THE PARADOX OF RISK BALANCING:
DO RISK-REDUCING POLICIES LEAD TO MORE RISK FOR FARMERS?

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Abstract:

The study presents stochastic optimal control/dynamic programming (SOC/DP) to derive the optimal debt level and consumption in farm models concerning two sources of uncertainty: the return on assets and interest rate. The SOC/DP analytic framework is used to analyze the impacts of risk-reducing farm policies on farm's financial and risk adjustments. The results show the violations of the risk-balancing concept, which theorizes that risk-reducing farm policies may lead to increases in financial leverage, total risk, and the expected returns. Also, this study examines the extent to which the estimates of the optimal debt level are biased when interest rate risk is ignored.

Keywords: Stochastic Optimal Control/Dynamic Programming, Financial Leverage, Uncertainty, Risk Balancing.

The Paradox of Risk Balancing: Do Risk Reducing Policies Lead to More Risk for Farmers?

Capital structure, or how the firm chooses to finance its operations, is a critical decision facing farm businesses. The use of financial leverage influences both the expected level of the return on equity and the risk associated with this return. Increasing the firm's borrowing increases the capital available for production, allowing expansion of the farm business. When the cost of debt is less than the returns it generates, adding leverage results in greater expected return on equity. However, before the owners of the equity capital can take their return, a share of the operating profit must be allocated to meeting the interest payment on the debt capital. Highly leveraged firms must be concerned about meeting their financial obligations and recognize uncertainty about future costs and availability of credit. As a result, the amount of financial leverage that a firm can utilize is dependent upon the riskiness or variability of the cash flows generated by the business. Therefore, producers' decisions of financial leverage must consider the business risks that confront their operation.

The risk-balancing hypothesis by Gabriel and Baker (1980) states that when an exogenous shock affects the level of farm's business risk, producers might make the offsetting financial adjustments leading to increased (or decreased) financial risk in response to a fall (or rise) in business risk. Business risk stems from the variability of the rate return on assets (ROA) and is independent of the firm's capital structure. The level of business risk is influenced by external factors such as uncertain market prices and yields, as well as by internal factors such as investment decisions and management skills.

While business risk is the aggregate effect of all the uncertainty concerning the profitability of the firm before leverage, financial risk is related to the way a farm enterprise uses debt. Financial risk arises from the fixed financial obligations and can be defined as the incremental increase in the variability of the rate of return on equity (ROE) due to financial leverage (Gabriel and Baker, 1980).

In further study of risk balancing, Collins (1985) indicated that risk-reducing farm policies would induce producers to increase their financial leverage to a level of desired financial risk. Featherstone, Moss, Baker, and Preckel (1988) showed that risk-reducing farm policies may lead to increases in financial leverage, total risk, and the expected returns on farm equity. Empirical evidence suggests that the increased financial risk associated with increased leverage adjustments to policy changes might increase the likelihood of farmers losing part of their equity or going bankrupt (Featherstone, Moss, Baker, and Preckel, 1988; Moss, Ford, and Boggess, 1989; Parcell, Featherstone, and Barton, 1998; Ramirez, Moss, and Boggess, 1997). Thus, the concept of risk balancing raises a paradoxical question that farm policies designed to reduce business risk may lead to more risk for farmers and increase the probability of farm financial failure (Featherstone, Moss, Baker, and Preckel, 1988). This is “the paradox of risk balancing”, which has been used as a theoretical argument about the futility of risk-reducing agricultural policies (Skees, 1999; Harwood, et al., 1999).

Results of the risk-balancing studies are essentially conducted under the assumption of one source of uncertainty: the rate of return on assets. Collins (1985) presented a generalized Mean-Variance (M-V) analysis (known as the Collins-Barry model) to solve for the optimal financial leverage that approximates expected utility maximization of wealth. This model has been used by many researchers to provide insights into agricultural finance and risk balancing (Collins and Karp, 1995). However, Ramirez, Moss, and Boggess (1997) pointed out that the Collins-Barry model provides little insight into the dynamic nature of the consumption/investment tradeoff. They argued that the consumption/investment decision in the dynamic model is central to the decision facing agricultural proprietors. Therefore, they applied the Merton model of lifetime portfolio-selection (Merton, 1969) to formulate a stochastic dynamic programming model to derive the optimal leverage and consumption. Capturing the same stochastic nature of the return on assets, the solution of the optimal leverage derived by Ramirez, Moss, and Boggess is essentially the same as the solution to the static Collins-Barry model (Collins and Karp, 1995).

These models typically assume that interest rates and borrowing costs are non-stochastic or known with certainty. Although many agricultural lenders may offer fixed interest rates over a short period of time, the assumption of non-stochastic interest rates is not consistent with financial theory. The level of the interest rate is a key input in capital structure models, and the optimal capital structure may be very sensitive to changes in the level of the interest rate (Leland, 1994; Goldstein, Ju, and Leland, 2001; Ju and Hui, 2006). Similarly, many lenders seek to shift interest rate risk to farmers through variable interest rates (Baker, 1984). The farm financial crisis of 1980s has provided a powerful lesson about the interactions between agricultural markets and financial markets. The period's highly volatile interest rates played a key role in the resulting farm debt crisis.

