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Optimal Length of Leys in an Area with Winter Damage Problems – Optimal Economic Lengths of Leys

Agnar Hegrenes, Gudbrand Lien, and J. Brian Hardaker

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Optimal economic length of leys

Gudbrand Lien, Agnar Hegrenes
Norwegian Agricultural Economics Research Institute,
Box 8024 Dep, 0030 Oslo, Norway
e-mail: gudbrand.lien@nilf.no

J. Brian Hardaker
Graduate School of Agricultural and Resource Economics, University of New England,
Armidale, NSW 2351, Australia

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Abstract

The optimal economic life cycle of grass leys with winter damage problems in northern Norway and the threshold of winter damage before it is profitable to re-seed grasses are investigated. The loss in profit of a sub-optimal strategy compared to an optimal strategy is briefly discussed. An infinite horizon stochastic dynamic programming model including a normally distributed yield process with possibilities for discrete downward jumps is developed. The jump process reflects the sudden drop in production after winter damage. Normally the yield of ley increases the first few years after a year with downward jump, and this dynamic is included in the model. The transition probabilities used in the model are estimated with Monte Carlo simulation.

Our result show that, in the case of winter damage of 50% or more compared to fields without winter damage, it is optimal to replace the ley immediately. If the winter damage is limited to 20-40% of the yield of fields without winter damage, the replacement decision depends on the age of the ley and the current yield level. It does not always pay to replace immediately mildly winter damaged fields that are still producing high yields. It does not pay to replace a ley if there has been no winter damage since establishment and if the relative yield level is 'satisfactory' at least until the first episode of winter damage occurs.

Introduction

Grass production is the main agricultural land use in many parts of Norway. In northern Norway, i.e., the three counties of Nordland, Troms and Finnmark, as much as 93 per cent of agricultural land in use was under grass in 1999 (Statistics Norway, 2000). The profitability of grassland depends strongly on choice of the appropriate length of ley periods (Hegrenes, 1991). Winter damage of the ley is one reason why the grass leys have to be re-seeded. In the years 1975, 1978, 1985, 1995, and 1998 the leys was severely damaged on many farms in northern Norway. Andersen (1960) reported a relatively high frequency of winter damage in some regions in northern Norway in the period 1922-59. Clearly, winter damage to grassland is a significant hazard in this area.

Young grass leys normally have higher yields than older leys (Nesheim, 1986). Therefore, even under 'normal' as well as under winter damage conditions, it might be beneficial to plough the fields and re-seed grasses. In this article we examine the replacement of leys from the farmer's point of view. Two main aspects are investigated: (1) What is the optimal replacement frequency when the probability of winter damage is accounted for? (2) How damaged should the ley be before it is profitable to re-seed it?

Deciding whether to re-establish grass leys is a typical replacement problem. Such problems are effectively handled by dynamic programming (for surveys see, e.g., Kennedy, 1986; Taylor, 1993). A farmer who cultivates grassland is always faced with uncertainty. For instance, the farmer will not know for sure what yields he will obtain in future. This uncertainty is accounted for using stochastic dynamic programming (SDP). Since we have sparse yield data, the transition probabilities used in the model are estimated using Monte Carlo simulation.

This article is structured as follows. First, a brief description of the problem is given. Then the dynamic programming model, where the transformation functions and the transformation probabilities estimated with Monte Carlo simulation are presented. Further, the empirical results are presented, and finally the last section contains some concluding comments.

Description of the problem

We assume a relatively flat area is sown with a grass ley on soil and under climatic conditions that lead to an unstable yield. In cold periods of the year when there is no snow cover, the soil freezes. In coastal areas the temperature often varies, and after a cold period there might be a milder period with rain. Frozen soil has low water permeability, so the water will not soak into the soil. If the mild, rainy period is followed by cold weather, the surface water freezes. An ice cover might damage the ley, and the yields can suddenly drop considerably between years because of winter damage.

The magnitude of the yield drop depends on the extent of the winter damage. If the winter damage is small, perhaps the farmer should take no action. The costs of re-seeding leys after minor winter damage may exceed the benefits of potentially higher yields in subsequent years. On the other hand, if there has been considerable winter damage, re-seeding normally is necessary. The years, especially the first, following the year with winter damage of the ley, normally show an increasing yield level (Haraldsen *et al*., 1995). Decision strategies to keep or replace the ley after a year with winter damage should account for this consideration.

The sharp yield drops caused by winter damage are in addition to 'normal' yield uncertainty. Moreover, young grass leys normally have higher yields than older leys (though not in the re-seeding year). Results given by Nesheim (1986) indicate higher yields from grass leys with an age of one to five years compared with older leys. This is also in accordance with

results from the 'Grassland Survey in Norway' described in Haraldsen and Waag (1991). The decision of whether or not to re-seed therefore also depends on years since last re-seeding.

