Analyzing Growth and Welfare Effects of Public Policies in Models of Endogenous Growth with Human Capital: Evidence from South Africa

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ABSTRACT

Since the abolition of its Apartheid regime in 1994, South Africa has launched a massive program of education, which has been financed through resources representing on average 21% of the national budget or 7% of GDP. Today, the GDP share of public spending on education is 1.3 times the average of industrialized countries (5.4%) and almost twice that of developing countries (3.9%).

In this paper, we simulate fiscal policy experiments to analyze the growth and welfare effects of a shift in the allocation of government expenditures between public spending on education and transfers as well as those of a change in the tax rate in a model of endogenous growth with human capital accumulation for the South African economy.

The results of simulations demonstrate that a shift in the allocation of fiscal resources between educational spending and transfers does not affect the long run allocation decisions. In the transition, however, this shift generates a negative effect on the rate of growth of GDP. In fact, a reallocation of expenditures shifts resources away from saving and toward consumption, and translate into lower rate of growth but higher welfare. Nonetheless, these growth and welfare effects are very small. On the other hand, a tax cut generates growth effects in the long run as well as in transition. In fact, reducing or cutting the tax rate in the long run lowers the interest rate, which in turn creates disincentives for saving and results in low rate of growth of GDP. However, in the transition, it reduces or removes distortions and translates into high work effort, high accumulation of human capital, and thus high rate of growth of GDP. Nonetheless, its welfare effect is negative.

1. Introduction

In recent years, there has been a growing interest in the analysis of the relationship between fiscal policies, growth, and welfare in models of endogenous growth. This analysis has two distinct strands in literature. The first strand is based on the view that government expenditures are unproductive consumption of economic resources and tax rates are distortionary. Studies in this line of research include, among others, King and Rebello (1990), Lucas (1990), Rebello (1991), Jones et. al (1993), Stokey and Rebello (1995), Razin and Yuen (1996), and Ortigueira (1998). These studies reach the conclusions that income taxation is negatively related to the long run rate of growth of

the economy, and that the welfare costs of taxation are larger in these models than they are in exogenous models.

The second strand in literature, however, is based on the view that not all government expenditures are unproductive and that some affect the productivity of the economy in different ways. For instance, public expenditures allocated to the maintenance and/or construction of public infrastructure improves the productivity of physical capital, whereas those allocated to education and health enhance the productivity of labor. Early contributions to this topic include Barro (1990) and Barro and Sala-i-Matin (1992). These papers show that public spending may be growth-promoting. Furthermore, they show that the size of the effect depends on the characteristics of the services and the policy design. Precisely, if the public services are publicly provided public or private goods, lump sum taxation creates high incentives for investment and growth. On the other hand, income taxation leads to higher growth if the public services are subject to congestion.

Most models built to study the effects of productive public spending on growth include public spending as an input of the final good production sector. This is the approach followed by Barro (1990), Alesina and Rodrik (1994), Barro and Sala-i-Martin (1992, 1995, 2005), Corsetti and Roubini (1996). These studies specify a Constant Returns to Scale (CRS) production function in both inputs but diminishing returns in each of them. However, there is no obvious reason to restrict public expenditures to affecting only the productivity of the final good production sector since they can serve different purposes. Jones et al (1993) takes a different approach by building them into the physical capital accumulation technology which they specify as a Constant Elasticity of

Substitution function. Corsetti and Roubini (1996) build them as either an input of a final good production sector or an input of human capital accumulation sector. In both cases, their production functions take the CRS form. Agenor (2005) and Greiner (2006) construct them to affect only the productivity of human capital accumulation sector using the CRS technology.

In this paper, we build on the two strands of the literature to analyze the growth and welfare effects of a shift in the allocation of public expenditures between spending on education and transfers as well as those of a reduction in the tax rate or of a 100% tax cut in a model of endogenous growth with human capital accumulation of the South African economy. South Africa has launched since the abolition of its Apartheid regime in 1994 a massive program of education. According to Schmidt (2003), the resources allocated to this program have represented on average 21% of the South Africa's national budget or 7% of its GDP for the period 1995-2001. The size of this share has made education the largest single item in the national budget of this country, and one of the highest proportions worldwide. Today, it is 1.3 times the average of industrialized countries (5.4%) and almost twice that of developing countries (3.9%). Does allocating more fiscal resources to education make any difference? What can possibly be the effects of reducing or eliminating them.

To answer these questions, we formulate first a simple model of endogenous growth with human capital accumulation of Lucas' type and solve it under the parameters estimated from the data and long run equilibrium conditions (baseline). Then, we simulate four fiscal policy experiments and compare their solutions to that of the baseline case to assess and analyze growth and welfare effects. The first and second experiments

3

consist of reducing the GDP share of spending on education to the averages of industrialized and developing countries, respectively; while the third and fourth consist of eliminating transfers and government from the model, respectively. In both the baseline and the simulated experiments, we restrict the analysis to services from government spending flows (exogenously given) rather than stocks of public expenditures.

The Lucas' model has well known long run properties. As documented in the seminal paper by Lucas (1988), the economy converges to the balanced growth path in which all variables grow at constant (possibly zero) rates. Furthermore, its transition properties, which were unclear in this original paper, have been described in a number of subsequent studies on this topic. Important contributions in this line of research include Mulligan and Sala-i-Martin (1992), Caballe and Santos (1993), Ortigueira (1998), Boucekkine and Tamarit (2004), Barro and Sala-i-Martin (1995, 2004), and Boucekkine et al (2007), among others. These studies show that the transition behavior of the economy is determined by the relative size of the parameter of substitution and the elasticity of physical capital. If the parameter of substitution is greater (less) than the elasticity of physical capital, an economy starting with higher physical-human capital ratio than that of the long run equilibrium will observe higher (lower) transition rates of growth of human capital than that of the long run equilibrium, and lower (higher) transition rates of growth of physical capital than that of the long run equilibrium. The transitions from low physical-human capital ratio are obtained by applying symmetrically the above results.

Our specification of the human capital accumulation technology with public spending on education differs slightly from the ones in the studies mentioned above. While the human capital accumulation technology in all these studies is CRS in all inputs

4

and diminishing in each of them, we adopt as Lucas (1998) a linear technology so that diminishing returns do not occur. As far as we know, this study is the first one to extend the Lucas framework to the one that includes government expenditures as an input of the human capital accumulation sector.

The remainder of this study is organized as follows. In Section 2, we analyze a simple model of endogenous growth with human capital and characterize its equilibrium. In section 3, we solve numerically the model. Section 4 includes the comparison of the predictions of the model to the data. In Section 5, we simulate four fiscal policy experiments. In Section 6, we analyze growth and welfare effects of fiscal policy experiments. Section 5 provides concluding remarks.

2. Model of Endogenous Growth with Human Capital and Policy

2.1 Setup

We consider a decentralized Ramsey model a la Lucas with unbounded horizon and continuous time. The economy is closed and populated by many infinitely-lived, rational, and identical agents with homothetic preferences, many competitive firms with identical technology, and a government. A single consumption good is produced in this economy from a technology that combines physical capital $(k(t))$ and effective labor $(u(t)h(t))$ - a combination of raw labor $(u(t))$ with human capital $(h(t))$, $t \geq 0$. We assume that population is constant over time.

A representative-agent derives her utility from consuming $c(t)$ units of the consumption good in each period (leisure does not enter the utility function). Let assume that agent's preferences are characterized by a twice continuously-differentiable utility

function $U(c(t))$ with $U' > 0$ and $U'' < 0$, for all $c(t) > 0$, and satisfies the Inada conditions, that is, $\lim_{c(t) \to 0} U'(c(t)) = \infty$ and $\lim_{c(t) \to \infty} U'(c(t)) = 0$, where U' and U'' are the first and the second derivatives of the utility function, respectively. The discounted sum of future utilities of the representative agent is given by:

$$\int_0^\infty U(c(t)) e^{-\rho t} dt \,^1, \tag{2.1}$$

where c is measured in units of final output, and r is the constant parameter of time preference. We assume that the representative agent supplies raw labor inelastically, where the supply of labor in each period is normalized to one. She has to decide on the fraction of labor to allocate to production (u) and the fraction to allocate to the accumulation of human capital $(1-u)$ in each period t. She receives a wage rate w in exchange for supplying one unit of effective labor (uh) and a rate of return r for renting one unit of physical capital k in each period t, where w and r are measured in units of consumption good c. She also receives a lump sum transfer T from government in each period t. Raw labor u and human capital h are perfect substitutes. In each period t, she allocates her total income to spending on consumption c and to saving $\dot{k}$. Thus her budget constraint is given by:

$$\dot{k} = (1 - t_u) wuh + (1 - t_k) rk - c + p + T, \tag{2.2}$$

where p is profit earned by the agent who holds $1/N$ fraction of the firm's shares, N is

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[1] From now on, we do not explicitly indicate the time dependence of variables if no ambiguity arises.

the size of population, t_k is a flat rate of tax on capital income , and t_u is a flat rate of tax on labor income. Human capital h is accumulated according to:

$$\dot{h} = j\,(1-u)(h + f(g_h)) - d_h h \qquad (2.3)\ ^2$$

where g_h is a stream of exogenously given government spending on education, $j\,(.)$ is a decreasing function in u, and $f(.)$ is an increasing function in g_h. We restrict the analysis to the educational services from government spending flows rather than a stock. (2.3) implies that the rate of accumulation of h in each period t is a function of the time spent on the learning field, the existing stock of h, and the public spending on education in that period. Furthermore, the representative agent starts with some positive endowments of physical and human capital, that is:

$$k(0) = k_0,\ h(0) = h_0,\ k_0\ \text{and}\ h_0\ \text{are given.} \qquad (2.4)$$

We also assume that all decision variables take on only non-negative values:

$$c \geq 0,\ k \geq 0,\ h \geq 0,\ 0 \leq u \leq 1. \qquad (2.5)$$

The supply side of the model consists of a representative competitive firm producing the consumption good y from a technology that combines physical capital k^f and effective labor z. The profit function of this firm in each period t is:

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[2] $\dot{k}$ is saving net of depreciation and $\dot{h}$ is the rate of accumulation of human capital net of depreciation. The above implies that there exist gross rates of accumulation of k and h, which are given by