This paper argues that that a rational analysis of what is “optimal leverage” must recognize that there is uncertainty and interaction among the future rates of return on assets and interest rates. The analytical techniques of risk balancing based upon the perfect foresight of interest rates may have led to biased or incorrect predictions regarding capital structure and the role of risk reducing agricultural policies.

In this paper, we use stochastic optimal control/dynamic programming (SOC/DP) to derive the optimal debt and consumption levels under two sources of uncertainty: the return on assets and the real interest rate. The model used in the paper is based upon the work of Fleming and Stein (2004) who initially analyzed optimal debt, consumption, and endogenous growth in models of international finance and then extended their analysis to other areas of inquiry (Stein, 2004; 2005; 2006; Stein and Zheng 2007). These studies generally consider economic units, which have productive capital and also incur debt. Thus, the approach could also be applied to the agricultural economy of a state, region, or sector within a country, as well as individual farm business enterprises.

The purpose of this paper is to analyze the relationship between farm policies and farmers’ leverage decisions under multiple sources of uncertainty. The analytical framework of SOC/DP is used to develop a better understanding of how leverage decisions respond to changes in the operating environment and farm policies. The study has three interrelated research objectives: (1) investigate bias in optimal financial leverage when interest rate risk is ignored; (2) analyze how risk-reducing farm policies impact the leverage decision and, (3) determine how leverage adjustments made in response to policy changes impact farm equity returns and the variability of returns.

Ignoring interest rate risk may lead to a negative or positive bias in estimating optimal debt. The results of a numerical analysis will demonstrate situations where these biases exist. Comparative statics analysis is used to analyze the effects of risk-reducing agricultural policies on farmer's optimal leverage decision, the mean of ROE, and the variance of ROE. The comparative statics effects derived from the SOC/DP model are compared to those produced by (A) the static M-V analysis under uncertainty of the return on assets, i.e. the Collins-Barry model (Collins, 1985; Featherstone, Moss, Baker, and Preckel, 1988), (B) the SOC/DP approach under uncertainty of the return on assets (Ramirez, Moss, and Boggess, 1997), and (C) the static M-V analysis under two sources of uncertainty: the return on assets and the interest rate on debt (Barry, Baker, and Sanint, 1981; Parcell, Featherstone, and Barton, 1998). These comparisons, along with a numerical example, identify conditions in which the concept of risk balancing is violated.

The remainder of this paper is organized as follows. The second section describes the SOC/DP approach. The third section shows the comparative statics analysis. The fourth section summarizes the comparison of the comparative statics effects among the four different models. The last section presents a numerical example and conclusion.

The Stochastic Optimal Control/ Dynamic Programming Model

The SOC/DP approach is used to derive the optimal debt and consumption for the farm sector. Farmers borrow to finance investment as well as consumption and face two sources of uncertainty, the return on agricultural investment and the variable interest rate on debt. Under such uncertainty, the productivity of investment and the real interest rate are stochastic and unpredictable. Decision makers cannot predict the future state of the economic system, because there are many possible paths that the system may take given the initial conditions and their past decisions. When the future is unpredictable, the dynamic programming (DP) method is generally used to derive the optimal decisions for inter-temporal optimization problems.

Closely following the model developed by Fleming and Stein (2004), we present the DP solution for inter-temporal optimization decision model under uncertainty over an infinite horizon. The stochastic control problem associated with the SOC/DP model is formulated by specifying the control and state variables, stochastic processes, the constraints, the dynamics of the state process, and the optimization criteria. Then, the economic implications and interpretations are discussed and related to the standard Mean-Variance model.

Assume that farmers maximize the expectation (E) of the discounted value ($\delta > 0$) of the utility $U(\bullet)$ of consumption $C(t)$ generated over an infinite horizon, equation (1). The maximization is expressed as the value function $V(X)$ in equation (1), where X is the initial equity.

(1) $$V(X) = \max E\left\{\int_0^\infty U(C(t))e^{-\delta t}dt\right\}$$

Consumption, $C(t)$, is required to be positive and is defined over a period of length dt in equation (2) as net income less interest payments on the external debt less investment plus the change in debt.

(2) $$C(t)dt = Y(t)dt - r(t)L(t)dt - I(t)dt + dL(t) > 0,$$

where $Y(t)$ is net income, $r(t)$ is the real interest rate, $L(t)$ is debt, and $I(t)$ is investment. Uncertainty is introduced to the model through the interest rate and the return on assets. The real interest rate over the short period *r(t)dt* in equation (3) is the sum of a deterministic term, the mean *rdt*, plus the Browonian motion term $\sigma_1 dw_1$, with a zero mean and the variance $\sigma_1^2 dt$ over the period.

(3) $r(t)dt = rdt + \sigma_1 dw_1$,

where $dw_1 = \varepsilon_1 \sqrt{dt}$, $\varepsilon_1 \sim N(0,1)$.

The stochastic component of the real interest rate may arise from variations in monetary policy, the business cycle, etc., and thus σ_1^2 is referred to interest rate risk, one of the major components of financial risk.

The production function in equation (4) states that net income $Y(t)$ is proportional to assets $K(t)$. The ratio of net income to assets $Y(t)/K(t)$ is the rate of return on assets $b(t)$, described by stochastic process in equation (5).

