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INTERNATIONAL TRADE IN EXHAUSTIBLE RESOURCES: A CARTEL-COMPETITIVE FRINGE MODEL

by

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Abstract

We characterize the open–loop and the Markov perfect Stackelberg equilibria for a differential game in which a cartel and a fringe extract a nonrenewable resource. Both agents have stock dependent costs. The comparison of initial market shares, across different equilibria, depends on which firm has the cost advantage. The cartel's steady state market share is largest in the open loop equilibrium and smallest in the competitive equilibrium. The initial price may be larger in the Markov equilibria, so a decrease in market power may make the equilibrium appear less competitive. The benefit to cartelization increases with market share.

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1. Introduction

Exhaustible resources are amongst the most important traded commodities of world trade over the past decade. Sustained, and at times successful attempts by exporters to exercise market power have increased the economic and political significance of these commodities. The effect of OPEC is still felt, and there have also been cartels in the mercury, uranium, diamond, copper and bauxite markets. Existing models do not give a plausible description of resource cartel–fringe markets because they typically assume that agents have constant production costs, and they use open–loop equilibria. The first assumption implies that the cartel and fringe do not extract simultaneously, which appears untrue, and the second assumption gives an equilibrium that is not subgame perfect.

We extend the literature by using stock–dependent extraction costs, and more importantly we solve both the open–loop and the Markov Perfect (subgame perfect) equilibria. We derive testable hypotheses concerning the effect of cartelization on the initial price and on the short– and long–run market shares. Comparison of the Open–Loop Stackelberg Equilibrium (OLSE) and the Markov Perfect Stackelberg Equilibrium (MPSE) also helps develop our intuition for how these models work. The MPSE allows the cartel to exercise less market power, so we might expect it to lie "between" the competitive equilibria and the OLSE. This intuition can be misleading. For example, after the cartel is formed the price level can be higher in the MPSE than in the OLSE.

Important early papers on the cartel–fringe model include Salant (1976), Gilbert (1978), Ulph and Folie (1980) and Newbery (1981).[1] These papers use OLSE and assume constant extraction costs, so the equilibria consist of different regimes in which only one firm produces. Ulph and Folie and Newbery emphasized that this equilibrium is time–inconsistent: the cartel at time $t>0$ would

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[1]Pindyck (1978) simulates oil, bauxite and copper markets using competitive and monopoly equilibria without introducing rational expectations for the fringe. Eswaran and Lewis (1985) offer numerical comparison for the open–loop and feedback Nash–Cournot equilibria. The open loop case is also studied by Loury (1986) and Polansky (1992). Griffin (1985) tests various theories on OPEC behaviour and finds support for viewing oil markets in cartel–competitive fringe framework. For a more detailed review, see Karp and Newbery, (1993).

(typically) like to deviate from the path that it announced at time 0, even if there had been no deviation in the past. Groot et al. (1989) show that price can be discontinuous between different regimes.

Introducing stock–dependent costs considerably changes the OLSE. The firms extract simultaneously during an infinite period of time, and price discontinuities no longer occur. The equilibrium is still time–inconsistent, but we obtain a useful characterization of the cartel's incentive to deviate. If the cartel begins with a cost disadvantage and produces little or nothing at the beginning of the OLSE, it would like the fringe to extract rapidly at the beginning. The cartel benefits in the future when it faces a rival with higher costs (lower stocks), and the current low price does not harm it, since it is (nearly) inactive. In order to induce the fringe to begin with rapid extraction, the cartel uses threats of high sales in the future. However, once the fringe's cost advantage has eroded, the cartel would like to sell less than it had threatened. If the cartel begins with a cost advantage, and therefore wants to sell early in the program, it would like to discourage fringe sales. To do this, it promises to extract slowly in the future. The resulting high future price trajectory induces the fringe to conserve its stock. The cartel wants to deviate from either type of plan, while the fringe wants to hold the cartel to a promise, but release it from having to carry out a threat. Since the cartel's relative cost advantage can change over time, an OLSE may change from being a threat to a promise, or it may remain a promise forever. This characterization of the OLSE is useful for understanding how the incentives differ in the MPSE.

The cartel's long run stationary market share is higher in the MPSE than in competitive equilibrium. The initial market share can be higher or lower, depending on whether the cartel begins with a cost advantage. Thus, we see that the exercise of market power can result in a higher market share in both the short run and the long run, which is contrary to what we expect from static cartel models. Of, course given the resource constraint, there must be some interval when the cartel's market share is lower. We also find that benefits to cartelization (in a MPSE) are large when the dominant firm has a cost advantage. The benefits decrease or increase over time, depending on whether the cartel's market share is decreasing or increasing.

Newbery (1992) constructed a MPSE for the special case in which the resource is worthless after an exogenous time, extraction costs are stock independent, and the cartel's resource constraint is never binding. These three assumptions imply that the game has only one state variable, the fringe's stock. We drop all of these assumptions, so our model has two state variables. Consequently, we cannot use the techniques described by Tsutsui and Mino (1990) for differential games with one state variable. Instead we use a linear–quadratic structure, which enables us to obtain closed–form solutions. Our model is more complex than Reynolds' (1987) linear–quadratic two–state variable model because we have asymmetric agents. In addition, for some initial states the non–negativity constraints are binding. Reynolds finessed this problem by restricting analysis to interior solutions. We consider the general case, and provide a simple characterization of the MPSE and a comparison with the OLSE. Hansen et al. (1985) construct a time–consistent equilibrium for a similar problem. However, that equilibrium is not subgame perfect: the decision–maker in the cartel today does not recognize that she is able to affect the incentives of future cartel decision–makers. Their paper also ignores non–negativity constraints on production.

The next section outlines the model and the competitive equilibrium. Section 3 analyzes the OLSE, and Section 4 presents the MPSE and a comparison of the two. Section 5 concludes the paper.

2 The model and competitive equilibria

The model consists of two agents, the cartel (c) and a representative resource owner (f) from the competitive fringe. Resource stocks evolve according to $\dot{X}_i=-q_i$, where $i=c,f$ and X_i and q_i are the resource stocks and extraction levels respectively. We drop the time variable where convenient. Demand is linear: $p=\bar{p}-q_c-q_f$, where $\bar{p}$ is the choke price. The costs are decreasing and linear with remaining stocks, and short run unit costs are constant, i.e. costs for agent i are $q_i(c_{0i}-c_iX_i)$, $i=_{c,f}$. We assume that $c_{0i}\geq\bar{p}$, implying that agent i will not leave less than $\hat{X}_i$ units in the ground, where $\hat{X}_i$ solves $c_{0i}-c_i\hat{X}_i=\bar{p}$; i.e., $\hat{X}_i=(c_{0i}-\bar{p})/c_i$. Define agent i's economically viable stock as $x_i\equiv X_i-\hat{X}_i$, and write i's unit costs as $\bar{p}-c_ix_i$, and the state equations as $\dot{x}_i=-q_i$. The states are now the economically viable stocks, rather than the physical stocks. Assume finally that δ is the rate of discount for both agents. The description of our model requires only three parameters, δ, c_f, and c_c.

In the competitive equilibrium, agent i maximizes $\int_0^\infty [pq_i-q_i(\bar{p}-c_ix_i)]e^{-\delta t}dt$, s.t. $\dot{x}_i=-q_i$, $x_i(0)=x_{i0}$, $\lim_{t\to\infty} x_i\geq 0$, and $q_i\geq 0$. This equilibrium can be solved as a social planner's problem. If the unit extraction costs are not equal ($c_fx_f\neq c_cx_c$), the agent with the lowest cost is the only producer. During an interval when both supply, $c_fx_f=c_cx_c$, in which case $q_f=c_cq_c/c_f$. We refer to the solution $x_f=c_cx_c/c_f$ as the "socially optimal stationary path" and denote it by $\Gamma_1(x_c)$. When $c_fx_f>c_cx_c$ the solution approaches Γ_1 along a vertical line, and when $c_fx_f<c_cx_c$ the solution approaches the stationary path along a horizontal line (Fig. 1). Thus Γ_1 is stable in the sense that if the state is off this path it approaches it. At the moment the solution hits Γ_1 there is a downward jump in the supply of the agent that had previously been extracting, and an upward jump in the supply of the other agent. However, the total supply and price are continuous.

The slope of any trajectory in x_f,x_c state space equals q_f/q_c. The fringe market share is $q_f/(q_f+q_c)\in[0,1]$, which is an increasing function of c_c/c_f. Along Γ_1, the fringe's market share equals $c_c/(c_c+c_f)$. A permanent decrease in the fringe costs caused by an increase in c_f, would lead to a reduction in its steady state market share. This is because along Γ_1 unit extraction costs are constant, so the increase in c_f must lead to a decrease in the fringe's steady state share of resources. In the steady state, market shares equal the share of resources.

3 The Open Loop Cartel–Fringe Equilibrium

We now assume that the cartel, at time 0, is able to announce an extraction trajectory. The competitive fringe behaves as a price–taker with rational expectations (perfect foresight). Its Hamiltonian is $H_f=(p-\bar{p}+c_fx_f-\lambda)q_f$, where λ is the fringe's rent. The necessary conditions include

$$-q_f-q_c+c_fx_f-\lambda \begin{Bmatrix} =0 \Rightarrow q_f\geq 0 \\ <0 \Rightarrow q_f=0 \end{Bmatrix}, \quad \dot{\lambda}=\delta\lambda-c_fq_f. \tag{3.1a, b}$$

To obtain equation (3.1a) we used $p=\bar{p}-q_f-q_c$. The cartel chooses an extraction path to maximize the present discounted value of its profits, $\int_0^\infty e^{-\delta t}[p-(\bar{p}-c_c x_c)q_c]dt$, subject to (3.1a,b), and the resource constraints. Thus the cartel regards the follower's rent as a state variable with a free initial condition. The complementary slackness relations in (3.1a) comprise three constraints. We could form the Lagrangian using these constraints. However, this leads to a control problem for which the constraint qualification (Seierstad and Sydsæter 1987, p. 278) does not hold at all admissible solutions, and the standard necessary conditions cannot be applied directly.

