Homogenous and Heterogenous Contestants in Piece Rate Tournaments: Theory and Empirical Analysis

Tomislav Vukina and Xiaoyong Zheng
North Carolina State University
NCSU Campus Box 8109
Raleigh NC 27695

Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Orlando, FL, July 27-29, 2008.

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Tomislav Vukina and Xiaoyong Zheng[†]

March 23, 2008

Abstract

In this paper we show that sorting different ability contestants in piece rate tournaments into more homogenous groups alters incentives for agents to exert effort. In particular we show that for a given mean of the tournament group's ability parameters, larger variance (more heterogenous agents) induces higher optimal effort. This implies that the principal can actually gain from heterogenizing the tournament groups. On the other hand, the effect of this change on growers' welfare is unclear because higher effort leads to higher productivity and hence higher payment, but also increases the cost of effort. Using broiler production contracts settlement data we empirically estimate a fully structural model of a piece rate tournament game with heterogenous players. Our counterfactual analysis shows that under reasonable assumptions the integrator's gain is actually larger than the growers' losses indicating that heterogenizing groups in piece rate tournaments may be efficient.

[†]Vukina and Zheng: Department of Agricultural and Resource Economics, North Carolina State University, Campus Box 8109, Raleigh, NC 27695-8109. Email addresses: vukina@ncsu.edu and xzheng@ncsu.edu.

1 Introduction

Tournaments are labor contracts where an individual's payoff depends on own performance relative to others. There are *rank-order (ordinal tournaments)* such as the ones considered by Lazear and Rosen (1981), where an individual player's payment depends on her rank within the group, and *cardinal tournaments* where the reward is a continuous function (typically linear) of the difference between an individual player's performance and the group average performance. Cardinal tournaments are also referred to as the *relative performance* compensation schemes (Nalebuff and Stiglitz, 1983) or the *yardstick competition* (Shleifer, 1985). Yet another type of tournaments frequently used in settlements of some production contracts (e.g. broiler chickens) are *piece rate tournaments* (Tsoulouhas and Vukina, 1999). Piece rate tournaments are in fact variable piece rate schemes where an individual player's piece rate varies in proportion with her performance relative to the group average performance.

Virtually all real world tournaments are contests among players with unequal abilities. When players have different abilities, rank-order tournaments are known to exhibit some undesirable properties. For example, asymmetries in the knowledge of abilities entail inefficiencies because contestants do not correctly self-sort into the leagues commensurate with their types. Correcting this problem may result in entry credentials and bigger prize spreads in leagues which target higher ability players. With full knowledge of abilities, rank-order tournaments with players of heterogeneous abilities still suffer from incentive problems as both high ability and low ability contestants tend to work less than their respective efficient effort levels. Handicapping and prize structures indexed by ability are required to correct for these inefficiencies (see McLaughlin 1988).

As for the cardinal tournaments, it has been widely believed in the literature that they exhibit no efficiency losses associated with mixing players of uneven abilities. When rewards are linearly related to performance, better players have no incentive to stop exerting effort once they realize that they are going to win and worse players have no incentive to surrender once they realize that they are going to lose. Consequently, in cardinal tournaments organizers have no incentives to sort players into more homogenous groups because the incremental

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reward for improved performance (penalty for worse performance) at the margin is the same whether a player is more or less able (Knoeber and Thurman 1994). However, as shown by Levy and Vukina (2002), regardless of whether the agents are risk-neutral or risk-averse, an optimal cardinal tournament is an individualized contract indexed by the ability of agents.

The welfare effects of mixing players of varying abilities in the piece rate tournaments has not been studied in the literature so far. The best known real business world example of piece rate tournaments is the settlement of the production contracts for broiler chickens. Our motivation comes from observing that unlike in many sport competitions, where organizers, in order to enhance competition, homogenize the contestants by placing them into similar ability leagues or divisions, poultry companies generally do not attempt to place their agents into more homogenous groups in which they compete for cost efficiency bonuses. This fact is even more puzzling knowing that via repeated contracting with the same pool of agents for a long period of time, the company production managers can precisely discern agents' abilities, yet they never exploit this information to their advantage.

The main tenet of this paper is that making groups of contestants in piece rate tournaments more homogenous or more heterogenous creates different incentives for agents to exert effort. Under certain assumptions, we show that for a given mean of the tournament group's heterogeneity parameters, larger variance (more heterogenous agents) induces higher optimal effort. Hence, the principal always wins by mixing contestants of different abilities rather then sorting them into more homogenous groups. The welfare effect on the growers, on the other hand, is indeterminate as higher optimal effort leads to both higher payment and higher cost of effort, leaving it as an empirical question.

In addition to its theoretical contribution, this paper also contributes to the growing literature on structural econometrics approach to estimating tournament models, which proves to be quite useful for conducting welfare analyses. Recently, several papers have estimated structural models of various type tournaments. Ferrall and Smith (1999) estimated a sequential tournament game for championship series in sports. Ferrall (1996) and Shum (2007) estimated elimination tournament models for workers competing for limited promotion slots.

Zheng and Vukina (2007) estimated a rank-order tournament model to quantify the efficiency gains of an organizational innovation that would replace an ordinal tournament with a cardinal one.

This paper represents the first attempt to structurally estimate a piece rate tournament model that captures the most important features of the production contracts observed in the broiler chickens industry. Using broiler production contracts settlement data, we empirically quantified the welfare effects of heterogenizing tournament groups, both for the company and the contract growers. Our counterfactual analysis show that under reasonable assumptions the principal's gain is actually larger than the agents' losses indicating that heterogenizing groups in piece rate tournaments, to the extent that it is practicable, may be efficient.

The rest of the paper is organized as follows. In the next section we describe the essential features of broiler production contracts and introduce the data set. In Section 3 we introduce the theoretical model of the piece rate tournament. Section 4 is devoted to the estimation methodology and the presentation of results. In Section 5, we simulate the welfare effects of heterogenizing tournament groups using estimated piece rate tournament model primitives. Finally, Section 6 concludes.