(4) $Y(t) = b(t)K(t)$

(5) $b(t) = bdt + \sigma_2 dw_2$

where $dw_2 = \varepsilon_2 \sqrt{dt}$, $\varepsilon_2 \sim iid \quad N(0,1)$.

The rate of return on assets $b(t)$ is the sum of a deterministic term *bdt* plus a stochastic term $\sigma_2 dw_2$. The deterministic term is the mean rate of return on assets *b,* with no time index, and the stochastic term is the Brownian motion $\sigma_2 dw_2$, with a zero mean and a variance $\sigma_2^2 dt$. The stochastic component of the rate of return on assets may result from variations in the prices of outputs and inputs as well as in yield and/or quality caused by weather, disease, pests, management, etc. The variance of the return on assets σ_2^2 is commonly referred to "business risk".

The critical feature of this model is that the stochastic processes of Brownian motion with drift are assumed to capture the uncertainty concerning the return on assets and interest payments. The formulation allows for correlation between the two stochastic terms dw_1, dw_2 in equations (4) and (5). The general case is considered in equation (6), where the two shocks are not necessarily independent. The correlation coefficient ρ could be positive, zero or negative, depending upon the economic situation.

(6) $$E[w_1 w_2] = E[\varepsilon_1 \varepsilon_2] = \rho dt, \quad 1 \geq \rho \geq -1,$$

The state variable of this model is the level of equity $X(t)$ defined as assets, $K(t)$, less debt, $L(t)$, in equation (7). Thus, the change in equity $dX(t)$ is given in equation (8). Also, the change in assets $dK(t)$ in equation (9) is defined as the investment over the period $I(t)dt$.

(7) $$X(t) = K(t) - L(t)$$

(8) $$dX(t) = dK(t) - dL(t)$$

(9) $$dK(t) = I(t)dt$$

The change of the state variable equity $X(t)$ can also be expressed as equation (10). Substitute $dK(t)$ in equation (9) and $dL(t)$ in equation (2) into equation (8), and then apply equations (3)-(5) to obtain equation (10).

(10) $$dX(t) = [bX(t) + (b-r)L(t) - C(t)]dt + [(X(t) + L(t))\sigma_2 dw_2 - L(t)\sigma_1 dw_1]$$

The dynamics of the state variable $X(t)$ in (10) has two components. The first set of terms in square bracket is deterministic, and the second set of terms in square bracket is stochastic. The state dynamics cannot be directly controlled but is conditional upon the level of the control variables and the realizations of the return on assets and interest rate.

The Dynamic Programming Solution

The objective is to maximize the expected present value of utility equation (1) subject to the dynamic equation (10) and the constraints $(C(t) > 0, \quad X(t) > 0)$. The utility function is assumed to exhibit hyperbolic absolute risk aversion (HARA), equation (11) as $\gamma < 1$ and $\gamma \neq 0$, or equation (12) as $\gamma = 0$ and the risk aversion coefficient $(1-\gamma)$ is assumed to be positive.

(11) $\quad U(t) = (1/\gamma)C^{\gamma}(t), \quad \gamma < 1, \quad \gamma \neq 0$

(12) $\quad U(t) = \ln(C(t)), \quad \gamma = 0$

Based on the assumption of HARA utility, one can use the ratios of consumption/equity $c(t) = C(t)/X(t)$ and debt/equity $f(t) = L(t)/X(t)$ as the control variables (Fleming and Stein, 2004). Thus, the dynamics of state variable in equation (10) can be written as equation (13) in terms of the control ratios $f(t)$ and $c(t)$.

(13) $\quad dX(t) = \left[(b-c) + (b-r)f\right]X(t)dt + \left[(1+f)X(t)\sigma_2 dw_2 - fX(t)\sigma_1 dw_1\right]$

The control variables $f(t)$ and $c(t)$ are chosen based upon information known up to time *t*. The optimal controls (f, c) are derived from the dynamic programming techniques in the same way as for the continuous time Merton portfolio optimization problem (Merton, 1969). The Merton problem corresponds to the special case where the real interest rate is constant, i.e. $\sigma_1 = 0$ (equation 3). The derivation of the DP solution is presented in Stein (2006, Appendix B in Chapter 3) for details. Our analysis begins with Stein's Hamilton-Jacobi-Bellman (HJB) stochastic DP equation (14), which is a necessary and sufficient condition for optimality in the given dynamic system.

$$\begin{aligned}\delta/\gamma = {} & Max_f \left[(b-r)f - 1/2(1-\gamma)\left(f^2\sigma_1^2 - 2\rho f(1+f)\sigma_1\sigma_2 + (1+f)^2\sigma_2^2\right)\right] \\ & + Max_c \left[(1/\gamma)c^\gamma / A \quad + (b-c) \quad \right]\end{aligned} \tag{14}$$

where $A > 0$ is the constant to be determined from the solution. The HJB equation (14) has two components: a maximum with respect to debt/equity, f, and a maximum with respect to consumption/ equity, c. The optimal debt ratio f^* in (15) and consumption ratio c^* in (16) are derived from the maximization of equation (14) with respect to f and c respectively.

$$f^* = \frac{(b-r)}{(1-\gamma)\sigma^2} + \frac{(\rho\sigma_1\sigma_2 - \sigma_2^2)}{\sigma^2} \tag{15}$$

where $\sigma^2 = (\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2) > 0$ is total variance.

$$c^* = \left[\delta - \gamma(b + \Lambda_\gamma^*) \quad \right]/(1-\gamma) \tag{16}$$

where $\Lambda_\gamma^* = \left[(b-r)f^* - (1/2)(1-\gamma)\left(f^{2*}\sigma_1^2 - 2\rho f^*(1+f^*)\sigma_1\sigma_2 + (1+f^*)^2\sigma_2^2\right)\right]$.