According to the Norwegian Farm Business Survey (NILF, 1994-2000a), in the period 1993 to 1999 in northern Norway the standard deviation of grass ley yields was 705 kilogram (kg) dry matter (DM) hectare^{-1} (ha) year^{-1} within farms. Results from this survey and earlier investigations (Stalleland, 1990; Hegrenes and Lien, 1999) indicate about the same standard deviation independent of yield level.

The length of the ley period has a strong influence on the profitability of the grassland. The optimal replacement of leys in northern Norway is investigated in this article. The decision involves selection of an optimal rotation strategy for grass leys that maximises expected present value of income over time. The decisions to replace or keep the ley are assumed to continue sufficiently far into the future to justify an assumption of an infinite time horizon. SDP problems with infinite time horizons can often be solved more easily than those with finite horizons, and are likely to give reliable results for real problems with time horizons of several years. The requirement for modelling with infinite horizon is that the problem must be stationary. A problem is stationary if the return and transformation functions are the same for all decision stages (Kennedy, 1986). An assumption of unchanging production relationships between decision stages may be reasonable for grassland from which an annual harvest is taken. An assumption of constant prices may be less satisfactory for an infinite planning horizon, but is assumed in this article for simplicity. An assumption of, say, declining product prices over time may have given less frequently replacement of leys than results estimated in this article.

At each stage or year, the farmer must decide whether to replace or keep the existing grass ley. The decision depends on yield level, grass quality and coarse fodder price, re-seeding cost, loss of production during the establishment phase, fieldwork and other variable

costs, etc. The yield level and grass quality depends on years since re-seeding and weather and soil conditions, among other things (Hopkins, 2000: 102).

Farmers, like all decision-makers, are faced with uncertainty. All of the above-mentioned factors affecting ley replacement decisions are in practice uncertain and so ideally should be modelled as stochastic variables in the model. For simplicity we express only yield, assumed to be the main uncertainty in the evaluation, in stochastic terms. We assume a truncated[1] normal distribution around the trend value of mean yield according to the years since last re-seeding. In addition to this dispersion, we also assume that in years with winter damage the yields suddenly drop significantly, and the first subsequent years the ley show an increasing yield trend. Such drops with following increase in yield are accounted for with five possible discrete downward jump processes.

The dynamic programming model

Dynamic programming is an optimisation technique to find best sequence of decisions over time or decision stages. Given that we assume a perfect capital market and a linear utility function (i.e. risk-neutrality), our problem can be represented by the general recurrence relation (Bellman, 1957):

$$V_t(x_t) = \max_{u_t} [r_t(x_t, u_t) + \beta V_{t+1}(\tau_t(x_t, u_t))] \qquad (t=T, \ldots, 1) \tag{1}$$

where:

$V_t(x_t)$ = maximal value of the objective function during the remainder of the planning horizon under optimal decisions given the state vector x_t,

$r_t(x_t, u_t)$ = stage return function: given the state vector x_t and the control vector u_t. This function measures the payoff earned at that stage, and is a function of the

[1] A lower limit truncation at zero is used in the normal distribution to avoid negative yields.

current state and decision made,

$\tau_t(x_t, u_t)$ = transformation function: given the state vector x_t and the control vector u_t at stage t the system moves in stage t+1 to state $x_{t+1} = \tau_t(x_t, u_t)$,

β = the discount factor. The interest is assumed to be 5 per cent p.a., which is approximately equal to the average over the past 20 years of the real interest rate on 10-year Government bonds,

T = length of the planning horizon. The planning horizon can also be infinite (∞).

We define the vector of state variables, x_t, to comprise: grassland yield, $y_t(c)$, at ley age (years since re-seeding) c in stage t; the price of coarse fodder per kg DM, $pr_t(c)$, at ley age c in stage t; variable costs and re-seeding costs (only if $c = 0$), $w_t(c)$, at ley age c in stage t.

We assume stochastic yield, $\tilde{y}_t(c)$ ha^{-1} year^{-1}, with associated probabilities, $P_t(\tilde{y}_t(c))$. The variable costs partly vary with the uncertain yield level. The recursive equation in the stochastic case with the state variables described above has the following form:

$$V_t(\tilde{y}_t(c), pr_t(c), w_t(c)) =$$

$$\max_{u_t} \left\{ \sum_{\tilde{y}=1}^{m} P_t(\tilde{y}_t(c)) [r_t(\tilde{y}_t(c), pr_t(c), w_t(c), u_t) + \beta V_{t+1}(\tau_t(\tilde{y}_t(c), pr_t(c), w_t(c), u_t))] \right\} \quad (2)$$

where:

$V_t(\tilde{y}_t(c), pr_t(c), w_t(c))$ = expected value of the objective function during the remainder of the planning horizon under optimal decisions given the current state, $(\tilde{y}_t(c), pr_t(c), w_t(c))$,

$r_t(\cdot)$ = function describing the expected impact of specific state and

control variables at stage t on the stage contribution to the objective function,

$\tau(\cdot)$ = transformation function: given the state and control level in stage t and realisation $\tilde{y}_t(c)$ of the stochastic variable, this function directs the system from $(\tilde{y}_t(c), pr_t(c), w_t(c))$ in stage t to state $(\tilde{y}_{t+1}(c), pr_{t+1}(c), w_{t+1}(c)) = \tau_t(\tilde{y}_t(c), pr_t(c), w_t(c))$ in stage $t+1$.

The other symbols have been defined previously.

As mentioned earlier, we assume an infinite time horizon for the decision problem. For sufficiently large T, $V_t(\cdot)$ is identical $V_{t+1}(\cdot)$ for the same $\tilde{y}(c)$, $pr(c)$ and $w(c)$, and the basic recursive equation for the stationary, infinite-stage stochastic problem with discounting is equation (2) but without stage subscript. This problem can be solved numerically with successive approximations. There are two basic methods of successive approximation, value iteration and policy iteration, described in, e.g., Kennedy (1986).[2] We used the latter for this analysis.

Decision alternatives

The decision alternatives considered are to re-establish ($u_t = 0$) or not re-establish ($u_t = 1$) the grass ley. The maximum possible age of ley is assumed to be 20 years, i.e., after 20 years the ley requires re-seeding. While this limit is somewhat arbitrary, leys are normally replaced at some stage, and an upper limit on the possible age is needed to make the problem computationally tractable.

[2] Kennedy's General Purpose Dynamic Programming (GPDP) software was used to solve the problem in this analysis. The software can be downloaded from the website <http://www.cornerstonecomputing.com/gpdp/gpdp.html>.

Transformation functions

Grassland yield

Grassland yields, $\tilde{y}_t(c)$, are assumed stochastic and dependent on the age, c, of the ley; ($c = 0,...20$), denoting the number of years since re-seeding. When $c = 0$ the grassland is re-seeded. After 20 years the model requires re-seeding, so $c = 20$ which implies that $u_t = 0$.

From the database 'Grassland Survey in Norway' (Haraldsen and Waag, 1991), which is a database of yield, soil and climate information in typical grassland districts in Norway, we calculated average yield in DM ha^{-1} dependent on age of the ley for fields in northern Norway. The number of observations for fields of each age varies between 109 for fields three years old to 8 for fields twelve years old. To find the "smoothed" yield curve dependent on age of the ley we first regressed estimated annual yields against age of ley. Assuming a quadratic function this gave the following equation (figures in parentheses indicate the parameters standard errors):

$$y(c) = \underset{(389.3)}{6483.5} - \underset{(135.8)}{152.8}\,c + \underset{(10.2)}{4.0}\,c^2 \tag{3}$$

where:

$y(c)$ = estimated grassland yield in kg DM ha^{-1} at the age c,

c = age of the ley, $c = 1,...,12$ for this database.

Equation (3) indicates that yields are falling until the ley is 19 years old, which is beyond the range of the available data. Moreover, the equation is not very satisfactory, first because the standard errors of the coefficients are rather high. Second, it seems likely that the data represent a situation in which some fields have an old grass ley because the yield is high, while fields with a more rapidly falling yield curve have been re-seeded at an earlier age and

so are not represented in the data for later ages. Therefore, there are reasons to believe that the estimated curve is too flat.

Given these limitations of the regressed yield function, we used expert advice to estimate a curve that is falling more steeply than indicated by equation (3) from years two, but which shows no decrease in annual yield in leys eight years old or older - see row 'no winter damage (WD)' yield in Table 1. This 'no WD' yield curve is assumed to apply provided no winter damage occurs during the life cycle of 20 years.

[Table 1 about here]

To reflect the stochastic nature of yields, historical data from the Norwegian Farm Business Survey (NILF, 1994-2000a) for farms in northern Norway over the period 1993 to 1999 was used to approximate the standard deviation (SD) of the yield-process. The estimated SD for this period was 705 kg DM ha^{-1} $year^{-1}$ within farms.