2   Industry and Data

The broiler industry is often considered a role model for the industrialization of agriculture. The industry is entirely vertically integrated from breeding flocks and hatcheries to feed mills, transportation divisions and processing plants. The final (finishing) stage of production where one day old chicks are brought to the farm and then grown to market weight is organized almost entirely through contracts between integrators and independent growers. Large national companies, such as Tyson Foods, Pilgrim's Pride, or Perdue Farms dominate broiler contract production. These companies run their operations through smaller divisions spread throughout the country, but mainly in the south-east.

Modern broiler production contracts are agreements between an integrator company and

growers that bind farmers to tend for company's chickens until they reach market weight by strictly following specific production practices in exchange for monetary compensation. According to a typical contract, the grower provides land, housing facilities, utilities (electricity and water) and labor and pays for operating expenses such as repairs and maintenance, clean-up, and manure and mortality disposal. The company provides chicks, feed, medication, and the services of field men. Most of the modern broiler contracts are settled using a two-part piece-rate tournament. In this type of tournament, the total payment R_i to grower i is the sum of the base rate and the bonus rate multiplied by the live pounds of poultry moved from the grower's farm:

$$R_i = \left[a + b \left(\frac{1}{N} \sum_{j=1}^{N} \frac{c_j}{y_j} - \frac{c_i}{y_i} \right) \right] y_i. \tag{1}$$

As seen from (1), individual grower's piece rate per pound of live poultry produced is the sum of a constant base rate a (e.g., 3.5 - 4.5 cents a pound), and a variable bonus rate determined by the grower's relative performance. The bonus rate is determined as a percentage b of the difference between group average performance $\frac{1}{N} \sum_{j=1}^{N} \frac{c_j}{y_j}$ and the producer's individual performance $\frac{c_i}{y_i}$. Performances are measured by $f_i = \frac{c_i}{y_i}$, that is, settlement cost per pound of live chicken produced. Settlement costs (c_i) are obtained by adding chicks, feed, medication, and other customary flock costs. The calculation of the group average performance includes all N growers whose flocks are settled on the same date. In order for all growers to be exposed to the same common production shock, the time between two settlement dates typically does not exceed two weeks. For the below average settlement cost per pound of chicken produced (above average performance), the grower receives a bonus and for the above average settlement cost per pound of chicken produced, she receives a penalty.

As is explained in Tsoulouhas and Vukina (1999), poultry tournaments are double-margin contests about who can produce more output (live poultry) with the smallest possible settlement cost. The growers' effort (husbandry practices) stochastically influence the settlement costs (feed utilization) and the quantity of output. Growers can economize with feed (and hence settlement costs c_i) by preventing spillage through proper maintenance of feeders and

storage bins and by maintaining a housing environment that is conducive to efficient feed conversion. Growers can also separately influence output (live poultry weight y_i) by undertaking actions aimed at preventing excessive animal mortality.

Different companies, or different profit centers within the same company, typically specialize in the production of a particular size (weight) birds and offer their own contracts to their growers. The contracts for growing different size birds usually differ only with respect to the base rate (parameter a in (1)) in that farmers growing heavier birds typically receive larger base rate than those growing smaller birds. Broiler contracts are always short-term (one flock of birds at a time) and explicitly uniform such that all growers, growing the same size birds for the same profit center, receive an identical contract regardless of their past performance, the length of tenure with the company, or any other specific attribute. An interesting feature of these contracts is that the composition of the tournaments (settlement groups) is governed by timing and logistics of the production process and not with an attempt to form more homogenous or more diverse groups of contestants. The random composition of tournament groups in broiler contracts was empirically confirmed by Levy and Vukina (2004).

The data set used in this study includes broiler production information gathered from the payroll data of one company's profit center whose production contract corresponds to the payment scheme described in (1). Each observation in the data set represents one contract settlement, i.e., the payment received and the grower performance associated with one grower and one flock of birds delivered to the integrator's processing plant. The data comes from the so called settlement sheets and contain the information on the quantities and costs of various inputs supplied by the integrator (chicks, feed, medication, vaccination etc.), the number of birds placed and harvested, the quantity of broiler meat (live weight) produced, the dates when production started and terminated, mortality rates, etc.

The tournaments are separated by the settlement date, which happened to be once a week. The settlement dates range from July 1995 to July 1997 totalling 104 tournaments. The total number of growers is 356 and the total number of usable observations is 3,247

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flocks. The average live weight of the fully grown broilers is 4.81 pounds with a maximum of 5.75 pounds and a minimum of 3.88 pounds, the average number of days that a grower needs to grow chickens to that weight is 53 with a maximum of 79 and a minimum of 43 and the average feed conversion ratio (pounds of feed necessary to produce one pound of live animal weight gain) is 2.03 with a maximum of 3.38 and a minimum of 1.83.

3   The Piece Rate Tournament Model

The exact modeling of a tournament game that would simultaneously take into account both the feed margin and the output margin is obviously quite complex, if not impossible, both in terms of theoretical modeling as well as econometric estimation. Earlier literature on broiler tournaments, such as Knoeber and Thurman (1994), Tsoulouhas and Vukina (1999) or Levy and Vukina (2004), fixed the output margin by assuming common mortality rate and the target weight of finished animals. This approach significantly simplifies the problem because, assuming fixed and common y_i reduces the payment mechanism in (1) from a piece-rate tournament to a standard cardinal tournament, where a is no longer a base piece rate but rather a simple salary. This way, the actual production contract is reduced into a contest of who can produce the target output with the lowest cost (feed utilization). Since the received theory predicts no welfare losses associated with mixing different ability contestants in standard cardinal tournaments, this model specification trivializes our problem. Integrators do not organize growers into more homogenous groups because this practice will have no effect on growers' performance and hence integrator's profits.