$\dot{k}_g = \dot{k} + d_k k$ and $\dot{h}_g = \dot{h} + d_h h$, respectively; where d_k and d_h are the rates of depreciation of

k and h, respectively. It follows from the above that $\dot{k} = \dot{k}_g - d_k k$ and $\dot{h} = \dot{h}_g - d_h h.$

$$p = y - rk^f - wz, \tag{2.6}$$

where k^f and z are the firm's demands of physical capital and effective labor, respectively, and y is its output produced according to:

$$y = F\left(k^f, z\right)^3, \tag{2.7}$$

where F is a Constant Returns to Scale (CRS) technology in k^f and z. This function is assumed to be twice continuously differentiable in each argument with $F' > 0$, $F'' < 0$, and satisfy the Inada Conditions:

$$\lim_{k^f \to \infty} F'(., z) = 0, \qquad \lim_{k^f \to 0} F'(., z) = \infty, \qquad \lim_{z \to \infty} F'(k^f, .) = 0, \quad \text{and} \quad \lim_{z \to 0} F'(k^f, .) = \infty.$$

We assume also that:

$$y \geq 0, \ k^f \geq 0, \ z \geq 0. \tag{2.8}$$

A government intervenes in this economy through a fiscal policy, that is, it collects taxes on incomes, and uses the proceeds to make lump sum transfers to consumers and to finance education. Its budget constraint, which by assumption must be balanced in each period, and the boundary conditions on the tax rates are respectively:

$$g = g_h + T = h_h g + h_T g = t_u wuh + t_k rk, \qquad \forall t \tag{2.9}$$

$$0 \leq t_u \leq 1, \ 0 \leq t_k \leq 1. \tag{2.10}$$

where h_h and h_T are respectively the constant budget shares of public spending on education and on transfers, $h_h + h_T = 1$. The assumption of a balanced budget for the

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[3] The production function in the Lucas' model has an external effect from human capital. We omit it here to ease the derivation of the Balanced Growth Path conditions.

government is intended to prevent it from running a deficit that it would finance by issuing debt (which it would pay by increasing the tax rates), or running a surplus by accumulating assets.

2.2 Equilibrium and its Characterization

A competitive equilibrium for this economy is a sequence of allocations of the representative agent $\{c, k, h, u\}_{t=0}^{\infty}$, a sequence of allocations of the representative firm $\{y, k^f, z\}_{t=0}^{\infty}$, a sequence of the rental rates of $\{r, w\}_{t=0}^{\infty}$ and a sequence of policies $\{t_u, t_k, g_h, T\}_{t=0}^{\infty}$ such that:

i) Given $\{r, w\}_{t=0}^{\infty}$ and $\{t_u, t_k, g_h, T\}_{t=0}^{\infty}$, $\{c, k, h, u\}_{t=0}^{\infty}$ maximizes (2.1) subject to $(2.2)-(2.5)$,

ii) The rental rates of physical capital and effective labor in each period t are given by:

$$r = \partial F(k^f, z)/\partial k^f , \quad w = \partial F(k^f, z)/\partial z, \tag{2.11}$$

iii) The government budget constraint (2.9) and the boundary conditions on the tax rates (2.10) hold in each period t,

iii) The following feasibility conditions hold in each period t:

$$c + g_h + \dot{k} = y , \; k^f = k , \; z = uh. \tag{2.12}$$

Let assume that the utility function, the production function, and the functions $j(1-u)$ and $f(.)$ take the following functional forms, respectively:

$$U(c) = c^{1-s}/1-s, \quad F(k^f, z) = A(k^f)^\alpha z^{1-\alpha}, \quad j(1-u) = f(1-u), \quad f(g) = xg, \quad (2.13)$$

where s is the inverse of the inter-temporal elasticity of substitution, A is the technology parameter, a is the output's share of physical capital, f is the human capital technology parameter, and x is a constant parameter. The Current Value Hamiltonian is:

$$J = \frac{c^{1-s}}{1-s} + 1_k \left[(1-t_u)wuh + (1-t_k)rk - c + (1-h_h)g\right] + 1_h \left[f(1-u)(h + xh_h g)\right]^{45},$$

where c and u are the control variables, k and h are the states variables, 1_k and 1_h are the co-state variables or the shadow prices of physical and human capital, respectively. 1_k and 1_h are derived from the co-state variables m_k and m_h of k and h in J^P as follows: $1_k = m_k e^{rt}$, $1_h = m_h e^{rt}$. The first-order conditions from maximizing J are:

$$c^{-s} - 1_k = 0, \qquad\qquad \forall t \qquad\qquad (2.14)$$

$$1_k w(1-t_u) - 1_h f(1 + xh_h(g/h)) = 0, \qquad \forall t \qquad\qquad (2.15)$$

$$\&= (1-t_u)wuh + (1-t_k)rk - c + (1-h_h)g, \qquad\qquad (2.16)$$

$$\&= f(1-u)(h + xh_h g), \qquad\qquad (2.17)$$

$$\&_k = -1_k\left[(1-t_k)r - r\right], \qquad\qquad (2.18)$$

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[4] We omit profit in J since it is zero in each period due to CRS specification of the production function.

[5] J is derived from the Present Value Hamiltonian, that is, $J = J^P e^{rt}$, where J^P is given by:

$$J^P = \frac{c^{1-s}}{1-s} e^{-rt} + m_k\left[(1-t_u)wuh + (1-t_k)rk - c + (1-h_h)g\right] + m_h\left[f(1-u)(h + xh_h g)\right]$$

$$\dot{l}_h = -l_k(1-t_u)wu - l_h[f(1-u)-r]. \tag{2.19}$$

The boundary conditions are the initial conditions (2.4) and the following transversality conditions (TVC):

$$\lim_{t\to\infty} l_k(t)e^{-rt}k(t) = 0, \ \lim_{t\to\infty} l_h(t)e^{-rt}h(t) = 0. \tag{2.20}$$

Taking the log and time-derivative of l_k in (2.14) and substituting the resulting expression into (2.18) yield the following equation of motion of c:

$$\dot{c} = s^{-1}c[(1-t_k)r - r]. \tag{2.21}$$

Manipulating $(2.14)-(2.19)$ we get the following equations of motion of u[6]:

$$\dot{u} = u[a + (1-a)xh_h(g/h)]^{-1}[f + fxh_h(g/h)u - (1-t_k)aAk^{a-1}(uh)^{a-1} +$$

$$a(1-xh_h(g/h))((\dot{k}/k)-(\dot{h}/h))] \tag{2.22}$$

Given the initial conditions (2.4), expressions $(2.16),(2.17),(2.21),(2.22)$, and the TVC (2.20) form the dynamic system describing the evolution of this economy over time.

2.3 Steady State

Let assume that the equilibrium paths converge to the balanced growth path (BGP) in which all variables grow at constant (possibly zero) rates. Let define these rates by:

$$g_c = \frac{\dot{c}}{c}, \ g_k = \frac{\dot{k}}{k}, \ g_h = \frac{\dot{h}}{h}, \ g_g = \frac{\dot{g}}{g}, \ g_w = \frac{\dot{w}}{w}, \ g_r = \frac{\dot{r}}{r}, \ g_u = \frac{\dot{u}}{u}. \tag{2.23}$$

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[6] See Appendix A1 for the derivation of the equations of motion of u.

From equation (2.21) and using (2.11)[7] and the definition of the BGP (2.23), we derive the following marginal products of k:

$$(1-t_k)aAk^{a-1}(uh)^{1-a} = r + sg_c, \tag{2.24}$$

which is constant along the *BPG*. Substitute (2.11) for r and w into (2.16), divide by k, and use the definition (2.23) for $\dot{k}/k$ to obtain:

$$c/k = (1-h_h[t_u(1-a)+t_ka])((r+sg_c)/a(1-t_k)) - g_k, \tag{2.25}$$

which is constant along the *BPG*. Taking the log and time-derivative of (2.25) yields $\dot{c}/c - \dot{k}/k = 0$, that is, c and k grow at the same rate $(g_c = g_k = g)$. Moreover, divide the feasibility condition (2.12) by k to get:

$$(\dot{k}+c+g_h)/k = Ak^{a-1}(uh)^{1-a}. \tag{2.26}$$

Multiply the LHS of (2.26) by $\dot{k}/\dot{k}$, use definition (2.23) for $\dot{k}/k$, inverse the resulting expression, and rearrange to obtain the savings rate (s):

$$s = a(1-t_k)g/(r+sg). \tag{2.27}$$

Divide the government budget constraint (2.9) by k and substitute (2.11) for r and w into the result to get:

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[7] The rental rates r and w are given by:

$$r = ak^{a-1}(uh)^{1-a}, \; w = (1-a)k^a(uh)^{-a}.$$

$$g/k = \left[t_u(1-a) + t_k a \right]\left((r+sg)/a(1-t_k) \right). \tag{2.28}$$

Differentiating (2.28) we get $\dot{g}/g - \dot{k}/k = 0$, that is, g and k grow at the same rate $\left(g_g = g_k = g \right)$. Furthermore, taking the log and time-derivative of (2.24) [8] yields $\dot{h}/h - \dot{k}/k = 0$, that is, h and k grow at the same rate $\left(g_h = g_k = g \right)$. Next, multiply both sides of (2.28) by k/h to obtain:

$$g/h = \left[t_u(1-a) + t_k a \right]\left((r+sg)/a(1-t_k) \right)(k/h). \tag{2.29}$$

Recovering k/h from (2.24) [9] and substituting its expression into (2.29) yields:

$$g/h = yu, \tag{2.30}$$

where $y = A^{\frac{1}{1-a}} \left[t_u(1-a) + t_k a \right]\left((r+sg)/a(1-t_k) \right)^{\frac{a}{a-1}}.$

Taking the log and time-derivative of (2.14) and of (2.15) yields:

$$\dot{\lambda}_k/\lambda_k = \dot{\lambda}_h/\lambda_h = -sg. \tag{2.31}$$

Divide (2.19) by λ_h, substitute (2.15) into the resulting expression, and rearrange to get:

$$\dot{\lambda}_h/\lambda_h = r - f - fxh_h(g/h)u. \tag{2.32}$$

Set (2.31) and (2.32) equal to obtain:

$$(g/h)u = (r-f+sg)/fxh_h. \tag{2.33}$$

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[8] To get this result we have assumed that $\dot{u}/u = g_u = 0$. We will show that this is indeed the case.

