Groundwater Use in Asymmetric Aquifer under Incomplete Information

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Abstract

This paper analyzes a game theoretic model of groundwater extraction in an asymmetric two-cell aquifer under incomplete information about the extent to which the local stock of groundwater depends on the extraction histories at nearby wells. A novel assumption is that the elevation of the bottom of the aquifer differs across, otherwise identical, cells. Asymmetry creates a strategic advantage (disadvantage) for the user in the deep (shallow) cell in "stealing" neighbor's water. The user with a larger initial stock actually benefits from the commonality of groundwater provided that the asymmetry is not too small or too great. Assuming that the asymmetry between users is sufficiently large, better informed, non-cooperative users attain a higher joint welfare when the prior belief about the rate of transmission is sufficiently dispersed. Moreover, better hydrologic information may allow non-cooperative users to achieve maximum social welfare even in the absence of groundwater use regulations. Yet, in an asymmetric aquifer there may be both winners and losers from better public information.

Keywords: common property resource, asymmetry, groundwater, information

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1. Introduction

Groundwater use has traditionally been analyzed under the assumptions of complete information about the hydrologic properties of an aquifer. This assumption is rarely questioned since most models of exploitation of groundwater as a common property resource rely on a rather stylized representation of groundwater hydrology. As is exemplified in Gisser and Sanchez (1980), it has become standard to (a) assume that the changes in the groundwater level are transmitted instantaneously to all users, and (b) describe an aquifer as a "bathtub", i.e. a basin with parallel sides and a flat bottom. Under these assumptions, the location of users in the area overlying the aquifer where the groundwater is mined is immaterial, and a representative user exists. However, the lateral movement of groundwater is, typically, not instantaneous and may be quite slow depending on the geologic conditions, and water level (the saturated thickness) varies across wells and users.[1,2]

We continue the line of inquiry initiated in Saak and Peterson (2007) (henceforth, SP) who relaxed assumption (a) that the rate of transmission is instantaneous, and studied groundwater use under incomplete information about its magnitude.[3] As SP pointed out, due to the complexity of natural hydrologic systems and the geologic variability, users are likely to have incomplete knowledge of the velocity of lateral flows in the aquifer. Typically, each user is aware that his neighbor's water use has some influence on his future water stock, but may be uncertain about the degree of this impact. SP analyzed a simple two-period, restricted access setting with a symmetric two-cell aquifer, and found that the lack of information in an unregulated (non-cooperative) equilibrium may either increase or decrease the average rate of water use and welfare depending on the curvature of the intertemporal marginal rate of substitution for water.[4]

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[1]Groundwater flows much faster in gravels and sands than in clay or in rock fractures. For example, typical groundwater velocity in a sandy or gravelly aquifer may range from 0.5 to 50 feet per day (Harter 2003).

[2] For example, in the High Plains aquifer the water-level changes from the time prior to substantial groundwater irrigation development (circa 1950) to 2005 range between a rise of 84 feet and a decline of 277 feet (McGuire 2007).

[3] A brief review of the previous literature on groundwater exploitation as a common property resource can be found in SP. Negri (1989), Dixon (1989), and Provencher and Burt (1993) developed dynamic game-theoretic models of groundwater use in a restricted access setting. Brozovic (2003) studied the social efficient allocation in a "non-bathtub" aquifer with spatially disbursed users and finite transmissivity.

[4] There is a large literature on the effects of information on equilibrium outcomes (e.g. Boyer and Kirhlstrom 1984). While more information enables decision-makers to better tailor their actions to circumstances, it may

This paper departs from the existing literature by relaxing both assumptions (a) and (b). Generalizing SP's setting, we analyze a model of groundwater use in an asymmetric two-cell aquifer ("shallow-cell-deep-cell"), where the elevation of the bottom (bedrock) differs across, otherwise identical, cells. The focus of this paper is on four questions: (1) whether in an asymmetric aquifer all users are made worse off by the exploitation of groundwater as a common property resource, (2) how better public information about the speed of lateral flows affects the average rates of pumping of individual users, (3) when better information raises overall welfare, and (4) whether better information makes some users better off and others worse off.

Following the SP's approach, we characterize non-cooperative equilibrium outcomes in complete and incomplete information regimes, and then compare ex ante welfare of individual users in each regime. The asymmetry in the initial stocks of groundwater (saturated thickness), combined with the possibility of lateral flows, introduces non-concavity into the profit function of the user in the shallow cell. It is a novel feature that cannot arise in a symmetric aquifer under the standard assumptions about the technology. Notably, in an asymmetric aquifer, the commonality of groundwater may be a source of not only intertemporal but also distributional inefficiencies. The user in the cell with a larger initial stock has a strategic advantage in appropriating her (smaller) neighbor's stock, and in fact, benefits (at the expense of her neighbor) from the lack of "full" ownership rights to groundwater as long as the extent of asymmetry is not too small or too great.[5]

We find that the asymmetry among users is an important determinant of the effect of public information on the equilibrium outcomes and profits of individual users. Consider an

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also constrain the set of feasible choices (Eckwert and Zilcha 2000). For example, Hirschleifer (1971) demonstrated that in an exchange economy the value of information may be negative because better information decreases the scope of ex ante risk-sharing opportunities among the agents. Our line of inquiry is also related to the literature on experimentation and learning in the multi-agent setting. For example, Harrington (1995) investigates a duopoly in which firms are uncertain about the degree of product differentiation but can learn from experimentation with prices. He finds that firms' incentive to acquire more information about the extent of product differentiation (i.e. a potential externality imposed by the firms on each other) depends on their prior beliefs. Although in our setting users do not experiment to learn more about the extent to which the resource is shared (since information has no value in the second period), the externality is dynamic and the willingness to pay for public information (and hence, an incentive to experiment) differs across agents.

[5] While groundwater users typically need to own or rent the overlying land as well as a water right, neighboring users usually do not compensate each other for the gain/loss of one's water stock stemming from lateral groundwater movements (Kaiser and Skiller 2001). Due to space constraints, a characterization of the socially efficient allocation as well as an examination of how the gains from optimal groundwater management are distributed across users under different information regimes are not reported here. It can be shown that the "tragedy of the commons effect" may be reduced or enhanced by the asymmetry among users.

aquifer with (i) a sufficiently large variation in the initial stocks (bottom elevation) across cells, and (ii) a sufficiently diffuse prior probability distribution of the transmissivity parameter. Our main result is that in such an aquifer non-cooperative users achieve a higher expected joint welfare under better public information for any concave production technology. But this gain is not equally distributed among users. In fact, while the user in the shallow cell is made better off, the user in the deep cell is made worse off by better public information.

Our model can be adapted to study the exploitation of other common property resources (such as fish, wildlife, or oil) by spatially distributed and heterogeneous users who are uncertain about the degree to which the resource is non-exclusive. For example, in fisheries exclusivity is determined by the rates of biomass dispersal across space, while asymmetry among users may arise due to the initial distribution of the stock and the asymmetric density-dependent migration (Sanchirico and Wilen 2005). In such settings, public information about the properties of the resource and environment may either enhance or reduce the overall welfare when the use of the resource is not regulated. Perhaps more importantly, it may hurt some users but benefit others if asymmetry among them is significant.

The rest of the paper is organized as follows. In Section 2, we extend the SP's model to the case of an asymmetric aquifer. In Section 3, we characterize equilibrium under incomplete information about the lateral flow velocity, of which equilibrium under complete information is a special case. In Section 4, we compare the equilibrium pumping rates and expected profits of individual users in the two information regimes. In Section 5, we offer some concluding remarks and policy implications.

2. Model

We follow SP's notation. There are two periods, $t = 1,2$, and two identical users (farmers), $i = 1,2$. The model of the "shallow-cell-deep-cell" aquifer is depicted in Figure 1. In the beginning of period 1, the stocks of groundwater on farm i is $x_{i,1}$, $i = 1,2$, where $x_{1,1} \leq x_{2,1}$.[6]

[6] $x_{i,1} = ASd_i$, where AS is cell land area times storativity (which is homogeneous throughout the aquifer), and d_i is the average vertical distance from the water table to the base of the saturated zone (the saturated thickness) in cell i, $d_1 \leq d_2$. For example, in the parts of the High Plains aquifer the estimated predevelopment (between

We normalize the average initial stock of groundwater to unity, so that $x_{1,1} = 1 - s$, and $x_{2,1} = 1 + s$, where $s \in [0,1)$ is the extent of asymmetry between the initial stocks. And so, SP's model is a special case of the present setting with $s = 0$. In what follows, the first symbol, *i*, in double subscripts on variables identifies the farm and the second, *t*, identifies the period; single subscripts of functions denote first derivatives. Let $u_{i,t}$ denote the amount of groundwater pumped on farm i in period t (in the case of mixed strategies, the notation is easily adapted). The amount that can be used for irrigation on each farm cannot exceed that farm's groundwater stock:

(1) $u_{i,t} \le x_{i,t}$ for $t = 1,2$ and $i = 1,2$.

According to condition (1), the individual groundwater stock is a private resource during each irrigation season, i.e. there is no *intra-seasonal* well interference, a reasonable assumption for most aquifers and typical spatial separation of wells.[7]

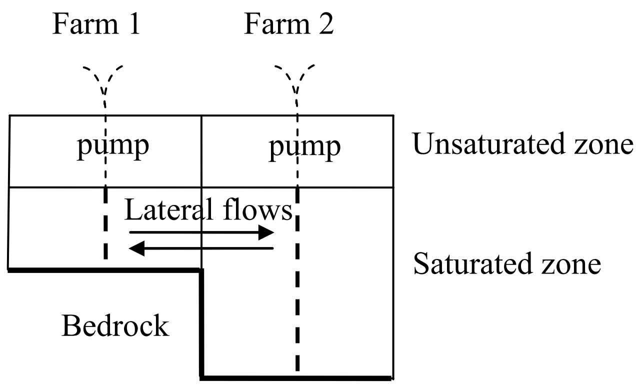

Figure 1. Hydrology of groundwater in a "shallow-cell-deep-cell" aquifer

2.1. Lateral groundwater flows

Between periods 1 and 2, groundwater will flow toward the well with the greater extraction in period 1. In particular, the inter-period flow of groundwater from farm 1 to farm 2 is given by

1940 and 1950) saturated thickness varied from less than 50 to over 300 feet (Schloss et al 2000). The initial spatial distribution of groundwater resources is determined by bedrock elevation along with land surface elevation (topography) and patterns of recharge and discharge.

[7] SP justify condition (1) by observing that, typically, groundwater flows too slowly for the extractions to interact during an irrigation season. Because the length of irrigation seasons is typically short, the "cones of depression" in the groundwater surface created by pumping at neighboring wells are not likely to intersect. Most lateral flows occur during a longer period of time that elapses between irrigation seasons. The extent to which the groundwater is an *inter-seasonal* common property depends on the velocity of lateral flows across farms.

Darcy's law:

$$Q(u_{1,1},u_{2,1})=\begin{cases} x_{1,1}-u_{1,1}, \text{ if } x_{1,1}-u_{1,1}<\alpha(u_{2,1}-u_{1,1}); \\ \alpha(u_{2,1}-u_{1,1}), \text{if -}(x_{2,1}-u_{2,1})\le\alpha(u_{2,1}-u_{1,1})\le x_{1,1}-u_{1,1}; \\ -(x_{2,1}-u_{2,1}), \text{if } \alpha(u_{2,1}-u_{1,1})<\text{-}(x_{2,1}-u_{2,1}), \end{cases}$$

where $\alpha\in[0,0.5]$ summarizes the hydrologic properties of the region, $u_{2,1}-u_{1,1}$ is the hydraulic gradient (the difference in hydraulic head between wells) in the end of period 1.[8] The flow of groundwater from farm 2 to farm 1 is $-Q$. The flow of groundwater between farms is bounded by the stocks left in each cell in the end of the irrigation season, $\text{-}(x_{2,1}-u_{2,1})\le Q\le x_{1,1}-u_{1,1}$. Because cell 1 is more shallow, $s\in[0,1)$, and the amount that can be used for irrigation on each farm in each period cannot exceed that farm's groundwater stock (see (1)), it must be that $\alpha(u_{1,1}-u_{2,1})\le x_{2,1}-u_{2,1}$, so that

$$Q(u_{1,1},u_{2,1})=\min[x_{1,1}-u_{1,1},\alpha(u_{2,1}-u_{1,1})].$$

The stocks of groundwater available in period 2 are[9]

(2) $\quad x_{1,2}(\alpha,u_{1,1},u_{2,1})=x_{1,1}-u_{1,1}-Q=\max[x_{1,1}-(1-\alpha)u_{1,1}-\alpha u_{2,1},0]$, and

$$x_{2,2}(\alpha,u_{1,1},u_{2,1})=x_{2,1}-u_{2,1}+Q=\min[x_{2,1}-(1-\alpha)u_{2,1}-\alpha u_{1,1},x_{2,1}-u_{2,1}+x_{1,1}-u_{1,1}].$$

While groundwater is always an intra-seasonally private property resource, $\alpha=0.5$ corresponds to the inter-seasonally common property resource because it implies that groundwater levels are equalized across farms in $t=2$, $x_{1,2}=x_{2,2}$, for any pumping in $t=1$, provided that $x_{1,1}>0.5(u_{1,1}+u_{2,1})$, while $\alpha=0$ corresponds to the purely private resource.

2.2 Benefits of groundwater use

The net benefits of water use on each farm is given by

(3) $\quad g(u,x)=v\big(py(u)-c(u,x)-k\big)$,

where p is the per unit price of the crop, y is yield, c is the cost of pumping groundwater, k is the cost of other farming inputs, and *v* is a utility-of-income function. An empirically

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[8] $\alpha=kZ/L$, where *k* is hydraulic conductivity, *Z* is the cross-sectional area of flow, *L* is the distance between wells on each farm (Freeze and Cherry 1979). We treat α and *s* as independent parameters. This can be justified by, for example, the distribution of the bedrock elevation within and across cells.
[9] For simplicity, we assume no aquifer recharge, although recharge could easily be incorporated in the analysis and would not change the qualitative nature of our results.

estimated specification of (3) is provided in Peterson and Ding (2005). Throughout, we assume that *g* is twice differentiable, strictly increasing, concave, and supermodular (i.e. $g_{ux} \geq 0$) over the relevant domain, $g(0,\cdot) = 0$, $g_u(0,\cdot) = \infty$, and $\lim_{u \to \infty} g_u(u,x) = 0$.[10]

2.3 Information about the hydrology of the region

Following SP, we distinguish between two information regimes. Under *complete* information, in period 1 farmers know with certainty the "speed" of lateral groundwater flow, α. Under *incomplete* information, in period 1 farmers view $\tilde{\alpha}$ as a random variable and only know its probability distribution, $\Pr(\tilde{\alpha} \leq \alpha) = H(\alpha)$, where *H* represents the variation in geologic conditions throughout the aquifer.[11] In the latter case, information is assumed to be symmetric across farmers, so that their subjective probabilities, *H*, are identical.

Farmers maximize the sum of discounted per period profits:

(4) $\quad g(u_{i,1}, x_{i,1}) + \beta g(u_{i,2}, x_{i,2}(\alpha, u_{1,1}, u_{2,1}))$ subject to (1) and (2),

where β is the discount factor, and $i = 1,2$. Let $\pi_i^c(\alpha)$ and π_i^n denote the maximum expected profits attained ex ante by the non-cooperating farmers, $i = 1,2$, respectively under complete and incomplete information about the hydrology of the region. Here superscripts "c" and "n" stand for, respectively, "complete" and "no information".

3. Equilibrium

We proceed by first characterizing equilibrium allocation under incomplete information. The equilibrium under complete information is then obtained as a special case of the incomplete information regime.

3.1. Incomplete information

In this section, we determine equilibrium pumping by both farmers when they know the probability distribution of the lateral flow speed but not the local realization of $\tilde{\alpha} \in [0, 0.5]$. For simplicity, we assume that $\tilde{\alpha}$ has a two-point probability distribution with $\Pr(\tilde{\alpha} = \alpha_L)$

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[10] The assumption that $g(u,x)$ is strictly increasing in u for all $u \in [0,x]$ reflects a situation of absolute water scarcity; all of the water remaining in period 2 will be consumed. If water is not scarce in this sense, so that $g(u,x)$ is decreasing in u for $u \in [\hat{u}, x]$, the analysis needs some modifications (e.g., a longer time horizon).

[11] There is a large variation in local hydrologic properties such as the aquifer's storativity and transmissivity values as well as well-spacing requirements that vary from 4 miles in parts of Kansas to less than 300 feet in Texas (Brozovic 2003, Kaiser and Skiller 2001). For example, the hydraulic conductivity typically ranges from 100 to 10,000 gallons per day (gpd) per square foot in sandy or gravelly aquifers (Harter 2003).

$= q$, $\Pr(\tilde{\alpha} = \alpha_H) = 1 - q$, $q \in (0,1)$, and $0 \le \alpha_L < \alpha_H \le 0.5$.[12]

In period 2, both farmers optimally exhaust the available stocks of underground water because g is increasing—i.e., $u^n_{i,2} = x_{i,2}(\tilde{\alpha}, u^n_{1,1}, u^n_{2,1})$ for $i = 1,2$. Farmer i's net benefits in t=2 depend on decisions made in t=1 by virtue of the binding (hydrologic) constraint (2), and as usual, we solve the game backwards. In t=1, farmer i chooses $u^n_{i,1}$ to maximize

(5) $\pi^n_i = \max_{u^n_{i,1}} g(u_{i,1}, x_{i,1}) + \beta E[b(x_{i,2}(\tilde{\alpha}, u^n_{1,1}, u^n_{2,1}))]$ subject to (1) and (2),

where $b(x) = g(x,x)$ is the periodic profit for a farmer who consumes his entire stock x. The cell asymmetry reveals itself in the pumping decision calculus of farmer 1 and 2. Farmer 1's optimal response to a higher pumping rate by his neighbor is, in general, non-monotone (with discontinuous jumps). However, farmer 2's optimal response to a higher pumping rate by her neighbor is invariably to lower her own pumping rate.