To avoid this problem we study the necessary conditions in the three possible regimes: (a) $q_f>0$, $q_c=0$; (b) $q_f>0$, $q_c>0$; and (c) $q_f=0$, $q_c>0$. When needed we denote these regimes by the superscripts a,b or c respectively. We first study regime (b), and obtain an explicit solution using standard methods. Next we consider the strategy that switches from regime (a) to (b) and stays in (b) forever. We can again use standard methods, since $q_f>0$ along the entire solution. Next, we formulate a control problem which determines regime (c) and an optimal switch from (c) to (b). Our procedure allows jumps in the costates and resource price at the entrance to regime (b). However, they turn out to be continuous. Finally, we show that depending on the initial resource levels, all other regime switches can be ruled out, and one of the above strategies constitute the equilibrium.

3A. Regime (b)

In Regime (b) $q_f=c_f x_f-\lambda-q_c$ by (3.1a), which after defining the switching function $\sigma\equiv x_c c_c-c_f x_f+c_f\rho+\lambda+\eta_f-\eta_c$ implies that the cartel's Hamiltonian[2] is

$$H_c^{ab}=\sigma q_c-\eta_f(c_f x_f-\lambda)+\rho\left[\lambda(\delta+c_f)-c_f^2 x_f\right]. \tag{3.2}$$

The costates η_c, η_f, and ρ are associated with the states x_c, x_f, and λ, respectively. In regime (b), the leader chooses the rate of extraction to maintain $\sigma=0$. In addition, necessary conditions include

$$\dot{\eta}_c=-c_c q_c+\delta\eta_c,\quad \dot{\eta}_f=c_f q_c+c_f\eta_f+c_f^2\rho+\delta\eta_f,\quad \dot{\rho}=-q_c-\eta_f-c_f\rho. \tag{3.3a--c}$$

The equation $\dot{\sigma}=0$, (3.2a–c), and $\dot{x}_i=-q_i$, imply that $\lambda=\eta_c-\eta_f$. Substituting this into $\sigma=0$ yields

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[2]We will denote this Hamiltonian by H^{ab} because it has the same form in regime (a).

$$c_m x_c - c_f x_f + c_f \rho = 0. \tag{3.4}$$

Because $\rho(0)=0$ is necessary for optimality, we obtain:

Remark 3.1. The open loop equilibrium begins in regime (b) only if the initial state is on the socially optimal stationary arc, i.e., $x_{c0}=x_{f0}c_f/c_c$.

In order to obtain an expression for q_c, we differentiate (3.4) with respect to time, use (3.3c) and the relation $\lambda=\eta_c-\eta_f$ to obtain $q_c=\gamma(c_c x_c-\eta_c)$; $\gamma\equiv c_f/(2c_f+c_c)$. Equations (3.3a) and $\dot{x}_c=-q_c$, where q_c is given above, comprise a pair of linear differential equations, which can be solved once we have boundary conditions. Because in regime (b) the Maximized Hamiltonian is linear in (x_c,x_f,λ), a solution that remains in regime (b) forever satisfies sufficient conditions for optimality. Thus there cannot exist solutions yielding higher profit for the cartel. Solving the system for $\dot{x}_c$ and $\dot{\eta}_c$ yields two roots with opposite sign. To find the solution that maintains $q_c>0$ and $x_c>0$ for $\forall t\in[T,\infty)$, we choose the stable root, which equals $r=\frac{1}{2}[\delta-(\delta^2+4\gamma\delta c_c)^{\frac{1}{2}}]<0$. By T we denote the moment when the trajectory enters regime (b). The stable path can be written in terms of the (unknown) values of x_c x_f at the moment system enters regime (b), which we denote as x_{c_T} and x_{f_T} respectively. This yields

$$x_c=x_{c_T}e^{r(t-T)},\quad \eta_c=(r/\gamma+c_c)x_{c_T}e^{r(t-T)},\quad q_c=-rx_{c_T}e^{r(t-T)}. \tag{3.5a–c}$$

Next (3.1a,b), $\dot{x}_f=-q_f$ and (3.5c) form a system in λ, x_f and t. The solution which maintains $q_f>0$ and $x_f>0$, is

$$x_f=(x_{f_T}-x_{c_T}c_c/2c_f)e^{v(t-T)}+x_{c_T}c_c e^{r(t-T)}/2c_f, \tag{3.6a}$$

$$\lambda=\dot{x}_f+x_{c_T}c_f r^2(2c_f+c_c)e^{r(t-T)}/2c_f^2\delta. \tag{3.6b}$$

where $v=\frac{1}{2}[\delta-(\delta^2+4\delta c_f)^{\frac{1}{2}}]<0$. In regime (b) q_c declines monotonically toward zero (3.5c). Fringe extraction decreases monotonically toward zero if $x_{f_T}>x_{c_T}(v^2-r^2)c_c/v^2 2c_f$ (note that $v<r<0$). This implies that fringe extraction is initially increasing only if the switch to regime (b) occurs when x_f/x_c is "low"; e.g. q_f is monotonically decreasing if the initial state in regime (b) is on or above Γ_1.

Using (3.5a) gives $(t-T)=\ln(x_c/x_{c_T})/r$. Next we can eliminate $(t-T)$ from (3.6a) and write x_f as a function x_c:

$$x_f=(x_{f_T}-x_{c_T}c_c/2c_f)(x_c/x_{c_T})^{v/r}+c_cx_c/2c_f. \tag{3.7}$$

Equation (3.7) specifies regime (b) in the x_c–x_f phase plane (Fig.1, path 1, and the nonlinear segments of paths 2–5).[3] The properties of the path depend crucially on whether $x_{f_T}-x_{c_T}c_c/2c_f\lesseqgtr 0$. When $x_{f_T}=x_{c_T}c_c/2c_f$ the path is linear. We denote the linear trajectory by Γ_2. Because $v<r<0$, $x_{f_T}>x_{c_T}c_c/2c_f$ ($<$) implies that the path is convex (concave). Because $\partial x_f/\partial x_c\big|_{x_c=0}=c_c/2c_f$, the paths converge toward the linear path independently of x_{c_T} and x_{f_T}. If fringe extraction is increasing in the beginning of regime (b), fringe market share is increasing implying that the path in x_c–x_f state space is concave. This case cannot be ruled out by studying regime (b) alone because x_{f_T} and x_{c_T} are determined by the switch to regime (b) from some other regime. However, using numerical simulations we have found that the switch to regime (b) always occur above Γ_3 which means that cartel's market share is always decreasing in regime (b).

Along the linear trajectory $q_f/q_c=c_c/2c_f$, i.e the market shares of the cartel and the fringe are constant. Because the market share in all other trajectories converge toward this market share, the linear trajectory can be designated an open–loop Stackelberg turnpike. Recall that along Γ_1 the fringe market share equals $c_c/(c_c+c_f)$, while along Γ_2 it is less and equals $c_c/(c_c+2c_f)$.

Given that the initial state lies on the socially optimal stationary path, there exist an equilibrium candidate in regime (b) for $\forall t\in[0,\infty)$ (Remark 3.1). Along such a path, the equilibrium x_f is a convex function of x_c (see Fig. 2, path 1) and by (3.4) $\rho<0$ for $\forall t\in(0,\infty)$. The market share of the cartel is first below but later above the socially optimal steady state market share. Because the OLSE path lies below Γ_1 it follows that given any level of x_f the cartel resource stock is higher in the OLSE than on the socially optimal stationary path.

The fact that $\rho\neq 0$ for $\forall t>0$ shows that this solution is time inconsistent. The costate variable ρ can be interpreted as the cartel's shadow price for the fringe's rent. The fact that ρ is negative below Γ_1 shows that the cartel would increase its profits if it were able to alter its supply trajectory

[3]Fig. 1 and all other figures except when stated otherwise, are computed assuming $\delta=1/20$, $c_c=c_f=1/2$.

in a way which decreases the fringe rent. Such a change requires the cartel to tilt its supply toward the present, i.e. to behave less conservatively. Thus, along regime (b) the cartel promises to follow a conservative supply policy in the future, in order to induce the fringe to save its resources. As a consequence, the fringe is more conservative in the beginning and the initial price is higher. In this sense, the cartel induces the fringe to "cooperate" with current supply restriction.

Above we have presented an explicit solution candidate to the cartel's open loop problem for $\forall t \in [0,\infty)$ given the initial state lies on the socially optimal singular path. After specifying the necessary conditions for regimes (a) and (c) we show that this is in fact the only equilibrium candidate with these initial states. We next consider cases where the initial state lies above the socially optimal stationary path.

<u>3B: Regime (a) and the switch (a)→(b)</u>

We postulate that when the initial state is above Γ_1, i.e., when the fringe has a cost advantage over the cartel, the cartel will not supply in the beginning. In regime (a) $q_c=0$ and the fringe supply equals $q_f=c_f x_f-\lambda>0$, implying that restrictions (3.1a,b) are met along a strategy (a)→(b).[4] The necessary conditions constitute a set of equations which can be solved together with the boundary conditions as second order ordinary differential equations (Appendix 3.1).

Equations (3.1a,b) and $q_c=0$ imply that $\dot{q}_f=-\delta\lambda$, i.e., in regime (a) the fringe supply decreases. In both regimes total supply is $c_f x_f-\lambda$ by (3.1a). By the continuity of x_f and λ this implies that total extraction and price are continuous when the solution switches from regime (a) to (b). However, when the cartel starts to supply, the fringe supply must jump downwards. Fig. 2 shows two such paths (2 and 3) in x_c–x_f space. The kink at the switching state shows that there is a discontinuous decrease in the fringe's market share. Note that because the switches occur above Γ_1, there is a set of states above Γ_1 where market power increases cartel's market share. This is in contrast to static models, where market power decreases the dominant firms market share.

[4]Note that along this strategy the restriction $(-q_f-q_c+c_f x_f-\lambda)q_f=0$ is always satisfied. Thus we have an ordinary control problem.

In Fig. 3 the dotted lines demonstrate the switch in extraction and price time paths. As shown analytically price path is monotonically increasing and continuous. At the switching moment ($T \simeq 2.95$) the decrease in fringe's supply just equals the upward jump in the cartel's supply.