[9] $\dot{k}/h = \left[(r+sg)/aA(1-t_k) \right]^{1/(a-1)} u$

13

Divide (2.17) by h, use definition (2.23), and rearrange the resulting expression to get:

$$u - xh_h(g/h) + xh_h(g/h)u = 1 - (g/f).$$
(2.34)

Substitute (2.30) and (2.33) into (2.34) and rearrange to obtain:

$$u = (1 - xh_h y_1)^{-1}[1 - ((g(1+s) + r - f)/f)].$$
(2.35)

Taking the log and time-derivative of (2.35) yields $\dot{u}/u = \gamma_u = 0$. Furthermore, it is obvious from (2.11) that the rates of growth of r and w are zeros $(g_r = g_w = 0)$. However, the growth rate of w augmented for skill growth is $g_w^* = g_w + g_h = g$. This is also the rate of growth GDP as can be verified by differentiating the expression of GDP in (2.13). From all of the above, it is obvious that in the BGP; c, k, h, g, and w augmented for skill growth grow at the same constant rate $(g_c = g_k = g_h = g_g = g_w^* = g)$; and u, r, w are constant $(g_u = g_r = g_w = 0)$. The common rate of growth g is recovered from (2.27). It is:

$$g = sr/(a(1 - t_k) - ss).$$
(2.36)

We use the above common rate of growth g to normalize variables as follows:

$$\hat{c} = ce^{-gt}, \quad \hat{k} = ke^{-gt}, \quad \hat{h} = he^{-gt}, \quad \hat{g} = ge^{-gt}.$$

The dynamic system of the normalized variables is formed by the following:

$$\dot{\hat{k}} = [1 - h_h(t_u(1-a) + t_k a)]A(\hat{k})^a (u\hat{h})^{1-a} - g\hat{k} - \hat{c},$$

$$\dot{\hat{h}} = f(1-u)(\hat{h} + xh_h\hat{g}) - g\hat{h},$$

14

$$\dot{c} = s^{-1}\hat{c}\left[(1-t_k)aA(\hat{k})^{a-1}(u\hat{h})^{1-a} - r - gs\right],$$

$$\dot{u} = u\left[a + (1-a)xh_h(\hat{g}/\hat{h})\right]^{-1}\left[f + fxh_h(\hat{g}/\hat{h})u - (1-t_k)aA\hat{k}^{a-1}(u\hat{h})^{a-1} + \right.$$

$$a\left(1-xh_h(\hat{g}/\hat{h})\right)\left(1-h_h(at_k + (1-a)(1-t_u))\right)A\hat{k}^{a-1}(u\hat{h})^{a-1} - (\hat{c}/\hat{k}) -$$

$$\left. f(1-u)\left(1 + xh_h(\hat{g}/\hat{h})\right)\right]$$

$$TVC: \lim_{t\to\infty} \lambda_k(t)e^{-rt}\hat{k}(t) = 0, \ \lim_{t\to\infty} \lambda_h(t)e^{-rt}\hat{h}(t) = 0. \tag{2.37}$$

The TVC in (2.37) imply that the vector $x(t)$ approaches its steady state, that is:

$$\lim_{t\to\infty} x(t) = x_{ss} < \infty \ \leftrightarrow \ \lim_{t\to\infty}(\dot{x}(t)/x(t)) = 0$$

where $x(t) = (\hat{c}, \hat{k}, \hat{h}, u)$. The steady state conditions are obtained by dividing the four equations in (2.37) by $\hat{c}$, $\hat{k}$, $\hat{h}$, and u, respectively, and by manipulating the resulting expressions. They are:

$$u = (1 - xh_h y_1)^{-1}\left[1 - ((g(1+s) + r - f)/f)\right],$$

$$\left(\frac{\hat{c}}{\hat{k}}\right)_{ss} = \left[1 + xh_h y u_{ss}\right]^{-1}\left[f + fxh_h y u_{ss}^2 - r - gs\right] + w\left(\frac{r + sg_c}{a(1-t_k)}\right) - g,$$

$$\left(\frac{\hat{k}}{\hat{h}}\right)_{ss} = \left[\frac{(r+gs)}{aA(1-t_k)}\right]^{1/(a-1)} u_{ss},$$

$$y = A^{\frac{1}{1-a}}\left[t_u(1-a) + t_k a\left(\frac{r+sg}{a(1-t_k)}\right)^{\frac{a}{a-1}}, \ g = \frac{sr}{a(1-t_k) - ss}. \tag{2.38}$$

The steady state values of the static variables are:

$$r_{ss} = (r + gs)/(1 - t_k), \quad w_{ss} = (1-a)A[(r+gs)/aA(1-t_k)]^{a/(a-1)},$$

$$g_{ss} = [t_u(1-a)+t_k a]Ak_{ss}^a (u_{ss}\hat{h}_{ss})^{1-a} \tag{2.39}$$

3. Numerical Solution

The dynamic system described in (2.37) does not admit a closed-form solution. Therefore we resort to the numerical solution. We use the relaxation algorithm[10] to solve numerically this dynamic system given the boundary conditions (initial and terminal conditions). The initial conditions include the initial conditions on the state variables (k,h) and some arbitrary guess on the control variables (c,u). The terminal conditions are the steady state conditions on the state and control variables. The algorithm transforms the infinite time variable into the time scale to facilitate the solution to the problem. It tries an arbitrary solution to both the state and control variables, assesses the deviation of the arbitrary solution to the true path by a multi-dimensional error function, and then uses the derivative of this function to boost the guess in an iteration of a Newton procedure type. At each point of the path, the adjustment is related to the incorrectness in slope and in the static equations' solution. The algorithm keeps adjusting the trial until it reaches an optimal solution, that is, the one for which the error becomes sufficiently small.

We use the relaxation algorithm as well as the estimated values of parameters summarized in Table 1 to solve numerically the dynamic system described (2.37). The numerical solution to the model is given by the time paths of variables as depicted in Figure 1. As can be noticed, the system is globally saddle point stable. In fact,

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[10] See Timborn, Koch, and Steger (2004) for the description and the implementation of the algorithm.

16

$\hat{c}, \hat{h}, \hat{k}, \hat{y}, \hat{g}, r, u$ and w converge to their steady state values, which are reached after 46 years or in 2040. Plus, in the steady state, a representative agent spends 67.70% of her time working and 32.30% accumulating human capital. Also, the interest and wage rates in the steady state are 15.84% and 1.2968, respectively. The steady state interest rate obtained from the model seems very close to data. For instance, in 1996, the Reserve Bank raised its rate to 17%, which induced commercial banks to increase theirs to 20%. The transition behaviors of variables are explained by the relationship between the initial and steady state ratios of physical to human capital. Graph v in this Figure 1 shows that
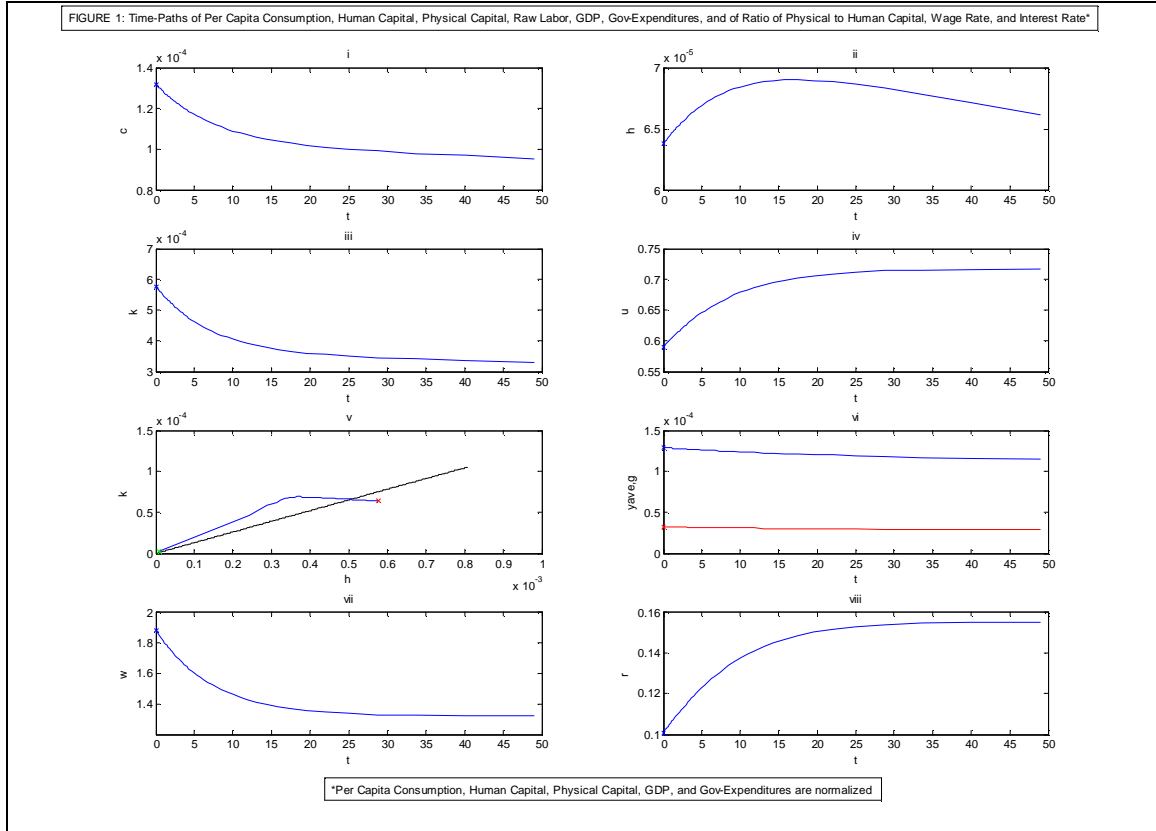
Table 1: Parameters' Values[11]

