Optimal Design of Weather Bonds

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Abstract

This paper investigates the optimal design of weather bonds for reinsurance purposes. The motivation for this task comes from an empirical study showing that German farmers are not willing to pay the premiums for weather insurance that insurers ask for. Since reinsurance costs constitute a major cost component of insurance premiums, minimizing these costs could decrease the observed gap between the willingness to pay and the willingness to accept the cost of the insurance. Against this background, we put forth the proposal to transfer weather risk directly to the capital market by issuing weather bonds. The structure of the weather bond is optimally designed in a utility maximizing framework that involves farmers, insurers, and capital market investors. The approach is illustrated by an example of securitizing draught risk in crop production in Germany.

Keywords: weather risk, weather bonds, reinsurance, securitisation

Agricultural production is highly exposed to weather risk, and in view of the climate changes, it is likely that extreme weather events will occur more frequently in the future. In that context, the development of weather insurance products plays an important role for farm income stabilization. Actually, the design of weather insurance is a subject of current research in agricultural economics (e.g., Turvey 2005). While the greater part of the literature on the subject considers the problem from the viewpoint of farmers, we focus in our paper on the supply side, i.e., the insurer. It is well known that weather fluctuations constitute a systemic risk for private insurance companies (Miranda and Glauber 1997). As a response to systemic risk private insurance companies purchase reinsurance contracts.

However, traditional reinsurance contracts have been criticized as being inefficient. Froot and O’Connell (1997) provide evidence for this statement by reporting price-loss-ratios of 1.6 to 1.7 for catastrophe reinsurance. In this context Doherthy (1997) pinpoints two problems that are inherent to traditional reinsurance. First, there is a moral hazard problem since reinsurance makes insures sloppy in their claim settlement practice. Second, additional transaction cost may arise from default risk of insurance and reinsurance companies. In order to overcome these problems, a direct transfer of weather risks to the capital market via weather bonds or catastrophe (CAT) bonds has been proposed as an alternative reinsurance tool for private insurance companies underwriting crop insurance (Skees et al. 2007, Mahul 2001). In brief, the issuer of a weather bond grants an investor an annual return in the form of a coupon and principal payments in exchange for paying the bond price. In the case of an unfavourable weather event, the issuer retains a certain share of the principal or the coupon as a compensation for his weather related losses. Due to high expected returns and a low correlation with stock market returns, weather bonds may appear attractive to capital market investors. Some applications of CAT bonds and weather bonds do already exist which underpin their potential as risk management tools in agriculture (e.g., Vedenov et al. 2006, Turvey 2007). However, these products are frequently specified on an ad hoc basis and some theoretical problems still remain unsolved. In particular, the pricing and the optimal design of weather bonds deserve further investigation.

The objective of this paper is to identify the optimal structure of a weather bond from the viewpoint of an insurer. This includes the determination of the bond price, the coupon payments and the contingent coupon reduction. The task is complicated by the fact that the optimal design of the weather bond depends on the risk position of the insurer which, in turn, is affected by offering weather insurance to farmers. Hence the securitization transaction and the insurance transaction have to be analyzed simultaneously.

Our modelling framework is in the sprit of Raviv (1979) who applies variational calculus for determining the optimal structure of an insurance contract. Barrieu and El Karoui (2002) extend this approach to three agents: a producer, an insurer, and an investor. Formally, their model consists of two interrelated constrained optimization problems, each showing the structure of a principal agent model. The first part, the insurance transaction, addresses the relation between the producer (farmer) and the insurer. The optimal compensation function and the optimal insurance premium are derived by maximizing the expected utility of the producer's terminal wealth under a participation constraint for the insurer. The second part, the securitization transaction, models the relation between the insurer and the investor. Herein, the parameters of the weather bond are determined so that the expected utility for the insurer is maximized under a participation constraint for the investor and for a given optimal insurance contract. We take up this model and generalize it by including idiosyncratic risks (basis risk) on the part of the producer.

The remainder of the paper is organized as follows: The subsequent section introduces an optimization model that supports the design of weather derivatives as a means to shift weather risk to the capital market. It is followed by an illustrative application of the model. The paper ends with a discussion on the potential of weather derivatives for the securitization of weather related risks.

Theoretical framework

Statement of the problem

In this section, we will use a modelling framework that was suggested by Barrieu and El Karoui (2002) in the spirit of Raviv (1979). Our model economy consists of three risk averse agents: a farm, an insurer, and an investor (figure 1**Fehler! Verweisquelle konnte nicht gefunden werden.**).