The sign of ρ tells whether OLSE can be characterized as a threat or as a promise. When the switch occur above Γ_1, as in Fig. 2, $\rho>0$ and the cartel has an incentive to change its supply in a way that would increase the fringe's resource rent. Such a change requires shifting supply to later dates, i.e. behaving more conservatively. When announcing its strategy, the cartel threatens to start extracting early, and rapidly. This induces the fringe to sell its stock more quickly along regime (a) because it anticipates that the price will be rather low in the future. However, when the fringe has sold part of its stock, the cartel would then like to be more conservative than originally announced. This incentive changes when the cartel obtains a cost advance. Below Γ_1, $\rho<0$, so the cartel would like to revise its original plan by extracting more quickly.

In regime (b) and (a) $\sigma=0$ or $q_c=0$ and the maximized Hamiltonian is linear in the state variables (see 3.2). This implies

Remark 3.2. (i) By Arrow's theorem (Seierstad and Sydsæter p. 236) the necessary conditions are sufficient for a global maximum. (ii) The value function in regime (a) or (b), which we denote as $J_c^{ab}(T,x_c,x_f,\lambda)$, *is differentiable, and its partial derivatives with respect to the states and T equal the corresponding costate variables and the Hamiltonian* $H_c^{ab}(0)$, *respectively (Seierstad and Sydsæter 1987, theorem 9, p. 213).*

We use Remark 3.2 when we study the entrance to regime (b) from (c).

3B. Regime (c) and the switch (c)→(b)

We next specify the cartel's optimal strategy for cases where the initial state is below the socially optimal stationary path. In these cases the cartel initially has a cost advantage over the fringe, and we hypothesize that initially the cartel is the only supplier. Consider the following form of the cartel's optimization problem:

$$\max_{q_f, q_c, T, x_c(T)} J_c^c = \int_0^T [(\overline{p}-q_f-q_c)q_c - q_c(\overline{p}-c_c x_c)] e^{-\delta t} dt + J_c^{ab}[T, x_c(T), x_f(T), \lambda(T)]$$

$$\text{s.t.} \quad \dot{x}_c = -q_c,\ x_c(0)=x_{c0},\ \dot{x}_f = -q_f,\ x_f(0)=x_{f0},\ \dot{\lambda} = \delta\lambda - c_f q_f,\ c_f x_f - \lambda - q_c - q_f \le 0,\ q_c \ge 0,\ q_f \ge 0.$$

This problem does not include the constraint $q_f[c_f x_f - \lambda - q_c - q_f] = 0$. However, any solution with the property $q_f = 0$ satisfies this constraint, and must thus be an optimal solution for the full problem. The necessary conditions lead to second and first order differential equations and their solutions and the boundary conditions are given in Appendix 3.2. Using these conditions together with the conditions for regimes (a) and (b) we can now prove the following:

Proposition 3.1. When $x_{c0}c_c - x_{f0}c_f > 0$ $[<0]$ *the cartel's optimal open loop strategy is (a)→(b) [(c)→(b)]. If* $x_{c0}c_c - x_{f0}c_f = 0$ *it is optimal to maintain regime (b) forever.* Proof, Appendix 3.1.

The proof shows that all strategies other than than those given by Proposition 3.1 contradict necessary conditions for optimality. Among other things we have shown that both agents exhaust their stocks. Newbery (1981) adopted this conclusion as a "Principle of Exhaustion" postulate. One possible strategy for the leader is to announce that it will not exhaust its stock. This would cause price to be higher in the future and discourage fringe production at present. However, by Proposition 3.1, such a promise is not optimal.

We can now study regime (c) knowing that the switch eventually occurs to (b). We define a limit pricing strategy as one in which the cartel supplies just enough so that the fringe is indifferent between producing and staying out of the market, i.e. $c_f x_f - \lambda = q_c$.

Proposition 3.2. During an interval before the switch (c)→(b) the cartel applies a limit pricing strategy. Proof, Appendix 3.2.

We next consider the case where the cartel applies limit pricing throughout regime (c). The cartel's extraction equals $q_c = c_f x_f - \lambda_0 e^{\delta t}$, where λ_0 is the fringe's initial rent. Thus, the cartel's extraction decreases exponentially, with $\ddot{q}_c < 0$. Note that in regime (b), total resource supply equals $c_f x_f - \lambda$, i.e., the cartel supply in regime (c). Thus, the total supply and the price are continuous, but the cartel supply jumps downwards at the switching moment.

Fig. 2 presents two examples of strategy (c)→(b) in x_f–x_c state space [paths (4) and (5)]. Note that when the initial cartel stock x_{c0}^5 lies on the open loop Stackelberg turnpike, Γ_2, the equilibrium does not start in regime (b) (ref. remark 3.1), but instead in regime (c). However, it later switches (path 5) to regime (b) and then converges toward the turnpike. Fig. 4 shows the resource supply as a function of time (dotted lines). At the switching moment ($T \simeq 7.1$), cartel supply jumps downwards but the total supply (not shown) and price are continuous.

Along the strategy (c)→(b), the cartel's costate for the fringe rent is negative. Thus, the cartel would like to increase its supply from the level originally announced. In regime (c) the cartel promises to be conservative in the future. This gives the fringe an incentive to save its resources to gain from the high future price and this allows the cartel to obtain high profits early in the program. However, since the fringe begins to extract below Γ_1, where it still has a cost disadvantage, the excertise of market power results in a decrease of cartel's market power early in the program. This is what happens in static models.

Above we considered the case where the constraint $c_f x_f - \lambda - q_f - q_c \leq 0$ is binding throughout regime (c), i.e. the cartel always applies limit pricing. When the solution is constructed under this assumption, the requirement $\mu \leq 0$ is violated for sufficiently large x_{c0}. This implies that regime (c) must start with a period where the constraint is slack and $\mu = \rho = 0$. This is the only time interval during which the cartel's supply plan is time consistent. Along this subregime we have $q_c = \frac{1}{2}(c_c x_c - \eta_c)$ and $c_f x_f - \lambda - q_c \leq 0$. The other equations for determing this regime are $\dot{x}_c = -q_c$, $\dot{\eta}_c = -q_c c_c + \delta \eta_c$ and $\dot{\lambda} = \delta \lambda$ (see Appendix 3.2). To piece this solution together with the regime where $\mu < 0$ we need four boundary conditions to determine the four unknowns, $\hat{T}$, $x_c(\hat{T})$, $\eta_c(0)$ and $\lambda(0)$, where $\hat{T}$ is the moment the cartel begins to apply the limit price. These conditions are: $x_c(0) = x_{c0}$, and continuity of x_c, η_c and λ. At $t = \hat{T}$ the constraint becomes binding, i.e., $c_f x_f - \lambda(\hat{T}) = q_c(\hat{T}) = \frac{1}{2}[c_c x_c(\hat{T}) - \eta_c(\hat{T})]$ implying that cartel's supply and thus also the resource price must be continuous.

The implications of our model are quite different than those of Newbery (1981) and Groot et al. (1991) where extraction unit costs are constant. Their formulation implies that agents never extract simultaneously. With increasing extraction costs, on the other hand, after the agent with the cost advantage has extracted its most economical stocks, both supply simultaneously. Newbery shows that if the cartel enjoys a substantial cost advance the equilibrium is time consistent. A similar circumstance occurs in our model. Here, however, the cost advantage must eventually disappear. Thus although a portion of the OLSE can be dynamically consistent, the cartel's supply plan must eventually become dynamically inconsistent. A third substantial difference is that price is continuous in our model, whereas discontinuities typically arise with constant costs. These discontinuities occur when the there is a switch from a regime with the fringe supplying, to a regime with the cartel supplying at the monopoly price (Groot et al. 1991). In our model this type of regime switch is ruled out, and with it also price discontinuities.

4 Markov perfect Stackelberg equilibrium

We restrict attention to linear equilibria. Tsutsui and Mino (1990) show that there exists a continuum of non–linear equilibria in linear–quadratic Nash games, and this is also true for Stackelberg games (Karp 1995). However, that result is driven by an "incomplete transversality condition", or the lack of a "natural boundary condition". In our model, the requirement that the states approach 0 in equilibrium imposes a natural boundary condition. Therefore we know that non–linear equilibria cannot arise for the reasons identified by Tsutui and Mino. There are likely to be non–Markov (reputational) equilibria in this short of game (Thomas 1992).

Our procedure for analyzing MPSE parallels that of the previous section. We first characterize the equilibrium in which both agents produce, and then consider the entrance to that regime.

4.1 Regime (b): $q_c>0$, $g_f>0$.

In regime (b) we construct equilibria where both players' supply is a linear function of the state variables. Thus, we postulate $q_f=\mu_1 x_f+\mu_2 x_c$. Using the conditions (3.1a,b) in the fringe's problem, differentiating (3.1a) with respect to time and eliminating λ and $\dot{\lambda}$, we obtain

$$\mu_1(\mu_1 x_f+\mu_2 x_c)+q_c(\mu_2+\delta)-\dot{q}_c+\delta\mu_1 x_f+\delta\mu_2 x_c-\delta c_f x_f=0. \qquad (4.1)$$

To proceed we must determine the feedback control for the leader, i.e. q_c as function of x_c and x_f. When the cartel takes the fringe resource consumption as a function of the states it solves:

$$\max_{q_c \geq 0} \quad V_c^b = \int_0^{\infty} [(\bar{p} - \mu_1 x_f - \mu_2 x_c - q_c) q_c - q_c(\bar{p} - c_c x_c)] e^{-\delta t} dt \qquad (4.2)$$
$$\text{s.t} \quad \dot{x}_c = -q_c,\ x_c(0) = x_{c0},\ \dot{x}_f = -\mu_1 x_f - \mu_2 x_c,\ x_f(0) = x_{f0}.$$