In an alternative specification, a double-margin tournament contract can be simplified by fixing the settlement cost margin. Under this assumption the actual tournament becomes a contest about who can produce more output (live weight) with a fixed amount of inputs. Let's consider a $N-$player piece rate tournament game in which N risk-neutral growers contract with a risk-neutral integrator the production of broiler chickens. Each grower i ($i = 1, 2, ..., N$) is given the same combination of inputs (chicks, feed, medication, etc.) worth

c_i dollars normalized to \$1.[1] Given these inputs, the performance of grower i is specified as

$$f_i = \frac{c_i}{y_i} = \frac{1}{y_i} = \frac{1}{\theta_i e_i u_i \eta} \tag{2}$$

where y_i indicates the pounds of live poultry produced, e_i is grower i's effort and θ_i is her idiosyncratic ability (efficiency) parameter. We define the grower ability in a broad sense as inherent or acquired skills resulting from experience, education, age, etc., as well as other grower-specific factors such as location, quality, and vintage of the production facilities and equipment.[2] Higher θ_i implies that a grower can combine inputs and effort more efficiently in the production of broiler meat. We assume that from grower i's perspective, θ_j, $\forall j \neq i$, the abilities of other growers in the same tournament, are random variables drawn from a distribution $G(\cdot)$ with support $[\underline{\theta}, \overline{\theta}]$ and that $\underline{\theta} \geq 0$. Distribution $G(\cdot)$ is twice continuously differentiable and has density $g(\cdot)$ that is strictly positive on the support. This specification captures the real-life situation where growers typically do not know who their opponents in a particular tournament are, but know the distribution of other growers' abilities through repeated participation in similar tournaments for an extended period of time.

The stochastic production technology is characterized by two types of shocks. Both grower i's idiosyncratic productivity shock u_i (equipment failure, sick child, etc.) and the common productivity shock η (outside temperature, humidity, feed formula, etc.) materialize slowly during the production process. Shocks u_i and η are assumed to be drawn from distributions $F(\cdot)$ with support $[\underline{u}, \overline{u}]$ and $\underline{u} \geq 0$ and $P(\cdot)$ with support $[\underline{\eta}, \overline{\eta}]$ and $\underline{\eta} \geq 0$, respectively. Both $F(\cdot)$ and $P(\cdot)$ are twice continuously differentiable and have densities $f(\cdot)$ and $p(\cdot)$ that are strictly positive on the support. Each grower only learns u_i and η after the production process is complete but it is common knowledge that the two shocks are drawn from the two densities. Finally, we assume that θ_i, u_i and η are independent of one another.

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[1]We implicitly assume constant returns to scale production technology and therefore this normalization is innocuous. We also assume that the combination of chicks and feed is feasible, i.e., it reflects the target weight of finished broilers and nutritionally meaningful feed-conversion ratio.

[2]For example, one grower may outperform her peers because she is younger and better educated but also because her chicken houses are equipped with tunnel ventilation. Tunnel ventilation is known to work better than the standard curtain ventilation in summer months when the climate is hot and humid.

The grower payment can be written as

$$R_i = \left[a + b \left(\frac{1}{N} \sum_j f_j - f_i \right) \right] \frac{1}{f_i},$$

and her payoff function is given by

$$\pi_i = R_i - C(e_i),$$

where R_i denotes the total revenue and $C(e_i)$ denotes the cost of effort. All standard assumptions regarding the cost function apply, that is, $C' > 0$ and $C'' > 0$. In particular, we assume $C(e_i) = \frac{\gamma}{2} e_i^2$ with $\gamma > 0$ such that the model has a closed form solution.

3.1 Characterization of the Equilibrium

When growers make decisions on how much effort to exert, the idiosyncratic productivity shocks u_i $(i = 1, ..., N)$ and the common productivity shock η have not yet been realized. Therefore, in this tournament game, ex ante, growers only differ in terms of their own ability and have the same information regarding other structural elements of the game. In such a case, a symmetric equilibrium is a natural outcome to analyze. The optimal strategy $e_i^* = s(\theta_i)$ is based on each grower's maximizing her ex-ante expected payoff with respect to e_i. After integrating out all the unknowns and assuming that all other growers adopt the same strategy $e_j^* = s(\theta_j)$ for $j \neq i$, the expected payoff function for grower i can be written as

$$
\begin{aligned}
E\pi_i &= \int ... \int (R_i - C(e_i)) \prod_{j \neq i} g(\theta_j) \prod_{i=1}^{N} f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^{N} du_i d\eta \\
&= \int ... \int \left\{ a\theta_i e_i u_i \eta + b\frac{1-N}{N} + b\frac{1}{N} \sum_{j \neq i} \frac{\theta_i e_i u_i}{\theta_j e_j u_j} - C(e_i) \right\} \\
&\quad \prod_{j \neq i} g(\theta_j) \prod_{i=1}^{N} f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^{N} du_i d\eta.
\end{aligned}
\tag{3}
$$

Now we are in the position to state the following result:

Proposition 1 *The unique symmetric pure-strategy Bayesian Nash equilibrium $e_i^* = s(\theta_i)$ $(i = 1, ...N)$ of this piece rate tournament game is*

$$e_i^* = s(\theta_i) = \theta_i \sqrt{\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N-1)}{N\gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)}
\tag{4}$$

where $E(\cdot)$ denotes the mean of the random variable in parenthesis.

Proof. The first order condition for (3) with respect to e_i is[3]

$$
\int \ldots \int \left[a\theta_i u_i \eta + b\frac{1}{N} \sum_{j \neq i} \frac{\theta_i u_i}{\theta_j e_j^*(\theta_j) u_j} - \gamma e_i^* \right]
$$
$$
\prod_{j \neq i} g(\theta_j) \prod_{i=1}^{N} f(u_i) p(\eta) \prod_{j \neq i} d\theta_j \prod_{i=1}^{N} du_i d\eta = 0.
$$

(5)

Using the independence assumption regarding θ_i, u_i and η, (5) can be written as

$$
a\theta_i E(u) E(\eta) + \frac{b\theta_i(N-1)}{N} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right) = \gamma e_i^*
$$

(6)

where we have used the fact that $E\left(\frac{1}{u_i}\right) = E\left(\frac{1}{u_j}\right) = E\left(\frac{1}{u}\right)$ and $E(u_i) = E(u_j) = E(u)$ for any i and j. Multiplying both sides by θ_i, taking expectations on both sides with respect to θ_i and rearranging terms, we have

$$
E\left(\frac{\gamma}{a\theta_i^2 E(u) E(\eta) + \frac{b\theta_i^2(N-1)}{N} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right)} \right) = E\left(\frac{1}{\theta_i e_i^*}\right).
$$