Best response by farmer 1

Next we characterize the best response by farmer 1 when his neighbor pumps $u_{2,1}$. Let

(6) $$\pi^{nK}_1(u_{2,1}) = \begin{cases} \max_{u \le x_{1,1}} \{g(u, x_{1,1}) + \beta E[b(x_{1,1} - (1-\tilde{\alpha})u - \tilde{\alpha}u_{2,1}) \mid \tilde{\alpha} \le \alpha_K] \Pr(\tilde{\alpha} \le \alpha_K)\}, \text{if } u_{2,1} < \dfrac{x_{1,1}}{\alpha_K}; \\ 0, \text{if } u_{2,1} \ge x_{1,1}/\alpha_K \end{cases}$$

denote the maximum expected profits achieved by farmer 1 when cell 1 has water in $t = 2$ for $\tilde{\alpha} \le \alpha_K$, $K = H, L$. Here the superscripts "*H*" and "*L*" stand for "high" and "low" speed of lateral flow.[13] Let $u^{nK}_{1,1}(u_{2,1})$ for $u_{2,1} \le x_{1,1}/\alpha_K$, $K = H, L$, denote the maximizer of (6), which is the (unique) solution to the FOC:

(7) $g_u(u^{nK}_{1,1}, x_{1,1}) - \beta E[(1-\tilde{\alpha})b'(x_{1,1} - (1-\tilde{\alpha})u^{nK}_{1,1} - \tilde{\alpha}u_{2,1}) \mid \tilde{\alpha} \le \alpha_K] \Pr(\tilde{\alpha} \le \alpha_K) \ge 0$, (=0 if $u^{nK}_{1,1} < x_{1,1}$).

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[12] A generalization where $\tilde{\alpha}$ has a finite support, $0 \le \alpha_1 < \ldots < \alpha_n \le 0.5$ and $\Pr(\tilde{\alpha} = \alpha_i) > 0$ for $i = 1, \ldots n$, is straightforward, but will complicate the notation.

[13] If in t=2 the stock in cell 1 is positive when the speed is high, $\tilde{\alpha} = \alpha_H$, it must also be positive when the lateral flow is slower, $\tilde{\alpha} = \alpha_L$, i.e. $x_{1,1} - (1-\alpha_H)u_{1,1} - \alpha_H u_{2,1} \ge 0$ implies that $x_{1,1} - (1-\alpha_L)u_{1,1} - \alpha_L u_{2,1} > 0$ since $u_{i,1} \le x_{i,1}$ and $x_{1,1} < x_{2,1}$. Superscript "*H*" symbolizes that even when the lateral outflow from cell 1 is fast, $\tilde{\alpha} = \alpha_H$, there is groundwater left in cell 1 in t=2. Superscript "*L*" symbolizes that only when the lateral flow is slow, $\tilde{\alpha} = \alpha_L$, there is groundwater left in cell 1 in t=2, but cell 1 is empty if the outflow is fast, $\tilde{\alpha} = \alpha_H$.

Differentiating (7) yields $du_{1,1}^{nK}(u_{2,1})/du_{2,1} \in (-1,0)$, $K = H, L$, because g is concave and $\alpha_K \in [0,0.5]$. Also, let $\pi_1^{nD}(u_{2,1}) = b(x_{1,1}) + \beta E[b(\tilde{\alpha}\max[x_{1,1} - u_{2,1}, 0])]$ denote the profits achieved by farmer 1 when he consumes his entire stock in period 1. Here superscript "*D*" stands for "dry well".

In addition, we define the following three threshold levels of pumping by farmer 2 that leave farmer 1 indifferent between pumping $u_{1,1}^{nH}(u_{2,1})$, $u_{1,1}^{nL}(u_{2,1})$, or $x_{1,1}$. Let $\hat{u}_{2,1}^{nKM}$ be (uniquely) determined by the equation

(8) $\qquad \pi_1^{nK}(\hat{u}_{2,1}^{nKM}) = \pi_1^{nM}(\hat{u}_{2,1}^{nKM})$ for $K = H, L$, $M = L, D$, $K \neq M$.

Lemma 1. (Best response by farmer 1) *Under incomplete information, the best response by farmer 1 is an upper hemicontinuous correspondence, and it is given by*

(9) $$u_{1,1}^{nBR}(u_{2,1}) = \begin{cases} u_{1,1}^{nH}(u_{2,1}), \text{if } u_{2,1} \le \min[\hat{u}_{2,1}^{nHL}, \hat{u}_{2,1}^{nHD}]; \\ u_{1,1}^{nL}(u_{2,1}), \text{if } \hat{u}_{2,1}^{nHL} \le u_{2,1} \le \hat{u}_{2,1}^{nLD}; \\ x_{1,1}, \text{ if } u_{2,1} \ge \max[\hat{u}_{2,1}^{nLD}, \hat{u}_{2,1}^{nHD}]. \end{cases}$$

Lemma 1 shows that farmer 1 may increase or decrease his pumping in response to an increase in pumping by his neighbor. To illustrate, suppose that either farmer 2's pumping rate, $u_{2,1}$ is sufficiently small (e.g. $u_{2,1} \le x_{1,1}$) and/or groundwater never flows laterally too fast (i.e., α_H is small). Then the (private) considerations of intertemporal efficiency prescribe that farmer 1 save enough of his stock in *t*=1 to have a positive stock in *t*=2 even when $\tilde{\alpha} = \alpha_H$. For a slightly higher neighbor's pumping, again based on the considerations of intertemporal efficiency, farmer 1's optimal response is to pump less since water becomes, on average, more scarce in *t*=2 due to a greater expected outflow. On the other hand, as his neighbor's pumping continues to increase, farmer 1 will eventually find it optimal to switch to a higher pumping rate, and let his well go dry in *t*=2 whenever the actual speed of lateral flow is high. If farmer 2's pumping continues to increase, farmer 1 will eventually find it optimal to consume his entire stock in *t*=1, and let his well go dry in *t*=2 no matter what the actual speed of lateral flow happens to be.

Note that $\hat{u}_{2,1}^{nHL}$, $\hat{u}_{2,1}^{nLD}$, and $\hat{u}_{2,1}^{nHD}$ depend on the probability distribution of $\tilde{\alpha}$ and the extent of asymmetry in initial stocks, s. Farmer 1 never depletes his stock in *t*=1 if $\alpha_L = 0$

and $q > 0$ because, by (8), $\lim_{\alpha_L \to 0} \hat{u}_{2,1}^{nLD} = \infty$. If there is a strictly positive probability that his stock is, in fact, fully private, farmer 1 always saves some groundwater for future use. On the other hand, suppose that $\alpha_L > 0$, but q and $\alpha_H - \alpha_L$ are small. Then, as his neighbor's pumping increases, farmer 1, who previously pumped $u_{1,1}^{nH}(u_{2,1})$ and saved enough stock to withstand any outflow, may start pumping $x_{1,1}$ and let his well go dry in *t*=2 with certainty.

Best response by farmer 2

Next we consider farmer 2's best response. Let $d(u,\alpha) = (x_{1,1} - (1-\alpha)u)/\alpha$ for $\alpha > 0$, denote the minimum pumping by farmer 2 that leaves his neighbor, who pumps u, with no groundwater in *t*=2. Let $u_{2,1}^{nH}(u_{1,1})$ denote the optimal pumping by farmer 2, when farmer 1 has a positive stock in *t*=2 for any $\tilde{\alpha}$. By (5), it is (uniquely) determined by the FOC

(10) $\quad g_u(u_{2,1}^{nH}, x_{2,1}) + \beta E[(1-\tilde{\alpha})b'(x_{2,1} - (1-\tilde{\alpha})u_{1,1} - \tilde{\alpha}u_{2,1}^{nH})] \geq 0$ (=0, if $u_{2,1} \leq \min[d(u_{1,1}, \alpha_H), x_{2,1}]$).

For $d(u_{1,1}, \alpha_H) \leq x_{2.1}$, let $u_{2,1}^{nL}(u_{1,1}) \in [d(u_{1,1}, \alpha_H), d(u_{1,1}, \alpha_L)]$ denote the optimal pumping by farmer 2 when cell 1 has water in *t*=2 only if $\tilde{\alpha} = \alpha_L$. By (5), it is (uniquely) determined by the FOC

$$(11) \quad g_u(u_{2,1}^{nL}, x_{2,1}) + \beta\{q(1-\alpha_L)b'(x_{2,1} - (1-\alpha_L)u_{2,1}^{nL} - \alpha_L u_{1,1}) \\ + (1-q)b'(x_{1,1} - u_{1,1} + x_{2,1} - u_{2,1}^{nL})\} \begin{cases} < 0, \text{if } u_{2,1}^{nL} = d(u_{1,1}, \alpha_H), \\ > 0, \text{if } u_{2,1}^{nL} = \min[d(u_{1,1}, \alpha_L), x_{2,1}], \\ = 0, \text{if otherwise.} \end{cases}$$

Finally, for $d(u_{1,1}, \alpha_L) \leq x_{2.1}$, let $u_{2,1}^{nD}(u_{1,1})$ denote the optimal pumping by farmer 2 when cell 1 is always empty in *t*=2. By (5), it is (uniquely) determined by the FOC

$$(12) \quad g_u(u^d, x_{2,1}) + \beta b'(x_{1,1} - u_{1,1} + x_{2,1} - u^d) \begin{cases} < 0, \text{if } u_{2,1}^{nD} = d(u_{1,1}, \alpha_L), \\ > 0, \text{if } u_{2,1}^{nD} = x_{2,1} \\ = 0, \text{if otherwise.} \end{cases}$$

Differentiation of the FOCs in (10)-(12) establishes that $du_{2,1}^{nK}(u_{1,1})/du_{1,1} \in (-1,0]$ for $k = H, L, D$. The next lemma shows that, provided that the asymmetry is sufficiently large, the best responses by farmer 1 and 2 are very different.

Lemma 2. (Best response by farmer 2) *The best response by farmer 2 is a continuous non-increasing single-valued function, and it is given by*

$$
(13) \quad u_{2,1}^{nBR}(u_{1,1}) = \begin{cases} u_{2,1}^{nH}(u_{1,1}), \text{if either (a) } x_{2,1} < d(u_1, \alpha_H), \text{or (b) } x_{2,1} \geq d(u_1, \alpha_H), \text{ and} \\ \qquad g_u(d(u_1, \alpha_H), x_{2,1}) \leq \beta E[(1-\tilde{\alpha})b'(x_{2,2}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))]; \\ u_{2,1}^{nL}(u_{1,1}), \text{if } g_u(d(u_{1,1}, \alpha_H), x_{2,1}) \geq \beta E[(1-\tilde{\alpha})b'(x_{2,2}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))] \\ \qquad \text{and either (a) } d(u_{1,1}, \alpha_H) < x_{2,1} < d(u_{1,1}, \alpha_L), \text{or (b) } x_{2,1} \geq d(u_{1,1}, \alpha_L), \\ \qquad \text{and } g_u(d(u_{1,1}, \alpha_L), x_{2,1}) \leq \beta(1-q\alpha_L)b'(x_{2,2}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L))); \\ u_{2,1}^{nD}(u_{1,1}), \text{if } x_{2,1} > d(u_{1,1}, \alpha_L) \text{ and } g_u(d(u_{1,1}, \alpha_L), x_{2,1}) \\ \qquad \geq \beta\{(1-q\alpha_L)b'(x_{1,1} - u_{1,1} + x_{2,1} - d(u_{1.1}, \alpha_L)). \end{cases}
$$

Unlike farmer 1, farmer 2 never pumps more in response to an increase in her neighbor's pumping. The best response by farmer 2 is a continuous non-increasing function that may exhibit flat sections where her pumping remains unchanged for different rates of extraction by her neighbor.

3.1.1 Characterization of non-cooperative equilibrium

First, we establish the existence and a basic property of equilibrium under incomplete information.

Lemma 3. (Existence) *Under incomplete information, the Nash equilibrium (possibly in mixed strategies) exists, and farmer 1 pumps less than farmer 2 in period 1, i.e.* $\Pr(\tilde{u}_{1,1}^n \leq u_{2,1}^n) = 1$ *(with strict inequality for all* $s \in (0,1)$*), where either* $\Pr(\tilde{u}_{1,1}^n < 1-s) = 1$ *or* $u_{2,1}^n < 1+s$ *(or both).*

Also note that in equilibrium farmer 2 earns higher profits than farmer 1, i.e. $\pi_{1,1}^n < \pi_{2,1}^n$ since asymmetry in bottom elevation bestows a "double" benefit (loss) on farmer 2 (1): a direct benefit (loss) due to a larger (smaller) initial stock, and an indirect benefit (loss) due to the strategic advantage (disadvantage) in "stealing" water from cell 1 (2) or preserving her (his) initial stock for her (his) own use in period 2.

Next we show that equilibrium under incomplete information is essentially always unique and offer a characterization. The equilibrium is described by a partitioning of the interval of asymmetry levels, $[0,1)$, into at most five sub-intervals. Within each sub-interval farmer 1 saves enough water to prevent the total loss of his stock in *t*=2 due to an outflow of any speed, or just of slow speed, or consumes his entire stock in *t*=1, or randomizes if his optimal response is non-unique.

Proposition 1. (Uniqueness and characterization) *Under incomplete information, equilibrium is unique except on a set of parameters of zero measure. Suppose that* $g_{uu}(u,x)+g_{ux}(u,x) \leq 0$ *for all* $u \leq x \leq 1$.[14] *There exists* $\alpha_0 \in (0,\alpha_H)$ *such that for any* $\alpha_L \in [0,\alpha_0]$*, there are at most four threshold levels of asymmetry,* $0 < s^{nH} \leq s^{nHL} < s^{nL} \leq s^{nLD} \leq 1$*, and the equilibrium pumping rates are given by*

- $u_{1,1}^n = u_{1,1}^{nH}(u_{2,1}^n) < 1-s$ *and* $u_{2,1}^n = u_{2,1}^{nH}(u_{1,1}^n) \in [0.5(1+s), 1+s]$*, if* $s \in [0, s^{nH}]$*;*
- $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})) = p^{nHL}$, $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})) = 1-p^{nHL}$, $u_{2,1}^n = \hat{u}_{2,1}^{nHL}$*, if* $s \in (s^{nH}, s^{nHL})$*;*
- $u_{1,1}^n = u_{1,1}^{nL}(u_{2,1}^n) < 1-s$ *and* $u_{2,1}^n = u_{2,1}^{nL}(u_{1,1}^n) \in [0.5(1+s), 1+s)$*, if* $s \in [s^{nHL}, s^{nL}]$*;*
- $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})) = p^{nLD}$, $\Pr(\tilde{u}_{1,1}^n = 1-s) = 1-p^{nLD}$ *, and* $u_{2,1}^n = \hat{u}_{2,1}^{nLD}$*, if* $s \in (s^{nL}, s^{nLD})$*;*
- $u_{1,1}^n = 1-s$ *and* $u_{2,1}^n = u_{2,1}^{nD}(1-s)$*, if* $s \in [s^{nLD}, 1]$.

For $s \in (0, s^{nLD})$, the strategic advantage of farmer 2 enables her to, at least sometimes, "steal" her neighbor's water, i.e. $E[\tilde{u}_{1,1}^n + x_{1,2}(\tilde{\alpha}, \tilde{u}_{1,1}^n, u_{2,1}^n)] < x_{1,1} < x_{2,1} < u_{2,1}^n + E[x_{2,2}(\tilde{\alpha}, \tilde{u}_{1,1}^n, u_{2,1}^n)]$, or forces farmer 1 to consume his entire stock in *t*=1, $u_{1,1}^n = x_{1,1}$, for $s \in [s^{nLD}, 1]$. Also, we remark that in equilibrium farmer 2's pumping rate is the same when either (a) groundwater is a fully private resource, i.e. $\Pr(\tilde{\alpha}=0)=1$, or (b) the differential in bottom elevation is large, i.e. $s \geq s^{nLD}$. Under either of these two circumstances, groundwater becomes a private resource from the point of view of farmer 2, albeit for distinct reasons. While lateral flows could occur in case (b), farmer 2 would incur a prohibitively high loss in the intertemporal efficiency of allocation of her own stock if she lowered her pumping enough to induce farmer 1 to pump less thus generating an inflow into cell 2.

Keeping everything else equal, as the extent of asymmetry increases, farmer 1 is more likely to take on the risk of letting his well run out of water in *t*=2 (see Table 1). For any $s \in [0, s^{nH}]$ (respectively, $s \in (s^{nH}, s^{nHL})$, $s \in [s^{nHL}, s^{nL}]$, $s \in (s^{nL}, s^{nLD})$, or $s \in [s^{nLD}, 1)$), farmer 1 saves enough water in *t* = 1 to guarantee access to groundwater in $t = 2$ for any speed of lateral flow $\tilde{\alpha}$ (respectively, he sometimes (always) takes on the risk of having an

[14] That is, in the zero probability events that either $s = s^{nH} = s^{nHL} = \hat{u}_{2,1}^{nHL} - 1$, or $s = s^{nL} = s^{nLD} = \hat{u}_{2,1}^{nLD} - 1$, there exist multiple probabilities with which farmer 1 randomizes between the profit-maximizing pumping rates in a mixed strategy equilibrium.

empty cell in $t = 2$ when $\tilde{\alpha} = \alpha_H$, $\tilde{\alpha} = \alpha_L$). As is demonstrated in the following example, the equilibrium pumping rates vary with the extent of asymmetry in a non-monotone manner.