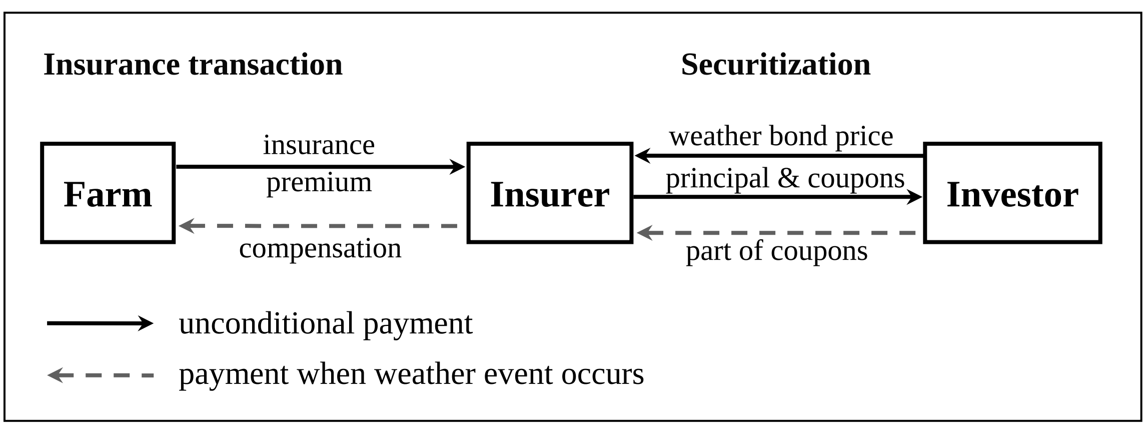

Figure 1. Description of the transaction structure

The economic relationship between these actors consists of two transactions: an insurance transaction between the firm and the insurer and a securitization transaction between the insurer and the investor via the weather bond. In the following, we will describe these transactions in more detail. It is assumed in this context that the firm has already decided as to its production. Contingent upon the weather conditions in year t, the firm suffers production losses $g(I_t, e_t)$. I denotes a well-defined weather index, and e_t captures other random factors which have an influence on the production loss. The function $g(\cdot)$ translates the stochastic factors into an actual loss. In a general setting, we consider a multi-period insurance contract where production losses are cumulated over a time period of n years resulting in a total loss Θ_h.

(1) $$\Theta_h = \sum_{t=1}^{n} \beta_{n-t} \cdot g(I_t, e_t),$$

Herein, $\beta_{n-t} = (1+r)^{n-t}$ represents an accumulation factor with a riskless discount rate r. In order to cover at least part of the losses, the producer buys an index based insurance for a price π and receives in exchange an indemnity payment $J(\Theta)$, $0 \le J(\Theta) \le \Theta$. The insured loss Θ is defined as

(2) $$\Theta = \sum_{t=1}^{n} \beta_{n-t} \cdot f(I_t),$$

The function *f*(.) translates the weather index into an insured loss. This specification is very flexible and comprises, for example, options-like insurance contracts which pay an indemnity if a predetermined strike level for the weather index is exceeded. Note that the indemnity payment solely depends on the weather index and not on other stochastic factors e_t. Hence Θ and Θ_h differ in general. This discrepancy causes a basis risk for the firm. To keep the exposition as simple as possible at the beginning, we subsequently assume that no basis risk is present, i.e. $\Theta = \Theta_h$. (This assumption is relaxed later on.) The values of the cash flows in period n resulting from this transaction are for the producer and the insurer, respectively:

(3) $$-\pi \cdot \beta_n - \Theta + J(\Theta)$$

and

(4) $\pi \cdot \beta_n - J(\Theta)$

At the same time, the insurer issues a weather bond at price Φ. An investor who pays the price Φ receives the principal N in year n and coupon payments s in each year t. However, the investor has to pay back a certain portion $\alpha(\Theta)$. Thereby, part of the loss risk that the insurer bears is transferred to the investor. The value of the cash flows associated with this transaction from the viewpoint of the investor is:

(5) $-\Phi \cdot \beta_n + s \cdot \sum_{t=1}^{n} \beta_{n-t} + N - \alpha(\Theta)$

The portfolio of the insurer is now

(6) $\pi \cdot \beta_n - J(\Theta) + \Phi \cdot \beta_n - s \cdot \sum_{t=1}^{n} \beta_{n-t} - N + \alpha(\Theta)$

Some simplifying assumptions underlie this modelling framework. First, we consider only the financial flows that are triggered by the insurance contract and the weather bond. This means that other stochastic or non-stochastic portfolio components of the three respective agents and possible diversification effects are neglected. Moreover, we suppose that no transaction costs occur. Finally it is assumed that there is no liquid secondary market for the insurance contract and the weather bond. Hence, it is not possible for the investor to build a replicating strategy. As a result, we cannot apply a risk-neutral pricing approach. Instead, the problem of pricing and designing the optimal insurance contract and the weather bond is solved in a utility maximization framework.

The optimization model and its solution

From the previous description of the two transactions, it is clear that the insurance company plays a double role: it offers the insurance contract and issues the weather bond. Both transactions are interdependent since the insurance contract influences the risk exposure of the insurer and, thus, the willingness to accept the weather bond. This relationship is taken into account by a two step procedure. First, the insurer determines the optimal structure of the weather insurance for the farm. Afterwards, the insurer specifies the parameters of the weather bond, conditional on the knowledge of the design of the insurance contract. The objective of all agents is to maximize the expected utility of their terminal wealth. We follow Barrieu and El Karoui (2002) and assume that the risk preference of the agents is captured by exponential utility functions

(7) $U(x) = -e^{-\gamma_i \cdot x},\ i = F, B, I$

where γ_F , γ_B and γ_I denote the risk aversion parameters of the farm, the insurer, and the investor, respectively.