A	α	k_0	h_0	ϕ	x	$t_k = t_u$	h_h	ρ	s	σ
1.00	0.451	11. 56	6.03	0.116	0.034	0.25	0.27	0.068	0.10	1.46

the path of the ratio of physical to human capital is a straight line through the origin (other paths of this ratio are possible). This shape is the result of the absence of external effects from human capital in the model. It suggests that the initial ratio of physical to human capital is greater than its steady state ratio, that is, physical capital is abundant relative to human capital in the initial period. Recall that the case we are exploring here is the same as the one described in Mulligan and Sala-i-Martin, that is, the share of physical capital a is less than the inverse of the inter-temporal elasticity of substitution s and $\hat{h}$ is scarce relative to $\hat{k}$. To correct for the imbalance between $\hat{k}$ and $\hat{h}$, $\hat{h}$ has to rise

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[11] The estimation procedure is explained in Appendix A2.

monotonically toward its steady state value, while $\hat{k}$ has to decrease monotonically

towards its steady state value so that the ratio $\hat{k}/\hat{h}$ is decreasing (Graphs ii and iii). While

FIGURE 1: Time-Paths of Per Capita Consumption, Human Capital, Physical Capital, Raw Labor, GDP, Gov-Expenditures, and of Ratio of Physical to Human Capital, Wage Rate, and Interest Rate*

*Per Capita Consumption, Human Capital, Physical Capital, GDP, and Gov-Expenditures are normalized

graph iii is consistent with the monotonic increase in $\hat{k}$, graph ii shows instead that $\hat{h}$

increases monotonically to reach its maximum, beyond which it starts to decrease until it

reaches its steady state. Further, an initial high ratio $\hat{k}/\hat{h}$ implies high wage (high

productivity of human capital), to which agents respond with high willingness to work.

As a result, u increases until it reaches its steady state. Also, a decrease in the ratio $\hat{k}/\hat{h}$

causes r to increase monotonically towards its steady state, and $\hat{w}$ to decrease

monotonically towards its steady state since this ratio is negatively related to the former

and positively related to the latter. Plus, from the expression of $\hat{y}$, it is obvious that a

18

decrease in $\hat{k}/\hat{h}$ is expected to cause a decrease in $\hat{y}$. However, the expression of $\hat{y}$ includes also $\hat{h}$ which is expected to impact positively $\hat{y}$. The shape of the time-path of $\hat{y}$ suggests that the negative effect of $\hat{k}/\hat{h}$ on $\hat{y}$ dominates the positive effect of $\hat{h}$ on $\hat{y}$ so that $\hat{y}$ decreases monotonically until it attains its steady state. Furthermore, the transition behavior of $\hat{c}$ can be inferred using the balance conditions. From these conditions, $\hat{c}$ can be expressed as the difference between output and the sum of educational spending and investment in physical capital, $\hat{c} = \hat{y} - h_h \hat{g} - \dot{\hat{k}} - g\hat{k}$. Thus, the shape of the path of $\hat{c}$ suggests that the sum of the second, third, and fourth terms dominates the first term during the transition so that $\hat{c}$ has to decrease monotonically towards its steady state.

The dynamic system of the normalized variables is a working device we used to obtain convergence. However, our variables of interest are the un-normalized ones. So we use the normalized variables to recover the rate of growth of the un-normalized variables. These rates of growth are depicted in Figure 2. In the BGP, in fact, this figure suggests that c, h, k, y, g and $w*$ grow at the same constant rate of 3.47%; while u, r and w do not grow. In the transition, on the other hand, c, k, y, g, w and $w*$ grow at increasing rates, while $h, u,$ and r grow at decreasing rates. The transition median rates of growth are 1.355% for c, -0.455% for k, 3.001% for y and g, -2.855% for w, 1.433% for $w*$, 1.568% for u, and 3.476% for r.

The above results are consistent with the predictions of the Lucas' model as concern the individual behaviors of variables in the transition as well as in the BGP. In fact, the
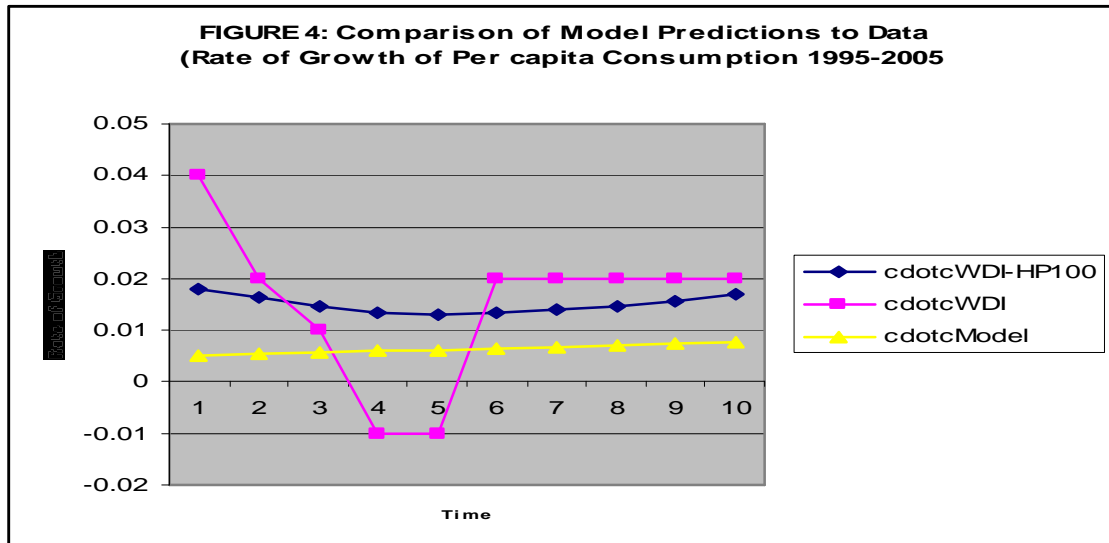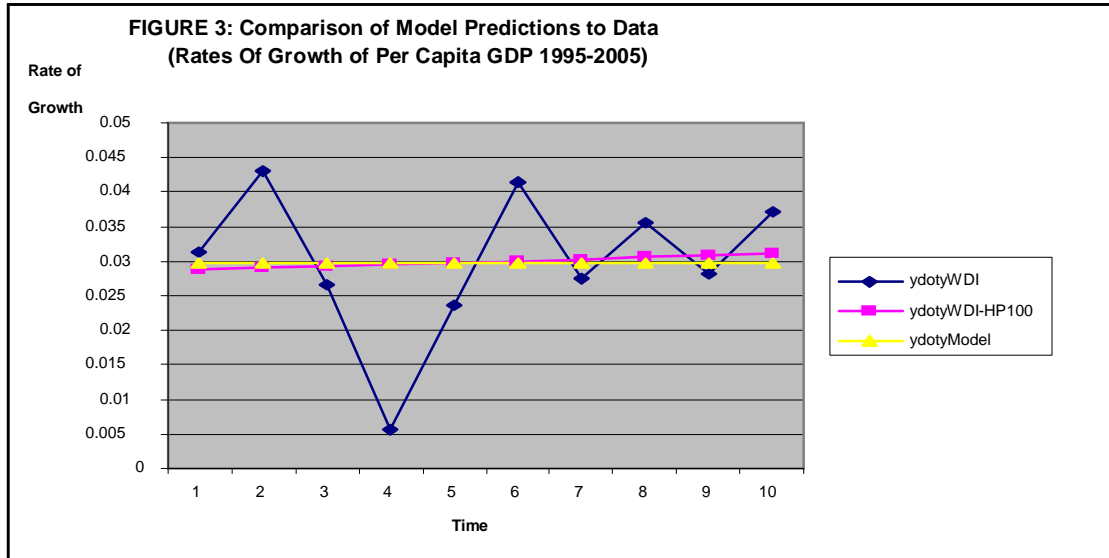
FIGURE 2: Time-Paths of Per Capita Consumption, Human Capital, Physical Capital, Raw Labor, GDP, Gv-Expenditures, and of Interest Rate, Wage Rate, and Wage Rate Augmented for Skill Growth
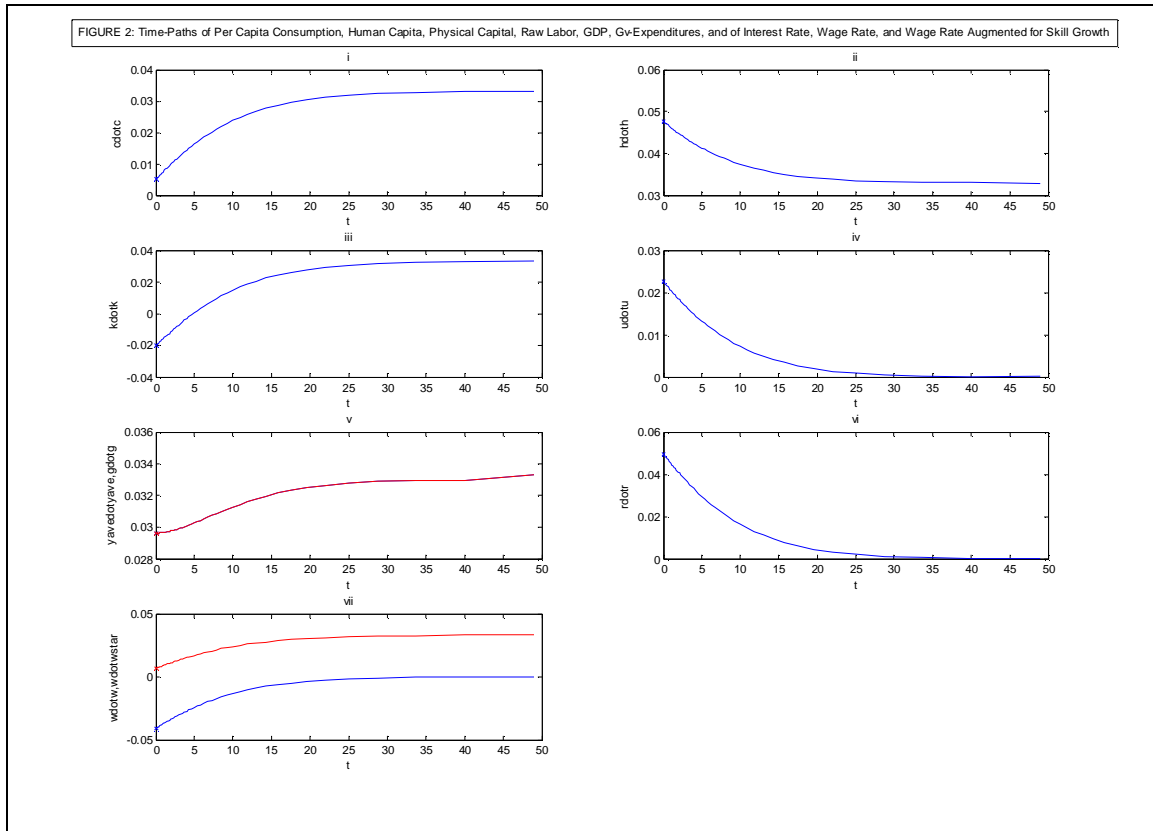
shapes of paths of variables as well as those of their rates of growth are similar to the ones predicted in the studies cited in the introduction.