Economic Implication and Interpretation

The optimal level of the debt/equity ratio, f^*, is positively related to the expected net return on assets $(b-r)$ and negatively related to the risk coefficient $(1-\gamma)$ and total risk σ^2. Total risk (σ^2) is the variance of the net return on assets, i.e. $Var[b(t)-r(t)]$ $=(\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2)$. The optimal debt level is positive if the expected rate of return on assets (b) exceeds the expected real interest rate (r) by an amount that depends on the variances of the return on assets and real interest rate and their correlation. The optimal debt ratio f^* in (15) is independent of the consumption/equity ratio c^* in (16), but consumption depends upon the optimal debt ratio f^*. The solution is also dependent upon the subjective parameters of risk aversion $(1-\gamma)$ and the discount rate δ. The level of risk aversion is dependent upon the individual's risk preference. Notice that the discount rate does not enter into the maximization with respect to the optimal debt ratio f^*, thus the optimal debt level is not impacted by time preferences. However, the optimal consumption ratio c^* is impacted by the discount rate.

The present SOC/DP results are generalizations of the Merton model, and also have a clear relationship to the Tobin-Markowitz Mean-Variance (M-V) analysis. M-V analysis has been widely used in the agricultural finance literature (Collins and Barry, 1986; Mapp, Hardin, Walker, and Persaud, 1979; Feldstein, 1980; Musser and Stamoulis, 1981; Yassour, 1982; Kahl, 1983; Collender and Zilberman, 1985). Fleming and Stein (2004) explain, and show graphically, how the inter-temporal dynamic programming equations can be given an interpretation in the static two-period M-V analysis. It turns out that the SOC/DP solution to the optimal debt ratio can be viewed as selecting the debt/equity ratio that maximizes the M-V expected utility. Therefore, we can apply the present SOC/DP solution to produce the optimal solution for the other three models, i.e.

the M-V model with one type of uncertainty, the SOC/DP model with one type of uncertainty, and the M-V model with two types of uncertainty. For example, setting $\sigma_1^2 = 0$, $\rho = 0$, and replacing the mean interest rate, r, with the spot interest rate, i, in equation (15) yields the optimal debt/equity ratio $f^\wedge$ in (17) for both the SOC/DP and M-V models with uncertainty concerning only the return on assets.

(17) $$f^\wedge = \frac{b-i}{(1-\gamma)\sigma_2^2} - 1$$

The solution in (17) is mathematically equivalent to Collins' equation (11) (1985, p. 629) and equation (10) of Ramirez, Moss, and Boggess (1997). In this manner, one can find that the optimal solution obtained from adjusting the parameters in the present SOC/DP model is exactly the same as the optimal solution solved for the other three models by the previous literature.

Comparative Statics

Comparative statics analysis of the SOC/DP model with two types of uncertainty is used to analyze how risk-reducing agricultural policies impact optimal leverage[1], the expected return on equity, and the variance of expected return. The following results can also be applied to analyze the comparative statics effects for the other three models associated with the risk-balancing hypothesis and "the paradox of risk-balancing". The comparative statics effects of the other three models derived from adjusting the

[1] Financial leverage in the present study is defined as the debt-to-equity ratio. The solution to optimal financial leverage in the models used to study risk balancing is most frequently defined as the debt-to-asset ratio (Collins, 1985; Featherstone, Moss, Baker, and Preckel, 1988; Ramirez, Moss, and Boggess, 1997). The results presented for all models are reported in terms of the debt to equity ratio.

parameters in the present SOC/DP model are consistent with those derived by the previous literature.

Effect of Shift in Business Risk on Leverage Choice

To review the risk-balancing hypothesis derived form the M-V model with one source of uncertainty, we differentiate the optimal debt $f^{\wedge}$ in (17) with respect to business risk (σ_2^2)

$$\text{(18)} \qquad \frac{\partial f^{\wedge}}{\partial \sigma_2^2} = -\left(\frac{b-i}{(1-\gamma)}\right)\left(\frac{1}{\sigma_2^2}\right)^2 < 0$$

The result in (18) is equivalent to Collins finding, and assuming that the mean return on assets (b) is greater than the spot interest rate (i), clearly illustrates a negative relationship between business risk and financial leverage[2]. Other things equal, farm policies designed to reduce business risk will lead to increased borrowing and financial risk.