In the first part of the transaction (the insurance transaction), it is assumed that the insurer is passive. That means the insurance contract is designed such that the expected utility of the farm is maximized subject to a participation constraint of the insurer. The design parameters of the insurance contract are the compensation function $J(\cdot)$ and the insurance premium π . The optimization problem can be formally stated as:

(8a) $$\max_{\pi, J} E\left[-\exp\left(-\gamma_F\left(-\pi \cdot \beta_n - \Theta + J(\Theta)\right)\right)\right]$$

s.t.

(8b) $$E\left[-\exp\left(-\gamma_B\left(\pi \cdot \beta_n - J(\Theta)\right)\right)\right] \geq E\left[-\exp\left(-\gamma_B(0)\right)\right] = -1$$

and

(8c) $$0 \leq J(\Theta) \leq \Theta$$

(8) constitutes an optimal control problem that can be solved with standard variational calculus techniques. For a detailed derivation of the subsequent results we refer the reader to Barrieu and El Karoui (2002) and Raviv (1979). The Hamiltonian for this problem is

(9) $$H = -\exp\left(-\gamma_F\left(-\pi \cdot \beta_n - \Theta + J(\Theta)\right)\right) - \lambda \cdot \exp\left(-\gamma_B\left(\pi \cdot \beta_n - J(\Theta)\right)\right)$$

where H denotes the Hamiltonian function and λ is a time-invariant co-state (a Lagrange multiplier). Three necessary conditions for an optimal solution to (8) arise from differentiating (9) with respect to $J(\Theta)$:

10a) $$\frac{\partial H}{\partial J(\Theta)} = \gamma_F \exp\left(-\gamma_F \left(J^*(\theta) - \theta - \pi^* \beta_n\right)\right) - \lambda \gamma_B \exp\left(-\gamma_B \left(\pi^* \beta_n - J^*(\theta)\right)\right) = 0 \text{ for } 0 < J^*(\theta) < \theta$$

10b) $$\frac{\partial H}{\partial J(\Theta)} = \gamma_F \exp\left(\gamma_F \left(\theta + \pi^* \beta_n\right)\right) - \lambda \gamma_B \exp\left(-\gamma_B \left(\pi^* \beta_n\right)\right) \leq 0 \text{ for } J^*(\theta) = 0$$

10c) $$\frac{\partial H}{\partial J(\Theta)} = \gamma_F \exp\left(\gamma_F \left(\pi^* \beta_n\right)\right) - \lambda \gamma_B \exp\left(-\gamma_B \left(\pi^* \beta_n - \theta\right)\right) \geq 0 \text{ for } J^*(\theta) = \theta$$

It is possible to confine the analysis to the first FOC if one determines two thresholds θ^+ and θ^-. $\theta^- \geq 0$ can be understood as a deductible. If the damage $\theta < \theta^-$ the optimal compensation is $J(\theta) = 0$. $\theta^+ \geq 0$ can be interpreted as an upper limit for a full compensation, i.e. $J(\theta) = \theta$ is valid only as long as $\theta < \theta^+$. With these thresholds at hand the optimal coverage between the two extremes $J(\theta) = 0$ and $J(\theta) = \theta$ can then be derived from (10a). Differentiating (10a) with respect to θ, substituting λ from (10a) and solving for $\frac{\partial J^*}{\partial \theta}$ yields:

(11) $$\frac{\partial J^*}{\partial \theta} = \frac{\gamma_F}{\gamma_B + \gamma_F}$$

A solution of this differential equation with a boundary condition $J(0) = 0$ is

(12) $$J^*(\theta) = \frac{\gamma_F}{\gamma_B + \gamma_F} \cdot \theta$$

Barrieu and El Karoui (2002) show that under the specified assumptions $\theta^- = \theta^+ = 0$. That means the optimal compensation function has no deductible and no upper limit and hence (9) constitutes the optimal compensation function for the whole range of possible losses. (12) can be interpreted as a sharing rule for the realized loss θ. Obviously, the optimal compensation depends on the relation of the risk aversion of the farmer and the insurer.