The task is to find a pair (μ_1, μ_2) such that the solution to problem (4.2) results in a control rule that satisfies equation (4.1). Applying the Maximum Principle gives the Modified Hamiltonian Dynamic System (MHDS): $\dot{x}_m = \frac{1}{2}[x_c(\mu_2 - c_c) + x_f\mu_1 + \eta_c]$, $\dot{x}_f = -\mu_1 x_f - \mu_2 x_c$, $\dot{\eta}_c = q_c(\mu_2 - c_c) + \eta_f \mu_2 + \delta\eta_c$, $\dot{\eta}_f = \mu_1 q_c + \eta_f(\mu_1 + \delta)$, where $q_c = \frac{1}{2}(c_c x_c - \eta_c - \mu_1 x_f - \mu_2 x_c)$ and η_c and η_f are the leaders costates for x_c and x_f respectively. The Jacobian matrix of the MHDS is:

$$\begin{bmatrix} \frac{1}{2}(\mu_2 - c_c) & \frac{1}{2}\mu_1 & \frac{1}{2} & 0 \\ -\mu_2 & -\mu_1 & 0 & 0 \\ -\frac{1}{2}(\mu_2 - c_c)^2 & -\frac{1}{2}\mu_1(\mu_2 - c_c) & -\frac{1}{2}(\mu_2 - c_c) + \delta & \mu_2 \\ \frac{1}{2}(c_c - \mu_2)\mu_1 & -\frac{1}{2}\mu_1^2 & -\frac{1}{2}\mu_1 & \mu_1 + \delta \end{bmatrix}.$$

According to theorem 1 by Dockner (1985), the four characteristic roots of this system equal $r_{1,2,3,4} = \frac{1}{2}\delta \pm [(\frac{1}{2}\delta)^2 - \frac{1}{2}\Omega \pm \frac{1}{2}(\Omega^2 - 4\Delta)^{\frac{1}{2}}]^{\frac{1}{2}}$, where $\Delta = \frac{1}{4}c_c\delta\mu_1(\delta + \mu_1)$ (the determinant of the MHDS) and $\Omega = -\frac{1}{2}[c_c\delta + \delta(2\mu_1 - \mu_2) + 2\mu_1(\mu_1 - \mu_2)]$.[5] We search for a saddle point stable equilibrium, which requires that two of the characteristic roots have positive and two have negative real parts. A sufficient condition for this is that $\Delta > 0$ and $\Omega < 0$ (Tahvonen 1989). To obtain the negative roots we choose $r_1 = \frac{1}{2}\delta - [(\frac{1}{2}\delta)^2 - \frac{1}{2}\Omega + \frac{1}{2}(\Omega^2 - 4\Delta)^{\frac{1}{2}}]^{\frac{1}{2}}$ and $r_2 = \frac{1}{2}\delta - [(\frac{1}{2}\delta)^2 - \frac{1}{2}\Omega - \frac{1}{2}(\Omega^2 - 4\Delta)^{\frac{1}{2}}]^{\frac{1}{2}}$, implying that the real part of r_1 is smaller than that of r_2. If $\Omega^2 - 4\Delta \geq 0$ (<0) the roots are real (complex). Without solving μ_1 and μ_2 explicitly, it is difficult to rule out the complex root case. However, because we have not been able to find any numerical example leading to complex roots, we restrict the analysis to the real root case

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[5] $\Omega \equiv \begin{vmatrix} \partial\dot{x}_c/\partial x_c & \partial\dot{x}_c/\partial\eta_c \\ \partial\dot{\eta}_c/\partial x_c & \partial\dot{\eta}_c/\partial\eta_c \end{vmatrix} + \begin{vmatrix} \partial\dot{x}_f/\partial x_f & \partial\dot{x}_f/\partial\eta_f \\ \partial\dot{\eta}_f/\partial x_f & \partial\dot{\eta}_f/\partial\eta_f \end{vmatrix} + 2\begin{vmatrix} \partial\dot{x}_c/\partial x_f & \partial\dot{x}_c/\partial\eta_f \\ \partial\dot{\eta}_c/\partial x_f & \partial\dot{\eta}_c/\partial\eta_f \end{vmatrix}.$

only. Along such a saddle point path the resource stocks and the leader's control are given by:

$$x_c(t)=e^{r_1(t-T)}[x_{c_T}(\mu_1\mu_2+r_1\mu_2)+x_{f_T}(\mu_1+r_1)(\mu_1+r_2)]/\mu_2(r_1-r_2)+$$
$$e^{r_2(t-T)}[x_{c_T}\mu_2(\mu_1+r_2)+x_{f_T}(\mu_1+r_1)(\mu_1+r_2)]/\mu_2(r_2-r_1), \quad (4.3)$$

$$x_f(t)=e^{r_1(t-T)}[x_{c_T}\mu_2+x_{f_T}(\mu_1+r_2)]/(r_2-r_1) + e^{r_2(t-T)}[x_{c_T}\mu_2+x_{f_T}(\mu_1+r_1)]/(r_1-r_2), \quad (4.4)$$

$$q_c(x_c,x_f)=-x_c(\mu_1+r_1+r_2) - x_f[\mu_1^2+\mu_1(r_1+r_2)+r_1r_2]/\mu_2. \quad (4.5)$$

We use (4.3)–(4.5) to write (4.1) as a linear equation in x_f and x_c. Equating coefficients of these variables to zero gives:

$$\mu_2(c_f\delta+r_1r_2)+(\delta-r_1-r_2)(\mu_1^2-\mu_1\mu_2+\mu_1r_1+\mu_1r_2+r_1r_2)=0, \quad (4.6)$$

$$(r_1+r_2-\delta)(\mu_1-\mu_2+r_1+r_2)-r_1r_2=0. \quad (4.7)$$

Because r_1 and r_2 are solutions for a polynomial of degree four, it is difficult to obtain explicit solutions for μ_1 and μ_2. However, note that in x_f, x_c state space the equilibrium satisfies $\dot{x}_f=-\mu_1x_f-\mu_2x_c$, $\dot{x}_c=x_f(\mu_1+r_1)(\mu_1+r_2)/\mu_2+x_c(\mu_1+r_1+r_2)$. Using this, (4.6) and (4.7) we obtain:

Lema 4.1. Given that a linear equilibrium exists, both agents' supply is increasing in his own stock levels and decreasing in the other agents' stock level, i.e., $\mu_1>0$, $\mu_2<0$, $\mu_1+r_1+r_2<0$ *and* $(\mu_1+r_1)(\mu_1+r_2)<0$. Proof, Appendix 4.1.

Lema 4.1, which is obtained without solving μ_1 and μ_2 explicitly, is intuitively appealing. The lema implies the phase diagram in Fig. 5. Note that $r_1<0, r_2<0$, $\mu_1>0$ and $\mu_2<0$ imply that the slope of $\dot{x}_c=0$ is greater than the slope of $\dot{x}_f=0$. The equilibrium paths define an interior equilibrium, given initial states between the isoclines. The trajectories approach the origin asymptotically.

Proposition 4.1. Between the isoclines there exists a separatrix or MPSE *turnpike, with the slope* $-\mu_2/(\mu_1+r_2)$. *All paths with initial state off the turnpike converge toward it as* $t\to\infty$. *An initial state above (below) the turnpike implies that the cartel market share is increasing (decreasing).*

Proof: For a system of two linear differential equations there must exist a separatrix between the isoclines along which the path in the state space is linear. This implies in (4.3) or (4.4) that either the coefficent for $e^{r_1(t-T)}$ or $e^{r_2(t-T)}$ must be zero. The former implies $x_f=-\mu_2x_c/(\mu_1+r_2)$, and the latter $x_f = -\mu_2 x_c/(\mu_1 + r_1)$. By lema 4.1, $(\mu_1+r_1)(\mu_1+r_2)<0$. Because $r_1<r_2$, the separatrix with positive slope is $\Gamma_3(x_f) \equiv -\mu_2 x_c/(\mu_1+r_2)$. All paths off the turnpike converge toward it when $t \to \infty$ because $r_1<r_2<0$. The convergence implies that the slope of paths with initial state above (below) the turnpike must decrease (increase). This implies that in the former case the cartel's market share increases, while it decreases in the latter.■

We next consider the time development of total extraction and price along regime (b).

Proposition 4.2. Along regime (b) in a MPSE, *the total extraction is* a *(monotonically) decreasing and resource price is* a *(monotonically) increasing function of time. When* x_{c0}/x_{f0} *is sufficiently low* [*high*], $q_c(t)$ [$q_f(t)$] *is initially increasing and has a unique maximum.* Proof, appendix 4.2.

Thus if the initial state is near the $\dot{x}_c=0$ isocline the cartel's extraction is initially increasing in regime (b). Recall that in regime (b) of the OLSE, cartel extraction monotonically decreases. This case is demonstrated in Fig. 3 (solid lines). The case where fringe extraction is initially increasing is shown in Fig. 4. We now turn to cases where the initial state of the game is not between the isoclines, i.e., it is outside the region where the linear MPSE exists.

4.2 Regime (c) and the switch (c)→(b)

Consider initial states below the $\dot{x}_f=0$ isocline ($x_{f0}<-\mu_2 x_{c0}/\mu_1$). We postulate that in these cases there exists an equilibrium where in the beginning the cartel is the only producer. This equilibrium must satisfy the necessary conditions for problem (4.2) when $\mu_1=\mu_2=0$. The necessary conditions include: $q_c=\frac{1}{2}(c_c x_c - \eta_c)$, $\dot{\eta}_c = -c_c q_c + \delta\eta_c$, and that the Hamiltonian is continuous. Denoting the switching moment by T and taking into account that $q_f^b(T)=0$, the continuity condition implies that $q_c^c(T)^2 = q_c^b(T)^2$, i.e., that cartel extraction must be continuous. This implies continuity of cartel resource rent and the producer price as well. The other necessary conditions imply $\dot{q}_c = -\delta\eta_c/2<0$, i.e., in regime (c) cartel extraction is a decreasing and producer price an increasing function of time. Solving the MHDS of the cartel problem and using conditions $x_c(0)=x_{c0}$, $x_c(T)=-\mu_1 x_{f0}/\mu_2$ and the continuity of η_c, it is possible to compute the length of this regime.