4. Comparison of the model's predictions to the data

In this section, we compare the model's solution to data to see whether or not it is capable of describing well the growth process of the South Africa's economy from 1995 on. We limit this comparison to variables whose data exist on the per year basis. Two variables meet this requirement, namely, Per Capita Consumption and Per Capita GDP. The data we use for this purpose are those on the rates of growth of these variables from the WDI as well as those on their levels from Summers-Heston Penn World Tables 6.2.

Starting with GDP, we can see from Figure 4 that the rates of growth of Real Per Capita GDP generated from the model are close to the filtered data on this variable over

20

the period 1995-2005. This conclusion is unchanged when we use the levels of Real Per Capita GDP. As is shown in Figure 5, the Real Per Capita GDP from the model tracks the data very well although the data start to deviate slightly after the 7th year.

FIGURE 3: Comparison of Model Predictions to Data (Rates Of Growth of Per Capita GDP 1995-2005)

FIGURE 4: Comparison of Model Predictions to Data (Rate of Growth of Per capita Consumption 1995-2005

On the other hand, the model series on Real Per Capita Consumption does not seem to mimic data closely. In fact, Figure 4 shows that the rate of growth of this variable from data is decreasing to reach the minimum and then starts to increase, while the one from

21

the model is increasing monotonically. Also, although the two rates are increasing after the 5th period, the gap between them is widening over time. Furthermore, this widening gap is also present between the levels of Real Per Capita Consumption from the model and from the data as can be seen in Figure 5.

We can supplement the above graphical analysis with a statistical one by using the Theil Inequality Coefficient (U). This statistics measures the predictive performance of a model and is bounded below by one and above by zero. Its expression is as follows:

$$U = \frac{\sqrt{(1/T)\sum_{t=1}^{T}\left(Y_t^M - Y_t^D\right)}}{\sqrt{(1/T)\sum_{t=1}^{T}\left(Y_t^M\right)^2} + \sqrt{(1/T)\sum_{t=1}^{T}\left(Y_t^D\right)^2}} \in [0,1]$$

where Y_t^M and Y_t^D are the values of the variable from the model and the data, respectively,

FIGURE 5: Comparison of Model Predictions to Data
(Real Per Capita consumption and GDP 1995-2005)
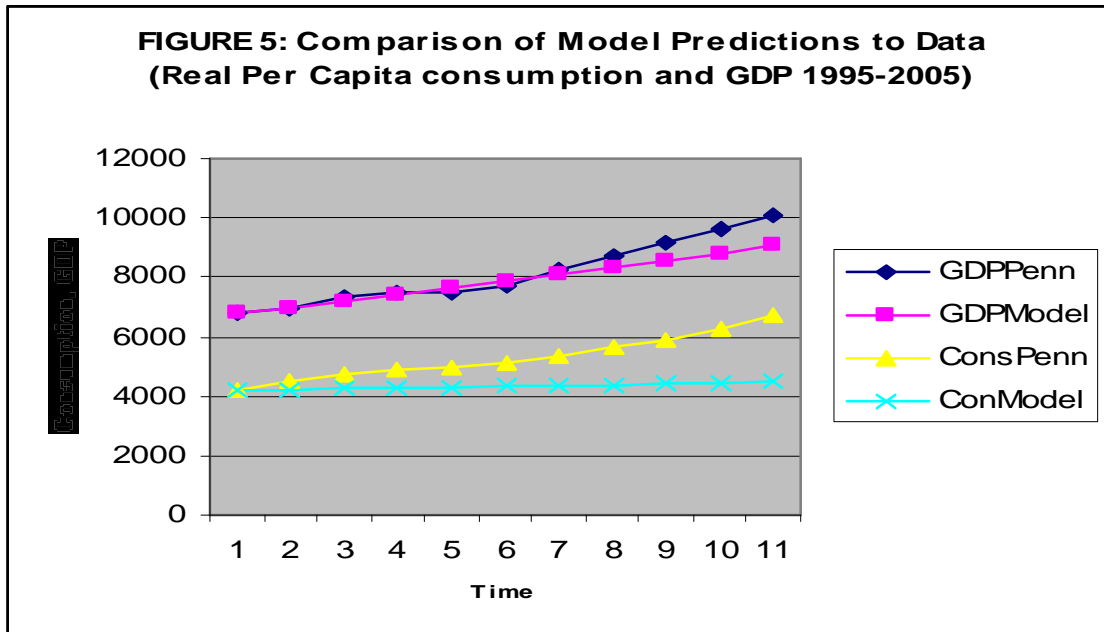
and T is the sample size. A value of U of zero indicates a perfect fit $\left(Y_t^M = Y_t^D\right)$ and a value of U of one indicates a bad predictive performance of the model. Using the series generated by the model and those from the data over 1995-2005 we find different values of U, which are summarized in table 5. As it was the case for the graphical analysis, the GDP series track the actual data very well. In fact, the values of U for the rate of growth (0.0133) as well as the level of Per Capita Real GDP (0,0293) are very close to zero. However, the consumption series seem to mimic the actual data but not so closely. In fact, the values of U for the rate of growth (0.4090) as well as the level of consumption

Table 2: Theil Inequality Coefficient *(U)*

	Rate of Growth	Levels
U (Real Per Capita GDP)	0.0133	0.0293
U (Real Per Capita Consumption)	0.4090	0.1250

(0.1250) are far from one but still not close to zero. This indicates that the predictive performance of the model with respect to the level of consumption is good but this performance with respect to rate of growth of consumption is just fair.

5. Simulation of Fiscal Policy Experiments

As indicated in the introduction, South Africa has allocated on average 7% of its GDP or 21% of its budget to spending on education since 1995. This share is 1.3 times the average of industrialized countries (5.4%) and almost twice that of developing countries (3.9%). Does allocating more resources to education make any difference? We simulate different fiscal policy experiments to answer this question. Specifically, we consider the cases of shifting the allocation of government expenditures between public

spending on education and transfer while maintaining the tax rate constant (Experiments 1 and 2), that of combining this shift with a tax reduction (Experiments 3), and that of cutting totally the tax rate (Experiment 4). These experiments are described in the next paragraph. Further, we compare the results of these experiments to the baseline case to analyze the growth and welfare effects. We should stress that the baseline case is the one associated with the parameters of the basic solution of the previous section, that is, $t_k = t_u = 0.25$, $h_h = 0.27$, and $h_T = 0.73$ [12].

Experiment 1 consists of reducing the GDP share of public spending on education to the share of industrialized countries (5.4% of GDP or 20.83 of national budget) while maintaining the tax rate constant. This shift represents a decrease of 22.857% in h_h or an increase of 8.454% in h_T. Experiment 2 is similar to Experiment 1 in its structure. However, h_h is reduced to the share of developing countries (3.9% of GDP or 15.043% of national budget). This shift is equivalent to a decrease of 44.286% in h_h or an increase of 16.308% in h_T. Experiment 3 consists of shifting the allocation of public expenditures between public spending on education and transfers and of decreasing the tax rate. In this experiment, we increase h_h to one, which is equivalent to decreasing h_T to zero. Also we decrease the tax rate from 0.25 to 0.07. This is equivalent to an increase in h_h of 270.370% or a decrease of 100% in h_T combined with a decrease of 72.000% in the tax rate. In other words, this experiment consists of eliminating transfers from the model while maintaining the same GDP share of educational spending. Experiment 4, finally, is about eliminating government from the model. In fact, in this experiment, we set the tax rate to zero. This is equivalent to decreases in the tax rate of 100% or to no government policy. The results of simulation of the four experiments are summarized in Tables 2 and

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[12] The value of h_h has been adjusted from 21% to 27% to satisfy the assumptions of balanced budget in closed economy. See Appendix A2 for details.

3 for the BGP and transition dynamics, respectively. Indeed, the BGP paths of Experiments 1 and 2 are the same as those of the baseline case (columns 2 and 3). The common rates of growth of $y, k, h, c, g, w*$ and of u, r, w are 3.47% and 0%, respectively. Also, the levels of un-normalized variables are also the same. In fact, the representative agent still spends 67.7% of her time to production and 32.3% to the accumulation of human capital. Also, the interest rate and the wage rate are the same as in the baseline case, that is, 15.84% and 1.297, respectively. However, the BGP of Experiments 3 and 4 differ from that of the baseline except for the common rate of growth of u, r, and w. Indeed, y, k, h, c, g, and $w*$ grow at lower rate than in the baseline case. Their rate of growth is 2.710% in Experiment 3 and 2.190% in experiment 4. Also, the interest rate is lower in each of those experiments. It is 11.58% in Experiment 3 and 10.00% in Experiment 4. Nonetheless, the labor supply and the wage rate are higher in each of them. The representative agent allocates 83.81% and 94.94% of her time to production activities and 16.19% and 5.06% to schooling in Experiment 3 and 4, respectively. Her wage rate is 1.6773 in Experiment 3 and 1.8922 in Experiment 4.