Since we focus on a symmetric equilibrium strategy and the growers' abilities θ_i $(i = 1, ..., N)$ are from the same distribution, $E\left(\frac{1}{\theta_j e_j^*(\theta_j)}\right) = E\left(\frac{1}{\theta_i e_i^*}\right)$ for all $j \neq i$. After labeling this term as M, we obtain

$$
E\left(\frac{\gamma}{a\theta_i^2 E(u) E(\eta) + \frac{b\theta_i^2(N-1)}{N} E(u) E\left(\frac{1}{u}\right) M} \right) = M.
$$

Rearranging, we get

$$
\left[\frac{\gamma}{aE(u) E(\eta) + \frac{b(N-1)}{N} E(u) E\left(\frac{1}{u}\right) M} \right] E\left(\frac{1}{\theta^2}\right) = M
$$

(7)

where we have used the fact that $E\left(\frac{1}{\theta_i^2}\right) = E\left(\frac{1}{\theta_j^2}\right) = E\left(\frac{1}{\theta^2}\right)$ for any i and j. (7) can be further written as a quadratic equation in M

$$
\frac{aE(u) E(\eta)}{\gamma} M + \frac{b(N-1)}{N\gamma} E(u) E\left(\frac{1}{u}\right) M^2 - E\left(\frac{1}{\theta^2}\right) = 0,
$$

that has a solution[4]

$$
M = \frac{-aE(\eta) + \sqrt{a^2 E^2(\eta) + \frac{4b(N-1)\gamma}{N} \frac{1}{E(u)} E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)}}{\frac{2b(N-1)}{N} E\left(\frac{1}{u}\right)}.
$$

(8)

---

[3] It is straightforward to show that the second order sufficient condition for maximization holds as well.

[4] The other root is automatically ruled out because $M \geq 0$ by construction.

Finally, plugging (8) into (6) completes the proof by obtaining the expression for optimal effort as

$$e_i^* = \theta_i \sqrt{\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N-1)}{N\gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)}.$$

∎

3.2 Comparative Statics

With the closed form solution for optimal effort e_i^*, it is easy to study various comparative statics results. First, optimal effort e_i^* is increasing in ability θ_i, the base rate a, the slope of the bonus piece rate b and the expectation of the common shock $E(\eta)$, and decreasing in the marginal cost of effort γ. On the other hand, the comparative static result with respect to $E(u)$ is ambiguous as $E\left(\frac{1}{u}\right)$ is a nonlinear function of $E(u)$.

Second, notice that

$$E\left(\frac{1}{\theta^2}\right) = E^2\left(\frac{1}{\theta}\right) + V\left(\frac{1}{\theta}\right)$$

where $V(\cdot)$ denotes the variance of the random variable in parenthesis. Therefore, for a constant $E\left(\frac{1}{\theta}\right)$, increasing $V\left(\frac{1}{\theta}\right)$, increases the optimal effort e_i^*. Since θ is defined as growers' ability (efficiency) parameter, $\frac{1}{\theta}$ can be thought of as inaptitude parameter. This implies that for a given mean of the growers' inaptitude parameters, larger variance (more heterogenous growers) produces higher optimal effort. This means that any grower i, given her own ability, when competing against a highly diversified group of growers will exert more effort than in situations where competing in a more homogenous group of contestants. The intuition for this result is difficult to grasp because in the piece rate tournament scheme the individual grower's payoff is a nonlinear function of her performance. However, it seems to be the case that if two players in the tournament are replaced by two new players, one with extremely high ability and one with extremely low ability, without the change in the average grower ability in the tournament group, then the increase in the expected group average performance due to the presence of a grower with extremely high ability seems to be outweighing the decrease in the expected group average performance due to the presence of a grower with extremely low

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ability, resulting in an overall increase in the expected group average performance. Given a grower's own expected performance, the higher the expected group average performance, the higher the incentives to exert effort either to close the gap (if the expected own performance is below the expected group average performance) or widen the gap (if the expected own performance is higher than the expected group average performance) as both situations lead to higher payments.

Finally, we also note that the optimal effort is increasing in the number of tournament contestants N. In our variable piece rate scheme, the magnitude of the piece rate depends on the difference between the group average performance and a grower's own performance. The larger the difference, the higher the payment, and hence the higher the incentives to exert effort. With small number of contestants, a grower's own performance can more readily influence the group average performance and hence the incentive to exert high effort is dampened because own high effort improves the benchmark for comparison. On the other hand, in groups with large number of contestants, one's own performance will not influence the group average performance by a lot, and hence exerting high effort would, ceteris paribus, produce larger increase in the difference between one's own performance and the group average performance.

4   Structural Estimation

As explained in detail in Section 2, our data set is an unbalanced panel where $\overline{N}$ growers that grow chickens for the same integrator compete in different tournaments of size $N < \overline{N}$. Denoting f_{kit} as the kth $(k = 1, ..., N_t)$ observation in tournament t $(t = 1, ..., T)$ that records grower i's $(i = 1, ...\overline{N})$ performance, we can rewrite (2) as

$$
\begin{aligned}
\frac{1}{f_{kit}} &= \theta_i e_{it} u_{kit} \eta_t \\
&= \theta_i^2 \sqrt{\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right)} u_{kit} \eta_t
\end{aligned}
$$

(9)

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where the second equality follows from (4). Taking logarithms of both sides of (9) yields

$$
\log \frac{1}{f_{kit}} = 2 \log \theta_i + 0.5 \log \left[\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right) \right]
$$
$$
+ \log u_{kit} + \log \eta_t
$$
$$
= 2 \log \theta_i + 0.5 \log z_t + \log \eta_t + \log u_{kit} \tag{10}
$$

where $z_t = \left[\frac{a^2 E^2(\eta) E^2(u)}{4\gamma^2} + \frac{b(N_t - 1)}{N_t \gamma} E(u) E\left(\frac{1}{u}\right) E\left(\frac{1}{\theta^2}\right) \right]$ varies only across tournaments due to its dependence on N_t. If N_t is fixed across tournaments, then z_t is also fixed across tournaments since other terms like $E(u)$, $E\left(\frac{1}{u}\right)$, $E\left(\frac{1}{\theta^2}\right)$ and $E(\eta)$ are all fixed constants. For estimation and identification purpose, we also assume that $E(\log u_{kit}) = 0$ and $\log \eta_t$ is normally distributed with mean 0 and variance σ_η^2. The zero mean assumptions for $\log u_{kit}$ and $\log \eta_t$ can be regarded as normalization assumptions as both u_{kit} and η_t are productivity shocks.