For the derivation of the optimal insurance premium π, we insert (12) into the constraint (8b) and realize that this constraint is binding at the optimum. Solving for π yields

$$(13)\quad \pi^* = \frac{1}{\beta_n} \cdot \frac{1}{\gamma_B} \cdot \ln E\left[\exp\left(\gamma_B \cdot \frac{\gamma_F}{\gamma_B + \gamma_F} \cdot \theta\right)\right]$$

Note that the pricing rule (13) differs from the actuarial fair price. Due to the concavity of the utility function, the insurance premium exceeds the discounted expected indemnity payments. That means a positive risk premium is included in π. The derivation of the pricing rule (13) can be understood as an application of the indifference pricing approach. This method became increasingly popular in the context of pricing contingent claims in incomplete markets (c.f. Xu, Odening and Mußhoff 2008).What happens in the presence of basis risk, i.e., $\Theta \neq \Theta_h$? It is convenient to introduce the conditional certainty equivalent $X(\Theta)$ of the actual farmer's loss Θ_h which is defined as

$$(14)\quad X(\Theta) = \frac{1}{\gamma_F} \cdot \ln E\left[\exp(\gamma_F \cdot \Theta_h) \mid \Theta\right]$$

To solve the modified problem we simply replace Θ by $X(\Theta)$ in the objective function of the program (8a). Carrying out similar steps as before yields the optimal compensation function

$$(15)\quad J^{*b}(X(\Theta)) = \frac{\gamma_F}{\gamma_F + \gamma_B} \cdot X(\Theta)$$

J^{*b} stands for the optimal compensation with basis risk. The only difference compared with the solution without basis risk is that the insured loss Θ is replaced by the certainty equivalent of the actual loss given a realization of the insured loss. For a better understanding of the implication of this modification we assume that the basis risk is additive, i.e. $\Theta_h = \Theta + \varepsilon$, $E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma$. In other words, the distribution of the actual loss Θ_h is a mean preserving spread of the insured loss. It can be easily seen that the following relation between Θ and $X(\Theta)$ holds:

(16) $X(\Theta) = \Theta + C$

where C denotes a constant. The immediate consequence is that the modified compensation function will in general not comply with the constraint $J(\Theta) \leq \Theta$. However, this can be ensured by introducing a deductible which amounts to *C*. Inserting this into eq. (15) reveals that the optimal compensation under additive basis risk is the same as without basis risk.

In the second part of the transaction (the securitization transaction), the insurer plays the active role. That means, the parameters of the weather bond (the price Φ, coupon value s, and the share of losses α) are chosen such that the expected utility of the insurer is maximized given his risk exposition from the insurance transaction, whereas the investor only has to decide to accept or refuse the contract offer. The formal structure of this optimization program is similar to (8a):

(17a) $$\max_{\Phi,s,\alpha} E\left[-\exp\left(-\gamma_B\left(\pi \cdot \beta_n - J(\Theta) + \Phi \cdot \beta_n - s \cdot \sum_{t=1}^{n} \beta_{n-t} - N + \alpha(\Theta)\right)\right)\right]$$

s.t.

(17b) $$E\left[-\exp\left(-\gamma_I\left(-\Phi \cdot \beta_n + s \cdot \sum_{t=1}^{n} \beta_{n-t} + N - \alpha(\Theta)\right)\right)\right] \geq -1$$

The Hamiltonian for this problem is

(18) $L = -\exp\{-\gamma_B \cdot (\pi \cdot \beta_n - J(\Theta) - \Lambda + \alpha(\Theta))\} - \lambda \cdot \exp(-\gamma_I(\Lambda - \alpha(\Theta)))$,

with $\Lambda = -\Phi \cdot \beta_n + s \cdot \sum_{t=1}^{n} \beta_{n-t} + N$. From the first order condition, we obtain

(19) $\alpha^* = \frac{\gamma_B}{\gamma_B + \gamma_I} \cdot J(\theta) + \Lambda - \frac{\gamma_B}{\gamma_B + \gamma_I} \cdot \pi \cdot \beta_n - \frac{1}{\gamma_B + \gamma_I} \ln \frac{\lambda \cdot \gamma_I}{\gamma_B}$

In order to simplify this expression, we impose the restriction that α^* must be zero if the weather event does not occur and, hence, no compensation takes place. It follows that

$$(20)\ \ \Lambda - \frac{\gamma_B}{\gamma_B + \gamma_I} \cdot \pi \cdot \beta_n - \frac{1}{\gamma_B + \gamma_I} \ln \frac{\lambda \cdot \gamma_I}{\gamma_B} = 0$$

Inserting (12) and (20) into (19) gives the optimal α:

$$(21)\ \ \alpha^* = \frac{\gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot \theta$$

Obviously the optimal repayment α^* is proportional to the insured loss and depends on the relation of the risk aversion of all involved agents.