This equilibrium must satisfy the fringe's necessary conditions (3.1a,b) with $q_f=0$. At the $\dot{x}_f=0$ isocline, $\alpha \equiv -q_f-q_c+c_f x_{f0}-\lambda=0$. Because q_c and q_f are continuous functions of time, α must also be continuous. We obtain $\dot{\alpha}=-\dot{q}_f-\dot{q}_c-\delta\lambda=\frac{1}{2}\delta\eta_c-\delta\lambda-\dot{q}_f$ and $\ddot{\alpha}=\delta(\frac{1}{2}\delta\eta_c-\delta\lambda)-\frac{1}{2}\delta c_c q_c=\dot{\alpha}+\dot{q}_f-\frac{1}{2}\delta c_c q_c$. For $q_f=0$ before T, (3.1a,b) imply that $\alpha \lesssim 0$. Suppose that our candidate does not satisfy (3.1a,b). In this case we must have $\alpha>0$ and $\dot{\alpha}<0$ before T. Consider two cases: (i) where $\dot{\alpha}$ is continuous at T and (ii) where $\dot{\alpha}$ is discontinuous at T. Case (i) and the hypothesis $\alpha>T$ for $t<T$ imply that α is convex before T. However, before T, $\ddot{\alpha}=\dot{\alpha}+\dot{q}_f<0$, $\Rightarrow\Leftarrow$ In case (ii) there must be an upward jump in $\dot{\alpha}$ at T. However, $\dot{\alpha}(T^-)=\frac{1}{2}\delta\eta\dagger(T)-\delta\lambda(T)>\frac{1}{2}\delta\eta_c(T)-\delta\lambda(T)-\dot{q}_f(T^+)=\dot{\alpha}(T^+)$, $\Rightarrow\Leftarrow$. Consequently, $q_f=0$ $\forall t<T$ satisfies the fringe's necessary conditions as well.

4.3 Regime (a) and the switch (a)→(b)

We postulate that regime (a) is the first regime when the initial state lies above the $\dot{x}_c=0$ isocline. This equilibrium must satisfy the necessary conditions for problem (4.2) when $q_c=0$, given that q_f is some function of the state variables. Denote this function by $q_f(x_f,x_c)$. For $q_c=0$ to be an optimal solution for problem (4.2), it is then necessary that $-q_f(x_f,x_c)+c_c x_c-\eta_c \lesssim 0$.

In addition, the equilibrium must satisfy simultaneously the fringe's necessary conditions (3.1a,b) with $q_f>0$. To piece this regime together with regime (b) and to determine the length of regime (a) we have as the boundary conditions: $x_f(0)=x_{f0}$, $x_f(T)=-x_{c0}(\mu_1+r_1+r_2)\mu_2/(\mu_1+r_1)(\mu_1+r_2)$ and the requirement that the fringe's rent must be continuous. The continuity of fringe's rent and the fact that $q_c=0$ in the beginning of regime (b) (i.e., on the isocline $\dot{x}_m=0$) implies that the fringe's extraction rate as well as the resource price must be continuous. The necessary conditions imply that $\dot{q}_f=-\delta\lambda$, i.e., the fringe's extraction is a decreasing function of time along regime (a). Using our results from sections 4.1–4.2 we can describe the MPSE as follows:

Remark 4.2. In the MPSE *both the cartel's and fringe's supply are continuous functions of time, total extraction is a decreasing and price an increasing function of time.*

The length of regime (a) and the fringe's feedback rule cannot be explicitly solved. As a consequence it is not possible to evaluate analytically whether $-q_f(x_f,x_c)+c_c x_c-\psi_m \lesssim 0$ holds. However,

Appendix 4.3 shows how to verify this condition numerically.

4.4 Comparisons of different equilibria

This section compares the OLSE and the MPSE and then discusses the benefits to cartelization. Since the inability to commit reduces the cartel's power, the MPSE may be expected to lie "between" the OLSE and competitive equilibrium. This intuition is correct if we are interested in long run market share, but we show that, in general, it is misleading.

We showed that the three equilibria can be described using stationary paths, toward which all solutions converge as $t \to \infty$. Moreover, $\Gamma_1' > \Gamma_2'$, so the cartel's stationary market share in the OLSE is higher than in the competitive equilibrium. Fig. 5 gives an example where $\Gamma_1' > \Gamma_3' > \Gamma_2'$, so the cartel's MPSE stationary market share is indeed between the stationary market shares in the other two equilibria, as intuition suggests. The generality of this result is seen from Fig.6, which plots the stationary fringe market shares in the three equilibria as functions of relative costs. Only for the MPSE does this function depend on the discount rate, but the two solid graphs (for $\delta=1/20$ and $\delta=9$) show that the dependence is negligible, and does not alter the ranking.

The comparison of cartelization's immediate effect on price and market share is less straightforward. The most interesting case is where the market was in a long–run competitive equilibrium prior to cartelization, i.e., the state begins on $\Gamma 1$. Figure 7 graphs the initial price as a function of x_c, for $x_f = \Gamma_1 x_c$. If the initial stock is small, price is near the choke price under competition, and cartelization has a negligible effect on the market. For large stocks, and low competitive prices, cartelization leads to a large percentage increase in price, which is larger in the MPSE than in the OLSE. Figures 3 and 4 suggest that this comparison also holds for states off Γ_1. When the cartel begins with a cost disadvantage (Figure 3) the price is only slightly higher in the MPSE than in the OLSE, but it is much higher when the cartel has a cost advantage (Figure 4).

The short–run market share effect of cartelization depends on the initial cost advantage. The comparison of initial market shares, for states that begin on Γ_1, is the same for all values of x_c. This is because in both cases the initial output levels are linear functions of the state (see equations

3.5c and 3.6a). For our parameters $c_c/c_f=1$ and $\delta=1/20$, we find that the cartel's initial market share is highest in the competitive equilibrium and lowest in the OLSE.

The comparison of initial market shares is simpler for values of the state off Γ_1. We know that there the cartel's initial market share is either 1 or 0, in both the competitive equilibrium and the OLSE, depending on relative costs (Proposition 3.1). Provided that the initial condition is in the cone formed by the isoclines, the cartel's initial market share in the MPSE is strictly between 0 and 1. Therefore, the initial MPSE market share is less than the share in the competitive equilibrium and the OLSE when the cartel has a cost advantage, and is greater when it has a cost disadvantage (Figures 3 and 4). Recall that our numerical example shows that the cartel's MPSE steady state market share exceeds the competitive level ($\Gamma_1 > \Gamma_3$). Thus, we see that if the cartel begins with a cost disadvantage, cartelization increases market share in both the short and the long run (although not, of course, during intervening periods). This contrasts to static models, in which the exercise of market power typically decreases market share.

In order to explain why the comparison of market shares depends on the initial relative costs, it helps to consider how the inability to commit erodes the cartel's power. In the MPSE the cartel's only leverage comes via control over its own stock, since this affects fringe production. The cartel is unable to use either threats or promises about future behavior to influence the fringe. The effect (on market share) of it's loss of power depends on whether it would have used a threat or a promise in the OLSE, and that, as we saw, depends on the relative costs.

Consider first the case where the cartel begins with a cost disadvantage, so in the OLSE it uses threats of high sales in the future to induce the fringe to extract rapidly in the initial periods. The cartel's inability to use threats (in the MPSE) increases the fringe's rent, causing their initial extraction trajectory to be lower and price to be higher. The higher price makes it more attractive for the cartel to enter the market, so its initial extraction trajectory is higher, increasing it's market share in the MPSE. Now consider the case where the cartel begins with a cost advantage, so in the OLSE it uses promises of a low sales path in the future in order to encourage conservation by the fringe. Its inability to use promises in the MPSE decreases the fringe's rent, increasing fringe supply and lowering the price. This makes it less attractive for the cartel to supply in the current period, causing its extraction path to fall. In this circumstance (where the cartel would like to reduce fringe supply) the cartel has an additional incentive to restrict its own supply. Fringe supply is decreased by a large cartel stock, so the cartel has a strategic incentive to keep its stock relatively large.

The argument is slightly different if the initial condition is on Γ_1. There, neither firm has a cost advantage, and since $\rho(0)=0$ the OLSE is exactly balanced between being a threat or a promise. However, we know that for these initial conditions the system moves immediately into the region of state space where the OLSE is characterized as a promise. Therefore we can apply the intuition described above, to explain why the initial cartel market share is lower in the MPSE, given initial conditions on Γ_1.

These remarks also help to understand how the initial stock level affects the magnitude of the difference in the initial price. We saw that regardless of initial relative costs, one agent decreases and the other increases its initial sales, when we move from the OLSE to the MPSE. Our simulations showed that the decrease in production more than offsets the increase, making initial price higher in the MPSE. We also noted that the cartel has a strategic incentive to reduce sales when it has a cost advantage, so in that situation we expect the price increase to be especially large. This is consistent with the simulation results. When the cartel has a cost disadvantage, it would like the fringe to accelerate sales, and this makes the maintenance of a large stock less attractive for the cartel.

Finally, Fig. 8 shows how the incentives for cartelization change, under the Markov assumption. Each point represents the percentage increase in cartel profits when moving from the competitive equilibrium to the MPSE. On path (1), along the Markov turnpike Γ_3, the gains are approximately constant (1.3%). On path (2), where the cartel initially has a cost disadvantage, the gains are very small, but they increase as cartel market share increases (and the cost disadvantage decreases). On path (3), where the cartel initially has a cost advantage, the gains from cartelization are larger, but they decrease as the market share decreases. The clear implication is that the gains from cartelization, like the cartel market share, increase with the cartel's cost advantage.

5. Conclusions

We modeled a nonrenewable resource market with a cartel and fringe, using both an open–loop and a Markov perfect equilibrium, under the assumption that costs are stock dependent. In the competitive equilibrium only one firm extracts until their costs are equal. In the OLSE both the fringe and the cartel begin to extract while they still have a cost disadvantage. The stationary cartel market share exceeds the competitive level. In the MPSE, both the cartel and the fringe extract in the first instant, unless the cost disadvantage of one is very large. The initial cartel market share is higher in the MPSE, relative to either the competitive or the OLSE, if and only if the cartel has a cost disadvantage. Cartelization increases the initial market price, but surprisingly this increase is greater in the MPSE than in the OLSE. In this sense, a smaller degree of market power is associated with what appears to be less competitive behavior, and may result in a larger loss to consumers.