Our estimation strategy consists of two steps. In the first step, with the assumption that $E(\log u_{kit}) = 0$, we propose the following OLS regression

$$
\log \frac{1}{f_{kit}} = \delta_0 + \sum_{i=2}^{\overline{N}} \mu_i d_{kit} + \sum_{t=2}^{T} \lambda_t g_{kit} + \epsilon_{kit} \tag{11}
$$

where δ_0 is the constant. To avoid multi-collinearity we excluded the dummy variable for the first grower and the dummy variable for the first tournament. Therefore, the coefficient on grower i's dummy $\widehat{\mu}_i$ ($d_{kit} = 1$ if the kth observation in tournament t records grower i's performance, $d_{kit} = 0$ elsewhere) is used to obtain an estimate of grower i's ability from (10) relative to the excluded first grower, that is, $\widehat{\mu}_i = 2 \log \theta_i - 2 \log \theta_1$. Similarly, the coefficient $\widehat{\lambda}_t$ associated with the tth tournament dummy g_{kit} is used to estimate the sum of the deterministic part of the output function (10) that only varies across tournaments and the common shock, all relative to the first (excluded) tournament, that is $\widehat{\lambda}_t = (0.5 \log z_t + \log \eta_t) - (0.5 \log z_1 + \log \eta_1)$. The constant $\widehat{\delta}_0$ is an estimate of $2 \log \theta_1 + (0.5 \log z_1 + \log \eta_1)$. It is clear that $2 \log \theta_1$ and $(0.5 \log z_1 + \log \eta_1)$ cannot be separately identified. Since our structural analysis requires estimates for growers' abilities (θ_i) instead of the differences in growers' abilities ($\widehat{\mu}_i$), a particular value for the ability of the first grower (θ_1), has to be

assumed. In our benchmark specification, θ_1 is assumed to be 1, or $\log \theta_1 = 0$. Estimates based on different values of θ_1 are discussed in subsection 5.1 below. With these assumptions, grower i's ability can be estimated using $\widehat{\theta}_i = \exp\left(\frac{\widehat{\mu}_i + 2\log \theta_1}{2}\right)$. Finally, the estimated residual term $\widehat{\epsilon}_{kit}$ can be used as an estimate of log of the idiosyncratic productivity shock $\log u_{kit}$ and hence $\widehat{u}_{kit} = \exp\left(\widehat{\epsilon}_{kit}\right)$.

In the second step, we exploit the following relationship

$$
\begin{aligned}
\lambda_t &= 0.5 \log z_t + \log \eta_t - \left(0.5 \log z_1 + \log \eta_1\right) \\
&= 0.5 \log z_t + \log \eta_t - \left(\delta_0 - 2\log \theta_1\right).
\end{aligned}
\tag{12}
$$

From the first step we obtained an estimate for λ_t. Furthermore, with $\widehat{u}_{kit}$ and $\widehat{\theta}_i$ estimated from the first step, estimates for $E\left(u\right)$, $E\left(\frac{1}{u}\right)$ and $E\left(\frac{1}{\theta^2}\right)$ can be easily constructed. Next, note that we assume $\log \eta_t$ is normally distributed with mean 0 and variance σ_η^2. This assumption leads to the result that $E\left(\eta\right) = \exp\left(\frac{\sigma_\eta^2}{2}\right)$. As a result, the only two unknowns in (12) are γ and σ_η^2 and we apply MLE to (12) to obtain estimates for these two unknown parameters. As this step of estimation uses variables generated from results of the first step estimation, standard errors of the second step will be obtained using the bootstrap method. This completes the structural estimation of the model.

4.1 Estimation Results

In the first step, we run the simple OLS regression of (11). The R^2 is 0.9408. After estimation, we can recover $\widehat{\theta}_i$ for each grower, $\widehat{\lambda}_t$ for each tournament and $\widehat{u}_{kit}$ for each observation. These results, together with the summary statistics for the dependent variable used in estimation, $\log \frac{1}{f_{kit}}$, are reported in Table I.

Using the results from the first step, we estimate $E\left(u\right)$ using $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\widehat{u}_{kit}$, $E\left(\frac{1}{u}\right)$ using $\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N_t}\sum_{i=1}^{N_t}\frac{1}{\widehat{u}_{kit}}$, and $E\left(\frac{1}{\theta^2}\right)$ using $\frac{1}{N}\sum_{i=1}^{\overline{N}}\frac{1}{\widehat{\theta}_i^2}$. In the second step, we maximize the following likelihood function

$$
L = \sum_{t=1}^{T} \phi \left[\frac{\widehat{\lambda}_t - 0.5\log \widehat{z}_t + \left(\widehat{\delta}_0 - 2\log \theta_1\right)}{\sigma_\eta} \right]
\tag{13}
$$

where $\widehat{\lambda}_t$ is taken directly from the first step estimation results,

$$\widehat{z}_t = \left[\frac{a^2 \exp\left(\sigma_\eta^2\right) \left[\widehat{E}\left(u\right)\right]^2}{4\gamma^2} + \frac{b\left(N_t - 1\right)}{N_t \gamma} \widehat{E}\left(u\right) \widehat{E}\left(\frac{1}{u}\right) \widehat{E}\left(\frac{1}{\theta^2}\right) \right]$$

and ϕ is the standard normal density. In the data, the base piece rate a is \$0.03, or 3 cents per pound and the marginal bonus rate b is 0.1. Estimation results are collected in the left panel of Table II.