Table 1. Water availability in *t*=2 and asymmetry under incomplete information

Asymmetry	$[0, s^{nH}]$	(s^{nH}, s^{nHL})	$[s^{nHL}, s^{nL}]$	(s^{nL}, s^{nLD})	$[s^{nLD}, 1]$
$\Pr(x_{1,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^n) > 0)$	1	1	1	p^{nLD}	0
$\Pr(x_{1,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^n) > 0)$	1	p^{nHL}	0	0	0

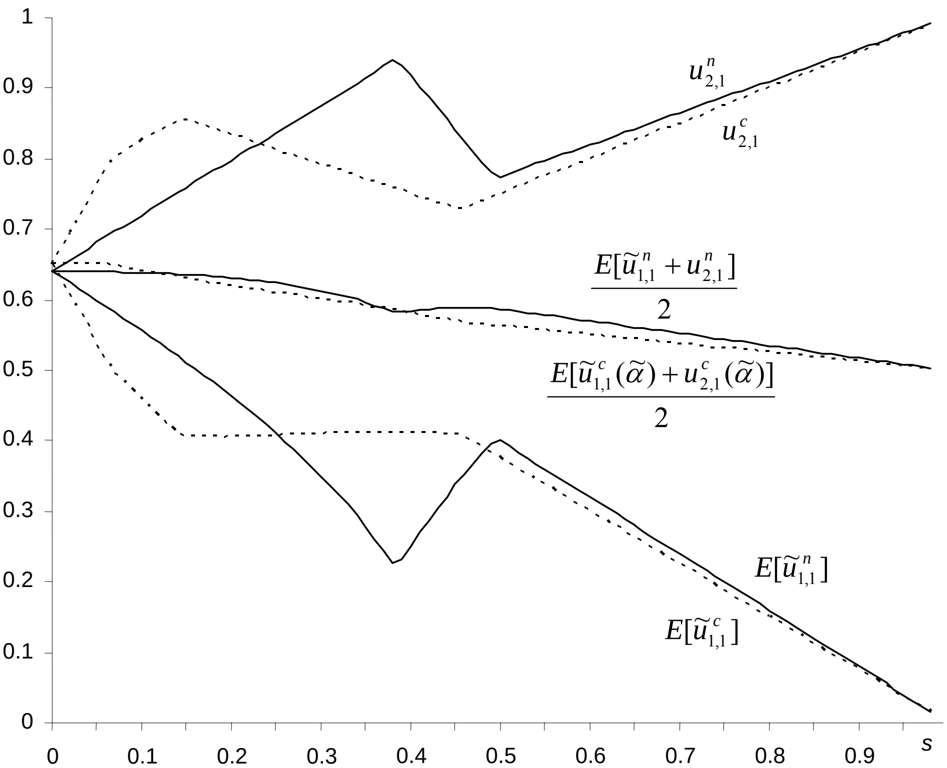

Figure 2. Pumping under incomplete and complete information in asymmetric aquifer

Example 1. (Pumping under incomplete information) Let $g(u) = \sqrt{u}$, $\alpha_L = 0$, $\alpha_H = 0.5$, $q = 0.5$, and $\beta = 1$. Applying Proposition 1, we find that $\hat{u}_{2,1}^{nLD} = \infty$, $\hat{u}_{2,1}^{nHL} < \hat{u}_{2,1}^{nHD}$, $s^{nH} = 0.38$, $s^{nHL} = 0.5$, and $s^{nL} = s^{nLD} = 1$. The expected equilibrium pumping rates under incomplete information for different levels of asymmetry are shown in Figure 2.

For $s \in [0, 0.38]$, as the asymmetry increases, farmer 2 capitalizes on her strategic advantage of having a larger initial stock, and aggressively raises her pumping rate. Farmer 1 is in the "accommodating mode" in which his consumption in $t=1$ declines with the asymmetry from 64% to 36% of his stock as a precaution against possible outflow in case $\tilde{\alpha}=\alpha_H$. However, when the asymmetry reaches the threshold level, $s^{nHL}=0.38$, farmer 1 is indifferent between the "accommodating mode" and the "cut-water-loss mode" in which he consumes 80% of his stock in $t=1$. As s increases in the interval $[0.38, 0.5]$, farmer 2's pumping drops from 67% to 52% of her stock, since her gains from a more efficient intertemporal allocation begin to dominate the gains from appropriating her neighbor's water (recall that the initial stock in cell 1 is shrinking). At $s=s^{nL}=0.5$, farmer 1 consumes 30% (= 80% - 50%) more of his stock than he would were the two cells hydrologically disconnected, while the analogous figure for farmer 2 is just 1.6% (= 51.6% - 50%). For $s \in [0.5,1)$, as s increases, farmer 2's pumping approaches 50% of her initial stock, while farmer 1's pumping remains unchanged at 80%. Because there is a positive probability that groundwater is fully private, i.e. $\Pr(\tilde{\alpha}=0)>0$, neither farmer consumes his or her entire stock in period 1 for any degree of asymmetry (this will not be the case in Example 2 in the next subsection).

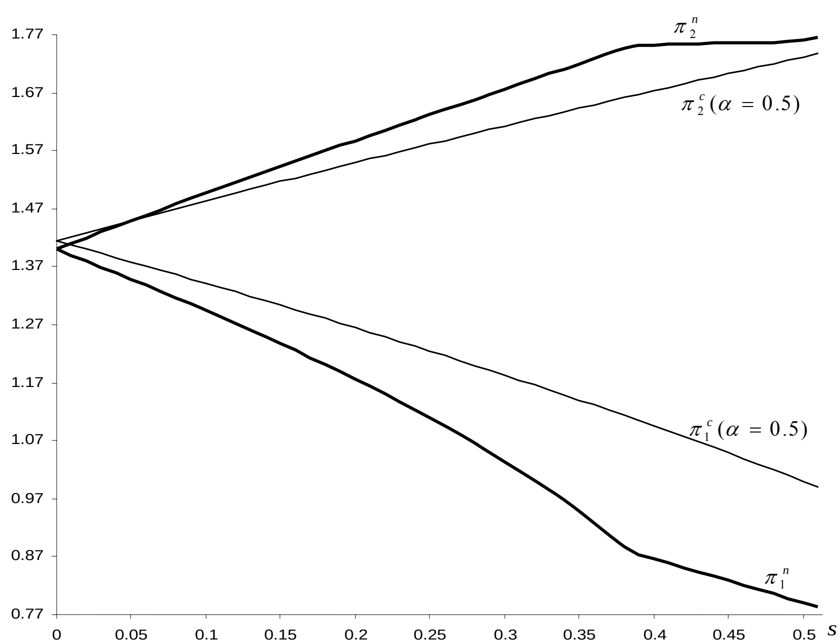

Figure 3. Profits under incomplete information and when it is known that $\tilde{\alpha}=0$

Additionally, let us examine how non-cooperative farmers 1 and 2 fare under uncertainty about $\tilde{\alpha}$ relative to the situation in which groundwater is a fully private resource (see Figure 3). When asymmetry is small, $s \in [0,0.05]$, both farmers suffer from the common pool nature of groundwater, i.e. their expected profits are smaller than the profits they would achieve were the hydrologic link between cells severed ($\tilde{\alpha} = 0$). However, for $s \in [0.05,1)$ the strategic advantage of farmer 2 allows her to appropriate, on average, enough of farmer 1's stock to compensate for inefficiency in the intertemporal allocation of her own stock due to uncertain inflow. Thus only farmer 1 always loses from the lack of "full" ownership rights to groundwater. ■

Next we consider the equilibrium under complete information.

3. 2. Complete information

Equilibrium pumping under complete information about the speed of lateral flow, $\Pr(\tilde{\alpha} = \alpha) = 1$ for some $\alpha \in [0,0.5]$, can be obtained as a special case of equilibrium characterized above by setting $\alpha_H = \alpha$ and $q = 0$.[15] Then, by Lemma 1, $u_{1,1}^{nL}(.) = 1 - s$, $\hat{u}_{2,1}^{nHL} = \hat{u}_{2,1}^{nHD} \equiv \hat{u}_{2,1}^{c}(\alpha)$, which, by (8), is (uniquely) determined by

(14) $$g(u_{1,1}^{cN}(\hat{u}_{2,1}^{c}), x_{1,1}) + \beta b(x_{1,1} - (1-\alpha)u_{1,1}^{cN}(\hat{u}_{2,1}^{c}) - \alpha\hat{u}_{2,1}^{c}) = b(x_{1,1}),$$

where $u_{1,1}^{cN}(u_{2,1}, \alpha) = u_{1,1}^{nH}(u_{2,1}; \alpha_H = \alpha, q = 0)$, and "*N*" stands for "not a dry well". From (14) it follows that $\hat{u}_{2,1}^{c}$ is decreasing in α. The best response by farmer 1, (9), becomes

(15) $$u_{1,1}^{cBR}(u_{2,1}^{c}, \alpha) = \begin{cases} u_{1,1}^{cN}(u_{2,1}, \alpha), \text{if } u_{2,1} \le \hat{u}_{2,1}^{c}(\alpha); \\ x_{1,1}, \text{if } u_{2,1} \ge \hat{u}_{2,1}^{c}(\alpha). \end{cases}$$

Turning to farmer 2's best response, by Lemma 2, we have $u_{2,1}^{nL}(.) = u_{2,1}^{nD}(.)$ when $\alpha_H = \alpha$ and $q = 0$, so that under complete information (13) becomes

(16) $$u_{2,1}^{cBR}(u_{1,1}; \alpha) = \begin{cases} u_{2,1}^{cD}(u_{1,1}), \text{if } d(u_{1,1}, \alpha) < x_{2,1} \text{ and } g_u(d(u_{1,1}, \alpha), x_{2,1}) \ge \beta b'(2 - u_{1,1} - d(u_{1.1}, \alpha)); \\ u_{2,1}^{cN}(u_{1,1}), \text{if otherwise,} \end{cases}$$

where $u_{2,1}^{cD}(u_{1,1}; \alpha) = u_{2,1}^{nD}(u_{1,1}; \alpha_H = \alpha, q = 0)$ and $u_{2,1}^{cN}(u_{1,1}; \alpha) = u_{2,1}^{nH}(u_{1,1}; \alpha_H = \alpha, q = 0)$.

By Lemma 3, the ordering of the pumping rates in *t*=1 and profits is preserved under complete information, i.e. $u_{1,1}^{c}(\alpha) < u_{2,1}^{c}(\alpha)$ and $\pi_1^{c}(\alpha) < \pi_2^{c}(\alpha)$ for $s \in (0,1)$. The

[15] Of course, we can also obtain equilibrium under complete information by setting $q = 1$ or $\alpha_L = \alpha_H$.

characterization of non-cooperative equilibrium under complete information is also obtained as a special case of Proposition 1.

Proposition 2. (Pumping under complete information) *Under complete information, equilibrium exists and it is unique. Suppose that* $g_{uu}(u,x)+g_{ux}(u,x)\le 0$ *for all* $u\le x\le 1$. *There exist at most two threshold levels of asymmetry,* $0<s^{cN}(\alpha)\le s^{cD}(\alpha)\le 1$, *such that the pumping rates are given by* $u_{1,1}^{c}=u_{1,1}^{cN}(u_{2,1}^{c}), u_{2,1}^{c}=u_{2,1}^{cN}(u_{1,1}^{c})$ *if* $s\in[0,s^{cN}(\alpha)]$;
$\Pr(\tilde{u}_{1,1}^{c}=u_{1,1}^{cN}(\hat{u}_{2,1}^{c}(\alpha)))=p^{c}$, $\Pr(\tilde{u}_{1,1}^{c}=1-s)=1-p^{c}$, *and* $u_{2,1}^{c}=\hat{u}_{2,1}^{c}$ *if* $s\in(s^{cN}(\alpha),s^{cD}(\alpha))$;
$u_{1,1}^{c}=1-s$, $u_{2,1}^{n}=u_{2,1}^{cD}(1-a)$ *if* $s\in[s^{cD}(\alpha),1]$, *where* $s^{cN}=\inf\{s\in[0,1]: s\ge\hat{u}_{2,1}^{c}-1,$
$g_u(\hat{u}_{2,1}^{c},1+s)\ge\beta\ (1-\alpha)b'(1+s-(1-\alpha)\hat{u}_{2,1}^{c}-\alpha u_{1,1}^{cN}(\hat{u}_{2,1}^{c}))]\}$, *and* $s^{cD}=\inf\{s\in[0,1]:$
$g_u(\hat{u}_{2,1}^{c},1+s)\ge\beta b'(1+s-\hat{u}_{2,1}^{c})\}$.

The following example illustrates.

Example 2. (Pumping under complete information) Consider the environment from Example 1, except now we assume that both farmers know the precise value of α. First, let $\alpha=\alpha_H$ $=0.5$. Applying Proposition 2, we find that $s^{cN}(\alpha_H)=\hat{u}_{2,1}^{c}-1\approx 0.14$ and $s^{cD}(\alpha_H)\approx 0.46$. When the extent of asymmetry is small, $s\in[0,0.07]$, the strategic advantage of farmer 2 is not a dominant factor, and both farmers save some water in $t=1$, $u_{1,1}^{c}=u_{1,1}^{cN}(u_{2,1}^{c})$ $=(4(1-s)-2u_{2,1}^{c})/3=0.8(1-5s)$ and $u_{2,1}^{c}=u_{2,1}^{cN}(u_{1,1}^{c})=(4(1+s)-2u_{1,1}^{c})/3\ =0.8(1+s)$. When the extent of asymmetry is greater, $s\in[0.07,0.14]$, farmer 2 fully capitalizes on her strategic advantage of having a larger initial stock as she consumes all of it in $t=1$, $u_{1,1}^{c}=u_{1,1}^{cN}(u_{2,1}^{c})=(2-6s)/3$, and $u_{2,1}^{c}=1+s$.[16] At *s*=0.14, the total consumption of farmer 2

[16] Note that the equilibrium outcome in which farmer 2 consumes her entire stock in *t*=1, and farmer 1 effectively saves a portion of his stock for both users, is not specific to this example. Applying Proposition 2, and using the best responses in (15) and (16), it follows that $u_{1,1}^{c}=u_{1}^{cN}(x_{2,1})<x_{1,1}$, and $u_{2,1}^{c}=u_{2,1}^{cN}(u_{1,1}^{c})=x_{2,1}$ is the unique equilibrium, if $x_{2,1}<\hat{u}_{2,1}^{c}$ and $g_u(x_{2,1},x_{2,1})\ge\beta(1-\alpha)\ b'(\alpha(x_{2,1}-u_{1}^{cN}(x_{2,1})))$. This is a noteworthy difference between the present setting and SP's model. As explained in footnote 9 in SP, in a symmetric aquifer, complete rent dissipation never arises in non-cooperative equilibrium due to restricted access. As long as the asymmetry across cells is small, movement of groundwater between cells is limited even when the velocity of lateral flow is (inter-seasonally) instantaneous. As a result, because each user benefits from a more

reaches 133% of her initial stock, while the total consumption of farmer 1 declines to just 56% of his initial stock. At this level of asymmetry, the strategic advantage of farmer 2 plateaus because the lateral flow cannot exceed the quantity of water that remains in cell 1 in the end of period 1. For $s \in (0.14, 0.46)$, farmer 1 randomizes between pumping $u_{1,1}^c = u_{1,1}^{cN}(\hat{u}_{2,1}^c) = (4 - 2\hat{u}_{2,1}^c(s))/3$ with probability $p^c(s) = [(1 + s - \hat{u}_{2,1}^c(s))^{-0.5} - (\hat{u}_{2,1}^c(s))^{-0.5}]$ $/[(1 + s - \hat{u}_{2,1}^c(s))^{-0.5} - 0.5((1+5s)/3 - \hat{u}_{2,1}^c(s)/6)^{-0.5}]$, and $u_{1,1}^c = 1 - s$ with probability $1 - p^c$, and $u_{2,1}^c = \hat{u}_{2,1}^c(s)$. Finally, for $s \in [0.46, 1)$, the considerations of intertemporal efficiency dominate farmer 2's pumping decision, which, given her large stock, implies that farmer 1 is not able to save any water for period 2 due to outflow, $u_{1,1}^c = 1 - s$ and $u_{2,1}^c = 0.5(1 + s)$.

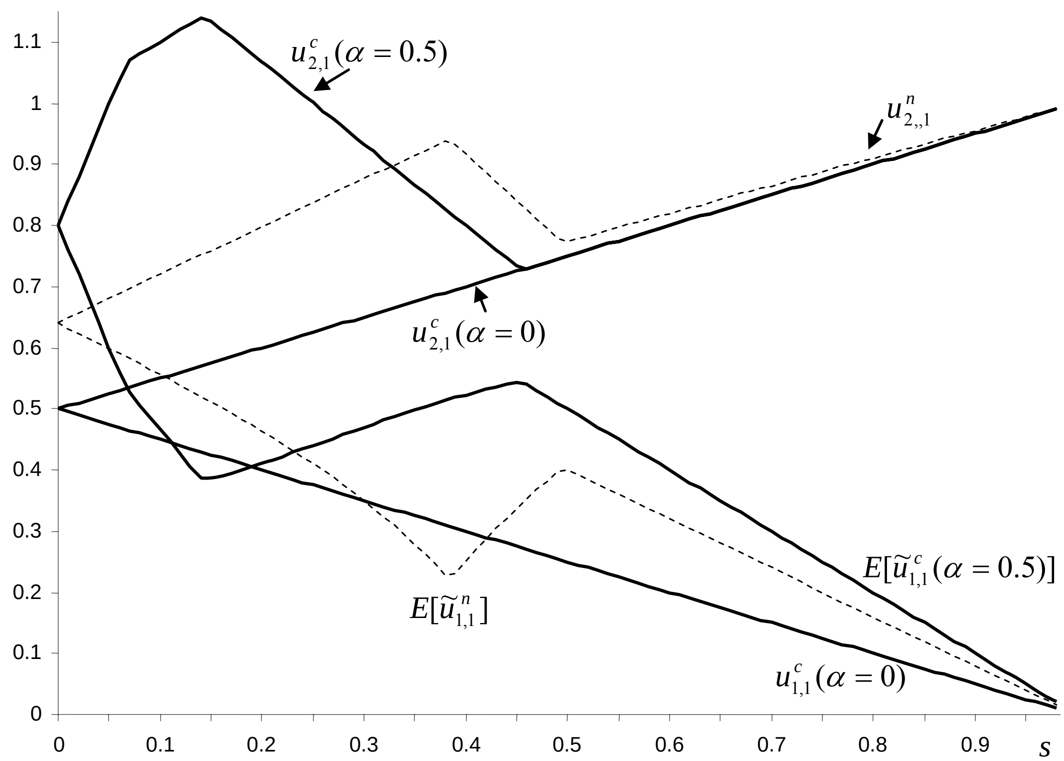

Figure 4. Equilibrium pumping under complete information

balanced usage of water across time, depletion in the first period cannot arise in either non-cooperative equilibrium or the socially efficient solution in a symmetric (or slightly asymmetric) aquifer.