To account for the risk of winter damage, we included discrete downward jump processes for yields. This was in addition to the 'no WD' yield process without any winter damage described earlier. It is not obvious that re-seeding always follows winter damage. If the winter damage is limited it could still be profitable to use the winter-damaged leys for some years longer before re-seeding. To investigate this aspect further we include five winter damage processes with stochastic yield reductions. These five processes reduce yields by 20% (20% WD), 30% (30% WD), 40% (40% WD), 50% (50% WD) and approximately[3] 75% (75% WD), respectively, compared to the 'no WD' yield process. The yield process after a winter damage is assumed stochastic, with the same SD for all processes as for the 'no WD' yield process, except the 75% WD process. For the 75% WD process the SD is reduced one-third and the yield is assumed constant until the sward is re-seeded (see Table 1). In this

[3] Approximately, since yield is assumed constant over time, and not treated as a percentage of 'no WD', see Table 1.

analysis it is assumed that the step down in yield following damage is reduced by 2/3 (based on expert advice) in the year following the year with winter damage due to a partial recovery of the damaged pasture. [4] Assumed mean yields in the years following the year with winter damage are reported in Table 1. After a yield drop and subsequent partial recovery, it is assumed for simplicity that the mean yield will remain at the level of the applicable process until re-seeded, unless there is another incident of winter damage causing a further proportionate yield decline.

It is assumed that there is a probability of 25% that one of the winter damage processes will apply in the model each year, i.e., that some degree of winter damage will occur on average one year in four. This is roughly in accordance with the frequency of winter damage observed over the past 25 years in northern Norway. The assumed probabilities for each of the winter damage processes are further described in Table 2.

The public crop disaster programme, which has several schemes, might reduce the farmers' financial burden due to winter damage of the grass ley. Two schemes are especially relevant for our analysis: The grants to repair and re-establish grass leys after winter damage, and grants to buy roughages in years of shortage of roughages in a region. The first mentioned is intended to stimulate farmers to repair the damaged areas as soon as possible. In the county of Finnmark and in part of Troms county the farmers own risk is equal to the cost of re-

[4] E.g., if the yield drops from 'No WD' with a mean yield of 7000 kg DM ha^{-1} in year one to 30% WD with a mean of 4900 kg DM ha^{-1} in year two, the downward jump in yield will be reduced from 30% to 20% (=30%*(2/3) in year three. This implies that the mean yield given no new winter damage is 5200 kg DM ha^{-1} (=6500*(1-0.2)) in year three.

How much the yield increases the first year (years) after a year after winter damage vary from case to case. May be the assumption of an increase of 2/3 of the drop to low. Yield on older leys may increase more than on younger leys the first years after a year with downward jump, but this condition is not included in the analysis. Our results then may show a to early replacement frequency on winter damaged leys.

establishing 15 per cent of the total roughage area on the farm. In other parts of the country the farmers own risk is equal to the cost of re-establishing 20 per cent of the total roughage area. The other grant is intended to finance extraordinary purchase of roughages in years of low crop yields. This scheme has an own risk of 22 per cent in Finnmark and part of Troms and 27 per cent in other parts of Norway. These grants are not included when estimating net costs of winter damage. The main reason is that our analysis is on a per ha basis while the crop disaster schemes are on a whole-farm basis. It would be possible to re-run the models to match the circumstances of any individual farmer, but it is not possible to generalise the effects of the grants in one single model. The omission of the crop disaster subsidies means that the farmers' net costs of winter damage have been over-estimated in this analysis.

Coarse fodder price

The coarse fodder is usually produced for the farmer's own animals but is valued in this analysis at market price. We have assumed a deterministic and constant price of Norwegian kroner (NOK) 1.65 kg^{-1} DM, taking into account the value of silage applied in the Norwegian Farm Business Survey (NILF, 2000b). There the price of roughages is based on the selling price of barley.

Costs

The estimated cost of re-seeding is NOK 8910 ha^{-1}. These re-seeding costs include seed, lime, labour input, and machinery costs. We have assumed, for simplicity, a deterministic grass yield in the re-seeding year of 4000 DM ha^{-1}. The income from production in the year with re-seeding is assumed to be NOK 6600 ha^{-1}, and the estimated net cost of re-seeding is therefore NOK 2310 ha^{-1}.

Assumed variable costs take account of yield-dependent uses of fertiliser, preservatives, fuel, other variable machinery costs and labour input. The soil is often acid, and rainfall and

use of fertilisers cause further acidification. Lime is often applied in the year of re-seeding grassland. However, in the model the re-seeding pattern is variable. Therefore, we have assumed that lime is applied annually, which, depending on the length of the ley, somewhat underestimates the present value of the cost of lime. Estimated variable cost is assumed to be proportional, or close to proportional, to yield. These variable cost are calculated for expected yield[5] based on Handbook in Farm Business Planning (NILF, 2000b).

Maintenance costs for drainage etc. are not included in the analysis since it is assumed these costs are independent of replacement decisions. There is an acreage subsidy for grassland and other roughages. As long as the fields are harvested, whether winter damaged or not, these subsidies are paid and so the provision of such payments has no influence on our solutions. They are therefore also excluded from the analysis.