As explained before, both the regressand and the regressors in the second step are constructed using results from the first step estimation. Consequently, standard errors in the second step estimation will be biased. To obtain the correct standard errors, we use the bootstrap method. Furthermore, to preserve the unbalanced panel structure of the data set, we use the nonparametric residual bootstrap method (e.g. Wooldridge 2002, pp. 380). The exact procedure can be detailed as follows. From the first step estimation, we recover the residuals $\widehat{\epsilon}_{kit}$ for each observation. Then, at each iteration of the bootstrap, we re-sample with replacement from the recovered residuals to obtain a new sample of residuals. The new sample of residuals are added back to $\widehat{\delta}_0 + \sum_{i=2}^{\overline{N}} \widehat{\mu}_i d_{kit} + \sum_{t=2}^{T} \widehat{\lambda}_t g_{kit}$ to obtain a new sample of $\log \frac{1}{f_{kit}}$. Finally, the entire estimation (both the first step and the second step) is repeated using the new sample of data. We repeat this procedure 200 times. The bootstrap standard deviation of the second step parameters estimates are used as their standard errors.

Next, using the structural estimates, we are able to compute all quantities of economic interest for each observation. The results are collected in Table III. We find that on average, a grower exerts 3.2487 unit of effort per \$1 worth of inputs and the cost associated with this effort level is 5.86 cents. As a result, on average, a grower earns 9.71 cents in total payment and her profit amounts to 3.85 cents per \$1 worth of inputs.

4.2 Specification Testing

As mentioned before, an alternative model of broiler production contracts is obtained by assuming that the output level is constant across all growers. More specifically, let's assume

that each grower has a contract to produce $y_i = 1$ pound of poultry, which makes this tournament a contest about who can produce a given output with smallest possible cost c_i. Under these assumptions, the performance of grower i becomes

$$f_i = \frac{c_i}{y_i} = c_i \tag{14}$$

and the grower payment (1) reduces to a standard form of cardinal tournament

$$R_i = a + b \left(\frac{1}{N} \sum_j f_j - f_i \right).$$

Using the same set of specifications for f_i, $C(e_i)$, as well as the distributions of shocks and grower abilities, the model yields a unique symmetric Nash equilibrium effort level

$$e_i^* = \left[\frac{b(N-1)}{N\gamma} \frac{1}{\theta_i} E\left(\frac{1}{\eta}\right) E\left(\frac{1}{u}\right) \right]^{\frac{1}{3}}. \tag{15}$$

Compared to the formula for equilibrium effort in the piece rate tournament (4), one can easily see that equilibrium effort in the cardinal tournament (15) is in fact independent of the variance of abilities. This result is in line with the earlier literature which claims that mixing players of uneven abilities in cardinal tournaments creates no problems for efficiency.

Regarding the estimation, based on (14) and (15), the kth observation in tournament t that records grower i's performance f_{kit} is linked to the model primitives as follows

$$\frac{1}{f_{kit}} = \theta_i^{\frac{2}{3}} \left[\frac{b(N_t - 1)}{N_t \gamma} E\left(\frac{1}{\eta}\right) E\left(\frac{1}{u}\right) \right]^{\frac{1}{3}} u_{kit} \eta_t. \tag{16}$$

Expression (16) further implies that structural parameters of the model can be estimated using a two-step estimation procedure very similar to the one implemented before. The two specifications share the same first step OLS estimation, but differ in the second step maximum likelihood estimation.

The first-step OLS regression is

$$\log \frac{1}{f_{kit}} = \delta_0 + \sum_{i=2}^{\overline{N}} \mu_i d_{kit} + \sum_{t=2}^{T} \lambda_t g_{kit} + \epsilon_{kit},$$

and the second-step likelihood function is

$$L = \sum_{t=1}^{T} \phi \left(\frac{\widehat{\lambda}_t - \frac{1}{3}\log \widehat{z}_t + \left(\widehat{\delta}_0 - 2\log \theta_1\right)}{\sigma_\eta} \right)$$

with $\widehat{\lambda}_t$ taken directly from the first step estimation results and $\widehat{z}_t = \left[\frac{b(N_t-1)}{N_t \gamma} E\left(\frac{1}{\eta}\right) \widehat{E}\left(\frac{1}{u}\right) \right]$. With the log normal assumption for η_t, $E\left(\frac{1}{\eta}\right)$ can be written as $\int_0^\infty \frac{1}{\eta^2} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left(-\frac{(\ln \eta)^2}{2\sigma_\eta^2}\right) d\eta$, which is a function of σ_η^2 only. Since the two specifications yield two different likelihood functions underlying two non-nested models, we use Vuong (1989) test to see which specification fits the data better.

The results of the alternative specification model are presented in the right panel of Table II. We notice that the estimate for σ_η^2 is very close to that of the base specification, and so is the log likelihood. However, the alternative specification yields a smaller estimate of the cost of effort parameter γ than the base specification. Denoting the likelihood function for the tth observation in the second stage estimation of the base specification as l_{1t} and that of the alternative specification as l_{2t}, the Vuong likelihood ratio test statistic can be computed as $T^{-1/2} \sum_{t=1}^{T} \left(\log \widehat{l}_{1t} - \log \widehat{l}_{2t} \right) / \widehat{w}_T$, where $\widehat{l}_{1t}$ and $\widehat{l}_{2t}$ are the likelihood functions evaluated at the parameter estimates, and $\widehat{w}_T = \frac{1}{T} \sum_{t=1}^{T} \left(\log \widehat{l}_{1t} - \log \widehat{l}_{2t} \right)^2 - \left[\frac{1}{T} \sum_{t=1}^{T} \left(\log \widehat{l}_{1t} - \log \widehat{l}_{2t} \right) \right]^2$. Under the null hypothesis that the two specifications are equivalent, the test statistic is distributed as a standard normal. Since the obtained test statistic is -0.9043, we cannot reject the null hypothesis at the conventional significance levels. This implies that we cannot empirically distinguish between the two specifications, which is actually not surprising given that the same set of information is used to estimate both models.