If the cells are hydrologically isolated, i.e. $\alpha = \alpha_L = 0$, both farmers split their stocks equally between periods 1 and 2, $u_{1,1}^c = 0.5(1-s)$ and $u_{2,1}^c = 0.5(1+s)$. The (expected) equilibrium pumping rates for $\alpha = 0$ and $\alpha = 0.5$ are depicted as solid lines in Figure 4. ■

Next we investigate the effect of public information about the speed of lateral flows on the expected pumping rates and producer welfare in non-cooperative equilibrium.

4. Complete versus incomplete information

To an observer who knows only the probability distribution of $\tilde{\alpha}$ over the aquifer, the expected water pumped in period 1 by farmer i is $E[u_{i,1}^c(\tilde{\alpha})]$, and expected profits attained by non-cooperative farmers with complete hydrologic information are $E[\pi_i^c(\tilde{\alpha})]$, $i = 1,2$. In the next section, we compare the ex ante equilibrium pumping rates in the two information regimes.

4.1. Pumping under complete and incomplete information

In the case of a symmetric aquifer, SP showed that the average equilibrium pumping rates may either increase or decrease under better public information depending on the curvature of the ratio of the marginal benefits of water in periods 1 and 2 , $f(u) = g_u(u,1)/[\beta b'(1-u)]$.[17] This result continues to hold in the case of an asymmetric aquifer provided that the asymmetry is sufficiently small.

Proposition 3. (Information and pumping in a slightly asymmetric aquifer) *Suppose that* $f''(u) > (<) 0 \;\; \forall u \in [0.5,1)$. *Then there exists an* $s_0 > 0$ *such that each non-cooperative farmer pumps, on average, more (less) groundwater in period 1 under complete information about the speed of lateral flows, i.e.* $E[u_{i,1}^c(\tilde{\alpha})] \geq (\leq)\, u_{i,1}^n$, *i=1,2, for all* $s \in [0, s_0]$.

To ascertain the effect of information on the pumping rates when the asymmetry is larger, we first establish the following.

Lemma 5. *For* $\alpha_L > 0$, (i) $s^{cD}(\alpha_H) < s^{cD}(\alpha_L)$, *and* (ii) $\max[s^{nLD}, s^{nHD}] < s^{cD}(\alpha_L)$.

[17] See SP for a discussion of the properties of f, intuition, and examples.

Suppose that in equilibrium farmer 1 depletes his stock in $t=1$, $u_{1,1}^{c}(\alpha_L)=1-s$, when it is (publicly) known that $\tilde{\alpha}=\alpha_L>0$. Then he also does so if either (i) it becomes (publicly) known that the actual speed of lateral flows is higher, or (ii) complete public information is no longer available, and both farmers only know the probability distribution of $\tilde{\alpha}$.

Using Lemma 5, and Propositions 1 and 2, we obtain

Proposition 4. (Irrelevant information) *Suppose that the asymmetry is sufficiently large and the lateral flow speed is always sufficiently close to instantaneous, i.e.* $s\in[s^{cD}(\alpha_L),1)$. *Then in either information regime in period 1 farmer 1 always consumes his entire stock, i.e.* $E[u_{1,1}^{c}(\tilde{\alpha})]=u_{1,1}^{c}(\alpha_L)=u_{1,1}^{c}(\alpha_H)=u_{1,1}^{n}=1-s$, *and farmer 2 treats her stock as fully private, i.e.* $E[u_{2,1}^{c}(\tilde{\alpha})]=u_{2,1}^{c}(\alpha_L)=u_{2,1}^{c}(\alpha_H)=u_{2,1}^{n}=u_{2,1}^{nD}(1-s)$.

Whenever the lateral flow velocity is *always* sufficiently high ($s^{cD}(\alpha_L)<1$, which implies that $\alpha_L>0$), and the asymmetry is sufficiently large ($s\geq s^{cD}(\alpha_L)$), the precise information about $\tilde{\alpha}$ is irrelevant for non-cooperative farmers, and the equilibrium pumping rates (as well as profits) are the same under both complete and incomplete information.[18]

For the rest of the analysis, we assume that the prior probability distribution of $\tilde{\alpha}$ is sufficiently dispersed. The following preliminary result will play a key role in ascertaining the demand for public information by individual users in the next subsection.

Lemma 6. *For any* $s\in[0,1)$ *there exists an* $\alpha_0>0$ *such that* $u_{2,1}^{c}(\alpha_L)\leq u_{2,1}^{n}$ *for all* $\alpha_L\in[0,\alpha_0]$.

In any aquifer, if α_L is sufficiently small, in equilibrium farmer 2 always pumps less when it is publicly known that $\tilde{\alpha}=\alpha_L$ than under incomplete information.[19] This is because the lack of information about $\tilde{\alpha}$ fosters farmer 2's strategic advantage in "stealing" her neighbor's water. Lemma 6 is trivially true for $s=0$ (see SP). In a symmetric aquifer, the equilibrium pumping rates (which are equal across farmers) in *t*=1 increase when it is publicly known that

[18] Nonetheless, it can be shown that information about $\tilde{\alpha}$ may have a strictly positive value for the social planner even when the equilibrium outcomes are exactly the same under complete and incomplete information.
[19] Note that, by Propositions 1 and 2, $\lim_{\alpha_L\to 0} s^{nLD}(\alpha_L)=s^{cD}(\alpha_L)=1$.

the speed of lateral flow is higher, i.e. $u^c_{i,1}(\alpha_L) \le u^c_{i,1}(\alpha_H)$, i=1,2. However, as shown in Section 3.2, in an asymmetric aquifer farmer 1's pumping rate in t=1 *may be greater* when it is known that $\tilde{\alpha} = \alpha_L$ than when it is known that $\tilde{\alpha} = \alpha_H$. This happens when farmer 1 is more concerned with preserving some of his stock for future use than avoiding the loss due to outflow. For instance, in Examples 1 and 2, $u^c_{1,1}(\alpha_L) = 0.5(1-s) \ge E[\tilde{u}^c_{1,1}(\alpha_H)]$ for $s \in [0.12, 0.2]$, and $u^c_{1,1}(\alpha_L) \ge E[\tilde{u}^n_{1,1}]$ for $s \in [0.3, 0.42]$. Using Lemma 6, Propositions 1 and 2, we establish

Corollary 1. *Suppose that* $s \in [s^{cD}(\alpha_H), 1)$. *There exists such a* $\alpha_0 \in (0, \alpha_H)$ *such that in equilibrium farmer 2 (ex post) pumps less under complete information for any realization of* $\tilde{\alpha}$, *i.e.* $u^c_{2,1}(\alpha_H) \le E[u^c_{2,1}(\tilde{\alpha})] \le u^c_{2,1}(\alpha_L) \le u^n_{2,1}$ *for all* $\alpha_L \in [0, \alpha_0]$.

Suppose that the asymmetry is sufficiently large and the range of the possible speeds of lateral flows is sufficiently wide, e.g. $s \in [s^{cD}(\alpha_H), 1)$ and $\alpha_L = 0$ (note that, by Lemma 5(i), $s^{cD}(\alpha)$ is non-increasing in α). Then farmer 2 (ex post) pumps less in t=1 under complete information. To intuitively see why, note that when it is known that groundwater does not flow across cells, both farmers treat their stocks as fully private. If the speed of lateral flow is known to be high, by assumption, farmer 1 consumes his entire stock in t=1, but farmer 2 still treats her stock as fully private since there is no possibility of stealing any water from farmer 1, who has already exhausted his resource. On the other hand, under incomplete information, farmer 1 always leaves some stock unused in t=1 to take advantage of the possible gains in intertemporal efficiency from a more even distribution of his stock across time. This strategy involves his taking on the risk that the unused portion of his stock will flow into cell 2. The possibility of "stealing" some of the resource from farmer 1 creates an incentive for farmer 2 to pump more than is called for by the (private) considerations of intertemporal efficiency alone. As a result, farmer 2 pumps more under incomplete information. The following example illustrates.

Example 3. (Information and pumping) Consider the same environment as in Example 1. The average (ex ante) pumping rates under complete (dotted lines) and incomplete information (solid lines) are shown in Figure 2. The effects of better information on the equilibrium pumping rates are summarized in Table 1. As shown in SP, if $s = 0$, $\alpha_L = 0$, $\alpha_H = 0.5$, $q = 0.5$, and $g(u, x) = \sqrt{u}$, better public information has a positive effect on the average pumping rate. This result continues to hold if the asymmetry is sufficiently small, $s \in [0, 0.12]$, or if it falls in some intermediate range, $s \in [0.38, 0.4]$. However, the effect of better public information on the average pumping rate is reversed if the asymmetry is larger, for $s \in [0.12, 0.38] \cup [0.4, 1)$. For $s \in [0.01, 0.23] \cup [0.26, 0.48]$, public information has the opposite effects on the pumping rates of farmers 1 and 2. Farmer 1 (2) pumps more under better information for $s \in [0, 0.01] \cup [0.26, 0.48]$ (respectively, $s \in [0, 0.23]$). ■

Table 1. The effect of better public information on equilibrium pumping

Asymmetry	Farmer 1 pumping	Farmer 2 pumping	Average pumping
$s \in [0, 0.01]$	+	+	+
$s \in [0.01, 0.12]$	−	+	+
$s \in [0.12, 0.23]$	−	+	−
$s \in [0.23, 0.26]$	−	−	−
$s \in [0.26, 0.38]$	+	−	−
$s \in [0.38, 0.4]$	+	−	+
$s \in [0.4, 0.48]$	+	−	−
$s \in [0.48, 1)$	−	−	−

Next we analyze the effect of better public information about the speed of lateral flows on the expected producer profits.

4.2. The effect of public information on profits

Keeping the neighbor's pumping rate unchanged, each farmer benefits from more information as it allows him or her to better predict the lateral flow and future stocks, and choose a more appropriate pumping rate. However, typically, *both* farmers adjust their pumping rates upon the public announcement. As a result, the expected profits may increase or decrease, as better information about the environment may, on average, exacerbate or alleviate the "mixed" tragedy of the commons effect.

In the case of a symmetric aquifer, SP identified the necessary and sufficient condition on the production technology under which welfare is reduced or enhanced by better information. The same condition (only slightly stronger) also determines the value of public information for non-cooperative farmers when the extent of asymmetry is sufficiently small.

Let $\pi(u) = g(u,1) + \beta b(1-u)$ denote the discounted profits attained by allocating water equally across farmers in a symmetric aquifer (with $s = 0$).

Proposition 5. (Information and welfare in a slightly asymmetric aquifer) *Suppose that* $\frac{f''(u)}{f'(u)} < (>) \frac{\pi''(u)}{\pi'(u)}$ $\forall u \in [f^{-1}(1), f^{-1}(0.5)]$.[20] *Then there exists an* $s_0 > 0$ *such that each non-cooperative farmer attains, on average, lower (higher) expected welfare under complete information about the speed of lateral flows, i.e.,* $E[\pi_i^c(\tilde{\alpha}, s)] \leq (\geq) \pi_i^n(s)$ *i=1,2, for all* $s \in [0, s_0]$.

SP provide examples and explain how the curvature conditions in Proposition 3 and 5 can be related to more basic properties of $g(u,x)$. We are now ready to state our main results that ascertain the effect of information on (joint and individual) welfare when the asymmetry is larger. First, we show that, provided that (i) the extent of asymmetry is sufficiently large, and (ii) the prior probability distribution on the hydrologic properties is sufficiently diffuse, non-cooperative farmers attain a higher expected aggregate welfare under better information about the lateral flow velocity.

Proposition 6. (Information and joint welfare in a sufficiently asymmetric aquifer) *There exist* $s_0 \in [s^{cD}(\alpha_H), 1)$ and $\alpha_0 \in (0, \alpha_H)$ *such that non-cooperative farmers attain, on average, higher (possibly maximum) expected joint welfare under complete information about the speed of lateral flows, i.e.* $\sum_{i=1,2} E[\pi_i^c(\tilde{\alpha})] \geq \sum_{i=1,2} \pi_i^n$ *for* $s \in [s_0, 1)$, $\alpha_L \in [0, \alpha_0]$.

The intuition is as follows. Under conditions (i) and (ii), the joint welfare in the non-cooperative equilibrium equals (or is close to) the maximum joint welfare attainable under complete information. In other words, in an asymmetric aquifer, distortions due to the exploitation of groundwater as a common property resource may be eliminated by better public information. If $\tilde{\alpha} = \alpha_L$ is common knowledge, and α_L is close to 0, the stocks of each farmer are (approximately) fully private, and hence, are, by default, efficiently allocated from the societal point of view. If $\tilde{\alpha} = \alpha_H$ is common knowledge, and the asymmetry is

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[20] $f^{-1}(\cdot)$ denotes the inverse of f.

sufficiently large, farmer 1 consumes his entire stock in $t = 1$ even under the socially efficient allocation. This is because any water that he saves in $t = 1$ will be lost due to outflow, but his marginal benefit of water in $t = 1$ exceeds his neighbor's discounted marginal benefit of water in $t = 2$. Thus, the privately optimal pumping rates of non-cooperative, informed users also maximize the joint welfare.[21] But the maximum attainable aggregate welfare cannot increase due to either the lack of information about the environment or non-cooperative behavior. This result has an important policy implication: better hydrologic information may allow non-cooperative users to achieve maximum social welfare even in the absence of groundwater use regulations.

Our next result states that under similar conditions (possibly, under weaker conditions since $s^{cD}(\alpha_H) \leq s_0$ in Proposition 6) farmer 1 (respectively, 2) attains higher (respectively, lower) expected profits under better public information.

Proposition 7. (Information and individual welfare in a sufficiently asymmetric aquifer) *Suppose that* $s \in [s^{cD}(\alpha_H),1)$. *There exists an* $\alpha_0 \in (0, \alpha_H)$ *such that farmer 1 (2) achieves higher (lower) expected profits under complete information about the speed of lateral flows for any* $\alpha_L \in [0, \alpha_0]$, *i.e.* $E[\pi_1^c(\tilde{\alpha})] \geq \pi_1^n$ *and* $E[\pi_2^c(\tilde{\alpha})] \leq \pi_2^n$.

To see intuitively why this is true, let $\alpha_L = 0$ (the speed of lateral flow is sometimes negligible). Also, we suppose that $s \in [s^{cD}(\alpha_H),1)$ (the aquifer is sufficiently asymmetric). Consider farmer 1's equilibrium pumping for different announcements about $\tilde{\alpha}$. If $\tilde{\alpha} = 0$ (respectively, $\tilde{\alpha} = \alpha_H$), farmer 1 consumes a portion of his stock (respectively, his entire stock) in period 1. In either outcome farmer 1 loses none of his water due to outflow. And so, farmer 1's ability to save some of his stock for future use while avoiding outflows is undermined by the lack of public information, and his expected profits decrease. Moreover, when the value of $\tilde{\alpha}$ is unknown, farmer 2 anticipates the possibility of an inflow, and raises her pumping rate in *t*=1 since her water supply becomes, on average, less scarce in *t*=2 (see

[21] Note that cell 1 will not be empty in $t = 2$ for $\tilde{\alpha} = \alpha_H$, only if farmer 2 consumes a small share of his (relatively large) stock in $t = 1$ in order to reduce the inter-period outflow from cell 1. But such an allocation of water brings about too onerous a loss in the intertemproal efficiency of allocation of farmer 2's stock compared with the gain in the intertemporal efficiency of allocation of farmer 1's stock.

Corollary 1). These effects are also at work for $\alpha_L > 0$, provided that farmer 2's (ex post) pumping rates decrease under better information. Therefore, farmer 1 is "hit twice" by the lack of information: (a) keeping his neighbor's pumping unchanged, his expected profits decrease because he cannot match a low (high) pumping rate with the low (high) lateral flow speed; and (b) farmer 1's profits is further reduced by a (weakly) greater outflow due to a greater drawdown in cell 2.

The lack of public information has the opposite (i.e. a positive) effect on the expected profits of farmer 2. When the value of $\tilde{\alpha}$ is unknown, farmer 2 sometimes appropriates a portion of her neighbor's stock, which she cannot do under complete information (if $\alpha_L = 0$). Furthermore, unlike farmer 1 whose pumping rate is very sensitive to the news about $\tilde{\alpha}$, farmer 2's pumping rate is invariant to the content of a public announcement since the best farmer 2 can do is treat her stock as fully private when farmer 1 knows whether $\tilde{\alpha} = 0$ or $\tilde{\alpha} = \alpha_H$. And so, better public information does not enhance farmer 2's ability to improve her own intertemporal efficiency given her neighbor's pumping flexibility in $t = 1$, while it inhibits her ability to "steal" water from cell 1. When α_L is strictly positive but sufficiently close to 0, the negative effect of public information on farmer 2's profits from a smaller expected inflow dominates the positive effect due to an improved ability to allocate her stock more efficiently across time. Example 4 illustrates.