Transition probabilities

To calculate the transition probabilities, i.e., the probabilities of moving from the kth to the jth state between stage t and stage t+1, the estimated standard deviation of yields from the Norwegian Farm Business Survey (NILF, 1994-2000a) in the period 1993 to 1999 in northern Norway is combined with subjective yield judgements in a Monte Carlo simulation. The approach involved the following steps.

- First, we made two important assumptions: 1) yield is (truncated) normal distributed, and 2) the variance is independent of the age of the ley. Neither of these assumptions is obviously true. Of course, a distribution other than the normal (e.g. empirical) could have been used, but the normal was used for simplicity. This implies that each expert based yield process in Table 1 is assumed to follow a normal distribution with the mean and standard deviation shown as reported in the last column in Table 1. In the Monte Carlo

[5] Expected budgeted variable costs depend of age of the ley and yield level (no WD, 20-75% WD), see Table 1.

simulation we simulated the yield distribution for each age of the 'no WD' yield process and each of the five winter damage processes. The relation between yield and ley age can be treated as an empirical question. From the grassland survey we found the average correlation of grass ley yield from one year to the following year to be 0.44. This stochastic dependency relationship is included in the simulation. To ensure stability, 25 000 sample simulation experiments were used.

- Second, the simulated yield series (no WD, 20-75%WD) for each year was divided in four intervals, very low (VL), low (L), high (H) and very high (VH) yield level with the 1st, 2nd and 3rd quartiles as the dividing yields. Then for every year we counted the numbers of observations within each interval dependent on within which interval the observation was the previous year. Since we use a normal distribution with the same correlation for every year, the transition probabilities will be approximately the same for every transition.
- Third, as noted, a probability of 25% of winter damage in a year is assumed. And, if damage occurs, the downward shift in the mean yield is assumed be reduced by 2/3 the first year after a year with downward jump and then be at this relative level of 'no WD' yield level until the grassland is re-seeded or further damaged. In other words, it is assumed that there is a probability of 75% that the yield will be either very low, low, high or very high of 'no WD', given that the yield was 'no WD' the preceding year. The 25% probability of winter damage is arbitrarily set to 10% probability for 75% WD, and the remaining 15% probability is divided equally across the remaining WD (20-50% WD) yield processes. If winter damage has occurred and the grassland is not re-seeded, a new downward jumps can happen, i.e., a winter damage on winter damaged land can occur. E.g., on a ley with 40% winter damage, it is assumed that there is 25% probability that a further downward jump may happen in any subsequent year, distributed 10% to 75% WD

and 15% to 50% WD. A ley that already has 75% damage is assumed not to drop more before it will be re-seeded. The estimated transition probabilities are reported in Table 2.

[Table 2 about here]

From the simulated yield series for each year the mean from each interval (under 1st quartile (VL), between 1st and 2nd quartile (L), between 2nd and 3rd quartile (H), above 3rd quartile (VH)) was calculated and used as the expected yield levels in the model, see Table 3.

[Table 3 about here]

Results

Two main aspects are investigated in this article. (1) What is optimal replacement frequency when the probability of winter damage is accounted for? (2) How damaged might the ley be to make it profitable to re-seed it? Even though it is not possible to give exact answers to these questions with a high degree of confidence, the results that are reported in Table 4 might illuminate the issues.

The optimal decision to replace a ley in an area with winter damage problems depends on age of the ley, the yield level and the extent of the winter damage. Table 4 show that, independent of age of the ley and the current yield level, in cases with 75% and 50% winter damage it is always best to replace the ley (except for high yield over the first three years in the case of 50% WD). If the winter damage is limited to 20-40% WD of 'no WD' yield, the replacement decision depends on the age of the ley and current yield level, see Table 4. In case with 40% WD (30% WD) the threshold before it is profitable to re-seed is zero to five (one to very old) years, depending on relative yield level. A field which suffers 20% WD and relative yield level below average should be re-seeded if the age of the ley is older than four

to five years.[6] A high yield on the same field should been replaced if the ley age is below fourteen years. A farmer who has a ley that is giving a very low yield but no winter damage should keep as it is eight years or younger. The optimal life cycle if no winter damage occurs and the relative yield level is not in the very low category is at least until any winter damage occur (see Table 4).

[Table 4 about here]

Finally, we look at differences in NPV between optimal and once-only (i.e. not in every cycle) sub-optimal choices, see Table 5. For simplicity, we only look at the percentage loss in NPV ha^{-1} caused of sub-optimal replacement strategy for some ley ages in cases where the keep strategy is optimal.