When two sources of risk are concerned, differentiating the optimal debt ratio f^* in equation (15) with respect to σ_2^2 yields

$$\text{(19)} \qquad \frac{\partial f^*}{\partial \sigma_2^2} = \frac{1}{\sigma^2}\left(\frac{1}{2}\rho\frac{\sigma_1}{\sigma_2}-1\right)-(\frac{1}{\sigma^2})^2\left(1-\rho\frac{\sigma_1}{\sigma_2}\right)B$$

[2] Note that set $\sigma_1^2 = 0$, $\rho = 0$, $r = i$ in equation (19) can also produce equation (18).

where $B = (\left(\frac{b-r}{1-\gamma}\right) + \rho\sigma_1\sigma_2 - \sigma_2^2)$. In this situation the sign of the derivative is not clear.

Assume the net return on assets $(b-r)$>0 and f^*>0. Then the positive optimal debt ratio f^* in (15) implies $B > 0$ in (19). This leaves four possible cases to consider.

Case 1: If $\rho \le 0$, then $\frac{\partial f^*}{\partial \sigma_2^2} < 0$.

If $\rho > 0$, then the sign of $\frac{\partial f^*}{\partial \sigma_2^2}$ is ambiguous.

Case 2: If $\rho > 0$ and $\sigma_2 > \rho\sigma_1$, implying $\left(1 - \rho\frac{\sigma_1}{\sigma_2}\right) > 0$ and $\left(\frac{1}{2}\rho\frac{\sigma_1}{\sigma_2} - 1\right) < 0$,

then $\frac{\partial f^*}{\partial \sigma_2^2} < 0$.

Case 3: If $\rho > 0$ and $2\sigma_2 > \rho\sigma_1 > \sigma_2$, implying $\left(1 - \rho\frac{\sigma_1}{\sigma_2}\right) > 0$ and

$\left(\frac{1}{2}\rho\frac{\sigma_1}{\sigma_2} - 1\right) > 0$, then $\frac{\partial f^*}{\partial \sigma_2^2}$ is ambiguous.

Case 4: If $\rho > 0$ and $\rho\sigma_1 > 2\sigma_2$, implying $\left(1 - \rho\frac{\sigma_1}{\sigma_2}\right) < 0$ and $\left(\frac{1}{2}\rho\frac{\sigma_1}{\sigma_2} - 1\right) > 0$,

then $\frac{\partial f^*}{\partial \sigma_2^2} > 0$.

Cases 3 and 4 both provide instances in which risk balancing may not describe how farmers adjust financial leverage in response to changes in business risk. Case 4 is the only situation that unambiguously violates risk balancing. Here, business risk and interest rate risk are positively correlated and interest rate risk is very large relative to business risk. In fact, interest rate risk would have to be at least twice as large as business risk for this situation to occur. Case 3 could also produce situations where producers would not respond to reductions in business risk by increasing financial risk.

Effect of Shift in Business Risk on the Mean of Return on Equity

The mean rate of return on equity for the SOC/DP model concerning two sources of uncertainty $(\overline{R}_E^*)$ can be defined as the expected growth of equity, i.e. $E[dX(t)/X(t)]$. The mean of ROE in (20) is derived from the state variable dynamics in equation (13)

(20) $$\overline{R}_E^* = E\left(\frac{dX(t)}{X(t)}\right) = (b-(c^*)+(b-r)\left(f^*\right)$$

Next, differentiating the expected mean of ROE in (20) with respect to σ_2^2 yields

(21) $$\frac{\partial \overline{R}_E^*}{\partial \sigma_2^2} = (b-r)\left(\frac{\partial f^*}{\partial \sigma_2^2}\right) + \frac{\partial c^*}{\partial \sigma_2^2}$$

The sign of equation (21) is obviously ambiguous, because $\partial f^* / \partial \sigma_2^2$ could be negative or positive and the sign of $\partial c^* / \partial \sigma_2^2$ is also unclear. Thus, this qualitative result suggests that risk-reducing farm programs would may or may not increase the expected mean of ROE after the firm reaches its new leverage position.

Effect of Shift in Business Risk on the Variance of Return on Equity

The variance of the return on equity $\sigma_E^{2^*}$ in equation (22), defined as the variance of the growth of equity, i.e. $Var\left[dX(t)/X(t)\right]$, can also be derived form from the state variable dynamics in equation (13).

$$(22) \qquad \sigma_E^{2^*} = Var\left(\frac{dX(t)}{X(t)}\right) = \sigma_1^2 f^* - 2\rho\sigma_1\sigma_2 f^* \left[1+f^*\right] + \sigma_2^2\left[1+f^*\right]^2$$

Differentiating the expected variance of ROE in equation (22) with respect to σ_2^2 yields:

$$(23) \qquad \frac{\partial\sigma_E^{2^*}}{\partial\sigma_2^2} = \left[1+(2-\rho\frac{\sigma_1}{\sigma_2})f^* + (1-\rho\frac{\sigma_1}{\sigma_2})(f^*)^2\right]$$

$$+\left[2\sigma_2^2\left\{(1-\rho\frac{\sigma_1}{\sigma_2}) + \frac{1}{2}(2-\rho\frac{\sigma_1}{\sigma_2})f^* + \frac{\sigma_1^2}{\sigma_2^2}f^*\right\}\left(\frac{\partial f^*}{\partial\sigma_2^2}\right)\right]$$

Let P and Q be the terms of the first and the second square bracket in equation (23) respectively, i.e. $\frac{\partial\sigma_E^{2^*}}{\partial\sigma_2^2} = P+Q$. This situation again produces four cases for consideration.