Example 4. (Information and welfare) The joint and individual producer profits under incomplete and complete information are shown, respectively, as "thick" and "thin" lines in Figure 5. The effect of incomplete information on profits is summarized in Table 2. As shown in SP, if $s = 0$ and $g(u, x) = \sqrt{u}$, the average welfare decreases under better public information about the lateral flow speed. This result continues to hold if the asymmetry is not too great, $s \in [0, 0.38]$ (see Proposition 5). But, for $s \in [0.38, 1)$, the joint welfare is higher under complete information (see Proposition 6). When $\tilde{\alpha} = 0.5$, better public information assuages both the distributional and temporal inefficiencies: Farmer 1 is better able to avoid water loss, and farmer 2 is more concerned with achieving an intertemporally efficient allocation of her initial stock rather than with "stealing" her neighbor's water. Of course, both farmers achieve maximum efficiency when it is common knowledge that $\tilde{\alpha} = 0$. Next we examine the effect of information on individual producer profits.

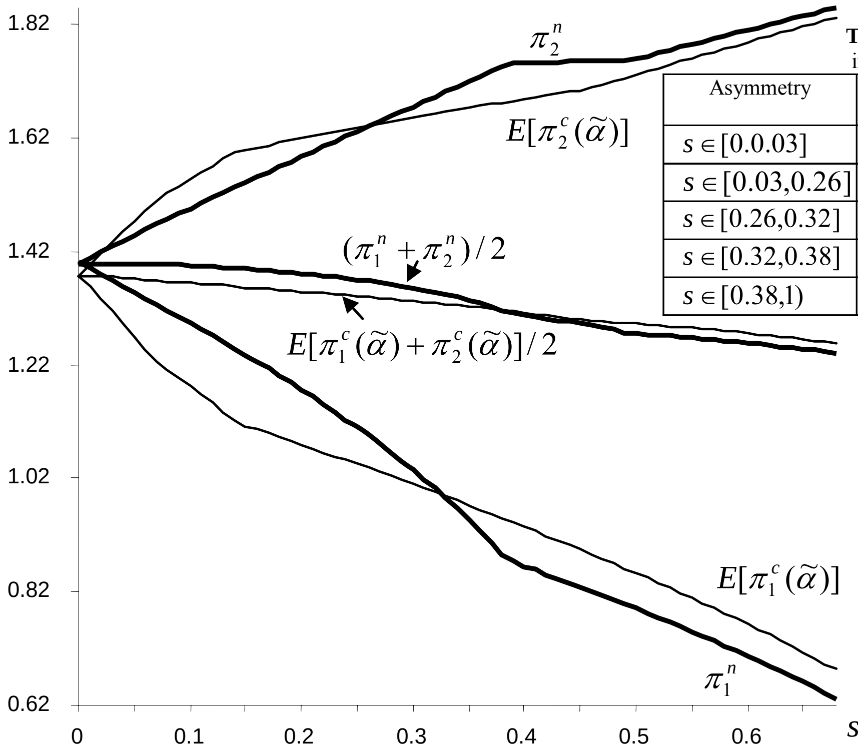

Table 2. The effect of better public information on producer profits

Asymmetry	Farmer 1's profit	Farmer 2's profit	Total profit
$s \in [0.0.03]$	-	-	-
$s \in [0.03, 0.26]$	-	+	-
$s \in [0.26, 0.32]$	-	-	-
$s \in [0.32, 0.38]$	+	-	-
$s \in [0.38, 1)$	+	-	+

Figure 5. Profits under incomplete and complete information

For $s \in [0.0.03]$, both farmers are worse off. As explained in SP, complete information, on average, aggravates the "tragedy of the commons effect" and the joint welfare falls because the average pumping rate is higher (see Example 3). However, for $s \in [0.03, 0.26]$, the effect of information differs across farmers: farmer 2 (1) is better off (worse off). Better information enables farmer 2 to better exercise her strategic advantage at the expense of farmer 1 who is in the "accommodating mode" and, on average, loses more of his stock due to the outflows. Better information exacerbates the distributional distortion stemming from the commonality of groundwater whereas farmer 1's (2's) total consumption, on average, decreases (increases). For $s \in [0.26, 0.32]$, both farmers are made worse off by information: farmer 1 is more often in the "cut water loss mode" in which he minimizes the outflow rather than strives to achieve a greater (expected) intertemporal efficiency. Here information exacerbates the temporal inefficiency associated with the unregulated water use whereas both farmers consume, on average, more water in period 1.

For $s \in [0.32, 0.38]$ and $s \in [0.38, 1)$, farmer 2's strategic advantage is further eroded by better public information. Only farmer 1 is better off since he frequently (in fact, always for $s \in [0.46, 1)$) consumes his entire stock in $t = 1$ when it is known that the inter-period lateral flow is instantaneous, $\tilde{\alpha} = 0.5$, while he consumes only a portion of his stock under uncertainty about $\tilde{\alpha}$ (see Proposition 7). ■

5. Conclusions and Implications

This paper analyzes a simple two-period model of groundwater exploitation in a two-cell aquifer under a novel assumption that the bottom elevation differs across cells. This asymmetry creates a strategic advantage (disadvantage) for the user in the deep (shallow) cell in "stealing" the groundwater lying beneath the neighboring farm at the end of the irrigation season. Asymmetry in bottom elevation, combined with the lack of "full" ownership rights to groundwater, aggravates the intrinsic inequality in income distribution caused by the difference in the initial water endowments. The user with a larger initial stock actually benefits from the exploitation of groundwater as a common property resource when the asymmetry is not too small or too great.

Because in reality producers do not have perfect knowledge of the local hydrologic properties, it is of interest to identify conditions under which incomplete information has an unambiguous effect on the equilibrium pumping rates and producer welfare in an asymmetric aquifer. Consistent with SP's conclusions, as long as asymmetry in bottom elevation is sufficiently small, production technology is the main determinant of the value of public information for non-cooperative users in the presence of distortions caused by the commonality of the resource.

However, this is not the case when the asymmetry in initial stocks is sufficiently large. Then, if the externality is always significant, information about its precise value is irrelevant for equilibrium outcomes. But, whenever the prior beliefs are sufficiently dispersed, better informed users achieve a higher joint welfare. Moreover, the welfare losses from unregulated resource use may stem *solely* from the lack of information. That is, in an asymmetric aquifer, non-cooperative equilibrium and socially efficient allocations may *coincide* under complete information but *differ* under uncertainty about the lateral flow velocity. In such cases, educating and informing users about the true resource dynamics is *all* that is needed to achieve maximum social welfare. Yet, even when the overall welfare increases, some users may be worse off under better public information.[22]

[22] As shown in SP, in the symmetric aquifer users always agree on the welfare ranking of equilibrium outcomes for different levels of precision of public information.

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Appendix

Proof of Lemma 1: To determine the best response correspondence by farmer 1, it is convenient to solve (5) for $i=1$ as a two-step optimization problem

(A1) $\pi_1^n(u_{2,1}) = \max[\pi_1^{nH}(u_{2,1}), \pi_1^{nL}(u_{2,1}), \pi_1^{nD}(u_{2,1})]$.

Note that farmer 1 never lets his well go dry in t=2 when $u_{2,1} \le x_{1,1}$, since

(A2) $\pi_1^{nD}(u_{2,1}) < \pi_1^{nL}(u_{2,1}) < \pi_1^{nH}(u_{2,1})$ for $u_{2,1} \le x_{1,1}$,

where the strict inequalities follow because $u_{2,1} \le x_{1,1}$ implies that $x_{1,2}(\alpha_L, x_{1,1}, u_{2,1}) \ge 0$ and $x_{1,2}(\alpha_H, u_{1,1}^L(u_{2,1}), u_{2,1}) > 0$, so that $u_{1,1}^{nH}(u_{2,1}) = u_{1,1}^{nL}(u_{2,1})$ and $u_{1,1}^{nH}(u_{2,1}) = x_{\cdot 1,1}$ are feasible but are not profit-maximizing. On the other hand, by (6)-(8), and because $g_u(0,\cdot) = \infty$ and $g(0,.) = 0$, we have

(A3) $\lim_{u_{2,1}\uparrow x_{1,1}/\alpha_H} \pi_1^{nH}(u_{2,1}) = \beta q b(x_{1,2}(\alpha_L, 0, x_{1,1}/\alpha_H)) < \lim_{u_{2,1}\uparrow x_{1,1}/\alpha_H} \pi_1^{nL}(u_{2,1})$, and

(A4) $\lim_{u_{2,1}\uparrow x_{1,1}/\alpha_L} \pi_1^{nL}(u_{2,1}) = 0 < \lim_{u_{2,1}\uparrow x_{1,1}/\alpha_L} \pi_1^{nD}(u_{2,1})$.

For $u_{2,1} \in (x_{1,1}, x_{1,1}/\alpha_L)$, by the envelope theorem, we have

$$\frac{d\pi_1^{nH}(u_{2,1})}{du_{2,1}} = -\beta\{q\alpha_L b'(x_{1,2}(\alpha_L, u_{1,1}^{nH}(u_{2,1}), u_{2,1})) + (1-q)\alpha_H b'(x_{1,2}(\alpha_H, u_{1,1}^{nH}(u_{2,1}), u_{2,1}))\}$$

$$= -\beta\frac{\alpha_L}{1-\alpha_L}\{q(1-\alpha_L)b'(x_{1,2}(\alpha_L, u_{1,1}^{nH}, u_{2,1})) + (1-q)\frac{(1-\alpha_L)\alpha_H}{\alpha_L}b'(x_{1,2}(\alpha_H, u_{1,1}^{nH}, u_{2,1}))\}$$

$$< -\frac{\alpha_L}{1-\alpha_L}\beta\{q(1-\alpha_L)b'(x_{1,2}(\alpha_L, u_{1,1}^{nH}, u_{2,1})) + (1-q)(1-\alpha_H)b'(x_{1,2}(\alpha_H, u_{1,1}^{nH}, u_{2,1}))\}$$

$$< -\frac{\alpha_L}{1-\alpha_L}\beta q(1-\alpha_L)b'(x_{1,2}(\alpha_L, u_{1,1}^{nL}(u_{2,1}), u_{2,1})) = \frac{d\pi_1^L(u_{2,1})}{du_{2,1}},$$

where the first inequality follows because $1-\alpha_H < \alpha_H(1-\alpha_L)/\alpha_L$, and the second inequality follows from the FOCs in (7) because $u_{1,1}^{nH}(u_{2,1}) < u_{1,1}^{nL}(u_{2,1})$ and $u_{1,1}^{nH}(u_{2,1}) < x_{1,1}$ for $u_{2,1} \in (x_{1,1}, x_{1,1}/\alpha_L)$. Therefore, by (A2)-(A4), and, because the profit functions are continuous in $u_{2,1}$, (note that $b(x_{1,1})$ is independent of $u_{2,1}$) there exist threshold values $\hat{u}_{2,1}^{nHL}$, $\hat{u}_{2,1}^{nLD}$, and $\hat{u}_{2,1}^{nHD}$ such that $\pi_1^{nH}(u) \ge (<) \pi_1^{nL}(u)$ for all $u \le \hat{u}_{2,1}^{nHL}$ (resp., $u \in (\hat{u}_{2,1}^{nHL}, x_{1,1}/\alpha_L)$), $\pi_1^{nL}(u) \ge (<) \pi_1^{nD}(u)$ for all $u \le (>) \hat{u}_{2,1}^{nLD}$, and $\pi_1^{nH}(u) \ge (<) \pi_1^{nD}(u)$ for all $u \le (>) \hat{u}_{2,1}^{nHD}$.

There are two cases to consider: (a) $\hat{u}_{2,1}^{nHL} \le \hat{u}_{2,1}^{nHD} \le \hat{u}_{2,1}^{nLD}$, and (b) $\hat{u}_{2,1}^{nLD} < \hat{u}_{2,1}^{nHD} < \hat{u}_{2,1}^{nHL}$. In case (a), by (A1), the best response for farmer 1 is given by

(A5) $$u_{1,1}^{nBR}(u_{2,1}) = \begin{cases} u_{1,1}^{nH}(u_{2,1}), \text{if } u_{2,1} \le \hat{u}_{2,1}^{nHL}; \\ u_{1,1}^{nL}(u_{2,1}), \text{if } \hat{u}_{2,1}^{nHL} \le u_{2,1} \le \hat{u}_{2,1}^{nLD}; \\ x_{1,1}, \text{ if } u_{2,1} \ge \hat{u}_{2,1}^{nLD}. \end{cases}$$

In case (b), by (A1), the best response for farmer 1 is given by

(A6) $u_{1,1}^{nBR}(u_{2,1}) = \begin{cases} u_{1,1}^{nH}(u_{2,1}), \text{if } u_{2,1} \le \hat{u}_{2,1}^{nHD}; \\ x_{1,1}, \text{ if } u_{2,1} \ge \hat{u}_{2,1}^{nHD}. \end{cases}$

Note that (A5) and (A6) are upper hemicontinuous correspondences since in case (a) $u_{1,1}^{nBR}(\hat{u}_{2,1}^{nHL}) \in \{u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}), u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})\}$ and $u_{1,1}^{nBR}(\hat{u}_{2,1}^{nLD}) \in \{u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}), x_{1,1}\}$, where $\lim_{u\uparrow\hat{u}_{2,1}^{nHL}} u_{1,1}^{nBR}(u) = u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})$, $\lim_{u\downarrow\hat{u}_{2,1}^{nHL}} u_{1,1}^{nBR}(u) = u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})$, $\lim_{u\uparrow\hat{u}_{2,1}^{nLD}} u_{1,1}^{nBR}(u) = u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})$, $\lim_{u\downarrow\hat{u}_{2,1}^{nLD}} u_{1,1}^{nBR}(u) = x_{1,1}$, and $u_{1,1}^{nBR}(u)$ is single-valued for $u \ne \hat{u}_{2,1}^{nHL}, \hat{u}_{2,1}^{nLD}$. Case (b) is analogous (just set $\hat{u}_{2,1}^{nHL} = \hat{u}_{2,1}^{nLD} = \hat{u}_{2,1}^{nHD}$ in case (a)). ■

Proof of Lemma 2: Consider (5) for $i = 2$ as a two-step optimization problem

(A7) $\pi_2^n(u_{1,1}) = \max[\pi_2^{nH}(u_{1,1}), \pi_2^{nL}(u_{1,1}), \pi_2^{nD}(u_{1,1})]$,

where

(A8) $\pi_2^{nH}(u_{1,1}) = \max_{u_{2,1}\le x_{2,1}} g(u_{2,1}, x_{2,1}) + \beta E[b(x_{2,2}(\tilde{\alpha}, u_{1,1}, u_{2,1}))]$ subject to $u_{2,1} \le d(u_{1,1}, \alpha_H)$,

(A9) $\pi_2^{nL}(u_{1,1}) = \max_{u_{2,1}\le x_{2,1}} g(u_{2,1}, x_{2,1}) + \beta E[b(x_{2,2}(\tilde{\alpha}, u_{1,1}, u_{2,1}))]$ subject to

$d(u_{1,1}, \alpha_H) \le u_{2,1} \le d(u_{1,1}, \alpha_L)$, and

(A10) $\pi_2^{D}(u_{1,1}) = \max_{u_{2,1}\le x_{2,1}} g(u_{2,1}, x_{2,1}) + \beta E[b(x_{2,2}(\tilde{\alpha}, u_{1,1}, u_{2,1}))]$ subject to $u_{2,1} \ge d(u_{1,1}, \alpha_L)$.

In each problem (A8)-(A10), the objective function is differentiable (and concave) on the constraint set $\{u_{2,1} : u_{2,1} \le \min[d(u_{1,1}, \alpha_H), x_{2,1}]\}$ (respectively, $\{u_{2,1} : u_{2,1} \in [d(u_{1,1}, \alpha_H), \min[d(u_{1,1}, \alpha_L), x_{2,1}]\}$, and $\{u_{2,1} : u_{2,1} \in [d(u_{1,1}, \alpha_L), x_{2,1}]\}$), and the maximizer for each problem $u_{2,1}^{nH}(u_{1,1})$ (respectively, $u_{2,1}^{nL}(u_{1,1})$, and $u_{2,1}^{nD}(u_{1,1})$) is a continuous function, which is given by equations (10) (respectively, (11) and (12)) in the text.