The magnitude of the benefits to cartelization (in a MPSE) are directly related to the magnitude of the cartel's cost advantage, and thus to its market share. This market share always approaches a stationary value, so whether the benefits of cartelization increase or decrease over time depend on whether the cartel's cost advantage – and market share – is increasing. Thus, we would expect that if there is a cost to forming cartels, potential cartels with cost advantages would be more likely to form. However, if there is a cost to maintaining cartels, our theory suggests that an initially powerful cartel may eventually fall apart, whereas an initially weak cartel may become more coherent.

Appendix 3.1. Time paths in regime (a) and the switch (a)→(b).

The cartel's Hamiltonian for regime (a) equals (3.2), and the hypothesis that $q_c=0$ requires $\sigma\leq 0$. In addition, necessary conditions include: $\dot{x}_f=-c_f x_f+\lambda$, $\dot{\lambda}=\delta\lambda-c_f^2 x_f+c_f\lambda$, $\dot{\eta}_f=\eta_f c_f+\rho c_f^2+\delta\eta_f$, $\dot{\rho}=-\eta_f-c_f\rho$, $\dot{\eta}_c=\delta\eta_c$ and $\rho(0)=0$. This yields:

$$x_f=(x_{f0}-A_1)e^{w_1 t}+A_1 e^{w_2 t},\quad \lambda=(c_f+w_1)(x_{f0}-A_1)e^{w_1 t}+(c_f+w_2)A_1 e^{w_2 t}, \qquad (3.1.1a,b)$$

$\rho=A_2(e^{w_1t}-e^{w_2t})$, $\eta_f=-(w_1+c_f)A_2e^{w_1t}+(w_2+c_f)A_2e^{w_2t}$, $\eta_c=\eta_{c0}e^{\delta t}$, (3.1.2a,b,c)

where $w_1=\frac{1}{2}(\delta-(\delta^2+4c_f\delta)^{\frac{1}{2}})<0$, $w_2=\delta-w_1>0$ and A_i is to be determined by the boundary conditions.

To find the optimal switch to regime (b) we need four conditions for the four unknowns $\lambda^a(0)$ (or A_1), $\eta_f^a(0)$ (or A_2), T and $x_f(T)$. A_1 is determined by the continuity of x_f, i.e., by $x_f^a(T)=x_f^b(T)$. The continuity of the state and costate variables imply that $\sigma(T)=\dot{\sigma}(T)=0$. By equations (3.4) and (3.1.2a) one has $\rho^a(T)=[c_fx_f(T)-c_cx_{c0}]/c_f$ which determines A_2. To find the optimal switching moment and $x_f(T)$ we have two conditions, the continuity of η_f and λ: $\eta_f^a(T)=\eta_c^a(T)-\lambda^b(T)$ [by $\sigma(T)=\dot{\sigma}(T)=0$] and $\lambda^a(T)=\lambda^b(T)$. The two last conditions determine two nonlinear equations for T and $x_f(T)$.

Appendix 3.2. Regime (c) and the switch (c)→(b).

The Lagrangian equals: $L=(\bar{p}-q_f-q_c)q_c-q_c(\bar{p}-c_cx_c)-\eta_cq_c-\eta_fq_f+\rho(\delta\lambda-c_fq_f)+\mu(c_fx_f-\lambda-q_c-q_f)$. For $q_f=0$, $q_c>0$ to be optimal it is necessary that

$$-2q_c+c_cx_c-\eta_c-\mu=0 \qquad (3.2.1)$$

$$-q_c-\eta_f-\rho c_f-\mu\leq 0, \qquad (3.2.2)$$

$$c_fx_f-\lambda-q_c\leq 0,\ \mu\leq 0,\ (c_fx_f-\lambda-q_c)\mu=0, \qquad (3.2.3a–c)$$

$$\dot{\eta}_c=-q_cc_c+\delta\eta_c, \qquad (3.2.4)$$

$$\dot{\eta}_f=-\mu c_f+\delta\eta_f, \qquad (3.2.5)$$

$$\dot{\rho}=\mu,\ \rho(0)=0, \qquad (3.2.6)$$

$$\eta_m^c(T)=\eta_c^b(T),\ n_f^c(T)=\eta_f^b(T),\ \rho^c(T)=\rho^b(T),\ H_c^c(T)=H^{ab}(T), \qquad (3.2.7a–d)$$

where $H_c^c=(\bar{p}-q_f-q_c)q_c-q_c(\bar{p}-c_cx_c)-\eta_cq_c-\eta_fq_f+\rho(\delta\lambda-c_fq_f)$.

Conditions (3.2.1), (3.2.4), and (3.2.6) yield $x_c=-c_fx_{f0}t+\lambda_0e^{\delta t}/\delta+A_3$, $\eta_c=A_4e^{\delta t}+c_cc_fx_{f0}/\delta+c_c\lambda_0te^{\delta t}$ and $\rho=2\lambda_0e^{\delta t}/\delta-2c_fx_{f0}t+c_c(\lambda_0e^{\delta t}/\delta^2-c_fx_{f0}t^2/2+V_3t)-V_4e^{\delta t}/\delta-(c_fx_{f0}c_ct/\delta-c_c\lambda_0e^{\delta t}(\delta t-1)/\delta^2)+V_5$. Note from (3.2.6) that along this solution ρ is decreasing and negative. Because in regime (b) $c_cx_c-c_fx_f+c_f\rho=0$, the switch must occur below Γ_1. To piece this solution together with regime (b), we need six conditions to determine six unknowns, namely T,V_1,V_2,V_3, λ_0, and $x_c(T)$. The conditions are: $x_c(0)=x_{c0}$, $x_c^a(T)=x_c^b(T)$, $\rho(0)=0$, $\eta_c^a(T)=\eta_c^b(T)$, $\lambda^a(T)=\lambda^b(T)$ $\rho^a(T)=[c_fx_{f0}-$

$c_c x_c(T)]/c_f$, where we have used (3.2.7a,b), the continuity of the state variables and the fact that $\sigma=\dot{\sigma}=0$ must hold in the beginning of regime (b). Because (3.2.7d) is equivalent to $\sigma(x_f c_f-\lambda)=0$, the switch satisfies the Hamiltonian continuity requirement.

Appendix 3.3. The proof for Proposition 3.1

Before proving the Proposition we begin with the following

Lema A3.3: Along regime (c) $\sigma\geq 0$.

Assume first that $\mu<0$. By (3.2.1) and (3.2.3a–c) $\mu=2\lambda-2c_f x_f+c_c x_c-\eta_c$. This yields by (3.2.2) that $c_f x_f+\eta_c-\lambda-\eta_f-\rho c_f-c_c x_c\leq 0$, i.e. $\sigma\geq 0$. Assume $\mu=0$. Then $q_c=\frac{1}{2}(c_c x_c-n_c)=c_c x_c-\eta c-q_c$. By (3.2.3a) $c_c x_c-c_f x_f+\lambda-\eta_c-q_c\geq 0$. Combining this with (3.2.2) implies that $\sigma\geq 0$.■

For proving proposition 3.1 we consider cases A and B, where the initial state is below and above Γ_1, respectively. For each of these cases, we show that all strategies other than those described in the Proposition violate a necessary condition for optimality. Let T denote the switching moment between two regimes.

Case A: $x_{c0}c_c-x_{f0}c_f\geq 0$.

A1 (a)→(b): In regime (a) $\sigma\leq 0$. If $\dot{\sigma}(0)=\lambda+\eta_f-\eta_f>0$ then $\sigma(0)=x_{c0}c_c-x_{f0}c_f+\lambda+\eta_f-\eta_c>0$, ⇒⇐. Consequently $\dot{\sigma}(0)\leq 0$. In this case there must exist t_1 such that $0\leq t_1<T$ and $\sigma(t_1)<0$, $\dot{\sigma}(t_1)=0$, $\ddot{\sigma}(t_1)>0$ for a regime (a) with a nonzero length to be possible. Using the necessary conditions for regime (a) this yields: $\sigma(t_1)=c_c x_{c0}-c_f x_f+\rho c_f<0$, $\dot{\sigma}(t_1)=\lambda+\eta_f-\eta_c=0$ and $\ddot{\sigma}(t_1)=\delta c_f(\rho c_f+\eta_f-q_f)>0$. $\sigma(t_1)<0$ requires that $\rho<0$ because $c_c x_{c0}-c_f x_f(t_1)>0$ (note that at t_1 the state must be below Γ_1). For $\ddot{\sigma}(t_1)>0$ to hold we must then have $\eta_f(t_1)>-\rho(t_1)c_f+q_f(t_1)>0$. At T it must hold in regime (a) that, $\sigma(T)=\dot{\sigma}(T)=0$ and $\ddot{\sigma}^a(T)<0$ because σ and $\dot{\sigma}$ are continuous. Thus the term $\eta_f+c_f\rho-q_f$ must change its sign from positive to negative along regime (a). Differentiation yields $\dot{\eta}_f+c_f\dot{\rho}-\dot{q}_f=\delta\eta_f-\dot{q}_f$. Recall that $\dot{q}_f<0$ in regime (a). Thus η_f must switch from positive to negative. Above we showed that $\eta_f(t_1)>0$ and $\eta_f(t_1)+\rho(t_1)c_f>0$, so $\dot{\eta}_f(t_1)=c_f[\eta_f(t_1)+\rho(t_1)c_f]+\eta_f(t_1)\delta>0$. Because $\dot{\eta}_f=\delta\eta_f>0$ we obtain that $\eta_f>0$ for $\forall t\in[t_1,T]$, ⇒⇐.