The optimal value of the net cash flow for the investor, Λ, is independent of the weather event and can be determined using similar arguments as in the case of the calculation of the optimal insurance premium π. Inserting (21) into the binding participation constraint in (17b) yields

$$(22)\ \ \Lambda^* = -\Phi^* \cdot \beta_n + s^* \cdot \sum_{t=1}^{n} \beta_{n-t} + N = \frac{1}{\gamma_I} \cdot \ln E\left[\exp\left(\frac{\gamma_I \cdot \gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot \theta\right)\right]$$

The optimal bond price is then given by

$$(23)\ \ \Phi^* = \frac{1}{\beta_n} \cdot \left\{ s^* \cdot \sum_{t=1}^{n} \beta_{n-t} + N - \frac{1}{\gamma_I} \cdot \ln E\left[\exp\left(\frac{\gamma_I \cdot \gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot \theta\right)\right]\right\}$$

(23) reveals an interesting feature of the optimal bond price. Recalling that the "fair price," Φ^*_{fair}, of a contingent claim is defined as the expected value of its discounted net cash flow, we find that

$$(24)\ \ \Phi^*_{fair} = \frac{1}{\beta_n} \cdot \left\{ s^* \cdot \sum_{t=1}^{n} \beta_{n-t} + N - \frac{\gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot E(\theta) \right\}$$

A comparison of (23) and (24) shows that the weather bond can be offered at a price which is lower than the "fair price."

Note that there is an indeterminacy concerning the optimal bond structure. The optimal repayment α^* determines the optimal net cash flow Λ^* uniquely, but the relation between the optimal bond price Φ^*

and the optimal coupon s^* can be chosen arbitrarily. We consider two common cases. In the first case, the bond is offered at a discount, i.e., the discounted principal payment is equal to the bond price, $\Phi^* = N \cdot \beta_n^{-1}$. In the second case, the bond is offered at par, i.e., the principal payment is equal to the bond price, $\Phi^* = N$.

The optimal coupon payments for the two cases are then given by

$$(25)\quad s^{1*} = \left(1 \Big/ \sum_{t=1}^{n} \beta_{n-t}\right) \cdot \frac{1}{\gamma_I} \cdot \ln E\left[\exp\left(\frac{\gamma_I \cdot \gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot \theta\right)\right]$$

and

$$(26)\quad s^{2*} \cdot \sum_{t=1}^{n} \beta_{n-t} = \Phi \cdot (\beta_n - 1) + \frac{1}{\gamma_I} \cdot \ln E\left[\exp\left(\frac{\gamma_I \cdot \gamma_B \cdot \gamma_F}{(\gamma_B + \gamma_F) \cdot (\gamma_B + \gamma_I)} \cdot \theta\right)\right]$$

In the exposition so far, the principal payment N was considered as an exogenous parameter. From a marketing viewpoint, however, it might be desirable to offer a certain return r_{bond} (before stochastic repayments) to the investor which should clearly exceed the riskless interest rate r. The definition $r_{bond} = s^* / \Phi$ then implies the bond price.

$$(27)\quad \Phi = \frac{s^*}{r_{bond}}$$

Securitization of weather risk in northeast Germany

Weber et al. (2008) conduct a survey among 249 farmers in northeast Germany on the economic consequences of draught risk and the willingness to pay for weather insurance. The poll held among the farmers revealed that every single one of the polled farmers had been affected by drought at least once during the previous decade: 50% of those polled indicated that they had been affected more than three times; 88% of the cases reported as high as between two and five occurrences, and only 3% of those polled stated that they had never been affected by drought. The farmers also estimated that the harvest risk due to drought was of corresponding importance. It represented the most important harvest-relevant weather event for 80% of those polled, followed by hail (55%), temperature (49%), and storms (25%). 69% of the farmers suffered harvest losses between 20% and 40%, and 10% indicated damages of 40-60% in the event of drought. Against that background it is not surprising that farmers indicated considerable interest in drought insurance. However, another finding from this survey was that farmers are less willing to pay than the insurers are willing to accept the insurance arrangement. Hence under the prevailing conditions, it is unlikely that a market for weather insurances will emerge in this region. Reinsurance costs constitute a major component of insurance premiums. This is the motivation for designing a weather bond which reduces those costs and, thereby, makes it possible to offer weather insurance at lower prices. In the following, we will apply the theoretical model of the previous section to the specific situation in northeast Germany.

Data and model assumptions

In our application, we focus on wheat production which is a major crop in northeast Germany, particularly in Brandenburg. Wheat yield data (in € per hectare) was collected from a representative cash crop farm during a period of time between 1993 and 2007. For the specification of the relationship between weather and revenues, we followed Vedenov and Barnett (2004) who suggest a model which is quadratic in deviations of temperature and rainfall. The weather variables are derived from the daily temperature and daily precipitation data recorded at the weather station in Berlin-Tempelhof. Significant parameter estimates were made for the following model specification:

(28) $Y_t = I_t + \varepsilon_t$

with

(29) $$I_t = \beta_0 + \beta_1 \cdot \Delta T_t^{June} + \beta_2 \cdot \left(\Delta T_t^{April}\right)^2 + \beta_3 \cdot \left(\Delta R_t\right)^2 + \beta_4 \cdot \Delta R_t \cdot \Delta T_t^{April} + \beta_5 \cdot \Delta R_t \cdot \Delta T_t^{June}$$

Herein, Y_t denotes wheat production revenues, I_t is a weather index at time t, $\varepsilon_t \sim N[0, \sigma_\varepsilon]$ is a normally distributed error term, and Δ measures the deviation of a weather variable from its long-term average. T_t^{April} and T_t^{June} represent the average monthly temperatures for April and June. R_t represents the cumulated precipitation in the period April 1 until June 30. The estimated parameters (p-values) are $\beta_0 = 65.73$ (0.0000), $\beta_1 = -3.97$ (0.0176), $\beta_2 = -3.09$ (0.0134), $\beta_3 = 0.0017$ (0.0039), $\beta_4 = -0.14$ (0.0052) and $\beta_5 = 0.14$ (0.0023). An R^2 of 0.85 indicates that the selected weather index can effectively explain the wheat yield in Brandenburg. The model specification is also supported by the corrected Akaike information criterion.