Table 2: BGP's Results of Simulation of Fiscal Policy Experiments

Variables	Policy Exp 1	Policy Exp 2	Policy Exp 3	Policy Exp 4
	Common Rates of Growth (in %)			
$y, k, h, c, g, w*$	3.470	3.470	2.710	2.190
u, r, w	0.000	0.000	0.000	0.000
	Levels of Un-Normalized Variables			
u	0.6769	0.6769	0.8381	0.9494
r	0.1584	0.1584	0.1158	0.1000
w	1.2968	1.2968	1.6773	1.8922

The transition dynamics in each of the 4 experiments look similar to the baseline ones as concerns the shapes of variables as well as concerns those of their rates of growth (we do not plot these variables to economize space). In fact, c, k, y, g, w and w^* grow at increasing rates, while $h, u,$ and r grow at decreasing rates. Their median rates of growth, which are displayed in Table 3 show variations across these experiments.

Table 3: Transition Dynamics's of Simulation of Policy Experiments

Variables	Policy Exp 1	Policy Exp 2	Policy Exp 3	Policy Exp 4
	Median Rates of Growth (in %)			
y	2.978	2.957	3.300	3.579
h	4.271	4.256	4.847	5.999
k	-0.483	-0.493	-1.491	-4.159
u	1.550	1.534	2.389	3.935
c	1.358	1.358	0.623	-1.064
r	3.461	3.450	4.791	7.738
w	-2.843	-2.834	-2.872	-6.356
w^*	1.428	1.422	0.991	-0.357

6. Growth and Welfare Effects of Fiscal Policy Experiments

In this section we assess and analyze the long run and transition growth effects (Tables 4 and 5) and the welfare effects (Table 6) in each of the four experiments.

First, a shift in the allocation of expenditures between public spending on education and transfers (Experiments 1 and 2) does not have any effects on the allocation decisions in the long run. Indeed, agents choose the same rates of growth of all variables in the baseline case. By contrast, a shift in the allocation of expenditures coupled with a reduction in the tax rate or a 100% tax cut does affect the long run allocation decisions. In fact, a decrease of 72% in the tax rate combined with the elimination of transfers (Experiment 3) or a tax cut of 100% (Experiment 4) results in low interest rate. Compared to the baseline case the rate of growth of the interest rate decreases by 26.895% in Experiment 3, and by 36.859% in Experiment 4. Also, a low tax rate induces agents to choose lower rate of growth of k, which is also the rate of growth of y, h, c, g, and w^*. With respect to the baseline, this growth rate decreases by 21.900% in Experiments 3 and by 36.869% in Experiment 4. At the same time, a decrease in the tax rate induces high wage rate. The wage rate increases by 29.341% in Experiment 3 and by 45.913% in Experiment 4 compared to the baseline. Further, a wage differential between the baseline and each of the two experiments makes working more attractive than learning. So agents respond to the high wage with high work effort. Indeed, they increase their supply of labor by 23.796 in Experiment 3 and by 40.263% in Experiment 4. In both of the two experiments, however, the effects are moderate since the tax is decreased by 72% and 100% in Experiments 3 and 4, respectively.

Turning now to the transition, we can see from Table 5 that a shift in the allocation of expenditures, a shift coupled with a tax reduction, and a 100% tax cut do affect the rate of growth of all variables. In fact a shift has a negative effect on the rate of growth of GDP in Experiment 1 (-0.776%) and in Experiment 2 (-1.466), whereas a shift coupled with a

tax reduction and a 100% tax cut have positive effects on this rate in Experiment 3 (9.963%) and in Experiment 4 (19.260%); respectively. These effects on the rates of growth of GDP are the results of the combined effects of public policies on h, k, and u in each of these experiments. These effects can be analyzed directly using the dynamic system formed by equations $(2.16),(2.17),(2.21),(2.22)$, and the static equations given in footnote 7.

In Experiment 1, a decrease of 22.857% in h_h or an increase of 8.454% in h_T shifts resources toward transfers, which agents use to increase their consumption. But an increase in consumption for a given level of income results in low saving and in low rate of growth of k compared to the baseline or a negative effect of -1.684%. Also, a decrease in h_h has negative effects on the accumulation of h and u. Indeed, the rates of growth of

Table 4: BGP's Growth Effects of Fiscal Policy Experiments

Variables	Baseline	Exp 1	Exp 2	Exp 3	Exp 4
	R G (%)	Growth Effects on Rates of Growth (in %)			
$y,k,h,c,g,w*$	3.470	0.000	0.000	-21.900	-36.889
u,r,w	0.000	0.000	0.000	0.000	0.000
	Levels	Growth Effect on Un-normalized variables			
u	0.6770	0.000	0.000	23.796	40.263
r	0.1584	0.000	0.000	-26.895	-36.869
w	1.2968	0.000	0.000	29.341	45.913

h and *u* decreases by 0.394% and by 1.173%, respectively. Plus, h_h does not affect directly y, r, and w, but through k, h, and u. The combined negative effects of a decrease in h_h on k, h and *u* translates into negative effects on the rates of growth of GDP and *r* of -0.776% and -0.432%, respectively, and in a positive effect on *w* of 0.420%. Intuitively, an increase in transfers shifts resources toward consumption but away from saving. This results in higher consumption but low rate of accumulation of *k* compared to the baseline case. At the same time, higher transfers create disincentives regarding labor supply and results in low work effort compared to the baseline. Moreover, a low supply of labor has a positive effect on the accumulation of *h*. But this positive effect on *h* is outweighed by a negative effect of a decrease in h_h on *h* so that the net effect is negative. This negative effect implies that *h* accumulates slowly compared to the baseline. We should stress that although a decrease of 22.857% in h_h generates growth effects, these effects are very small. Put differently, decreasing the share of public spending to the proportion of industrialized countries does have negligible effects on the rate of growth of GDP as well as on those of the other variables in the economy as long as the resources generated are used to increase transfers.

In Experiment 2, a decrease of 44.286% in h_h does generate negative growth effects on the rates of growth of *k* (-0.400%), *u* (-2.149%), *h* (-0.732%), *r* (-0.742%), but a positive growth effect on that of *w* (0.385%). Additionally, the combined negative effects of this decrease on k, h and *u* translate into an negative effect on the rate of growth of GDP of -1.446%. The results of this experiment are the same as those of Experiment 1 as concern the direction of the effects but slightly different as concern their sizes. In fact, a

decrease in h_h in this experiment is almost twice that of Experiment 1. Coincidently, the effects of this decrease on $y, u, h,$ and r are almost twice those of the corresponding effects in Experiment 1. However, that on k is one fourth and that on w is almost the same as the corresponding effect in Experiment 1. Compared to Experiment 1, this is an indication that a large shift in the allocation of expenditures amplifies the effects on $y, u, h,$ and r, but not k and w. Also, since the effects in this experiment move in the

Table 5: Transition Growth effects of Fiscal Policy Experiments

Variables	Baseline	Policy Exp 1	Policy Exp2	Policy Exp 3	Policy Exp4
	Rates of Growth %	Growth Effects on Rates of Growth (in %)			
y	3.001	-0.776	-1.500	9.968	19.260
h	4.289	-0.394	-0.732	14.638	39.848
k	-0.475	-1.684	-0.400	-215.368	-775.579
u	1.568	-1.173	-2.149	52.360	150.982
c	1.355	0.044	0.221	-54.050	-174.759
r	3.476	-0.432	-0.742	37.846	122.617
w	-2.885	0.420	0.385	-0.603	-122.627
$w*$	1.433	-0.384	-0.712	-36.343	-124.920

same direction as those in Experiment 1, the intuition behind the behaviors of variables is the same as the one described in the previous paragraph (Experiment 1). As it was the case in Experiment 1, decreasing the share of spending on education to the proportion of the developing countries generates very small growth effects as long as the resources generated are allocated to transfers.

Moving now on to Experiment 3, we can see from Table 4 that eliminating transfers while maintaining the share of the educational spending to 7% of GDP, which is the same as decreasing the tax rate by 72.000% and increasing h_h by 270.370%, results in a positive effect on the rate of growth of 9.963%. Indeed, increasing h_h by 270.370% lower the income available to the agent while reducing the tax rate by 72.000% increases it. The end result is that the effect of the former dominates that of the latter so that the net effect is low saving and thus low rate of growth of k or a negative effect of -215.368%. Also, the elimination of transfer has a negative effect of -52.022% on the rate of growth of consumption. Plus, a decrease in the tax rate and an increase in budget share of educational spending have opposing effects on the supply of labor. Although the effect of the former is negative, that of the latter is positive and dominates. In fact, the net effect is an increase in the rate of growth of labor supply of 52.360% compared to the baseline. Indeed, reducing the tax rate creates more incentives for high work effort. Also, high labor supply combined with high h_h results in high accumulation of h compared to the baseline or a positive effect on its rate of growth of 14.638%. The combination of the positive effects on the rates of growth of h and u dominates the negative effect on k and translates into a positive effect on the rate of growth of GDP of 9.963%. This dominance also implies that wage rates grow slowly compared to the baseline or a negative growth

effect of -0.603%. However, the combined positive effects of this shift and the tax reduction on the rates of growth of h and u, and their negative effect on k result in a positive effect of 37.846% on the interest rate. So the interest rate grows faster in this experiment than it does in the baseline case. The effects of this fiscal policy experiment vary from small (w) to moderate (y, h, u, c, r) to large (k).

The last experiment –Experiment 4- is similar to the previous one as concern the direction of the effects. Indeed, a tax cut of 100% has positive effects on y (19.260%), h (38,869%), u (150.982%), and r (122.617%); but negative effect on k (-775.579%), c (-174.579%) and w (-122.627%). In fact, cutting tax removes all the distortions and improves the efficiency of the allocation of labor between working and schooling. In other words, cutting tax creates incentives for high supply of labor and results in a positive effect of 150.982% on its rate of growth compared to the baseline case. Also, high supply of labor means less time spent on the learning field, which would suggest that human capital accumulates slowly compared to the baseline. However, this is not the case. Indeed, although labor supply grows faster in this experiment compared to the baseline, it starts and stays below that of the baseline for a quite amount of periods[13]. Consequently, time spent on the learning field is higher in this experiment for a certain number of periods, implying that human capital grows faster or a positive effect of 38.869% with respect to the baseline. Meanwhile, efficiency requires that the rate of accumulation of physical capital be reduced since it is abundant compared to human capital. This results in a low rate of growth of physical capital or a negative effect of -

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[13] The labor supply in initial period is 0.5904 and 0.4121 in the baseline and experiment 4, respectively.