Case 1: If $\rho \le 0$, implying $\left(\frac{\partial f^*}{\partial\sigma_2^2}\right) < 0$, $P>0$, and $Q<0$, then the sign of $\frac{\partial\sigma_E^{2^*}}{\partial\sigma_2^2}$ is unclear.

Case 2: If $\rho > 0$ and $\sigma_2 > \rho\sigma_1$, implying $\left(\frac{\partial f^*}{\partial\sigma_2^2}\right) < 0$, $P>0$, and $Q<0$, then the sign of

$\frac{\partial\sigma_E^{2^*}}{\partial\sigma_2^2}$ is unclear.

Case 3: If $\rho > 0$ and $2\sigma_2 > \rho\sigma_1 > \sigma_2$, the signs of $\left(\frac{\partial f^*}{\partial \sigma_2^2}\right)$, P, and Q are ambiguous and

the sign of $\frac{\partial \sigma_E^{2^*}}{\partial \sigma_2^2}$ is unclear.

Case 4: If $\rho > 0$ and $\rho\sigma_1 > 2\sigma_2$, implying $\left(\frac{\partial f^*}{\partial \sigma_2^2}\right) > 0$, the sign of P and Q ambiguous,

then the sign of $\frac{\partial \sigma_E^{2^*}}{\partial \sigma_2^2}$ is unclear.

The sign of equation (23) is ambiguous in all the cases, in contrast to the unequivocally negative sign of $\frac{\partial \sigma_E^2}{\partial \sigma_2^2} = -\left(\frac{b-i}{(1-\gamma)\sigma_2^2}\right)^2$ derived from the models with uncertainty concerning the return on assets only (Featherstone, Moss, Baker, and Preckel, 1988).[3] It implies that the paradox of risk balancing is not necessarily true for all possible conditions when both interest rate and returns are uncertain. Thus, the qualitative result derived from equation (23) reveals that even though risk-reducing policies lead to increased optimal leverage, the induced financial risk does not necessary more than offset the decrease in business risk, bringing more total risk to farmers. For example, given the condition of $\rho = 0$ and $\sigma_1^2 > (\frac{b-r}{1-\gamma}) > \sigma_2^2$, implying $\frac{\partial f^*}{\partial \sigma_2^2} < 0$, one can verify that $\frac{\partial \sigma_E^{2^*}}{\partial \sigma_2^2}$ in equation (23) is greater than zero. In other words, a situation can arise where farmers would increase optimal leverage in response to a reduction in business risk, but would

[3] This partial derivative is equation (7) of Featherstone, Moss, Baker, and Preckel, (1988, p.574). We also can produce it by plugging $\sigma_1^2 = 0$, $\rho = 0$, and $r = i$ into equation (23)

arrive at a level of leverage that would leave them with a lower level of variability in return on equity than before the reduction in business risk occurred.

Summary of Comparative Statics

Table 1 summarizes the sign of the comparative statics results to provide with a snapshot comparison of the four models. For the M-V model with uncertainty concerning the return on assets only, the sign of three partial derivatives ($\partial f / \partial\sigma_2^2$, $\partial\overline{R}_E / \partial\sigma_2^2$, and $\partial\sigma_E^2 / \partial\sigma_2^2$) is negative. These theoretical results state the risk balancing hypothesis and the paradox of risk balancing. Thus, farm policies designed to reduce business risk increase the optimal debt ratio, the expected value of ROE, and the variance of ROE. However, the comparative statics results derived from the other three models provide some situations that contradict the risk-balancing concept. Three important points arise from the results in Table 1.

First, the effect of changes in business risk on optimal leverage adjustments becomes ambiguous when considering uncertainty associated with interest rates and the return on assets. The impact of changes in business risk on optimal leverage ($\partial f / \partial\sigma_2^2$) for both the M-V and SOC/DP models with two sources of uncertainty in Table 1 is either uncertain or definite, depending upon the magnitudes of the interest rate risk σ_1^2, business risk σ_2^2, and the correlation between the two ρ. Given the condition $\rho > 0$ and $\rho\sigma_1 > 2\sigma_2$, the comparative statics result does not support the concept of risk balancing. Thus, contrary to risk balancing, farm policies designed to reduce business risk may not always lead to higher levels of borrowing by farmers. This would occur under occasions of relatively high interest rate risk and strong positive correlation between the interest rate risk and business risk.

Second, both the M-V and SOC/DP models with one source of uncertainty unequivocally predict that risk-reducing farm policies may lead to increased variance of the return on equity. In other words, even as the government removes business risk, farmers add back enough financial leverage that their overall risk position is increased. In contrast, when both interest rates and return on assets are uncertain, the M-V and SOC/DP models show that the effect of changes in business risk on the variance of return on equity is ambiguous in all the cases (Table 1). The ambiguous risk effect depends on the complicated relationship among the net return on assets $(b-r)$, risk aversion ($1-\gamma$), interest rate risk (σ_1^2), business risk (σ_2^2) and correlation between the risks (ρ). Furthermore, this implies that even though risk-reducing policies may lead to increased optimal leverage, the induced financial risk does not necessary more than offset the decrease in business risk, bringing more risk to farmers.