Subsequent to the estimation of the model (29), we fit a parametric distribution for the weather index using standard test procedures. The best fit is attained by a Weibull distribution which is shown in Figure 3. The expected value for the weather index amounts to 62.6 points, and the standard deviation is 11.8 points. According to (28), this distribution may also be interpreted as a yield distribution. Multiplying the stochastic yield with a constant wheat prices of 11.22 €/dt gives the wheat production revenues, *Rev* (I_t).

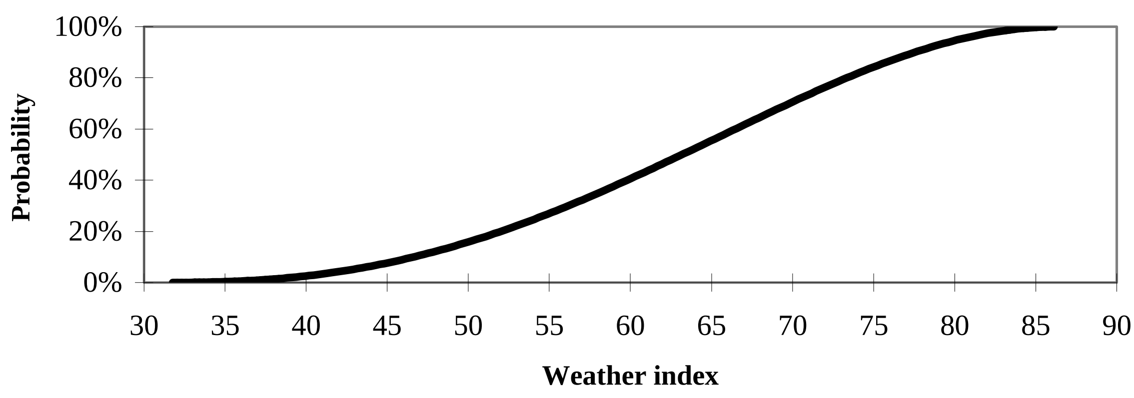

Figure 2. Cumulative probability distribution of the weather index

In our model, we assume that the farmer can buy the insurance contract on July 1 and will receive an indemnity payment on June 30 next year if unfavourable weather occurs. That is, we consider a two-date insurance contract with a contract period of 1 year ($n = 1$). A contract refers to one hectare.

Next, we have to specify the indemnity trigger (strike-level) $\bar{I}$. We assume that the farmer will receive an insurance payoff if the weather index falls below its expected value which amounts 62.6 points. This corresponds to revenues of € 701 per hectare. In a second scenario, we define another trigger level which equals a 30% percentile of the weather index distribution and amounts to 56.5 points (€ 634 per ha). This is equivalent to introducing a deductible amount of € 67 per ha.

The revenue loss θ equals the positive difference between the actual revenues and the revenues at the predetermined indemnity trigger $\bar{I}$ (see eq. (2)):

(30) $\theta = f\left(I_t\right) = \max\left(0, Rev(I_t) - \overline{R}ev(\bar{I})\right)$

Figure 4 depicts the insured revenue losses in accordance with the predetermined indemnity triggers for the observation period 1993 – 2007. Apparently, in some years considerable losses occurred that could jeopardize the farmers' liquidity.