5   Counterfactual Analysis

The most interesting theoretical result obtained in Section 3 shows that for a fixed $E\left(\frac{1}{\theta}\right)$ (the mean of the growers' inaptitude parameters), an increase in its variance $V\left(\frac{1}{\theta}\right)$ generates an increase in the equilibrium effort e_i^*. This implies that the principal can gain from hetero-

genizing the tournament groups as long as he can sell the product at a price higher than the payment to the contract growers and as long as he does not violate growers' participation constraint. On the other hand, the effect of this policy change on growers' welfare is unclear. This is because higher effort leads to higher productivity and hence higher payment, but also higher cost of effort. One advantage of the structural econometrics approach is that it enables us to use the model estimates to quantify the welfare effects of such regime shifts or policy proposals.

To quantify these welfare effects, we run a counterfactual experiment by increasing $E\left(\frac{1}{\theta^2}\right)$ by 10%, holding $E\left(\frac{1}{\theta}\right)$ fixed. Then, using the structural estimates and holding the productivity shocks at the level before the policy change, we can first compute the new optimal effort level for each grower, and subsequently compute their outputs, payments, and profits under the new scenario. The results from such an experiment are collected in Table IV. We find that with the more heterogenous groups, growers' equilibrium efforts (and hence their outputs and payments), would increase by 4.06% on average. On the other hand, growers' costs of effort would increase on average by 8.28%, and as a result, on average, their profits would decrease by about 2.91%. More specifically, the percentage changes in growers' profits range from -22.78% to 9.86% and only in 33 out of 3274 observations a positive profit change is recorded. The results clearly show that if the principal makes the tournaments more heterogenous, the average contract grower effort, output and payment would go up but the majority of growers would be worse off than before the change.

As mentioned briefly before, worsening of the anticipated profits may violate some growers' participation constraint, causing them to drop out of the contract by not signing the new contract when contracts are up for renewal. This problem can be retroactively fixed by keeping the growers *ex-post* at the same level of profits (utility) as before the change by paying them a lump-sum payment equal to the welfare loss caused by the proposed change. This will also enable us to quantify the effect of heterogenizing the tournament groups on social welfare (the sum of the principal's and growers' profits). Based on the result from Table IV, we see that for $1 worth of inputs, on average, a grower's output increases from

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3.2341 pounds of chicken to 3.3653 pounds. Assuming the industry is perfectly competitive, the principal can sell all his output at the prevailing market price. Therefore, the principal gets an additional revenue of 7.71 cents by selling the additional 0.1312 pounds of chicken meat.[5] On the other side, on average, growers' profits decrease from 3.85 cents to 3.76 cents, for a 0.09 cents loss. If the principal pays back this amount as a lump sum transfer to the growers, he would still end up better off than before the change. Consequently, the net social welfare gain associated with this policy change is positive. Heterogenizing tournament groups by 10% results in 7.62 cents (per \$1 worth of inputs) increase in social surplus. On a per grower basis, the average settlement cost per grower in the data is 31.26 cents per pound of chicken produced and the average number of pounds produced per grower is 240,390 pounds, implying an average value of inputs (feed and chicks and other chargeable costs) per grower at \$75,145.91. Therefore, on a per grower basis, the increase in social welfare equals \$5,726.12 $(0.0762 \times \$75,145.91)$.

5.1 Robustness

As mentioned above, our structural analysis requires a particular value for θ_1, the ability of the first grower, to be assumed. In our benchmark specification, we assumed $\theta_1 = 1$ or $\log \theta_1 = 0$. Therefore, it is appropriate to investigate how would the results change if we assume different values for θ_1. Since $\delta_0 = 2 \log \theta_1 + (0.5 \log z_1 + \log \eta_1)$ is estimated to be 1.2515 from the first stage regression and our baseline specification corresponds to the case where $2 \log \theta_1 = 0$, we now assume $2 \log \theta_1 = 1$ or $\theta_1 = \exp(0.5)$. The new results are presented in Table V. Several things are worth mentioning. First, assuming a higher value for the first grower's ability, the estimates of other growers' abilities, θ_i, become larger. Correspondingly, the tournament specific components, λ_t, become smaller. Second, the estimate of the log variance of the common shock (σ_η^2) does not change. This is not surprising since it is clear from (13) that different values of θ_1 only affect the mean of dependent variable $\widehat{\lambda}_t$, but not

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[5] The 12-city broiler price average for the period covered by our data set (July 1995 - July 1997) is 58.78 cents per pound.

its variance. Third, the new specification yields a larger cost of effort parameter, γ, leading to a smaller value for the average optimal effort. Finally, the alternative value for θ_1 leaves the earlier welfare effects due to an increase in the heterogeneity of the tournament groups almost intact. Hence, from the perspective of welfare calculations, assuming a particular value for θ_1 is innocuous.

6   Conclusions

In many sporting events we observe the formation of leagues or divisions structured by the approximately even ability of teams or individual contestants. For example, in European football (soccer), England's Football Association divides clubs into different leagues depending on their strength. The best league is called the Premier League where 20 best teams such as Manchester United, Arsenal and Liverpool compete for the national title. At the end of each season, typically two teams exchange leagues such that the worst teams drop to the league immediately below and the best placed teams from the league below advance to the league immediately above. The enhancement of competition is an intuitively obvious reason for such a prevalent practice. The motivation for this study came from observing that poultry integrators, unlike the Football Association, generally do not attempt to homogenize the settlement groups of contract growers for the purpose of creating more fierce competition among them. Instead, the composition of tournaments (settlement groups) seem to be governed by the timing and logistics of the production process and the membership in those groups seem to change quite randomly.

Poultry production contracts are double-margin tournaments about who can produce more live poultry weight at the smallest possible cost. Since the exact modeling and structural estimation of these contests is extremely complex, if not impossible, the standard simplifying assumption in the literature so far has been to fix the output margin. This assumption reduces the payment scheme in these contracts to a standard cardinal tournament where the competition is about who can produce the targeted output at the smallest possible cost. This

approach trivializes our problem because the equilibrium effort in this game ends up being independent of the variance of growers' abilities, and therefore homogenizing tournament groups is irrelevant for efficiency.