If $x_{2,1} < d(u_{1,1}, \alpha_H)$, the constraint sets in (A9) and (A10) are empty, and $\pi_2^n(u_{1,1}) = \pi_2^{nH}(u_{1,1})$. If $x_{2,1} \ge d(u_{1,1}, \alpha_H)$ and $g_u(d(u_1, \alpha_H), x_2) \le \beta E[(1-\tilde{\alpha}) b'(x_{2,2}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))]$, none of the constraints in (A8) bind, and therefore, $u_{2,1}^{nH}(u_{1,1})$ is the unique global maximizer (note that the objective function in (5) is strictly concave). Now suppose that $g_u(d(u_1, \alpha_H), x_2) > \beta E[(1-\tilde{\alpha}) f(x_{2,2}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))]$. Then $\pi_2^{nH}(u_{1,1}) = g(d(u_{1,1}, \alpha_H), x_{2,1}) + \beta E[g(x_{2,2}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))] \le \pi_2^{nL}(u_{1,1})$, where the inequality follows because $u_{2,1} = d(u_{1,1}, \alpha_H)$ satisfies the constraints in (A9), and re-optimization cannot decrease profits. There are two cases to consider. If $d(u_1, \alpha_H) < x_{2,1} < d(u_1, \alpha_L)$, the constraint set in (A10) is empty, and $u_{2,1}^{nL}(u_{1,1})$ must be the global maximizer. If $x_{2,1} \ge d(u_{1,1}, \alpha_L)$ and $g_u(d(u_{1,1}, \alpha_L), x_2) \le \beta(1 - q\alpha_L) b'(x_{2,1}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L)))$, we have

$\pi_2^{D}(u_{1,1}) = g(d(u_{1,1}, \alpha_L), x_{2,1}) + \beta\, b(x_{1,1} - u_{1,1} + x_{2,1} - d(u_{1,1}, \alpha_L)) \le \pi_2^{L}(u_{1,1})$,

where the inequality follows because $u_{2,1} = d(u_{1,1}, \alpha_L)$ satisfies the constraints in (A9), and re-optimization cannot decrease profits. And so, under any of these conditions, $u_{2,1}^{nL}(u_{1,1})$ is the (unique) global maximizer. Finally, if $x_{2,1} \geq d(u_{1,1}, \alpha_L)$ and $g_u(d(u_{1,1}, \alpha_L), x_{2,1}) \geq \beta(1-q\alpha_L)\ b'(x_{2,1}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L)))$, we have $\pi_2^{nL}(u_{1,1}) = g(d(u_{1,1}, \alpha_L), x_{2,1}) + \beta\ b(2-u_{1,1} - d(u_{1,1}, \alpha_L)) \leq \pi_2^{nD}(u_{1,1})$, where the inequality follows because $u_{2,1} = d(u_{1,1}, \alpha_L)$ satisfies the constraints in (A10), and re-optimization cannot decrease profits. The proof that the best response for farmer 2 is given by (13), is completed by noting that $u_{2,1}^{nBR}(u_{1,1})$ is a continuous single-valued function since $u_{2,1}^{nH}(u_{1,1}) = u_{2,1}^{nL}(u_{1,1}) = d(u_{1,1}, \alpha_H)$ if $x_{2,1} \geq d(u_{1,1}, \alpha_H)$ and $\beta E[(1-\tilde{\alpha})\ b'(x_{2,1}(\tilde{\alpha}, u_{1,1}, d(u_{1,1}, \alpha_H)))] \leq g_u(d(u_{1,1}, \alpha_H), x_2) \leq \beta\{q(1-\alpha_L) b'(x_{2,1}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L))) + (1-q) f(x_{2,1}(\alpha_H, u_{1,1}, d(u_{1,1}, \alpha_H)))\}$, and $u_{2,1}^L(u_{1,1}) = u_{2,1}^D(u_{1,1}) = d(u_{1,1}, \alpha_L)$ if $x_2 \geq d(u_1, \alpha_L)$ and $\beta(1-q\alpha_L)\ b'(x_{2,2}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L))) \leq g_u(d(u_{1,1}, \alpha_L), x_{2,1}) \leq \beta\ b'(x_{2,2}(\alpha_L, u_{1,1}, d(u_{1,1}, \alpha_L)))$. ■

Proof of Lemma 3: The existence of equilibrium in mixed strategies follows by Theorem 3 in Dasgupta and Maskin (1986) (or by Lemmas 1, 2, and Kakutani fixed point theorem). Because $g_u(0,\cdot) = \infty$ (and restricted access), farmers never simultaneously deplete their stocks in period 1 in equilibrium. By Lemma 2, farmer 2 cannot randomize. To show that $\Pr(\tilde{u}_{1,1}^n < u_{2,1}^n) = 1$, suppose to the contrary that $\Pr(\tilde{u}_{1,1}^n \geq u_{2,1}^n) > 0$. Then it must be that $1-s-(1-\alpha)\tilde{u}_{1,1}^n - \alpha u_{2,1}^n \geq 0$ for any α since $u_{2,1}^n \leq u_{1,1}^n \leq 1-s$ for some $u_{1,1}^n$, i.e. $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^{nH}(u_{2,1}^n)) = 1$ and $u_{2,1}^n = u_{2,1}^{nH}(u_{1,1}^n)$ in any equilibrium with $\Pr(\tilde{u}_{1,1}^n \geq u_{2,1}^n) > 0$.
Hence, by Lemmas 1 and 2, we have

(A11) $g_u(u_{1,1}^n, 1-s) - \beta E[(1-\tilde{\alpha})b'(1-s-(1-\tilde{\alpha})u_{1,1}^n - \tilde{\alpha}u_{2,1}^n)] \geq 0$, (=0 if $u_{1,1}^n < 1-s$);

(A12) $g_u(u_{2,1}^n, 1+s) - \beta E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})u_{2,1}^n - \tilde{\alpha}u_{1,1}^n)] = 0$.

From (A11) and (A12), it follows that

$$g_u(u_{2,1}^n, 1+s) = \beta E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})u_2^n - \tilde{\alpha}u_1^n)]$$
$$< \beta E[(1-\tilde{\alpha})b'(1-s-(1-\tilde{\alpha})u_{1,1}^n - \tilde{\alpha}u_{2,1}^n)] \leq g_u(u_{1,1}^n, 1-s),$$

which yields a contradiction because g is concave and $g_{ux} \geq 0$. ■

Proof of Proposition 1: First, we state the necessary and sufficient conditions for different types of the best responses to be played in equilibrium, and prove the (a.e.) uniqueness of equilibrium. Then we show the existence of the threshold levels of asymmetry that partition $s \in [0,1)$ into sub-intervals where the type of equilibrium is the same.

The following lemma will be used to characterize equilibrium under incomplete information.

Lemma 4. (Asymmetry and threshold pumping rates) Suppose that $g_{uu}(u,x) + g_{ux}(u,x) \leq 0$ for all $u \leq x \leq 1$. Then $d\hat{u}_{2,1}^{nLD}/ds < -1$, and there exists such a $\alpha_0 \in (0, \alpha_H)$ such that for any $\alpha_L \in [0, \alpha_0]$, $d\hat{u}_{2,1}^{nHL}/ds < -1$.

Proof of Lemma 4: Using the envelope theorem to differentiate (8) for $K = L, M = D$ yields $d\hat{u}_{2,1}^{nLD} / ds = -[\partial(\pi_1^{nL} - \pi_1^{nD}) / \partial s]/[\partial(\pi_1^{nL} - \pi_1^{nD}) / \partial \hat{u}_{2,1}^{nLD}] < -1$, where $\partial(\pi_1^{nL} - \pi_1^{nD}) / \partial \hat{u}_{2,1}^{nLD}$ $= -\beta\alpha_L q b'(1 - s - (1-\alpha_L)u_{1,1}^{nL} - \alpha_L \hat{u}_{2,1}^{nLD}) < 0$, and $\partial(\pi_1^{nL} - \pi_1^{nD}) / \partial s$ $= -g_x(u_{1,1}^{nL}, 1-s) + b'(1-s) - \beta q b'(1 - s - (1-\alpha_L)u_{1,1}^{nL} - \alpha_L \hat{u}_{2,1}^{nLD}) = b'(1-s)$ $-(g_u(u_{1,1}^{nL}, 1-s) + g_x(u_{1,1}^{nL}, 1-s)) + \partial(\pi_1^{nL} - \pi_1^{nD}) / \partial \hat{u}_{2,1}^{nLD} < 0$. The last equality follows by the FOC in (7) for $K = L$. The inequality follows because $u_{1,1}^{nL}(u_{2,1}) < 1-s$ and $g_{uu}(u,x) + g_{ux}(u,x) \le 0$.

Using the envelope theorem to differentiate (8) for $K = H, M = L$ yields $d\hat{u}_{2,1}^{nHL} / ds$ $= -[\partial(\pi_1^{nH} - \pi_1^{nL}) / \partial s]/[\partial(\pi_1^{nH} - \pi_1^{nL}) / \partial \hat{u}_{2,1}^{nHL}] < -1$, where $\partial(\pi_1^{nH} - \pi_1^{nL}) / \partial \hat{u}_{2,1}^{nHL}$ $= -\beta(1-q)\alpha_H b'(1 - s - (1-\alpha_H)u_{1,1}^{nH} - \alpha_H \hat{u}_{2,1}^{nHL}) + \beta q \alpha_L (b'(1 - a - (1-\alpha_L)u_{1,1}^{nL} - \alpha_L \hat{u}_{2,1}^{nHL})$ $- b'(1 - a - (1-\alpha_L)u_{1,1}^{nH} - \alpha_L \hat{u}_{2,1}^{nHL})) < 0$ when α_L is close to 0, because $\beta(1-q)\alpha_H b'(1 - s - (1-\alpha_H)u_{1,1}^{nH} - \alpha_H \hat{u}_{2,1}^{nHL}) > 0$ is bounded away from zero as $\alpha_L \to 0$, and $\partial(\pi_1^{nH} - \pi_1^{nL}) / \partial s = -g_x(u_{1,1}^{nH}, 1-s) + g_x(u_{1,1}^{nL}, 1-s) - \beta(1-q)b'(1 - s - (1-\alpha_H)u_{1,1}^{nH} - \alpha_H \hat{u}_{2,1}^{nHL})$ $- \beta q b'(1 - s - (1-\alpha_L)u_{1,1}^{nH} - \alpha_L \hat{u}_{2,1}^{nHL}) + \beta q b'(1 - s - (1-\alpha_L)u_{1,1}^{nL} - \alpha_L \hat{u}_{2,1}^{nHL})$ $= g_u(u_{1,1}^{nL}, 1-s) + g_x(u_{1,1}^{nL}, 1-s) - (g_u(u_{1,1}^{nH}, 1-s) + g_x(u_{1,1}^{nH}, 1-s)) + \partial(\pi_1^{nH} - \pi_1^{nL}) / \partial \hat{u}_{2,1}^{nHL} < 0$. The last equality follows by the FOCs in (7). The inequality follows because $u_{1,1}^{nH}(u_{2,1})$ $< u_{1,1}^{nL}(u_{2,1})$ and $g_{uu}(u,x) + g_{ux}(u,x) \le 0$. ■

Step 1. Suppose that $\hat{u}_{2,1}^{nHL} < \hat{u}_{2,1}^{nLD}$. Then farmer 1 may find it optimal to risk saving some of his stock for later use even though the fast outflow will leave his well dry. For example, this must be the case whenever α_L is sufficiently small since, by (8), $\lim_{\alpha_L \to 0} \hat{u}_{2,1}^{nLD} = \infty$. There are five cases that need to be considered as possible equilibrium outcomes.

Case 1. Suppose that in equilibrium $x_{1,2}(\alpha_H, u_{1,1}^n, u_{2,1}^n) > 0$. Then $u_{i,1}^n = u_{i,1}^{nH}(u_{j,1}^n)$, and by Lemmas 1 and 2, the necessary and sufficient conditions for $u_i^{nBR}(u_{j,1}^n) = u_i^{nH}(u_{j,1}^n)$, $i, j \in \{1,2\}$, $i \ne j$ are

(A13) $u_{2,1}^n < \hat{u}_{2,1}^{nHL}$, and either (a) $1 + s < d(\alpha_H, u_1^n)$, or (b) $1 + s \ge d(\alpha_H, u_1^n)$ and

$$g_u(d(\alpha_H, u_{1,1}^n), 1+s) \le \beta E[(1-\tilde{\alpha})b'(1 + s - (1-\tilde{\alpha})d(\alpha_H, u_{1,1}^n) - \tilde{\alpha}u_{1,1}^n).$$

Note that (A13) is implied by

(A14) either (a) $\hat{u}_{2,1}^{nHL} \ge 1 + s$, or (b) $\hat{u}_{2,1}^{nHL} \le 1 + s$ and

$$g_u(\hat{u}_{2,1}^{nHL}, 1+s) \le \beta \; E[(1-\tilde{\alpha})b'(1 + s - (1-\tilde{\alpha})\hat{u}_{2,1}^{nHL} - \tilde{\alpha}u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}))].$$

First, note that (A14) implies that $u_{2,1}^n = u_{2,1}^{nH}(u_{1,1}^n) \le \hat{u}_{2,1}^{nHL}$. To see why, suppose that $u_{2,1}^n = u_2^{nH}(u_{1,1}^n) > \hat{u}_{2,1}^{nHL}$ and (A14) holds. Then, by (A14b), we have

$$g_u(u_{2,1}^n, 1+s) < g_u(\hat{u}_{2,1}^{nHL}, 1+s) \le \beta \; E[(1-\tilde{\alpha})b'(1 + s - (1-\tilde{\alpha})\hat{u}_{2,1}^{HL} - \tilde{\alpha}u_{1,1}^{nH}(\hat{u}_2^{HL}))]$$

$< E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})u_{2,1}^{n}-\tilde{\alpha}u_{1,1}^{nH}(u_{2,1}^{n}))] = g_u(u_{2,1}^{n},1+s)$,

where the inequality follows because $du_{i,1}^{nH}(u)/du \in (-1,0)$ (the best responses are piece-wise differentiable). Hence, we obtained a contradiction. Second, since $u_{2,1}^{n} < d(\alpha_H, u_{1,1}^{n})$, it must be that either (A13a) or (A13b) holds because $g_u(d(\alpha_H,u_{1,1}^{n}),1+s) < g_u(u_{2,1}^{n},1+s)$ $= \beta E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})u_{2,1}^{n}-\tilde{\alpha}u_{1,1}^{n}) \le \beta E[(1-\tilde{\alpha})\, b'(1+s-(1-\tilde{\alpha})d(\alpha_H,u_{1,1}^{n})-\tilde{\alpha}u_{1.1}^{n})$.

Also, it must be that $u_{1,1}^{n} < x_{1,1}$. If $u_{1,1}^{n} = x_{1,1}$, then cell 1 must be empty in $t=2$ because, by Lemma 3, in equilibrium $u_{1,1}^{n} < u_{2,1}^{n}$, which contradicts the assumption that $x_{1,2}(\alpha_H, u_{1,1}^{n}, u_{2,1}^{n}) > 0$. Furthermore, this equilibrium is locally unique in the sense that there is no other equilibrium $u_{i,1}' \ne u_{i,1}^{n}$, $i=1,2$ with $u_i^{nBR}(u_{j,1}') = u_i^{nH}(u_{j,1}')$. This is because the mapping $u_{1,1}^{nH}(u_{2,1}^{nH}(u))$ has a unique fixed point on $(0,1-s)$ since $du_i^{nH}(u)/du \in (-1,0)$.

Case 2. Suppose that in equilibrium farmer 1 randomizes: $\Pr(\tilde{u}_{1,1}^{n} = u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})) = p^{nHL}$ and $\Pr(\tilde{u}_{1,1}^{n} = u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})) = 1-p^{nHL}$, and farmer 2 pumps $\hat{u}_{2,1}^{nHL}$. By Lemmas 1 and 2, the necessary and sufficient conditions are

(A15) $\hat{u}_{2,1}^{nHL} \le 1+s$, and $\beta\ E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})\hat{u}_{2,1}^{nHL}-\tilde{\alpha}u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}))]$

$$< g_u(\hat{u}_2^{nHL},1+s) < \beta\{\ q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})) + (1-q)g_u(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL})\}.$$

To see why, observe that $\hat{u}_{2,1}^{nHL}$ must be the unique maximizer of

(A16) $\max_{u \le 1+s} g(u,1+s) + \beta E[p^{nHL}b(x_{2,2}(\tilde{\alpha},u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),u)) + (1-p^{nHL})b(x_{2,2}(\tilde{\alpha},u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}),u))]$.

Because $p^{nHL} > 0$, it must be that farmer 1 always has a positive stock in *t*=2 if he pumps $\tilde{u}_{1,1}^{n}$ $= u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})$, i.e. $x_{1,2}(\alpha_H, u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL}) > 0$. Also, because $p^{nHL} < 1$, it must be that farmer 1 has no water in *t*=2 for $\tilde{\alpha} = \alpha_L$, when he pumps $u_{1,1}^{n} = u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})$. And so, the FOC for (A16) is

(A17) $$g_u(\hat{u}_{2,1}^{nHL},1+s) - \beta\{qp^{nHL}(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL}-\alpha_L u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})) + (1-q)p^{nHL}(1-\alpha_H)b'(1+s-(1-\alpha_H)\hat{u}_{2,1}^{nHL}-\alpha_H u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})) + q(1-p^{nHL})(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2.1}^{nHL}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})) + (1-q)(1-p^{nHL})b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL})\} \ge 0 \ (=0, \text{ if } \hat{u}_{2,1}^{nHL} < x_{2,1}).$$

Hence, the probability that farmer 1 pumps $u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})$ is $p^{nHL} \ge \lambda/\mu$ (with equality if $\hat{u}_{2,1}^{nHL} < x_{2,1}$), where $\lambda \equiv g_u(\hat{u}_{2,1}^{nHL},1+s) - \beta\{\ q(1-\alpha_L)b'(x_{2,2}(\alpha_L,u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL}) + (1-q)g_u(2-u_1^{n,L}(\hat{u}_2^{n,HL})-\hat{u}_2^{n,HL})\}$, and $\mu \equiv \beta\{q(1-\alpha_L)[b'(x_{2,2}(\alpha_L,u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL})) - b'(x_{2,2}(\alpha_L,u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL}))] + (1-q)[(1-\alpha_H)b'(x_{2,2}(\alpha_H,u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nL})) - b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL})]\} < 0$. The inequality follows because $u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}) < u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})$ and $x_{2,2}(\alpha_H,u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL}) \ge x_{2,2}(\alpha_H,u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2.1}^{nHL}) \ge 2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL}$, and b' is positive and decreasing. Therefore, (A15) guarantees that $\lambda / \mu \in (0,1)$. Note that, by (A17), this equilibrium is locally unique (i.e., p^{nHL} is unique) if $\hat{u}_{2,1}^{nHL} < 1+s$.