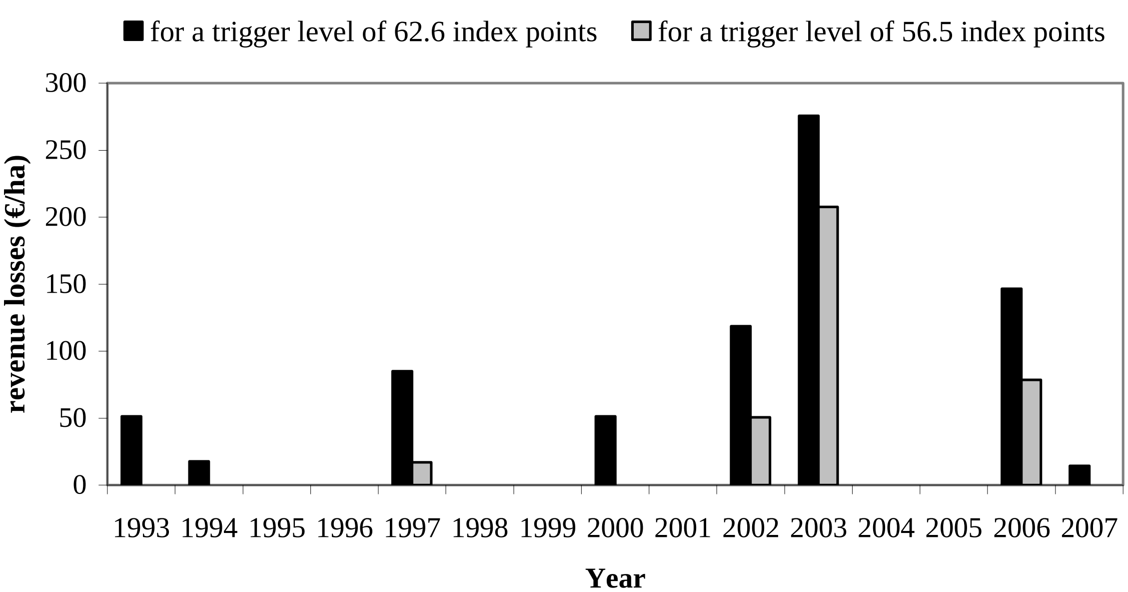

Figure 3. Insured revenue losses for different indemnity triggers.

The optimal risk transfer structure depends on the risk aversion of the involved contract partners and hence, we have to make assumptions about those parameters. In accordance with Xu, Odening, and Mußhoff (2007), we assume an absolute risk aversion parameter of $8 \cdot 10^{-6}$ for the base scenario. Since the risk aversion is a crucial parameter, we carry out different calculations with alternative values. Furthermore, we assume a risk free interest rate r of 3.12% (the four year average of the return on German state bonds). The return for the investor without damage occurrence ($= s/\Phi$) should exceed the risk free interest and is fixed at 5%.

Results and Discussion

Table 1 presents the optimal risk transfer structure. The results are based on a stochastic simulation with 10,000 random draws from the estimated Weibull distribution of the weather index. The columns in Table 2 represent different degrees of the three agents' risk aversion. In case 1, all the market participants have the same absolute risk aversion. In that case, the compensation ratio (J/θ) amounts to 50%. That means that the insurer is willing to compensate for half of the actual (discounted) loss of the farmer (26.30 €). It is interesting to note that the insurance premium π is only slightly higher than the discounted expected loss $E(J) \cdot \beta_1^{-1}$, which can be interpreted as the actuarially fair price. In other words, the risk loading is negligible. This can be explained by the fact that 25% of the insured losses, i.e., €13.15 per contract, are transferred to the investor. In order to define the cash flow structure between the insurer and investor, one has to decide whether the bond is offered at a discount or at par. First, we consider the former case in which the principal payment N equals the compound bond price $\Phi \cdot \beta_1$ at the end of the contract period. The bond price Φ amounts to €271.57 which is virtually the same as the actuarially fair price. The coupon, s^1, is determined according to eq. (25) and amounts to €13.58. One can easily verify the fact that the return without repayments matches the desired value of 5%. The investor's expected net return (i.e., the return after a correction for the repayment α) is only 0.16%. For the interpretation of this return, one should recall the fact that the bond was sold at a discount and, hence, the 0.16% represents an (expected) return above the risk free interest rate of 3.12%. On the average, the coupon payments are sufficient to cover the investor’s repayment. However, in the (unlikely) event of a total loss, the investor has to repay €175.25 and is left with a negative return of -0.60%

Next, we consider the case in which the bond is offered at par, i.e., the principal N at the time of expiration equals the bond price Φ. Compared with the previous case, the principal payment, the bond price, and the coupon are about 2.5 times higher. According to eq. (26), the coupon s^2 is equal to s^1 plus $\Phi \cdot r$. Hence, s^2 must be higher in order to satisfy the constraint in (17a). The resulting expected net return for the investor is 3.27%.

In case 2, the risk aversion parameters of all market participants are increased by a factor of one thousand. Compared to the base case, both the losses that are transferred from the farmer to the insurer and from the insurer to the investor remain unchanged. However, the insurance premium π is significantly higher than in case 1 due to the increased risk aversion of the insurer. For the securitization part, we observe an increase in the coupon s^1 and a decrease in the cover rate. In view of the higher risk aversion of the