775.579%. The combined effects on h, k and u translate into positive effects on y, r, of 19.260%, 122.617%, respectively; and on a negative effect of -122.627% on w. Unlike the previous experiments, this one generates moderate to large growth effects.

Regarding the welfare effects of the four fiscal policy experiments, we can see from Table 6 that Experiments 1 and 2 are associated with positive welfare effects but these effects are very small (less than 1%). Nonetheless, Experiments 3 and 4 have negative effects on welfare. In fact, the welfare decreases by 4.888% in Experiment 3 and by 16.568% in Experiment 4.

Table 6: Welfare Effects of Public Policies (in %)

Variable	Baseline	Exp 1	Exp2	Exp 3	Exp4
Welfare Value	-0.58245	-0.58235	-0.58225	-0.61092	-0.67895
Welfare Effect	-	0.025	0.035	-4.888	-16.568

To sum up, a shift in the allocation of government expenditures between public spending on education and transfers (Experiments 1 and 2) generates negligible growth and welfare effects. Plus, a shift coupled with a tax reduction (Experiment 3) or a 100% tax cut (Experiment 4) generates small welfare effects. Nonetheless, it has mixed growth effects, that is, it generates small effects on some variables but large effects on some other variables.

Now, how are the results of this study compared to those of the previous ones. To our knowledge, there do not exit results from previous studies to which we can directly confront ours. However, some share a great deal of issues explored here. For instance,

Agenor (2005) explores the growth effects of a shift in allocation of public spending between education and public infrastructure to find that a reallocation of resources has a large impact on long run growth for some choice of parameters but limited effects for some other choice of parameters. This finding is totally different from the results obtained here. In fact, a shift does not have any long run effect on growth. The difference in the results of our study and his resides in the specification used and in the nature of policies. Their specification of the human capital accumulation function includes public spending on education and public infrastructure while the latter variable is absent in our model. Plus, public infrastructure as well as spending on education is considered as a productive spending while transfers are considered as unproductive expenditures. The different nature of the two spending makes the comparison even harder. The transition behaviors of variable are also un-comparable. Indeed, this paper uses the ratio of variables while we use the rates of growth of the levels of variables.

Other papers that provide a basis for comparison are John et al. (1993), Corsetti and Roubini (1996), and Greiner (2006). The last two papers not only build spending on education in the human capital function but also predict the long run behaviors of variables that are consistent with our results of Experiments 3 and 4, that is, an tax on capital income distorts allocation decisions and does not correct for the externalities generated by public good. However, these papers do not quantify the effects, making thus the comparison more difficult. The paper by John et al. (1993) provides insight into the issue since it includes productive spending into the physical capital accumulation function and transfers. The results of simulations from this paper, unlike ours, show large welfare effects of change in fiscal policy. Additionally, the growth effects are larger in

this paper while they are mixed in our. We should stress that the difference in the results is explained by the difference in the structure of our models as well as the procedure used to assess the effects. In fact, building productive spending in human capital function is not the same as building it in the physical capital accumulation function. Also, while this paper builds on the optimal taxation, we use the flat tax rate as reflected in the data.

7. Conclusion

In this paper, we have formulated a model of endogenous growth with human capital accumulation to analyze the growth and welfare effects of a shift in the allocation of public expenditures between spending on education and transfers as well those of a shift coupled with a tax reduction or those of a 100% tax cut. South Africa has launched since the abolition of its Apartheid regime in 1995 a massive program of education, which has been financed exclusively through fiscal resources representing on average 7% of its GDP or 21% of its national budget. This share is 1.3 times the average of industrialized countries (5.4%) and almost twice that of developing countries (3.9%).

To analyze the growth and welfare effects mentioned above, we have simulated four fiscal policy experiments. The first and second experiments (Experiments 1 and 2) have consisted of reducing the budget share of spending on education to the averages of industrialized and developing countries, respectively, while using the generated resources to increase transfers. In the third experiment (Experiment 3), we have eliminated transfers in the model while maintaining the GDP share of spending on education constant. In the last experiment (Experiment 4), we have eliminated government in the model.

The numerical solution to the model (baseline case) has demonstrated that the South Africa's economy converges after 46 years from 1995 or in 2040. In the long run or BGP,

physical capital, human capital, consumption, government expenditures, wage augmented for skill growth, and GDP grow at a constant rate of 3.47%; while the interest rate, wage rate, and labor supply do not grow. The transition dynamics to the BGP are characterized by two different patterns, that is, physical capital, consumption, wage rate, wage rate augmented for skill growth, government expenditures, and GDP grow at increasing rates, whereas human capital, the interest rate, and labor supply grow at decreasing rates. Additionally, the graphical as well as the statistical analyses have indicated that the model describes the growth process of the South Africa' economy pretty well.

In the next step, we have simulated a solution to each of the four fiscal policy experiments and then compared it to the baseline's solution to assess and analyze growth and welfare effects. The solution to each experiment is qualitatively similar to that of the baseline. In fact, not only the economy converges after 46 years, but also the BGP as well as the transition behaviors are similar.

The comparison of the solutions of these experiments has revealed what follows. First, reallocating public expenditures between spending on education and transfers does not have any effects on the BGP, whether the reallocation is operated through the reduction of the GDP share of educational spending to the average of industrialized countries (Experiment 1) or to that of developing countries (Experiment 2). However, an elimination of transfers coupled with a tax reduction (Experiment 3) or a 100% tax cut (Experiment 4) does indeed affect the BGP but the effects are moderate. The effects on the common rate of growth and on the interest rate are negative while those on the supply of labor and the wage rate are positive. Intuitively, in the absence of labor-leisure choice a labor income tax is neutral, that is, reducing a labor income tax does not modify agents'

36

behaviors. However, a reduction of a capital income tax (Experiment 3) or a 100% tax cut (Experiment 4) does affect negatively the return on physical capital, which in turn creates disincentives for high accumulation of physical capital and high rate of growth.

Second, in transition, a reallocation of government expenditures whether operated through a reduction of the GDP share of educational spending to the average of industrialized countries (Experiment 1) or that of developing countries (Experiment 2) does generate growth and welfare effects but these effects are very small. Indeed, this reallocation shifts resources away from saving and toward consumption and results in low rate of growth of GDP. As an implication, the rate of growth of consumption as well as the welfare is higher compared to the baseline. Also, it is associated with high rate of growth of the wage rate. However, its rate of growth of the interest rate is low. Also it creates disincentives for accumulation of physical and human capital and induces low work effort since agents receive additional income from transfers. On the other hand, a tax reduction coupled with the elimination of transfers (Experiment 3) or a 100% tax cut (Experiment 4) generates positive growth effects on the rates of growth of GDP, human capital, labor supply, and the interest rate but negative growth effects on the rates of growth of consumption (welfare), physical capital, and the wage rate. These effects vary from small to moderate to large. Indeed, a tax reduction or a 100% tax cut reduces or removes distortions and creates incentives for high rates of growth of labor supply, human capital, and the interest rate. Nonetheless, its associated rates of growth of the wage rate and physical capital are low.

37

Third, the welfare effects associated with Experiments 1 and 2 are positive but very small. By contrast, those associated with Experiments 3 and 4 are negative and vary in size. They are small in Experiment 3 but moderate in Experiment 4.

In terms of policy recommendation, the concern now is which one of the four experiments is recommendable? There does not exist a clear cut answer. Indeed, Experiments 1 and 2 are good on the welfare ground but bad when it comes to growth. On the other hand, Experiments 3 and 4 perform well on the growth ground but are worse on the welfare ground. Clearly, growth and welfare do not go one to one in each of the four experiments. This is to say that the choice will depend on the objective pursued by the government.

This study can be extended in several directions. First, transfers can be disaggregated in order to identify the other expenditures that qualify as productive so as to include them in the appropriate sectors. Our choice to treat expenditures other than educational ones as transfers was motivated by the concern to ease the derivation of the BGP. The disaggregation may provide new insights and lead possibly to different results. For instance, these transfers include expenditures such as health, social infrastructures (housing, special development initiatives), promotion of industrial development, research and technology development, and competitiveness fund and sectoral partnership facility. Some of these expenditures can enter the production function while other can enter the physical capital accumulation function. Second, we have compared the results of each experiment to the baseline case. This, however, does not prevent us from comparing the results of Experiment 4 to that of Experiment 3. In other words, if we consider Experiment 3 as our baseline and try to assess the effects of moving from this experiment

to Experiment 4, we can see that the effect on the rate of growth of GDP is negative in the BGP but positive in transition, while the effect on welfare is negative. The BGP effect on the rate of GDP is consistent with the predictions in Barro (1990) and Barro and Sala-i-Martin (1992). However, the transition effects, which are not explored in these studies, suggest that educational spending is not productive. The question is whether these spending are not really productive or the policy design used in this study is not appropriate. As indicated in the mentioned studies, a publicly provided private good as education must be primordially financed through a consumption tax. Thus, Experiment 3 can be re-specified by replacing the labor income tax and capital income tax by a consumption tax, and solved numerically before to draw a definitive conclusion.