In this paper, we propose an alternative specification whereby, instead of fixing the output margin, we fix the cost margin. This assumption gives rise to a piece rate tournament scheme (variable piece rate) where the competition is about who can produce more output at a given fixed cost. Our most interesting theoretical result shows that for a fixed mean of the growers' inaptitude parameters, an increase in its variance generates an increase in the equilibrium effort. This implies that the principal can actually gain from heterogenizing the tournament groups. On the other hand, the effect of this change on growers' welfare is unclear because higher effort leads to higher productivity and hence higher payment, but also increases the cost of effort.

We estimated both models structurally and conducted the specification testing which indicated that we could not empirically distinguish between the two models. These results show that there is actually a very good reason for why poultry integrators never attempt to homogenize the growers settlement groups. Under the cardinal tournament specification, we obtain a trivial result that homogenizing groups would accomplish absolutely nothing. Under the piece rate tournament specification we obtain a somewhat unexpected result that hetero-genizing the tournament groups would in fact benefit the integrator whereas homogenizing them would hurt him because the average equilibrium grower effort would decline and the production would suffer. Moreover, our counterfactual analysis shows that under reasonable assumptions the integrator's gain could be larger than the growers' losses and that he can *ex-post* compensate growers and still be better off.

The above result suggests that heterogenizing groups in piece rate tournaments, to the extent that it is practicable, may be efficient in the sense of increasing social surplus. Now, instead of the original puzzle which we successfully solved, we are faced with a new puzzle, i.e., why poultry integrators do not try to assemble more heterogenous groups of growers. Aside from the practicability of the scheme and potentially high transactions costs associated

with its implementation, which could eliminate the formation of more heterogenous groups from the integrator's feasible option set, we can offer two additional possible explanations for the discrepancy between our results and the observed phenomena. At the same time, these explanations provide directions for valuable, albeit very complex, extensions of the current paper. First, in our current model, we ignore a possible violation of the agents' participation constraints that heterogenizing the groups can lead to, and we dealt with the problem *ex-post* via a lump-sum transfer from the principal to the agents that would keep the agents at the same level of utility as before the change. A substantially more complicated approach as well as an assumption regarding growers' utility from outside choice would address this problem by making sure that the participation constraints are satisfied *ex-ante*.

The second possible extension of this analysis would be to explore whether the obtained results carry over to the case of risk averse agents. With risk averse agents, the piece rate tournament model does not have a closed form solution and the comparative statics results are difficult to evaluate. Intuitively, when agents are risk averse, they care about both the mean and the variance of the returns to their efforts. Since the mean of the returns is already a function of the variance of the abilities, as we show in the model with risk neutral agents, the variance of the returns is likely to be a function of the variance of the abilities as well. Hence, the equilibrium effort will depend on the variance of the abilities in a more complex way, either positively or negatively.

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Table I: Estimates from the First Stage Estimation

	Mean	Standard Deviation	Number of Observations
$\log \frac{1}{f_{kit}}$	1.1683	0.1040	3247
$\widehat{\lambda}_t$	1.1855	0.0988	104
$\widehat{\theta}_i$	0.9918	0.0092	356
$\widehat{u}_{kit}$	1.0003	0.0247	3247

Table II: Estimates from the Second Stage Estimation[6]

	Estimate	Std. Err.	t-Stat	Estimate	Std. Err.	t-Stat
	Base Specification			Alternative Specification.		
σ^2_η	$9.7349*10^{-3}$	$9.1918*10^{-5}$	105.91	$9.4289*10^{-3}$	$1.4471*10^{-4}$	65.16
γ	$1.1100*10^{-2}$	$1.0385*10^{-4}$	106.88	$2.7961*10^{-3}$	$3.2817*10^{-5}$	85.20
Log Likelihood	188.8668			188.9245		

Table III: Quantities of Economic Interest

	Mean	Standard Deviation
R_{kit} (\$)	0.0971	0.0114
e_{it}	3.2487	0.0286
$C(e_{it})$ (\$)	0.0586	0.0010
$R_{kit} - C(e_{it})$ (\$)	0.0385	0.0112

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[6]Standard errors are based on 200 iterations of boostrap.

Table IV: Effect of Heterogenizing Tournament Groups

	Before		After		Change (%)	
	Mean	Stan. Dev.	Mean	Stan. Dev.	Mean	Stan. Dev.
$y_{kit}\left(\frac{1}{f_{kit}}\right)$	3.2341	0.3409	3.3653	0.3547	4.0570	0.0041
R_{kit} (\$)	0.0971	0.0114	0.1011	0.0118	4.0629	0.2018
e_{it}	3.2487	0.0286	3.5805	0.0298	4.0570	0.0041
$C(e_{it})$ (\$)	0.0586	0.0010	0.0634	0.0011	8.2785	0.0085
$R_{kit} - C(e_{it})$ (\$)	0.0385	0.0112	0.0376	0.0116	-2.9093	2.1141

Table V: Estimation and Counterfactual Results when $\theta_1 = \exp(0.5)$

Estimates

	Mean	Stan. Dev.		Estimate	Std. Err.	t-Stat
$\widehat{\lambda}_t$	0.1855	0.0988	σ_η^2	$9.7349*10^{-3}$	$9.2131*10^{-5}$	105.66
$\widehat{\theta}_i$	1.6352	0.0152	γ	$3.0178*10^{-2}$	$2.9146*10^{-4}$	103.54
			Log Likelihood	188.8667		

Counterfactual

	Before		After		Change (%)	
	Mean	Stan. Dev.	Mean	Stan. Dev.	Mean	Stan. Dev.
$y_{kit}\left(\frac{1}{f_{kit}}\right)$	3.2341	0.3409	3.3653	0.3547	4.0571	0.0041
R_{kit} (\$)	0.0971	0.0114	0.1011	0.0118	4.0630	0.2018
e_{it}	1.9702	0.0173	2.0502	0.0180	4.0571	0.0041
$C(e_{it})$ (\$)	0.0586	0.0010	0.0634	0.0011	8.2788	0.0085
$R_{kit} - C(e_{it})$ (\$)	0.0385	0.0112	0.0376	0.0116	-2.9088	2.1138