Table 1. Optimal risk transfer structure of yield losses*

		Case 1	*Case 2*	*Case 3*	*Case 4*	*Case 5*	*Case 6*
Varied parameters	Indemnity trigger (index points)			62.6			56.5
	Risk aversion of producer γ_F			$8\cdot10^{-6}$	$8\cdot10^{-6}$	$8\cdot10^{-6}$	$8\cdot10^{-6}$
	Risk aversion of insurer γ_B	$8\cdot10^{-6}$	$8\cdot10^{-3}$	$8\cdot10^{-8}$	$8\cdot10^{-6}$	$8\cdot10^{-8}$	$8\cdot10^{-8}$
	Risk aversion of investor γ_I			$8\cdot10^{-6}$	$8\cdot10^{-8}$	$8\cdot10^{-8}$	$8\cdot10^{-8}$
Insurance	Compensation ratio J/θ (%)	50	50	99	50	99	99
	Discounted expected loss $E(\theta)\cdot\beta_1^{-1}$ (€)			52.59			23.57
	Discounted expected compensation $E(J)\cdot\beta_1^{-1}$ (€)	26.30	26.30	52.07	26.30	52.07	23.34
	Insurance premium π (€)	26.30	31.17	52.07	26.30	52.07	23.34
Securitization	Risk transfer ratio α/θ (%)	25	25	1	50	50	50
	Discounted expected repayment of investor $E(\alpha)\cdot\beta_1^{-1}$ (€)	13.15	13.15	0.52	26.04	26.04	11.67
	Specification 1:			$\Phi\cdot\beta_1 = N$			
	Principal N (€)	280.44	304.43	11.00	555.29	555.29	248.90
	Bond price Φ (€)	271.57	294.81	10.65	537.73	537.73	241.03
	"Fair bond price" Φ^*_{fair} (€)	271.71	296.08	10.65	538.00	538.00	241.15
	Coupon s^1 (€)	13.58	14.74	0.53	26.89	26.89	12.05
	Expected net return of the investor $\left(s^1 - E(\alpha)\cdot\beta_1^{-1}\right)/\Phi$ (%)	0.16	0.54	0.16	0.16	0.16	0.16
	Cover rate $E(\alpha)\cdot\beta_1^{-1}/s^1$ (%)	97	89	97	97	97	97
	Specification 2:			$\Phi = N$			
	Principal N (€)	760.28	825.32	29.81	1505.39	1505.39	674.77
	Bond price Φ (€)	760.28	825.32	29.81	1505.39	1505.39	674.77
	"Fair bond price" Φ^*_{fair} (€)	760.28	826.45	29.81	1505.39	1505.39	674.77
	Coupon s^2 (€)	38.01	41.27	1.49	75.27	75.27	33.74
	Expected net return of the investor $\left(s^2 - E(\alpha)\cdot\beta_1^{-1}\right)/\Phi$ (%)	3.27	3.41	3.27	3.27	3.27	3.27
	Cover rate $E(\alpha)\cdot\beta_1^{-1}/s^2$ (%)	35	32	35	35	35	35

* Constant parameters: risk free interest rate r = 3.12%, and return for an investor without damage occurrence s/Φ = 5%.

investor, the insurer has to pay a higher coupon and receives on the average only 89% of the coupon back from the investor as a compensation for the losses from the insurance transaction. The wedge between the optimal bond price and the fair price becomes visible now, but it is still rather small.

In scenarios 3 to 5, we change the relations of the risk aversion parameters of the market participants. More precisely, the risk aversion parameters of the insurer (case 3), of the investor (case 4), and of both (case 5) are reduced. In case 3, the production losses are almost completely compensated for by the insurer, while only 1% of production losses are transferred to the capital market. In case 4, the farmer's compensation ratio as well as the risk transfer ration of the insurer amount to 50%. That means that the insurer completely transfers the weather risk to the capital market. In case 5, almost all of the farmer's revenue losses are transferred to the insurer.

Finally, we reduce the indemnity trigger from 62.6 index points to 56.5 index points (case 6). The risk aversion parameters are the same as in scenario 5. Accordingly, the compensation ratio J/θ and the risk transfer ratio α/θ are unchanged. However, due to the lower indemnity trigger, we observe a lower discounted, expected loss of €23.57 per ha. All other values change proportionally.

Conclusions

In this paper we look for the optimal structure of a weather bond in which the production weather risk is transferred to the capital market, and three representative market participants (producer, insurer, and investor) are involved. We show that the indemnity payments made to the farmers are a linear function of the insured loss which depends on the stochastic weather event. The cost of insurance is a non-linear function of the indemnity payments capturing a risk premium for the insurer. The size of the risk premium depends on the risk aversion of the insurer and the farmer. These findings are in line with previous work on optimal risk sharing rules. Moreover, it is shown that the optimal price of the weather bond is lower than its actuarially fair price which is defined as the expected value of the bond's discounted cash flows. This surprising finding is important from a marketing perspective. However, in our application the difference is not very pronounced. Another finding is that the determination of the structure of the weather bond is ambiguous since there is an infinite amount of optimal combinations of coupon levels and bond prices. This fact can also be exploited for marketing purposes.

Apart from these theoretical insights, our paper is relevant for the ongoing discussion on how to hedge weather related risks. We argue that weather derivatives constitute efficient instruments for transferring risks to the capital market. This has two implications. First of all, the existence of systemic weather risk does not provide a justification for governmental intervention in insurance markets. Second, since reinsurance costs are a major component of insurance premiums, a reduction of those costs might help to reduce the frequently observed gap between the willingness to pay and the willingness to accept weather insurance.

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