[Table 5 about here]

The loss from a one-time sub-optimal replacement strategy, compared to the optimal keep the ley strategy is clearly largest the first year after re-seeding, and also depend strongly on current yield level (Table 5). A sub-optimal strategy on an old ley has almost no effect on the NPV ha^{-1}, while a sub-optimal choice in the first year after re-seeding with high yield means a NPV ha^{-1} loss of 14.1%. Although results are not shown, the opposite situation to that presented here applies when we consider a sub-optimal keep strategy compared to an optimal replacement strategy. The NPV loss of a sub-optimal keep strategy the first year after re-seeding is low compared with the sub-optimal keep strategy for old leys for which replacement is more urgent.

[6] In the case of mild winter damage, partial re-seeding of the field could be optimal (only areas of the field with considerably winter damage being re-seeded). Such a partial re-seeding strategy, without ploughing, is not investigated in this analysis.

Concluding comments

This analysis is based on many uncertain assumptions. It is extremely difficult to find reliable data for the yield curve. In addition, many other factors than yield are uncertain. Further research should include more stochastic variables, if relevant data can be obtained.

The assumption of a normal distributed yield may or may not be appropriate (Just and Weninger, 1999). However, we do not believe that the results in this analysis are much affected by this assumption. That yield may increase relatively more on older leys than on younger leys the first years after a year with downward jump (winter damage) is not included in the analysis. On winter damaged leys our results may then be over-estimated. This aspect should be accounted for in future analysis.

This article is based on climatic and growing conditions in northern Norway, and so the results can only be relevant for this region. However, our conclusion that it may not always pay to replace immediately mildly damaged fields that are still producing high yields is likely to be widely applicable to other areas where winter damage may occur.

Even if the empirical results have limited relevance, the dynamic programming model used in this analysis is applicable to all replacement problems of leys in areas with winter damage problems. With some modification, our model may be used for other similar replacement problems with jumps in the transformation functions. It can also be modified to more closely match the circumstances and assumptions of individual farmers facing a ley replacement problem with winter damage.

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Table 1 Assumed mean yields in kg DM ha^{-1} and standard deviation (SD). 'No WD' yield is grass without any winter damage (WD). 20% WD first year (the year winter damage occur), 30% WD first year, 40% WD first year, 50% WD first year and 75% WD are 20%, 30%, 40%, 50% and 75% below 'no WD' yield, respectively. 'later years' means yield level subsequent years after a years with winter damage (given no new winter damage occur).

Year of rotation	1	2	3	4	5	6	7	8-20	SD
No WD	7 000	7 000	6 500	6 000	5 500	5 300	5 100	5 000	705
20% WD, first year	5 600	5 600	5 200	4 800	4 400	4 240	4 080	4 000	705
20% WD, later years		6 067	5 633	5 200	4 767	4 593	4 420	4 333	705
30% WD, first year	4 900	4 900	4 550	4 200	3 850	3 710	3 570	3 500	705
30% WD, later years		5 600	5 200	4 800	4 400	4 240	4 080	4 000	705
40% WD, first year	4 200	4 200	3 900	3 600	3 300	3 180	3 060	3 000	705
40% WD, later years		5 133	4 767	4 400	4 033	3 887	3 740	3 667	705
50% WD, first year	3 500	3 500	3 250	3 000	2 750	2 650	2 550	2 500	705
50% WD, later years		4 667	4 333	4 000	3 667	3 533	3 400	3 333	705
75% WD	1 400	1 400	1 400	1 400	1 400	1 400	1 400	1 400	465

Table 2 Transition probabilities. Yields are assumed to fall into one of for four possible states, very low (VL), low (L), high and very high (VH) yields for the states of nature 75% winter damage (WD), 50% WD, 40% WD, 30% WD, 20% WD and No WD.

Age of ley c+1 (rows: Age of ley c)