Finally, when explicitly incorporating the consumption decision in the SOC/DP models, consumption may be particularly important in determining the farms' expected return on equity. Thus, the effect of the expected return on equity to changes in business risk may partly depend on how farmers adjust consumption in response to changes in risk reducing agricultural policies. The fifth column of Table 1 shows that the sign of ($\partial \overline{R}_E / \partial \sigma_2^2$) is unclear in the SOC/DP models that consider either one or two sources of uncertainty. As the consumption/investment tradeoff is taken into account dynamically, the effect of changes in business risk on the mean of ROE cannot be determined without knowing subjective risk aversion and the magnitude of other parameters. On the other hand, the sign of ($\partial \overline{R}_E / \partial \sigma_2^2$) of the M-V models in the fourth column of Table 3 is either definite or uncertain, following the sign of ($\partial f / \partial \sigma_2^2$) of the M-V models in the second column of Table 3.

Conclusion: A Numerical Example

A numerical example is offered to demonstrate some results of the comparative statics analysis. Specifically, this example shows that the effect of changes in business risk on the variance of ROE may be negative or positive, and thus risk-reducing farm policies may not necessarily lead to more risk for farmers. In addition, this example examines the extent to which the estimates of the optimal debt level are biased when interest rate risk is ignored. This empirical study utilizes aggregated data, although the conceptual framework of the present SOC/DP can be macro or micro in nature.

Farm sector income and balance sheet data for the period of 1985 to 2003 were obtained from the Economic Research Service (ERS) of USDA. The data for value added ($Y(t)$), value of farm assets ($K(t)$), debt ($L(t)$), interest expenses ($r(t)L(t)$), and equity ($X(t)$) are all measured in constant dollars on a base 1996= 100. The estimates of the rate of return on assets ($b(t)$) and the real interest rate, ($r(t)$), are calculated by $Y(t)/K(t)$ and $r(t)L(t)/L(t)$ for each period respectively. Note that the real interest rate $r(t)$ is denoted as the spot interest rate i when no interest rate risk is assumed.

Observe that over a long horizon, parameters b and r in the model vary considerably depending upon the economic environment for agriculture (Table 2). The variances and correlations also vary over time. The analysis relies upon ten-year moving averages of $b(t)$ and $r(t)$ as the true means b and r for each period. The parameter estimates of the standard deviation of $b(t)$, σ_2, the standard deviation of $r(t)$, σ_1, and the correlation of the return on assets and real interest rate, ρ, are computed over a rolling ten-year time period. Table 2 presents the parameter estimates during the period of 1994 to 2003. The subjective parameter of risk aversion was chosen as $(1-\gamma)$ =287, which minimizes the mean squared error between the estimated optimal debt/equity ratio f^* and the actual aggregated farm debt/equity available on the ERS website.

These parameter estimates were used to calculate the optimal debt ratio and analyze the sign of the comparative statics effects for each period. Table 2 also shows the optimal debt ratios derived from the models with two sources of uncertainty (f^*) and the models with one source of uncertainty ($f^\wedge$). The numerical analysis demonstrates the substantial bias in the optimal debt estimation, caused by the perfect foresight of interest payments. The models with one source of risk predict higher leverage ratios for years 1994-1997, and 2001. In 1994, the optimal debt ratio estimates are f^*=0.303 and $f^\wedge$ =3.815. In this year, the amount of bias in the estimated debt level, calculated as $(f^\wedge - f^*)/f^*$, was 1160% (Table 3). The optimal debt ratios produced by the model with two sources of uncertainty can also be larger than those produced from the model with one source of uncertainty, such as the years 1998-2000, and 2002-3. In general, the bias comes from the difference between the net return on assets $(b-i)$ of $f^{\wedge}$ and $(b-r)$ of f^{*}, and magnitudes and interactions of multiple sources of uncertainty.

When the correlation coefficient between interest rate and business risk is nonpositive ($\rho \le 0$), the comparative statics analysis indicates that a decline in business risk warrants an increase in both the financial leverage f^{*} and $f^{\wedge}$. Hence, farm policies designed to reduce business risk will lead to an increase in financial risk, i.e. risk balancing. However, the values of the correlation coefficient estimated in Table 2 are all positive. In the macro economy, there is often a positive correlation between the return on investment and the interest rate. An increase in the return on investment stimulates an economic growth, which leads to an increase in the interest rate. A negative correlation between the return on capital and the interest rate may exist when there has been a change in monetary policy, or a financial crisis (Friedman and Schwartz, 1963).

Given the condition of $\rho > 0$ and $\sigma_2 > \rho\sigma_1$, and $1-\gamma$ =287, the empirical study through 1994-2003 does not have a situation in which the theoretical result of $(\partial f / \partial \sigma_2^2) > 0$ is found in practice (Table 3). In addition, when there is a change in business risk, the empirical example shows that the effect on the mean of ROE moves in exactly the same direction as the effect on financial leverage (Table 3). In these empirical results, risk-reducing farm policies may induce increases in financial leverage that increase the expected farm return.

