necessary and sufficient for agents with greater productive wealth to have a non-empty feasible contract set while agents with less productive wealth will be quantity rationed.

Finally, we take up the existence of W^*. From the above argument, it is clear that if the poorest agent is *not* quantity rationed, then no agent will be quantity rationed. To demonstrate the existence (and uniqueness) of W^* we need to find a condition such that the poorest agent is quantity rationed. Given the discussion above, this condition holds if and only if the following inequality holds:

$$s_g^{min}(\underline{W};T) > s_g^{max}(\underline{W};T) \tag{3}$$

Using the definition of $s_g^{\max}$ given by equation 1b and the fact that, holding s_b at $-W/T - p_T$, any contract with $s_g < s_g^{min}(W;T)$ would violate the ICC, it is easy to show that inequality 3 is equivalent to:

$$u\left(\frac{T(\overline{x}^H - rk) + \underline{W} + p_T T}{\phi^H}\right) < \frac{d(H) - d(L)}{\phi^H - \phi^L} + u(0)$$

which is the necessary and sufficient condition in the proposition.

Proof of Proposition 2

To prove Proposition 2, we need to show that if $P > 3A$ then agents with financial wealth greater than $\widehat{W}$ will strictly prefer the entrepreneurial activity financed with their optimal contract, while agents with financial wealth less than $\widehat{W}$ will prefer the low return, safe reservation activity, or $\Delta^W(\widehat{W};T) > 0$, where $\Delta^W(\widehat{W};T)$ is defined by equation 10 of the text. Productive wealth is held constant and for notational simplicity, we set $T = 1$ and define $V'(W) = V_W\ (W;T)$. We now derive an expression for $V'(W)$.

The Lagrangian of the contract design problem is:

$$\begin{aligned}(4)\qquad \pounds(W,\lambda,\mu) \;&=\; EU(W + p_T + s_j, H)\\ &-\lambda\left\{d(H) - d(L) - [u(W + p_T + s_g) - u(W + p_T + s_b)]\,(\phi^H - \phi^L)\right\}\\ &-\mu[-\phi^H(X_g - s_g) - (1-\phi^H)(X_b - s_b) + rk]\end{aligned}$$

where λ and μ are the multipliers associated with the incentive compatibility and participation constraints. Applying the envelope theorem yields:

$$\begin{aligned}(5)\qquad V'(W) \;&=\; \phi^H u'(W + p_T + s_g^*) + (1-\phi^H)u'(W + p_{Tt} + s_b^*)\\ &+\lambda^*(\phi^H - \phi^L)[u'(W + p_T + s_g^*) - u'(W + p_T + s_b^*)]\end{aligned}$$

Both the lender's participation and the incentive compatibility constraints are binding at the optimum so that λ^* and μ^* are strictly positive, and the first order necessary conditions for an optimum are:

$$\text{(6a)} \quad \frac{\partial \pounds}{\partial S_g} = \phi^H u'(W + p_T + s_g^*) + \lambda^*(\phi^H - \phi^L)u'(W + p_T + s_g^*) - \mu^* \phi^H = 0$$

$$\text{(6b)} \quad \frac{\partial \pounds}{\partial S_b} = (1 - \phi^H)u'(W + p_T + s_b^*) - \lambda^*(\phi^H - \phi^L)u'(W + p_T + s_b^*) - \mu^*(1 - \phi^H) = 0$$

Solving equations 6a and 6b for λ^* yields:

$$\text{(7)} \qquad \lambda^* = \frac{\phi^H(1 - \phi^H)[u'(W + p_T + s_b^*) - u'(W + p_T + s_g^*)]}{(\phi^H - \phi^L)[\phi^H u'(W + p_T + s_b^*) + (1 - \phi^H)u'(W + p_T + s_g^*)]}$$

Substituting for λ^* in equation 5 and simplifying yields:

$$\text{(8)} \qquad V'(W) = \frac{u'(W + p_T + s_b^*)u'(W + p_T + s_g^*)}{\phi^H u'(W + p_T + s_b^*) + (1 - \phi^H)u'(W + p_T + s_g^*)}.$$

Thus $\Delta^W(\widehat{W}; 1) > 0$ is equivalent to:

$$\text{(9)} \qquad \frac{u'(\widehat{W} + p_T + s_b^*)u'(\widehat{W} + p_T + s_g^*)}{\phi^H u'(\widehat{W} + p_T + s_b^*) + (1 - \phi^H)u'(\widehat{W} + p_T + s_g^*)} > u'(\tau + p_T + \omega + \widehat{W})$$

Next, assume the utility function $\frac{1}{u'(.)}$ exhibits greater absolute risk aversion than $u(.)$. By definition of the indifferent agent:

$$\text{(10)} \qquad u(\tau + p_T + \omega + \widehat{W}) = \phi^H u(\widehat{W} + p_T + s_g^*) + (1 - \phi^H)u(\widehat{W} + p_T + s_b^*)$$

If presented with the same contract, $(s_g^*(\widehat{W}), s_b^*(\widehat{W}))$, an agent with the same wealth, but with utility function $\frac{1}{u'(.)}$ would strictly prefer the certainty equivalent:

$$\text{(11)} \qquad \frac{1}{u'(\tau + p_T + \omega + \widehat{W})} > \frac{\phi^H}{u'(\widehat{W} + p_T + s_g^*)} + \frac{1 - \phi^H}{u'(\widehat{W} + p_T + s_b^*)}$$

Inverting both sides of this inequality yields the inequality in equation 9.

The final step is to demonstrate that $P > 3A$ is equivalent to an agent with utility function $\frac{1}{u'(.)}$ being more risk averse than an agent with utility u. Using the definition of the coefficient of absolute risk aversion, $\frac{1}{u'(.)}$ is more risk averse than u if and only if:

$$\text{(12)} \qquad -\frac{(\frac{1}{u'})''}{(\frac{1}{u'})'} > -\frac{u''}{u'} \leftrightarrow P > 3A$$

Following similar steps it can be shown that an agent with utility function $\frac{1}{u'(.)}$ is *less* risk averse than an agent with utility u if and only if $P < 3A$.

To summarize, we have shown that, if an indifferent agent exists, and provided $P > 3A$, any financially poorer agent would strictly prefer the certain reservation activity to the risky activity with her optimal credit contract so that - under this preference condition - risk rationing is biased against the poor. Note that since both the value function, V, and the agent's reservation utility are monotonically increasing in W, this result is a global result so that the indifferent agent, if she exists, is unique. To see this, assume that two indifferent agents – and thus two crossing points – exist. Monotonicity implies that the relative magnitudes of the slopes of the two functions would be inverted at the two crossing points. This cannot occur, however, since we have just seen that at a crossing point the value function is steeper than reservation utility. A symmetric proof can be used to show that $P < 3A$ implies that risk rationing is biased against richer agents.