State of nature		75% WD VL	L	H	VH	50% WD, first year VL	L	H	VH
75% WD	VL	0.449	0.370	0.103	0.083				
	L	0.280	0.288	0.254	0.179				
	H	0.181	0.266	0.283	0.280				
	VH	0.090	0.181	0.281	0.448				
50% WD, first year	VL	0.112	0.069	0.046	0.023				
	L	0.070	0.072	0.063	0.045				
	H	0.045	0.064	0.071	0.070				
	VH	0.023	0.045	0.070	0.112				
50% WD, following year	VL	0.112	0.069	0.046	0.023				
	L	0.070	0.072	0.063	0.045				
	H	0.045	0.064	0.071	0.070				
	VH	0.023	0.045	0.070	0.112				
40% WD, first year	VL	0.045	0.027	0.018	0.009	0.067	0.041	0.027	0.014
	L	0.028	0.029	0.025	0.018	0.042	0.043	0.038	0.027
	H	0.018	0.026	0.028	0.028	0.027	0.038	0.042	0.042
	VH	0.009	0.018	0.028	0.045	0.014	0.027	0.042	0.067
40% WD, following year	VL	0.045	0.027	0.018	0.009	0.067	0.041	0.027	0.014
	L	0.028	0.029	0.025	0.018	0.042	0.043	0.038	0.027
	H	0.018	0.026	0.028	0.028	0.027	0.038	0.042	0.042
	VH	0.009	0.018	0.028	0.045	0.014	0.027	0.042	0.067
30% WD, first year	VL	0.045	0.027	0.018	0.009	0.034	0.021	0.014	0.007
	L	0.028	0.029	0.025	0.018	0.021	0.022	0.019	0.013
	H	0.018	0.026	0.028	0.028	0.014	0.019	0.021	0.021
	VH	0.009	0.018	0.028	0.045	0.007	0.014	0.021	0.034
30% WD, following year	VL	0.045	0.027	0.018	0.009	0.034	0.021	0.014	0.007
	L	0.028	0.029	0.025	0.018	0.021	0.022	0.019	0.013
	H	0.018	0.026	0.028	0.028	0.014	0.019	0.021	0.021
	VH	0.009	0.018	0.028	0.045	0.007	0.014	0.021	0.034
20% WD, first year	VL	0.045	0.027	0.018	0.009	0.022	0.014	0.009	0.005
	L	0.028	0.029	0.025	0.018	0.014	0.014	0.013	0.009
	H	0.018	0.026	0.028	0.028	0.009	0.013	0.014	0.014
	VH	0.009	0.018	0.028	0.045	0.005	0.009	0.014	0.022
20% WD, following year	VL	0.045	0.027	0.018	0.009	0.022	0.014	0.009	0.005
	L	0.028	0.029	0.025	0.018	0.014	0.014	0.013	0.009
	H	0.018	0.026	0.028	0.028	0.009	0.013	0.014	0.014
	VH	0.009	0.018	0.028	0.045	0.005	0.009	0.014	0.022
No WD	VL	0.045	0.027	0.018	0.009	0.017	0.010	0.007	0.004
	L	0.028	0.029	0.025	0.018	0.010	0.011	0.010	0.007
	H	0.018	0.026	0.028	0.028	0.007	0.010	0.011	0.011
	VH	0.009	0.018	0.028	0.045	0.003	0.007	0.011	0.017

State of nature		50% WD following years VL	L	H	VH	40% WD, first year VL	L	H	VH
75% WD	VL								
	L								
	H								
	VH								
50% WD, first year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
50% WD, following year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
40% WD, first year	VL								
	L								
	H								
	VH								
40% WD, following year	VL								
	L								
	H								
	VH								
30% WD, first year	VL					0.034	0.021	0.014	0.007
	L					0.021	0.022	0.019	0.013
	H					0.014	0.019	0.021	0.021
	VH					0.007	0.014	0.021	0.034
30% WD, following year	VL					0.034	0.021	0.014	0.007
	L					0.021	0.022	0.019	0.013
	H					0.014	0.019	0.021	0.021
	VH					0.007	0.014	0.021	0.034
20% WD, first year	VL					0.022	0.014	0.009	0.005
	L					0.014	0.014	0.013	0.009
	H					0.009	0.013	0.014	0.014
	VH					0.005	0.009	0.014	0.022
20% WD, following year	VL					0.022	0.014	0.009	0.005
	L					0.014	0.014	0.013	0.009
	H					0.009	0.013	0.014	0.014
	VH					0.005	0.009	0.014	0.022
No WD	VL					0.017	0.010	0.007	0.004
	L					0.010	0.011	0.010	0.007
	H					0.007	0.010	0.011	0.011
	VH					0.003	0.007	0.011	0.017

State of nature		40% WD, following years VL	L	H	VH	30% WD, first year VL	L	H	VH
75% WD	VL								
	L								
	H								
	VH								
50% WD, first year	VL								
	L								
	H								
	VH								
50% WD, following year	VL								
	L								
	H								
	VH								
40% WD, first year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
40% WD, following year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
30% WD, first year	VL								
	L								
	H								
	VH								
30% WD, following year	VL								
	L								
	H								
	VH								
20% WD, first year	VL					0.022	0.014	0.009	0.005
	L					0.014	0.014	0.013	0.009
	H					0.009	0.013	0.014	0.014
	VH					0.005	0.009	0.014	0.022
20% WD, following year	VL					0.022	0.014	0.009	0.005
	L					0.014	0.014	0.013	0.009
	H					0.009	0.013	0.014	0.014
	VH					0.005	0.009	0.014	0.022
No WD	VL					0.017	0.010	0.007	0.004
	L					0.010	0.011	0.010	0.007
	H					0.007	0.010	0.011	0.011
	VH					0.003	0.007	0.011	0.017