A2 (a)→(c)→any regime: At both switching moments $\sigma=0$, implying by lema A3.3 that along regime (c) there must be a moment of time, t_1, where $\sigma(t_1)>0$, $\dot{\sigma}(t_1)=0$ and $\ddot{\sigma}(t_1)<0$. Assume that $\mu(t_1)=0$. We obtain $\dot{\sigma}(t_1)=\lambda+\eta_f-\eta_c=0$ and $\ddot{\sigma}(t_1)=\delta\dot{\sigma}+\delta q_c c_c>0$, $\Rightarrow\Leftarrow$. Assume next that $\mu(t_1)<0$. In this case $\dot{\sigma}(t_1)=\lambda+\eta_f-\eta_f=0$ and $\ddot{\sigma}(t_1)=-\mu c_f+q_c c_c>0$, $\Rightarrow\Leftarrow$.

A3 (a)→(c). If $x_f>0$ for $\forall t$ then the transversality problem for the fringe problem requires $\lambda=0$ at the end of regime (a). It must then hold that $c_f x_f-q_c\leq 0$ $\forall t>T$ (conditions 3.2.3a–c). However, q_c must finally converge to zero, $\Rightarrow\Leftarrow$. If $x_f=0$ at the end of regime (a) then $q_f=-\lambda$ at the end of regime (a). Since both q_f and λ are nonnegative $x_f=\lambda=q_f=0$ at the end of regime (a). However, such a candidate contradicts the solution (3.1.1a,b), $\Rightarrow\Leftarrow$.

A4 (c)→(a)→(b): This implies that in regime (a) there must be moment of time $t_1<T$ (where T is the switching moment to b) such that $\ddot{\sigma}(t_1)>0$ and $\ddot{\sigma}(T)<0$, which was shown in part (A1) of the proof to contradict necessary conditions.

A5 (c)→(a): In this case $\sigma<0$ for $\forall t>T$. Because $\dot{\sigma}(T)\leq 0$ and $\rho(T)\leq 0$ (by 3.2.6) the switch cannot occur above Γ_1 implying that $x_c>0$ $\forall t$. By the transversality condition $\lim_{t\to\infty} e^{-\delta t}x_c\eta_c\geq 0$ we obtain that $\eta_c=0$ in the beginning of regime (a). As $t\to\infty$, $\sigma\to c_f\rho+c_c x_c+\eta_f$, implying that $c_f\rho+\eta_f$ must remain negative. In regime (a) $c_f\dot{\rho}+\dot{\eta}_f=\delta\eta_f$. If $\eta_f>0$, it must hold for $c_f\rho+\eta_f$ to remain negative, that $\eta_f\to 0$ as $t\to\infty$. However, in regime (a) $\dot{\eta}_f=c_f(\eta_f+\rho c_f)+\delta\eta_f$. It must thus hold that $\eta_f<0$. But then $\dot{\eta}_f=c_f(\eta_f+\rho c_f)+\delta\eta_f<\delta\eta_f<0$ and $\lim_{t\to\infty} e^{-\delta t}\eta_f(t)<0$, i.e. transversality condition is violated, $\Rightarrow\Leftarrow$.

Case B. $x_c c_c-x_f c_f\leq 0$.

B1 (c)→(b): At the switching moment $\rho\leq 0$ implying that $x_c c_c-x_f c_f+c_f\rho<0$, $\Rightarrow\Leftarrow$.

B2 (c)→(a)→(b): By lema 3.3 $\sigma(0)=x_c c_c-x_f c_f+\lambda+\eta_f-\eta_c\geq 0$. Thus $\dot{\sigma}(0)>0$. This implies that in regime (c) there must be $t_1<T$ [where T is the switch from (c) to (a)] such that $\sigma(t_1)>0$, $\dot{\sigma}(t_1)=0$, $\ddot{\sigma}(t_1)<0$. However, in A2 this is shown to contradict with optimalty.

B3 (c)→(a): At the switching moment $\sigma(T)=x_c c_c-x_f c_f+\rho c_f+\dot{\sigma}=0$ but $\dot{\sigma}<0$ (by lema A3.3), $\rho\leq 0$, $x_c c_c-x_f c_f<0$, $\Rightarrow\Leftarrow$.

B4 (a)→(c): Apply A3.

B5 (a)→(c)→any regime: Along regime (c) there must be a moment of time, say t_1 such that $\dot{\sigma}(t_1)=0$

and $\ddot{\sigma}(t_1)=\delta\dot{\sigma}-\mu c_f+q_c c_c<0$, $\Rightarrow\Leftarrow$.

We are left with the strategies: $x_{c0}c_c-x_{f0}c_f=0 \Rightarrow$(b) for $\forall t$, $x_{c0}c_c-x_{f0}c_f>0 \Rightarrow$ (c)$\rightarrow$(b) and $x_{c0}c_c-x_{f0}c_f<0$ $\Rightarrow$(a)$\rightarrow$(b).■

Appendix 3.4 Proof of Proposition 3.2

Assume the reverse, i.e., that $\mu(T)=0$. By (3.2.1) $q_c=\frac{1}{2}(x_c c_c-\eta_c)$ and by (3.2.7d) $-q_c^2+q_c(x_c cc-\eta_c)=\frac{1}{2}(x_c c_c-\eta_c)^2=-(n_f+\rho c_f)(x_f c_f-\lambda)$ at T. By (3.2.2) and (3.2.3a–c) this implies that (3.2.7d) is satisfied only if $\frac{1}{2}(x_c c_c-\eta c)=-(\eta_f+\rho c_f)=x_f c_f-\lambda$.

Consider first the case that $\mu=0$ for $\forall t<T$. Define $\tau\equiv\frac{1}{2}(x_c c_c-\eta_c)-x_f c_f+\lambda$. By (3.2.1) and (3.2.3a–c) $\tau>0$ when $t<T$. Because $\tau(T)=0$, $\dot{\tau}=\delta(\lambda-\frac{1}{2}\eta_c)<0$ before T. By (3.2.6), $\mu=0$ for $\forall t<T$ implies $\rho(T)=0$. By (3.4) we obtain that the switch must occur at Γ_1, i.e., $x_c(T)c_c-x_{f0}c_f=0$. Thus $\tau=\frac{1}{2}x_c c_c-x_f c_f+\lambda-\frac{1}{2}\eta_c<0$ before the switch, $\Rightarrow\Leftarrow$.

Consider next the case that $\mu(T)=0$ but $\rho(T)<0$. By (3.2.1) and (3.2.3a–c) q_c is continuous in regime (c). Thus there must be a moment of time, $t_1<T$, at which $\tau(t_1)=0$. When $t<t_1$ (3.2.2) and (3.2.3a–c) imply that $\tau<0$. Accordingly when $t\in(t_1,T)$ we obtain $\tau>0$. Because $\tau(T)=0$ is necessary for optimality there must be a moment of time, say T_2 such that $T_1<T_2<T$, $\dot{\tau}(T_2)=0$ and $\ddot{\tau}(T_2)<0$. Differentiating yields $\dot{\tau}=\delta(\lambda-\frac{1}{2}\eta_c)$ and $\ddot{\tau}(T_2)=\delta q_c c_c>0$, $\Rightarrow\Leftarrow$.■

Appendix 4.1: Proof for Lema 4.1.

The existence of a linear equilibrium requires that $r_1,r_2<0$ because otherwise there do not exist bounded solutions converging toward the steady state $x_c=x_f=\eta_c=\eta_f=0$. By (4.6) and (4.7) μ_2 must satisfy the relationship: $\mu_2=-r_1r_2[\delta^2-\delta(r_1+r_2)+r_1r_2]/c_c\delta(\delta-r_1-r_2)$. Using $r_1,r_2<0$ we obtain $\mu_2<0$. If $\mu_1<0$ we obtain $q_f<0$ $\forall x_f>0$, $\forall x_c>0$ implying that a linear equilibrium cannot exist. Thus $\mu_1>0$. Using the expressions for r_1 and r_2 one obtains:

$$(\mu_1+r_1)(\mu_2+r_2)=[(A^{\frac{1}{2}}+B)^{\frac{1}{2}}-2\mu_1-\delta]\{[B-(A)^{\frac{1}{2}}]^{\frac{1}{2}}-2\mu_1-\delta\},$$

where $A=\mu_2^2(2\mu_1+\delta)^2-2\mu_2(2\mu_1+\delta)(2\mu_1^2+2\delta\mu_1+c_c\delta)+4\mu_1^4+8\delta\mu_1^3+4\delta\mu_1^2(\delta-c_c)-4c_c\delta^2\mu_1+c_c^2\delta^2$ and $B=-\mu_2(2\mu_1+\delta)+2\mu_1^2+2\delta\mu_1+\delta(c_c+\delta)$. When $\mu_2=0$ the term $[(A^{\frac{1}{2}}+B)^{\frac{1}{2}}-2\mu_1-\delta]$ can be developed to the form $[|2\mu_1{}^2+2\delta\mu_1-c\mu\delta|+2\mu_1{}^2+2\delta\mu_1+\delta(c_c+\delta)]^{\frac{1}{2}}-2\mu_1-\delta$, which is always positive. Because A and B are decreasing functions of μ_2, and $\mu_2<0$, it follows that $(A^{\frac{1}{2}}+B)^{\frac{1}{2}}-2\mu_1-\delta>0$ when $\mu_2<0$. We next show that $[B-(A)^{\frac{1}{2}}]^{\frac{1}{2}}-2\mu_1-\delta<0$. Note that $A>(2\mu_1^2-c_c\delta+2\mu_1\delta)^2>0$ and that $B>0$. It can be shown that $B^2-A>0$ implying $B-(A)^{\frac{1}{2}}>0$. Thus $[B-(A)^{\frac{1}{2}}]^{\frac{1}{2}}-2\mu_1-\delta<0 \Leftrightarrow A^{\frac{1}{2}}>B-(2\mu_1+\delta)^2$. If $B-(2\mu_1+\delta)^2\leq0$ our claim is verified. When $B-(2\mu_1+\delta)^2>0$ we obtain $A^{\frac{1}{2}}>B-(2\mu_1+\delta)^2 \Leftrightarrow [B-(2\mu_1+\delta)]^2-A<0$. Because $(B-4\mu_1^2-4\mu_1\delta-\delta^2)^2-A=4\mu_2\mu_1[\mu_1(2\mu_1+\delta)+\delta(2\mu_1+\delta)]+8\mu_2(\mu_1^3+\delta\mu_1^2)+\ 4\mu_2\delta(\mu_1^2+\mu_1\delta)<0$ we obtain that $(\mu_1+r_1)(\mu_2+r_2)<0$ as claimed. Finally, if $\mu_1+r_1+r_2\geq0$ the LHS of (4.7) is always positive, $\Rightarrow\Leftarrow$.■

Appendix 4.2. Proof of proposition 4.1.