In the years of 1994-1996, a positive relationship between changes in the variance of the return on equity and changes in business risk ($\partial \sigma_E^2 \ / \partial \sigma_2^2$) is observed in the model with two source of uncertainty (Table 3). The realizations of two sources of uncertainty in years 1994-1996 show that a decrease in business risk will lead to an increase in financial leverage and the mean of ROE, but a decrease in the variance of ROE. Thus, the effect of reducing business risk indirectly leads to an increase of financial risk through the leverage increase. After the farm reaches its new leverage position, the decline in total risk due to risk-reducing farm policies more than offsets the induced financial risk due to increased financial leverage, thus finally resulting in a decrease in total risk. These three cases contradict "the paradox of risk balancing". Thus, by considering multiple sources of uncertainty in the analysis, the results suggest that risk-reducing farm policies may help farmers use debt efficiently to reduce income volatility and increase the expected value of returns.

Overall, the comparative statics effects in Table 3 are consistent with the risk balancing hypothesis for all the years, and also consistent with the paradox of risk balancing for all other years except years 1994-1996. Since risk-reducing farm policies may mostly induce income volatility due to increased leverage adjustments, researchers may be concerned that government interventions may increase the risk of farm failure. Nevertheless, the risk-return tradeoff can be central to farm capital structure and managerial decision-making. When there is a leverage effect, an increase in return volatility may in turn raise the risk premium. Government interventions may possibly aid farmers to achieve the mean-variance efficiency with an optimal mix of reward and risk, providing the highest prospective return for a given level of risk or the lowest risk for a given level of expected return.

Table 1: Signs of the Comparative Statics Effect of Changes in Business Risk

	$\frac{\partial f}{\partial \sigma_2^2}$ [a]		$\frac{\partial \overline{R}_E}{\partial \sigma_2^2}$ [b]		$\frac{\partial \sigma_E^2}{\partial \sigma_2^2}$ [c]	
The Models [d]	Mean-Variance	Dynamic Programming	Mean-Variance	Dynamic Programming	Mean-Variance	Dynamic Programming
One Sources of Uncertainty						
All Cases	−	−	−	+ / −	−	−
Two Sources of Uncertainty						
Case 1: If $\rho \le 0$	−	−	−	+ / −	+ / −	+ / −
Case 2: If $\rho > 0$ & $\sigma_2 > \rho\sigma_1$	−	−	−	+ / −	+ / −	+ / −
Case 3: If $\rho > 0$ & $2\sigma_2 > \rho\sigma_1 > \sigma_2$	+ / −	+ / −	+ / −	+ / −	+ / −	+ / −
Case 4: If $\rho > 0$ & $\rho\sigma_1 > 2\sigma_2$	+	+	+	+ / −	+ / −	+ / −

a. Effect of changes in business risk (σ_2^2) on optimal leverage.

b. Effect of changes in business risk (σ_2^2) on mean of ROE ($\overline{R}_E$).

c. Effect of changes in business risk (σ_2^2) on variance of ROE (σ_E^2).

Table 2: Parameter Estimates and the Optimal Debt/Equity Ratio, 1994-2003

Year	***b(t)***	***i***	***b***	***r***	σ_2	σ_1	ρ	(f^*)	$(f^\wedge)$
1994	0.113	0.085	0.113	0.099	0.0045	0.0111	0.195	0.303	3.815
1995	0.099	0.088	0.112	0.097	0.0062	0.0111	0.465	0.468	1.170
1996	0.116	0.088	0.113	0.095	0.0063	0.0106	0.472	0.600	1.135
1997	0.106	0.084	0.112	0.093	0.0067	0.0100	0.517	0.752	1.158
1998	0.099	0.082	0.111	0.090	0.0077	0.0089	0.638	1.093	0.695
1999	0.096	0.082	0.108	0.088	0.0081	0.0066	0.566	0.729	0.361
2000	0.096	0.083	0.106	0.085	0.0081	0.0038	0.467	0.374	0.225
2001	0.095	0.073	0.104	0.083	0.0087	0.0045	0.600	0.432	0.441
2002	0.079	0.068	0.101	0.081	0.0107	0.0063	0.775	0.111	0.000
2003	0.090	0.064	0.099	0.080	0.0108	0.0084	0.774	0.426	0.037

Table 3: The Effect of Changes in Business Risk

	Debt Bias	**The Models with Two Sources of Uncertainty**			**The Models with One Source of Uncertainty**		
Year	$\frac{f^\wedge - f^*}{f^*}$	$\frac{\partial f}{\partial \sigma_2^2}$	$\frac{\partial \overline{R}_E}{\partial \sigma_2^2}$	$\frac{\partial \sigma_E^2}{\partial \sigma_2^2}$	$\frac{\partial f}{\partial \sigma_2^2}$	$\frac{\partial \overline{R}_E}{\partial \sigma_2^2}$	$\frac{\partial \sigma_E^2}{\partial \sigma_2^2}$
1994	1160%	-	-	+	-	-	-
1995	150%	-	-	+	-	-	-
1996	89%	-	-	+	-	-	-
1997	54%	-	-	-	-	-	-
1998	-36%	-	-	-	-	-	-
1999	-51%	-	-	-	-	-	-
2000	-40%	-	-	-	-	-	-
2001	2%	-	-	-	-	-	-
2002	-100%	-	-	-	-	-	-
2003	-91%	-	-	-	-	-	-

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