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Appendix A: Equation of motion of u in Model 1

Let rewrite the first order conditions -conditions $(2.14)-(2.19)$ in the text- as:

$$c^{-s} - 1_k = 0, \qquad\qquad \forall t \qquad\qquad (A.1)$$

$$1_k w(1-t_u) - 1_h f(1+xh_h(g/h)) = 0, \qquad \forall t \qquad (A.2)$$

$$\dot{k} = (1-t_u)wuh + (1-t_k)rk - c + (1-h_h)g, \qquad (A.3)$$

$$\dot{h} = f(1-u)(h + xh_h g), \qquad (A.4)$$

$$\dot{1_k} = -1_k[(1-t_k)r - r], \qquad (A.5)$$

$$\dot{1_h} = -1_k(1-t_u)wu - 1_h[f(1-u) - r], \qquad (A.6)$$

Plus the boundary conditions, that is, the following *TVC* and initial conditions:

$$\lim_{t\to\infty} 1_k(t)e^{-rt}k(t) = 0, \ \lim_{t\to\infty} 1_h(t)e^{-rt}h(t) = 0, \qquad (A.7)$$

$$k(0) = k_0, \ h(0) = h_0, \ k_0 \text{ and } h_0 \text{ are given.} \qquad (A.8)$$

Taking the log of $(A.2)$ yields:

$$\ln 1_k - \ln 1_h + \ln w + \ln(1-t_u)f^{-1} = \ln(1+x(g/h)) \approx x(g/h), \qquad (A.9)$$

In $(A.9)$ we have approximated $\ln(1+x(g/h))$ by $x(g/h)$ following a similar approximation used in Enders (2006), p.107. Taking the time-derivative of $(A.9)$ after substituting (2.11) for w, and rearranging yields:

$$\left[a + xh_h(1-a)\left(\frac{g}{h}\right)\right]\frac{\dot{u}}{u} = \frac{\dot{1_k}}{1_k} - \frac{\dot{1_h}}{1_h} + a\left(1 - xh_h\left(\frac{g}{h}\right)\right)\left(\frac{\dot{k}}{k} - \frac{\dot{h}}{h}\right) \qquad (A.10)$$

Divide $(A.5)$ by l_k and substitute for r to obtain:

$$\dot{P}_k / l_k = r - (1-t_k)aAk^{a-1}(uh)^{1-a}. \tag{A.11}$$

Divide $(A.6)$ by l_h, substitute $(A.2)$ into the resulting expression, and rearrange yields:

$$\dot{P}_h / l_h = r - f - fxh_h(g/h)u. \tag{A.12}$$

Substituting (2.11) for r and w, and (2.9) for g into $(A.3)$ and dividing by k yields:

$$(\dot{k}/k) = [1 - h_h(t_u(1-a) + t_k a)]Ak^{a-1}(uh)^{1-a} - (c/k). \tag{A.13}$$

Dividing $(A.4)$ by h it comes:

$$\dot{h}/h = f(1-u)(1 + xh_h(g/h)). \tag{A.14}$$

Substitute $(A.11)-(A.14)$ into $(A.10)$ and rearrange to get:

$$\dot{u} = u[a + (1-a)xh_h(g/h)]^{-1}[f + fxh_h(g/h)u - (1-t_k)aAk^{a-1}(uh)^{a-1} +$$

$$a(1 - xh_h(g/h))\{(1 - h_h(at_k + (1-a)(1-t_u)))Ak^{a-1}(uh)^{a-1} - (c/k) -$$

$$f(1-u)(1 + xh_h(g/h))\}]. \tag{A.15}$$

We can see from $(A.15)$ that the expression inside the braces is $\dot{k}/k - \dot{h}/h$. So we rewrite $(A.15)$ as:

$$\dot{u} = u[a + (1-a)xh_h(g/h)]^{-1}[f + fxh_h(g/h)u - (1-t_k)aAk^{a-1}(uh)^{a-1} +$$

$$a(1 - xh_h(g/h))((\dot{k}/k) - (\dot{h}/h))], \tag{A.16}$$

which is exactly the equation (2.22) in the text.

Appendix B: Parameters' Estimation

To solve the dynamic system described in (2.37) given (2.4) and $(2.38) - (2.39)$ for the Post-Apartheid South African economy $(initial - period = 1995)$, we need to obtain the estimated values of the parameters as well as those of initial conditions.

We obtain the parameters of the production function as follows. First, we normalize technology parameter A to 1, and then estimate a using the following formula:

$$a = 1 - (CSUL/GDP), \qquad (B.1)$$

where *CSUL* is the compensation of skilled plus compensation of unskilled labor -the equivalent of Z in the *CRS* production function given in (2.13) - and *GDP* is the aggregate output. South Africa's data on $CSUL/GDP$ is obtained from the Version 5 of GTAP Aggregate Database 2004.

To estimate the initial per capita physical capital stock (k_0), we use the South Africa's data on per capita real investment (i_t) from Summer and Heston's Penn Tables Version 6.2 to construct a series of the capital stock according the following rule:

$$k_{t+1} = (1 - d_k)k_t + i_t, \qquad (B.2)$$

$$k_{T_0} = \bar{k}_{T_0}, \qquad (B.3)$$

where $T_0 = 1995$, and d_k is the depreciation rate of capital stock, which is calculated from the South Africa's data from the Version 5 of GTAP Aggregate Database 2004 using the following expression:

$$d_k = (VDEP/VKB),\qquad\qquad (B.4)$$

where *VKB* and *VDEP* are the Value of Capital Stock at the Beginning of period and the Value of Depreciation of Capital Stock, respectively. We choose $\bar{k}_{T_0}$ such that[14]:

$$k_{T_0+1}/k_{T_0} = \left(k_{T_0+10}/k_{T_0}\right)^{1/10}. \qquad\qquad (B.5)$$

The initial per capita human capital is the average years of schooling of population aged 15 year old and over for the year 1995 from Barro and Lee (2000), and the human capital technology parameter f is obtained from the following formula:

$$f = \left(\frac{1}{5}\frac{h_{t+1}^{25} - h_t^{25}}{h_t^{25}}\right)\left(\frac{1}{5}\right)^{15}, \qquad\qquad (B.6)$$

where h_t^{25} is the average years of schooling of the population aged 25 and over. Furthermore, ξ is obtained in the following way. First, note that if all effort is allocated to the accumulation of human capital $(u = 0)$, the marginal product of $\dot{h}$ with respect to g is:

$$(\Delta h_t - \Delta h_{t-1})/(g_t - g_{t-1}) = fx$$

where the numerator is the change in investment in human capital and the denominator is the change in expenditures on education. Using data on Human capital (measured by

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[14] This rule is taken from "Econ 8107 Macroeconomics", Spring 2005, University of Minnesota.

[15] Using the average years of schooling for the population aged 15 and over yields a value of 0.02 for this parameter. This value yields in turn a negative value for the $BGP's$ rate of growth g. This is the reason why we decide to use the average years of schooling for the population aged 25 and over.

average years of schooling from Barro and Lee (2000)) and expenditures on education and training over 1995-2000, we obtain fx, which we divide by f to get x.

The tax rates as well as the GDP's share of spending on education are obtained from the South Africa's national budgets over 1995-2006. The average national budget share of GDP over 1995-2006 have represented 30.1%. Since we have assumed a balanced budget, we reduce this share to the tax revenue share of GDP, which is 29%. Recall also that we have assumed that the economy is closed. This assumption implies that tax revenue does not include excise duties. The average excise duties' share of GDP is 4%. Subtracting this average excise duties' share of GDP from the tax revenue share of GDP yields the expenditure's share of GDP of 25%. For simplicity, we assume that capital and labor incomes are taxed at the same rate. This implies that $t_k = t_u = 0.25$. Also, the average GDP share of spending on education h_h from these national budget data is 7%. In terms of the balanced budget with closed economy, this share represents 27% of budget. The budget share of transfers h_T is the complement to unity of h_h, that is $h_T = 73\%$.

The preference parameters (r, s) and the savings rate (s) are determined jointly from the SS conditions $(2.38) - (2.39)$. Recall from these conditions that r_{ss} is given by:

$$r_{ss} = (r + gs)/(1 - t_k) \text{ or } r_{ss} = aA\hat{k}_{ss}^{a-1}\left(u_{ss}\hat{h}_{ss}\right) \tag{B.7}$$

Also, the SS expression of $\left(\dot{\hat{k}}/\hat{k}\right)_{ss}$ from (2.38) is given by:

$$\left(\dot{\hat{k}}/\hat{k}\right)_{ss} = 0 = [1 - h_h(t_u(1-a) + t_k a)]A\left(\hat{k}_{ss}\right)^{a-1}\left(u_{ss}\hat{h}_{ss}\right)^{1-a} - g - \hat{c}/\hat{k}, \tag{B.8}$$

which we can rewrite as:

$$0 = \left[1 - h_h\left(t_u(1-a) + t_k a\right)\right] r_{ss}/a - g - \left(\hat{c}/\hat{k}\right)_{ss}, \quad \text{or} \tag{B.9}$$

$$r_{ss} = a\left(g + \hat{c}/\hat{k}\right)/\left[1 - h_h\left(t_u(1-a) + t_k a\right)\right]. \tag{B.10}$$

Setting $(B.7) = (B.10)$ and rearrange yields:

$$g\left\{\left[1 - h_h\left(t_u(1-a) + t_k a\right)\right]s - a(1 - t_k)\right\} = a(1 - t_k)\left(\hat{c}/\hat{k}\right)_{ss} - \left[1 - h_h\left(t_u(1-a) + t_k a\right)\right]r. \tag{B.11}$$

Substitute (2.39) for $\left(\hat{c}/\hat{k}\right)_{ss}$ into $(B.11)$ and rearrange to get:

$$f + fxh_h y u_{ss}^2 = r + gs \tag{B.12}$$

Substitute (2.39) for y and u_{ss} into $(B.12)$ and rearrange to obtain:

$$\Phi(r,s,s) = -\frac{ra(1 - t_k)}{a(1 - t_k) - ss} + \left(f + fxh_h\left[t_u(1-a) + t_k a\right]\right)\left[\frac{r}{a(1 - t_k) - ss}\right]^{\frac{a}{a-1}} x$$

$$\left[\frac{\left\{(2f - r)(a(1 - t_k) - ss) - sr(1+s)\right\}(a(1 - t_k) - ss)^{\frac{1}{a-1}}}{f(a(1 - t_k) - ss)^{\frac{a}{a-1}} - fxh_h\left[t_u(1-a) + t_k a\right]A^{\frac{1}{1-a}} r^{\frac{a}{a-1}}}\right]^2 \tag{B.13}$$

For a given value of r, $(B.13)$ is solved for s and s using Newton iteration method.

The values of parameters from this exercise are summarized in the following table:

Table 1: Parameters' Values

A	α	d_k	k_0	h_0	ϕ	x	$t_k = t_u$	h_h	ρ	s	σ
1.00	0.451	0.04	11. 56	6.03	0.116	0.034	0.25	0.27	0.068	0.10	1.46