Case 3. Suppose that cell 1 is (resp., not) empty in *t*=1 when $\alpha = \alpha_H$ (resp., $\alpha = \alpha_L$). Then $u_{i,1}^n = u_{i,1}^{nL}(u_{j,1}^n)$, and by Lemmas 1 and 2, the necessary and sufficient conditions for $u_{i,1}^{nBR}(u_{j,1}^n) = u_{i,1}^{nL}(u_{j,1}^n)$, $i, j \in \{1,2\}$, $i \neq j$ are

(A18) $\hat{u}_{2,1}^{nHL} \le u_{2,1}^n \le \hat{u}_{2,1}^{nLD}$ and $g_u(d(u_{1,1}^n, \alpha_H), 1+s) \ge \beta E[(1-\tilde{\alpha})b'(x_{2,2}(\tilde{\alpha}, u_{1,1}^n, d(u_{1,1}^n, \alpha_H)))$,

(A19) and either (a) $d(u_{1,1}^n, \alpha_H) < 1+s < d(u_{1,1}^n, \alpha_L)$, or (b) $1+s \ge d(u_{1,1}^n, \alpha_L)$ and

$$g_u(d(u_{1,1}^n, \alpha_L), 1+s) \le \beta(1-q\alpha_L)b'(1+s-(1-\alpha_L)d(u_{1,1}^n, \alpha_L) - \alpha_L u_{1,1}^n).$$

Note that (A18) and (A19) are implied by

(A20) (a) $\hat{u}_{2,1}^{nLD} \le 1+s$, (b) $g_u(\hat{u}_{2,1}^{nHL}, 1+s) \ge \beta\{\, q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL} - \alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}))$
$+ (1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2.1}^{nHL}) - \hat{u}_{2.1}^{nHL})\}$, and (c) $g_u(\hat{u}_{2,1}^{nLD}, 1+s)$
$\le \beta\{\, q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nLD} - \alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})) + (1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}) - \hat{u}_{2,1}^{nLD})\}$

.

To see why, note that since $du_{1,1}^{nL}(u)/du \in (-1,0)$, (A20b) and (A20c) imply that $\hat{u}_{2,1}^{nHL} \le u_{2,1}^n \le \hat{u}_{2,1}^{nLD}$. Since $u_{2,1}^n = u_{2,1}^{nL}(u_{1,1}^n) > d(u_{1,1}^n, \alpha_H)$, by (11) in the text, we have

$$\beta E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})d(u_{1,1}^n, \alpha_H) - \tilde{\alpha}u_{1,1}^n)] < \beta\{q(1-\alpha_L)b'(1+s-(1-\alpha_L)u_{2,1}^n - \alpha_L u_{1,1}^n) + (1-q)b'(2-u_{1,1}^n - u_{2,1}^n)\} \le g_u(u_{2,1}^n, 1+s) \le g_u(d(u_{1,1}^n, \alpha_H), 1+s).$$

This verifies that (A20) implies (A18). To verify that (A20) also implies (A19), suppose that $1+s \ge d(u_{1,1}^n, \alpha_L)$ and note that since $u_{2,1}^n = u_{2,1}^{nL}(u_{1,1}^n) < d(u_{1,1}^n, \alpha_L) \le 1+s$, by (11), we have

$$g_u(d(u_{1,1}^n, \alpha_L), 1+s) < g_u(u_{2,1}^n, 1+s) = \beta\{q(1-\alpha_L)b'(1+s-(1-\alpha_L)u_{2,1}^n - \alpha_L u_{1,1}^n) + (1-q)b'(2-u_{1,1}^n - u_{2,1}^n)\} \le \beta(1-q\alpha_L)b'(1+s-(1-\alpha_L)d(u_{1,1}^n, \alpha_L) - \alpha_L u_{1,1}^n).$$

Furthermore, this equilibrium is locally unique by the same argument that is used to show local uniqueness in Case 1.

Case 4. Suppose that in equilibrium farmer 1 randomizes: $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})) = p^{nLD}$ and $\Pr(\tilde{u}_{1,1}^n = x_{1,1}) = 1 - p^{nLD}$, and farmer 2 pumps $\hat{u}_{2,1}^{nLD}$. By Lemmas 1 and 2, the necessary and sufficient conditions are

(A21) $\hat{u}_{2,1}^{nLD} \le 1+s$, $\beta\{\, q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nLD} - \alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}))$
$+ (1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}) - \hat{u}_{2,1}^{nLD})\} < g_u(\hat{u}_{2,1}^{nLD}, 1+s) < \beta\; b'(1+s-\hat{u}_{2,1}^{nLD})$.

To see why, observe that $\hat{u}_{2,1}^{nLD}$ must be the unique maximizer of

(A22) $\max_{u \le 1+s} g(u, 1+s) + \beta E[p^{nLD} b(x_{2,2}(\tilde{\alpha}, u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}), u)) + (1-p^{nLD}) b(x_{2,2}(\tilde{\alpha}, 1-s, u)))]$.

Because $p^{nLD} > 0$, it must be that cell 1 is not empty in *t*=2 if $\tilde{\alpha} = \alpha_L$, when farmer 1 pumps $u_{1,1}^n = u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})$. Also, because $p^{nLD} < 1$, it must be that cell 1 is always empty in *t*=2, when $u_{1,1}^n = 1-s$. Hence, because $g_u(0,\cdot) = \infty$, it must be that $\hat{u}_{2,1}^{nLD} < 1+s$, and the FOC becomes

$$g_u(\hat{u}_{2,1}^{nLD}, 1+s) - \beta\{qp^{nLD}(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nLD} - \alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}))$$

$$+(1-q)p^{nLD}b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})-\hat{u}_{2,1}^{nLD})+(1-p^{nLD})b'(1+s-\hat{u}_{2,1}^{nLD})\}=0.$$

By using the same arguments as in Case 2, it follows that (A21) guarantees that $p^{nLD}\in(0,1)$. Furthermore, this equilibrium is also locally unique (i.e., p^{nLD} is unique).

Case 5. Suppose that in equilibrium $u_{1,1}^{n}=1-s$, $u_{2,1}^{n}=u_{2,1}^{nD}(x_{1,1})$. Then, by Lemmas 1 and 2, the necessary and sufficient conditions for $u_{1,1}^{nBR}(u_{2,1}^{n})=1-s$ and $u_{2,1}^{nBR}(1-s)=u_{2,1}^{nD}(1-s)$, are

(A23) $\hat{u}_{2,1}^{nLD}\le u_{2,1}^{n}$ and $g_u(1-s,1+s)\ge\beta(1-q\alpha_L)b'(2s)$.

Note that (A23) is implied by

(A24) $\hat{u}_{2,1}^{nLD}<1+s$ and $g_u(\hat{u}_{2,1}^{nLD},1+s)\ge\beta\, b'(1+s-\hat{u}_{2,1}^{nLD})$.

To see why, observe that (A24) implies that $u_{2,1}^{n}=u_{2,1}^{nD}(1-s)\ge\hat{u}_{2,1}^{nLD}\ge1-s$, and

$$g_u(1-s,1+s)>g_u(\hat{u}_{2,1}^{nLD},1+s)\ge\beta b'(1+s-\hat{u}_{2,1}^{nLD})\ge\beta b'(2s)>\beta(1-q\alpha_L)b'(2s).$$

Clearly, this equilibrium is locally unique.

Because the necessary and sufficient conditions in Cases 1-5 are mutually exclusive, equilibrium is (a.e.) globally unique. The analysis in the case with $\hat{u}_{2,1}^{nHL}\ge\hat{u}_{2,1}^{nHD}$ is similar, except that in equilibrium cell 1 is either always empty or never empty in period 2, i.e. farmer 1 avoids any gamble whose worst outcome is zero stock in period 2.[23]

Step 2. Let $s^{nH}=\inf\{s\in[0,1]:s\ge\hat{u}_{2,1}^{nHL}-1,\ g_u(\hat{u}_{2,1}^{nHL},1+s)\ge\beta\ E[(1-\tilde{\alpha})$ $b'(x_{2,2}(\tilde{\alpha},\hat{u}_{2,1}^{nHL},u_1^{H}(\hat{u}_2^{HL})))]\}$, $s^{nHL}=\inf\{s\in[0,1]:s\ge\hat{u}_{2,1}^{nHL}-1,\ g_u(\hat{u}_{2,1}^{nHL},1+s)$ $\ge\beta(q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}))+(1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL}))\}$, $s^{nL}=\inf\{s\in[0,1]:s\ge\hat{u}_{2,1}^{nLD}-1,\ g_u(\hat{u}_{2,1}^{nLD},1+s)\ge\beta(q(1-\alpha_L)b'(x_{2,2}(\alpha_L,u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}),\hat{u}_{2,1}^{nLD}))$ $+(1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD})-\hat{u}_{2,1}^{nLD}))\}$, $s^{nLD}=\inf\{s\in[0,1]:s\ge\hat{u}_{2,1}^{nLD}-1,\ g_u(\hat{u}_{2,1}^{nLD},1+s)\ge\beta$ $b'(1+s-\hat{u}_{2,1}^{nLD})\}$. Note that $dg_u(\hat{u}_{2,1}^{nHL},1+s)/ds=g_{uu}d\hat{u}_{2,1}^{nHL}/ds+g_{ux}>0$, $dg_u(\hat{u}_{2,1}^{nLD},1+s)/ds$ $=g_{uu}d\hat{u}_{2,1}^{nLD}/ds+g_{ux}>0$, because, by Lemma 4, $d\hat{u}_{2,1}^{nHL}/ds<0$ and $d\hat{u}_{2,1}^{nLD}/ds<0$. Also, observe that $dE[(1-\tilde{\alpha})b'(x_{2,2}(\tilde{\alpha},u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL})))]/ds=E[(1-\tilde{\alpha})\{1-(1-\tilde{\alpha}$ $+\tilde{\alpha}du_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})/d\hat{u}_{2,1}^{nHL})\ d\hat{u}_{2,1}^{nHL}/ds\}b''(x_{2,2}(\tilde{\alpha},u_{1,1}^{nH}(\hat{u}_{2,1}^{nHL}),\hat{u}_{2,1}^{nHL}))]<0$ because $du_{1,1}^{nH}(\hat{u}_{2,1}^{nHL})$ $/d\hat{u}_{2,1}^{nHL}\in(-1,0)$, $d[q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}))+(1-q)$ $b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL})]/ds=q(1-\alpha_L)b''(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}))(1-(1-\alpha_L$ $+\alpha_L du_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})/d\hat{u}_{2,1}^{nHL})d\hat{u}_{2,1}^{nHL}/ds)-(1-q)b''(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})-\hat{u}_{2,1}^{nHL})(1+du_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})/d\hat{u}_{2,1}^{nHL})$ $d\hat{u}_{2,1}^{nHL}/ds<0$ because $du_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})/d\hat{u}_{2,1}^{nHL}\in(-1,0)$, $d[q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nLD}$

[23] In the zero probability event that $\hat{u}_{2,1}^{nHL}=\hat{u}_{2,1}^{nHD}\equiv\hat{u}_{2,1}^{0}$, the equilibrium may not be unique in the sense that farmer 1 may randomize between $u_{1,1}^{nH}(\hat{u}_{2,1}^{0})$ and either $u_{1,1}^{nL}(\hat{u}_{2,1}^{0})$ or $x_{1,1}$, if $\beta E[(1-\tilde{\alpha})b'(x_{2,2}(\tilde{\alpha},\hat{u}_{2,1}^{0},u_{1,1}^{nH}(\hat{u}_{2,1}^{0})))]$ $<g_u(\hat{u}_2^{0},x_{2,1})<\beta\{\ q(1-\alpha_L)b'(x_{2,2}(\alpha_L,u_{1,1}^{nL}(\hat{u}_{2,1}^{0}),\hat{u}_{2,1}^{0}))+(1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{0})-\hat{u}_{2,1}^{0})\}$, or farmer 1 may randomize between $x_{1,1}$ and either $u_{1,1}^{nH}(\hat{u}_{2,1}^{0})$ or $u_{1,1}^{nL}(\hat{u}_{2,1}^{0})$, if $\beta\{\ q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{0}-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{0}))$ $+(1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{0})-\hat{u}_{2,1}^{0})\}<g_u(\hat{u}_{2,1}^{0},1+s)<\beta b'(1+s-\hat{u}_{2,1}^{0})$.

$-\alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}))) + (1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nLD}) - \hat{u}_{2,1}^{nLD})]/ds < 0$, and $db'(1+s-\hat{u}_{2,1}^{nLD})/ds$
$= b''(1+s-\hat{u}_{2,1}^{nLD})(1-d\hat{u}_{2,1}^{nLD}/ds) < 0$.

Therefore, because $\hat{u}_{2,1}^{nHD} < \hat{u}_{2,1}^{nLD}$, $b'' < 0$ and $g_{uu} < 0 \le g_{ux}$, from the definitions of s^{nK} and s^{nKM}, $K = H, L$, $M = L, D$, $K \neq M$, it follows that $0 < s^{nH} \le s^{nHL} < s^{nL} \le s^{nLD}$. By Step 1, conditions that are necessary and sufficient for Case 1 (respectively, Case 2, 3 4, and 5) are satisfied for any $s \in [0, s^{nH}]$, (respectively, $s \in (s^{nH}, s^{nHL})$, $s \in [s^{nHL}, s^{nL}]$, $s \in (s^{nL}, s^{nLD})$, and $s \in [s^{nLD}, 1]$). ■

Proof of Lemma 5: Because, by (8) $\hat{u}_{2,1}^{nLD}(q)$ is increasing in q and $\hat{u}_{2,1}^{nLD}(q=1) = \hat{u}_{2,1}^{c}(\alpha_L)$, it follows that $\hat{u}_{2,1}^{nLD} < \hat{u}_{2,1}^{c}(\alpha_L)$. Because, by (14), $\hat{u}_{2,1}^{c}(\alpha)$ is decreasing in α, it follows that $\hat{u}_{2,1}^{c}(\alpha_H) < \hat{u}_{2,1}^{c}(\alpha_L)$, which implies, by (8), that $\hat{u}_{2,1}^{c}(\alpha_H) \le \hat{u}_{2,1}^{nHD}(q) \le \hat{u}_{2,1}^{c}(\alpha_L)$. The result follows by observing that $s(u)$ that is defined by the equation, $g_u(u, 1+s(u)) - \beta b'(1+s(u)-u) = 0$, is increasing in u since $g_{ux} \ge 0$ and g is concave, and, by Step 2 in Proposition 1, we have $s^{nLD} = s(\hat{u}_{2,1}^{nLD})$, $s^{nHD} = s(\hat{u}_{2,1}^{nHD})$, and $s^{cD}(\alpha) = s(\hat{u}_{2,1}^{c}(\alpha))$. ■

Proof of Lemma 6: Because $\lim_{\alpha_L \to 0} \hat{u}_{2,1}^{nLD} = \infty$, it follows that $\lim_{\alpha_L \to 0} s^{nLD} = 1$. Hence, by continuity, there exists such a $\alpha_1 > 0$ such that $\hat{u}_{2,1}^{nHL} < \hat{u}_{2,1}^{nHD} < \hat{u}_{2,1}^{nLD}$ and $s \le s^{LD}$ for all $\alpha_L \in [0, \alpha_1]$. Then, by Proposition 1, we need to consider equilibrium in four cases depending on whether $s \in [0, s^{nH}]$, $s \in (s^{nH}, s^{nHL})$, $s \in [s^{nHL}, s^{nL}]$, or $s \in (s^{nL}, s^{nLD})$.

Case 1. $s \in [0, s^{nH}]$. Here $u_{2,1}^{n} = u_{2,1}^{nH}(u_{1,1}^{n}) < d(u_{1,1}^{n}, \alpha_H)$. If $u_{2,1}^{n} = 1+s$ then we are done. So suppose that $u_{2,1}^{n} < 1+s$. Then, by (10), we have

$$g_u(u_{2,1}^{n}, 1+s) = \beta E[(1-\tilde{\alpha})b'(1+s-(1-\tilde{\alpha})u_{2,1}^{n} - \tilde{\alpha}u_{1,1}^{n}) < \beta b'(1+s-u_{2,1}^{n})$$

which implies that $u_{2,1}^{n} > u_{2,1}^{c}(\alpha_L = 0)$ since in equilibrium under complete information with $\tilde{\alpha} = \alpha_L = 0$, $u_{2,1}^{c}(0)$ is determined by the first-order condition:

(A25) $g_u(u_{2,1}^{c}(0), 1+s) = \beta b'(1+s-u_{2,1}^{c}(0))$.

Then, by continuity, there exists a $\alpha_2 > 0$ such that $u_{2,1}^{n} \ge u_{2,1}^{c}(\alpha_L)$ for all $\alpha_L \in [0, \alpha_2]$.

Case 2. $s \in (s^{nH}, s^{nHL})$. Since $p^{nHL} > 0$, by (A15),

$$g_u(\hat{u}_{2,1}^{nHL}, 1+s) < \beta\{\, q(1-\alpha_L)b'(1+s-(1-\alpha_L)\hat{u}_{2,1}^{nHL} - \alpha_L u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL})) + (1-q)b'(2-u_{1,1}^{nL}(\hat{u}_{2,1}^{nHL}) - \hat{u}_{2,1}^{nHL})\} < \beta b'(1+a-\hat{u}_{2,1}^{nHL}),$$

which, by (A25), implies that $u_{2,1}^{n} > u_{2,1}^{c}(\alpha_L = 0)$. Then, by continuity, there exists a $\alpha_3 > 0$ such that $u_{2,1}^{n} = \hat{u}_{2,1}^{nHL} \ge u_{2,1}^{c}(\alpha_L)$ for all $\alpha_L \in [0, \alpha_3]$.