State of nature		30% WD, following years VL	L	H	VH	20% WD, first year VL	L	H	VH
75% WD	VL								
	L								
	H								
	VH								
50% WD, first year	VL								
	L								
	H								
	VH								
50% WD, following year	VL								
	L								
	H								
	VH								
40% WD, first year	VL								
	L								
	H								
	VH								
40% WD, following year	VL								
	L								
	H								
	VH								
30% WD, first year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
30% WD, following year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
20% WD, first year	VL								
	L								
	H								
	VH								
20% WD, following year	VL								
	L								
	H								
	VH								
No WD	VL					0.017	0.010	0.007	0.004
	L					0.010	0.011	0.010	0.007
	H					0.007	0.010	0.011	0.011
	VH					0.003	0.007	0.011	0.017

State of nature		20% WD, following years VL	L	H	VH	No WD VL	L	H	VH
75% WD	VL								
	L								
	H								
	VH								
50% WD, first year	VL								
	L								
	H								
	VH								
50% WD, following year	VL								
	L								
	H								
	VH								
40% WD, first year	VL								
	L								
	H								
	VH								
40% WD, following year	VL								
	L								
	H								
	VH								
30% WD, first year	VL								
	L								
	H								
	VH								
30% WD, following year	VL								
	L								
	H								
	VH								
20% WD, first year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
20% WD, following year	VL	0.337	0.206	0.137	0.070				
	L	0.210	0.216	0.190	0.134				
	H	0.136	0.192	0.212	0.210				
	VH	0.068	0.136	0.210	0.336				
No WD	VL					0.337	0.206	0.137	0.070
	L					0.210	0.216	0.190	0.134
	H					0.136	0.192	0.212	0.210
	VH					0.068	0.136	0.210	0.336

Table 3 Example of simulated mean yields (DM) ha^{-1} $year^{-1}$ for 20% WD first year, 20% WD following years and 'no WD' for each interval.

	State of nature											
	20% WD, first year				20% WD, following years				No WD			
Ley age	VL	L	H	VH	VL	L	H	VH	VL	L	H	VH
1	4 710	5 382	5 847	6 510					6 093	6 768	7 223	7 883
2	4 709	5 364	5 823	6 495	5 164	5 837	6 297	6 962	6 101	6 768	7 233	7 901
3	4 317	4 982	5 434	6 091	4 743	5 395	5 854	6 527	5 599	6 266	6 722	7 378
4	3 908	4 567	5 025	5 691	4 310	4 972	5 429	6 096	5 105	5 769	6 232	6 886
5	3 518	4 166	4 622	5 293	3 856	4 530	4 990	5 661	4 607	5 271	5 728	6 392
6	3 350	4 014	4 467	5 137	3 716	4 383	4 833	5 500	4 406	5 064	5 530	6 210
7	3 168	3 842	4 300	4 969	3 521	4 180	4 642	5 298	4 406	5 064	5 530	6 210
8-20	3 113	3 772	4 217	4 894	3 433	4 099	4 559	5 224	4 099	4 771	5 232	5 904

Table 4 Optimal age for replacement of a ley in an area with winter damage. The decision alternatives are replace (2) or keep (1) the ley.

	State of nature																							
Ley	75% WD				50% WD				40% WD				30% WD				20% WD				No WD			
age	VL	L	H	VH	VL	L	H	VH	VL	L	H	VH	VL	L	H	VH	VL	L	H	VH	VL	L	H	VH
1	2	2	2	2	2	2	2	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1
3	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1	1	1	1	1	1	1	1	1
4	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1	1	1	1	1
5	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	2	1	1	1	1	1	1
6	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	1	1	1	1
7	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	1	1	1	1
8	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
9	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
10	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
11	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
12	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
13	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
14	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	1	1	2	1	1	1
15	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1
16	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1
17	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1
18	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1
19	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	1	2	1	1	1
20	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 5 NPV loss ha^{-1} caused of one sub-optimal replace decision rather than optimal keep decision.

	State of nature							
	30% WD				No WD			
Ley age	VL	L	H	VH	VL	L	H	VH
1	0.2 %	2.5 %	4.0 %	6.0 %	9.3 %	11.2 %	12.4 %	14.1 %
2		1.5 %	2.9 %	4.9 %	7.5 %	9.5 %	10.7 %	12.5 %
3		0.1 %	1.5 %	3.5 %	5.0 %	7.1 %	8.4 %	10.2 %
5				1.5 %	1.3 %	3.6 %	5.0 %	7.0 %
10				0.7 %		2.1 %	3.6 %	5.7 %
15				0.7 %		1.9 %	3.3 %	5.4 %
19				0.2 %		0.2 %	1.3 %	3.0 %