Case 3. $s \in [s^{nHL}, s^{nL}]$. Then $u_{2,1}^{n} = u_{2,1}^{nL}(u_{1,1}^{n}) \in (d(u_{1,1}^{n}, \alpha_H), d(u_{1,1}^{n}, \alpha_L))$, and by (11),

$$g_u(u_{2,1}^{n}, 1+s) = \beta\{q(1-\alpha_L)b'(1+s-(1-\alpha_L)u_{2,1}^{n} - \alpha_L u_{1,1}^{n})$$

$$+(1-q)b'(2-u_{1,1}^n-u_{2,1}^n)\} < \beta b'(1+s-u_{2,1}^n),$$

which, by (A25), implies that $u_{2,1}^n > u_{2,1}^c(\alpha_L = 0)$. Then, by continuity, there exists a $\alpha_4 > 0$ such that $u_{2,1}^n \geq u_{2,1}^c(\alpha_L)$ for all $\alpha_L \in [0,\alpha_4]$.

Case 4. $s \in (s^{nL}, s^{nLD})$. Since $p^{nLD} > 0$, (A21) and (A25) imply that $u_{2,1}^n > u_{2,1}^c(\alpha_L = 0)$. Then, by continuity, there exists a $\alpha_5 > 0$ such that $u_{2,1}^n \geq u_{2,1}^c(\alpha_L)$ for all $\alpha_L \in [0,\alpha_5]$

The proof is completed by setting $\alpha_0 = \min_{i=1,\ldots,5}[\alpha_i]$. ■

Proof of Proposition 3: By Propositions 1 and 2, the equilibrium pumping rates under incomplete (respectively, complete) information are continuous in s for $s \in [0, s^{nH}]$ (respectively, $s \in [0, s^{cN}(\alpha)]$, $\alpha = \alpha_L, \alpha_H$). Hence, by continuity, there exists a $s_0 > 0$ such that $E[u_{i,1}^c(\tilde{\alpha}, s)] \geq (\leq) u_{i,1}^n(s)$ for all $s \in [0, s_0]$ depending on whether $f'' > (<) 0$ because, by Proposition 3 in SP, $E[u_{1,1}^c(\tilde{\alpha}, s)] = E[u_{2,1}^c(\tilde{\alpha}, s)] > (<) u_{2,1}^n(s) = u_{1,1}^n(s)$ for $s = 0$. ■

Proof of Corollary 1: Because, by assumption, $s \geq s^{cD}(\alpha_H)$, applying Proposition 2, we have $u_{2,1}^c(\alpha_H) = u_{2,1}^{cD}(1-s) \leq u_{2,1}^c(\alpha_L)$. By Lemma 6, there exists an $\alpha_0 > 0$ such that $u_{2,1}^c(\alpha_L) \leq u_{2,1}^n$ for all $\alpha_L \in [0, \alpha_0]$. ■

Proof of Proposition 5: The proof is analogous to the proof of Proposition 3. ■

Proof of Proposition 6: Consider the social planner's problem under complete information

(A26) $W^{sc}(\alpha, s) = \max_{\{u_{i,t}^{sc}\}_{i=1,2,t=1,2}} \sum_{i=1,2} g(u_{i,1}^{sc}, x_{i,1}) + \beta g(u_{i,2}^{sc}, x_{i,2}(\alpha, u_{1,1}^{sc}, u_{2,1}^{sc}))$.

It is convenient to solve (A26) as a two-step optimization program, as follows

$$W^{sc}(\alpha, s) = \max[W^{cN}(\alpha, s), W^{cD}(\alpha, s)],$$

where $W^{cN}(\alpha, s) \equiv \max_{\{u_{i,1}^{sc}\}_{i=1,2}} \sum_{i=1,2} g(u_{i,1}^{sc}, x_{i,1}) + \beta b(x_{i,2})$ subject to (1), (2), and $1-s-(1-\alpha)u_{1,1}^{sc} - \alpha u_{2,1}^{sc} \geq 0$, and $W^{cD}(\alpha, s) \equiv \max_{\{u_{i,1}^{sc}\}_{i=1,2}} \sum_{i=1,2} g(u_{i,1}^{sc}, x_{i,1}) + \beta b(x_{i,2})$ subject to (1), (2), and $1-s-(1-\alpha)u_{1,1}^{sc} - \alpha u_{2,1}^{sc} \leq 0$.

Also, let $W^{sn}(s) = \max_{\{u_{i,t}^{sn}\}_{i=1,2,t=1,2}} \sum_{i=1,2} g(u_{i,1}^{sn}, x_{i,1}) + \beta E[g(u_{i,2}^{sn}, x_{i,2}(\tilde{\alpha}, u_{1,2}^{sn} u_{2,2}^{sn}))]$ denote the maximum attainable welfare under incomplete information. Note that for $\alpha \in (0, 0.5]$, we have

$$\lim_{s\to 1} W^{cN}(\alpha, s) = \beta b(2) < \max_{u_{2,1}^{sc}} g(u_{2,1}^{sc}, 2) + \beta b(2 - u_{2,1}^{sc}) = \lim_{s \to 1} W^{cD}(\alpha, s).$$

Hence, by continuity, for any $\alpha_H \in (0, 0.5]$, there exists an $s_0(\alpha_H) \in (0,1)$ such that $W^{cN}(\alpha_H, s) \leq W^{cD}(\alpha_H, s)$, and the socially efficient allocation is given by $u_{1,1}^{sc}(\alpha_H, s) = 1-s$ and $u_{2,1}^{sc}(\alpha_H, s) = u_{2,1}^{cD}(1-s; \alpha_H)$ for all $s \in [s_0(\alpha_H), 1)$. But, by Proposition 2, $u_{1,1}^c(\alpha_H, s)$

$=1-s$ and $u^c_{2,1}(\alpha_H, s) = u^{cD}_{2,1}(1-s;\alpha_H)$ are also the (unique) equilibrium pumping rates when both farmers know that $\tilde{\alpha} = \alpha_H$. Hence, it must be that $s^{cD}(\alpha_H) \le s_0(\alpha_H)$. In addition, the socially efficient and equilibrium allocations (trivially) coincide for $\alpha = 0$ (since the cells are hydrologically disconnected). Hence, $u^c_{i,1}(\alpha) = u^{sc}_{i,1}(\alpha)$ for $i = 1,2$ and $\alpha = 0, \alpha_H$, and we obtain

$$\sum\nolimits_{i=1,2} \pi^n_i < W^{sn}(s) < E[W^{sc}(\tilde{\alpha}, s)] = \sum\nolimits_{i=1,2} E[\pi^c_i(\tilde{\alpha})].$$

The first inequality follows because non-cooperative farmers cannot attain a higher joint welfare than the social planner (in fact, from the optimality conditions it follows that they attain a strictly lower joint welfare under incomplete information). The second inequality follows because the lack of information cannot possibly increase the maximum attainable joint welfare, i.e. $W^{sn}(s) < E[W^{sc}(\tilde{\alpha}, s)]$ (because the social planner can always discard more precise information about $\tilde{\alpha}$). The inequality is strict because the socially efficient allocation is responsive to the realization of $\tilde{\alpha}$ since $\alpha_L = 0 < \alpha_H$. By continuity, there exists α_0 $\in (0, \alpha_H)$ such that $\sum\nolimits_{i=1,2} \pi^n_i \le \sum\nolimits_{i=1,2} E[\pi^c_i(\tilde{\alpha})]$ for any $\alpha_L \in [0, \alpha_0]$. ■

Proof of Proposition 7: Because $s \ge s^{cD}(\alpha_H)$, by Corollary 1, there exists an α_1 such that

(A27) $u^c_{2,1}(\alpha_H) \le u^c_{2,1}(\alpha_L) < u^n_{2,1}$ for all $\alpha_L \in [0, \alpha_1]$.

Hence, by (5), for any $\alpha_L \in [0, \alpha_1]$, we have

$$\begin{aligned}
\pi^n_1 &= E[g(\tilde{u}^n_{1,1}, 1-s) + \beta b(x_{1,2}(\tilde{\alpha}, \tilde{u}^n_{1,1}, u^n_{2,1}))] \\
&< E[g(\tilde{u}^n_{1,1}, 1-s) + \beta b(x_{1,2}(\tilde{\alpha}, \tilde{u}^n_{1,1}, u^c_{2,1}(\tilde{\alpha})))] \\
&< E[g(\tilde{u}^{cBR}_{1,1}(u^c_{2,1}(\tilde{\alpha})), 1-s) + \beta b(x_{1,2}(\tilde{\alpha}, \tilde{u}^{cBR}_{1,1}(u^c_{2,1}(\tilde{\alpha})), u^c_{2,1}(\tilde{\alpha})))] \\
&= E[g(\tilde{u}^c_{1,1}(\tilde{\alpha}), 1-s) + \beta b(x_{1,2}(\tilde{\alpha}, \tilde{u}^c_{1,1}(\tilde{\alpha}), u^c_{2,1}(\tilde{\alpha})))] = E[\pi_1(\tilde{\alpha})].
\end{aligned}$$

The first inequality follows by (A27). The second inequality follows because re-optimization by farmer 1 conditional on the new information cannot decrease profits, where we assume that he randomizes in an equilibrium manner if $u^c_{2,1}(\alpha_L) = \hat{u}^c_{2,1}$ (recall that farmer 1 is indifferent between pumping $u^{cN}_{1,1}(\alpha_L)$ and $1-s$, if $u^c_{2,1}(\alpha_L) = \hat{u}^c_{2,1}$).

To show that farmer 2 achieves a lower expected profit under better public information, we first establish the validity of the following claims.

Claim 1. There exists an $\alpha_2 > 0$ such that $\forall \alpha_L \in [0, \alpha_2]$

(A28) $\Pr(1 - s - (1-\alpha_L)\tilde{u}^n_{1,1} - \alpha_L u^c_{2,1}(\alpha_K) \ge 0) = 1$ for $K = L, H$.

Note that, by (1), (A28) trivially holds for $\alpha_L = 0$. Hence, by continuity, one can always pick α_2 sufficiently close to 0 such that (A28) continues to hold, because, by Proposition 2, $s \ge s^{cD}(\alpha_H)$ implies that $u^c_{2,1}(\alpha = 0) = u^c_{2,1}(\alpha_H)$.

Claim 2. There exists an $\alpha_3 > 0$ such that $\forall \alpha_L \in [0, \alpha_3]$

(A29) $E[b(1 + s - (1-\alpha_L)u^c_{2,1}(\alpha_L) - \alpha_L \tilde{u}^n_{1,1}) - b(1 + s - (1-\alpha_L)u^c_{2,1}(\alpha_H) - \alpha_L \tilde{u}^n_{1,1})$

$$- (b(\min[1 + s - (1-\alpha_H)u^c_{2,1}(\alpha_L) - \alpha_H \tilde{u}^n_{1,1}, 2 - \tilde{u}^n_{1,1} - u^c_{2,1}(\alpha_L)])$$

$-b(\min[1+s-(1-\alpha_H)u_{2,1}^c(\alpha_H)-\alpha_H\tilde{u}_{1,1}^n, 2-\tilde{u}_{1,1}^n-u_{2,1}^c(\alpha_H)]))] < 0$.

To show (A29), first suppose that $\Pr(\tilde{u}_{1,1}^n = u_{1,1}^n) = 1$ for some $u_{1,1}^n$. One can pick α_3 sufficiently close to 0 such that $d(\alpha_H, u_{1,1}^n) \notin (u_{2,1}^c(\alpha_H), u_{2,1}^c(\alpha_L))$ $\forall \alpha_L \in [0, \alpha_3]$ since, by Proposition 2, $u_{2,1}^c(\alpha_L) \geq u_{2,1}^c(\alpha_H) = u_{2,1}^c(\alpha = 0)$. Hence, there are only two cases to consider (a) $u_{2,1}^c(\alpha_L) \leq d(\alpha_H, u_{1,1}^n)$ and (b) $u_{2,1}^c(\alpha_H) \geq d(\alpha_H, u_{1,1}^n)$. Suppose that $u_{2,1}^c(\alpha_L) \leq d(\alpha_H, u_{1,1}^n)$. Then, because $u_{2,1}^c(\alpha_L) \geq u_{2,1}^c(\alpha_H)$, (A29) becomes

$$b(1+s-(1-\alpha_L)u_{2,1}^c(\alpha_L)-\alpha_L u_{1,1}^n) - b(1+s-(1-\alpha_L)u_{2,1}^c(\alpha_H)-\alpha_L u_{1,1}^n)$$
$$-[b(1+s-(1-\alpha_H)u_{2,1}^c(\alpha_L)-\alpha_H u_{1,1}^n) - b(1+s-(1-\alpha_H)u_{2,1}^c(\alpha_H)-\alpha_H u_{1,1}^n)] \leq 0,$$

where the inequality follows because the function $r(\alpha,u) = b(1+s-(1-\alpha)u-\alpha u_{1,1}^n)$ is supermodular for $u \geq u_{1,1}^n$ since $\frac{\partial^2 r_1(\alpha,u)}{\partial\alpha\partial u} = -b''(1+s-(1-\alpha)u-\alpha u_{1,1}^n)(1-\alpha)(u-u_{1,1}^n)$ $+b'(1+s-(1-\alpha)u-\alpha u_{1,1}^n) > 0$.

(b) Suppose that $u_{2,1}^c(\alpha_H) \geq d(\alpha_H, u_{1,1}^n)$. Then, because $u_{2,1}^c(\alpha_L) \geq u_{2,1}^c(\alpha_H)$, (A29) becomes

$$b(1+s-(1-\alpha_L)u_{2,1}^c(\alpha_L)-\alpha_L u_{1,1}^n) - b(2-u_{1,1}^n-u_{2,1}^c(\alpha_L))$$
$$-(b(1+s-(1-\alpha_L)u_{2,1}^c(\alpha_H)-\alpha_L u_{1,1}^n) - b(2-u_{1,1}^n-u_{2,1}^c(\alpha_H))) \leq 0,$$

where the inequality follows because the function $r^*(\alpha,u) = b(1+s-(1-\alpha)u-\alpha u_{1,1}^n)$ $-b(2-u_{1,1}^n-u)$ is decreasing in u when $\alpha \to 0$. But taking the expectation over $\tilde{u}_{1,1}^n$ cannot change the sign of the inequality, which proves (A29).

By Claims 1 and 2, for $\alpha_L \in [0, \min[\alpha_2, \alpha_3]]$, we have

$$\text{(A30)}\quad E[q^2 b(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + (1-q)qb(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))$$
$$+(1-q)qb(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + (1-q)^2 b(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))]$$
$$\geq E[(q^2+\varepsilon)b(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + (q(1-q)-\varepsilon)qb(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))$$
$$+((1-q)q-\varepsilon)b(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + ((1-q)^2+\varepsilon)b(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))],$$

where $\varepsilon \in [0, q(1-q)]$.

Turning to the effect of information on the farmer 2's expected profits, by (5), there exists an $\alpha_0 > 0$ such that for any $\alpha_L \in [0, \alpha_0]$, we have

$$\text{(A31)}\quad \pi_2^n = g(u_{2,1}^n, 1+s) + \beta E[b(x_{2,2}(\tilde{\alpha}, \tilde{u}_{1,1}^n, u_{2,1}^n))]$$
$$\geq qg(u_{2,1}^c(\alpha_L), 1+s) + (1-q)g(u_{2,1}^c(\alpha_H), 1+s)$$
$$+\beta\{E[q^2 b(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + q(1-q)b(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))$$
$$+(1-q)qb(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + (1-q)^2 b(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))]\}$$
$$\geq qg(u_{2,1}^c(\alpha_L), 1+s) + (1-q)g(u_{2,1}^c(\alpha_H), 1+s)$$
$$+\beta\{E[qb(x_{2,2}(\alpha_L, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_L))) + (1-q)b(x_{2,2}(\alpha_H, \tilde{u}_{1,1}^n, u_{2,1}^c(\alpha_H)))]\}$$
$$\geq E[g(u_{2,1}^c(\tilde{\alpha}), 1+s) + \beta b(x_{2,2}(\tilde{\alpha}, \tilde{u}_{1,1}^c(\tilde{\alpha}), u_{2,1}^c(\tilde{\alpha})))] = E[\pi_2^c(\tilde{\alpha})].$$

The first inequality follows because $u^n_{2,1}$ is the (unique) maximizer of the farmer 2's expected profits under incomplete information. In particular, farmer 2's expected profits cannot increase if she randomizes between the pumping rates that are her equilibrium choices under complete information (note that the pumping rates are assumed to be independent from $\tilde{\alpha}$). The second inequality follows because, by (A30), her expected profits decrease further if her pumping rates: $\tilde{u}^c_{2,1} = \begin{cases} u^c_{2,1}(\alpha_L), \text{with prob.}\, q \\ u^c_{2,1}(\alpha_H), \text{with prob.}\, 1 - q \end{cases}$, become perfectly correlated with $\tilde{\alpha}$, i.e. $\Pr(\tilde{u}^c_{2,1} = u^c_{2,1}(\alpha_k) \mid \tilde{\alpha} = \alpha_k) = 1$ for $k = L, H$, provided that α_L is sufficiently close to 0. The third inequality in (A31) holds strictly for $\alpha_L = 0$ because then we have

$$\Pr\{x_{2,2}(\alpha_L, \tilde{u}^n_{1,1}, u^c_{2,1}(\alpha_L)) = x_{2,2}(\alpha_L, u^c_{1,1}(\alpha_L), u^c_{2,1}(\alpha_L))\} = 1, \text{ and}$$

$$\Pr\{x_{2,2}(\alpha_H, \tilde{u}^n_{1,1}, u^c_{2,1}(\alpha_H)) > x_{2,2}(\alpha_H, u^c_{1,1}(\alpha_H), u^c_{2,1}(\alpha_H))\} = 1,$$

since, by assumption, $s \geq s^{cD}(\alpha_H)$ so that $u^c_{1,1}(\alpha_H) = 1 - s > u^{nBR}_{1,1}(u^n_{2,1})$. Hence, by continuity, there exists an $\alpha_4 > 0$ such that the third inequality in (A31) is satisfied for any $\alpha_L \in [0, \alpha_4]$ sufficiently close to 0. The proof is completed by setting $\alpha_0 = \min_{i=1,..,4} \alpha_i$. ■