Proof: Along Γ_3 both extraction levels are defined by one exponential term with negative root, implying that $\dot{q}_c,\dot{q}_f<0$ and price price is increasing. When x_{c0}/x_{f0} is low enough, $\dot{x}_c(0)=0$ and we obtain $\dot{q}_f(0)=\mu_1\dot{x}_f(0)<0$. Because $q_f(t)$ is given by two exponential terms, $\dot{q}_f(t)<0$ for $\forall t$. Accordingly $q_c(0)=0$ and $\dot{q}_c(0)=r_1r_1x_{c0}>0$. Because $q_c(t)\to0$ as $t\to\infty$, q_c must have a unique maximum. Using (4.3) and (4.4) we obtain $\dot{q}_c+\dot{q}_f\big|_{\dot{x}_c=0}=x_{c0}[r_1r_2-\mu_1\mu_2r_1r_2/(\mu_1+\mu_2)(\mu_1+r_2)]$. This implies that $\text{sign}(\dot{q}_c+\dot{q}_f\big|_{\dot{x}_c=0})=\text{sign}-(\mu_1^2+\mu_1\mu_2+\mu_1r_1+r_1r_2-\mu_1\mu_2)$. Because $(\mu_1^2+\mu_1\mu_2+\mu_1r_1+r_1r_2-\mu_1\mu_2)<0$ contradicts equation (4.6), total extraction must decrease in the beginning of the path starting at the $\dot{x}_m=0$ isocline. Because the total extraction is determined by two exponential terms, we obtain $\dot{q}_c+\dot{q}_f<0$ and $\dot{p}>0$ for $\forall t$ along any path starting above (or on) Γ_3.

By parallel arguments, a large enough x_{c0}/x_{f0} implies $\dot{x}_f(0)=0$ and that $q_f(t)$ has a unique maximum, while the cartel supply is monotonically decreasing. Using (4.3) and (4.4) we obtain $\text{sign}(\dot{q}_m+\dot{q}_f\big|_{\dot{x}_f=0})=\text{sign}(\mu_1-\mu_2+r_1+r_2)$. Because $\mu_1-\mu_2+r_1+r_2>0$ contradicts equation (4.7), it follows that $\dot{q_c}+\dot{q}_f<0$ and $\dot{p}>0$ for $\forall t$.■

Appendix 4.3: The evaluation of condition $-q_f(x_f,x_c)+c_cx_c-\eta_c\leq0$ in regime (a).

In regime (a) the leader's Hamilton–Jacobi–Bellman equation is $\delta V=-q_f\partial V(x_c,x_f)/\partial x_f$, where V is the value function. To develop a differential equation for $\partial V/\partial x_f$ in x_f differentiate the HJB function with respect to x_f and apply Young's theorem. This yields: $\delta\partial V/\partial x_c=-q_f[\partial^2V/\partial x_c\partial x_f]-[\partial V/\partial x_f]\,\partial q_f/\partial x_c$. These equations include q_f and $\partial q_f/\partial x_c$ as unknown functions. Using the necessary conditions (3.1a,b) and the fact $\dot{q}_f=-q_f\partial q_f/\partial x_f$ we obtain an equation for determing q_f: $\partial q_f/\partial x_f=(\delta c_f x_f-\delta q_f)/q_f$. Next, writing this equation as $\partial q_f/\partial x_f q-\delta(c_f x_f-q)=0$, differentiating with respect to x_c and applying Young's theorem, yields a differential equation for $\partial q_f/\partial x_c$: $\partial^2 q_f/\partial x_c \partial x_f=-\partial q_f/\partial x_c(\delta+\partial q_f/\partial x_f)/q_f$. Let us denote the derivatives $\partial V/\partial x_c$ and $\partial q_f/\partial x_c$ by $\gamma_1(x_f)$ and $\gamma_2(x_f)$ respectively. Recall that in regime (c) $q_c=0$, and thus x_c enters the above differential equations as a constant. Thus we obtain the following set of ordinary ordinary differential equations in x_f:

$$dV(x_f)/dx_f=-\delta V(x_f)/q_f(x_f),$$

$$d\gamma_1(x_f)/dx_f=-[\delta\gamma_1(x_f)+\gamma_2(x_f)dV(x_f)/dx_f]/q_f(x_f),$$

$$dq_f(x_f)/dx_f=\delta[c_f x_f-q_f(x_f)]/q_f(x_f),$$

$$d\gamma_2(x_f)/dx_f=-\gamma_2(x_f)[dq_f(x_f)/dx_f+\delta]/q_f(x_f).$$

To solve this system of nonlinear nonautonomous equations we must have the initial level of x_f and four boundary conditions. Given any x_{c0}, we obtain the corresponding level of x_f on the $\dot{x}_m=0$ isocline, i.e., $x_f=-x_{c0}(\mu_1+r_1+r_2)\mu_2/(\mu_1+r_1)(\mu_1+r_2)$. Given this initial state in regime (b) we know the value function V, its derivative with respect to x_c, i.e. η_m, the level of fringe extraction q_f, and its dependence on x_c i.e. μ_2. Using these initial levels it is possible to compute the solution for the differential equation system forward in x_f and to evaluate whether the necessary condition $-q_f(x_f,x_c)+c_c x_c-\eta_m\leq 0$ holds along regime (c). We have verified that using the example $\delta=1/20$, $c_c=c_f=1/2$, various initial level for the states and the fourth–order Runge–Kutta method.

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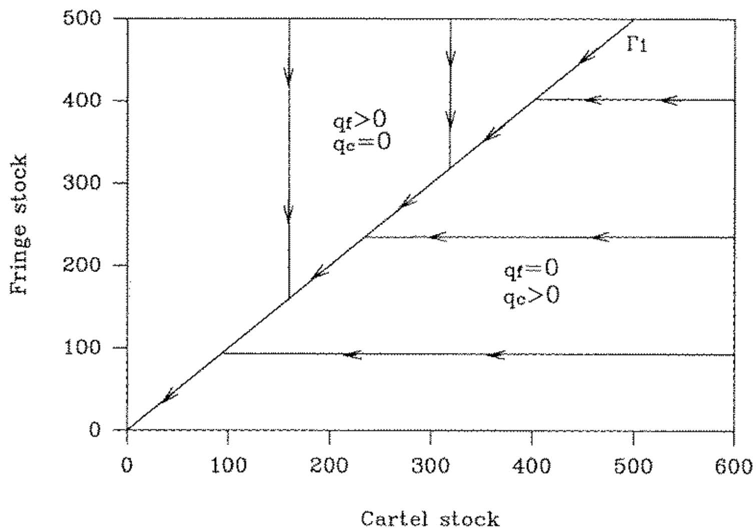

Figure 1. The competitive equilibrium.

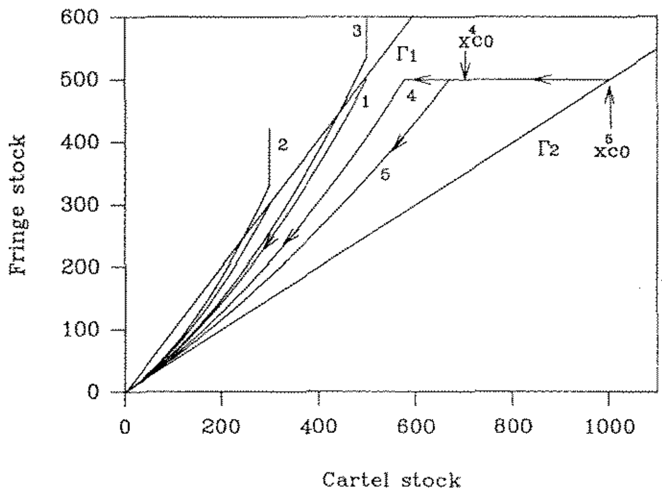

Figure 2. Stackelberg open loop equilibria

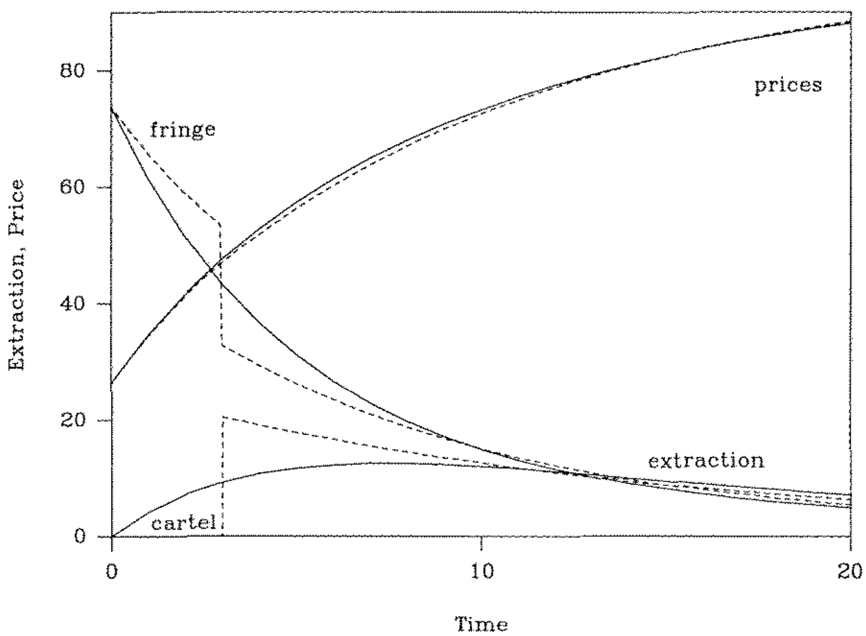

Figure 3. OLSE and MPSE time paths and OLSE switch (a)–(b).

Solid lines: MPSE

Dotted lines: OLSE

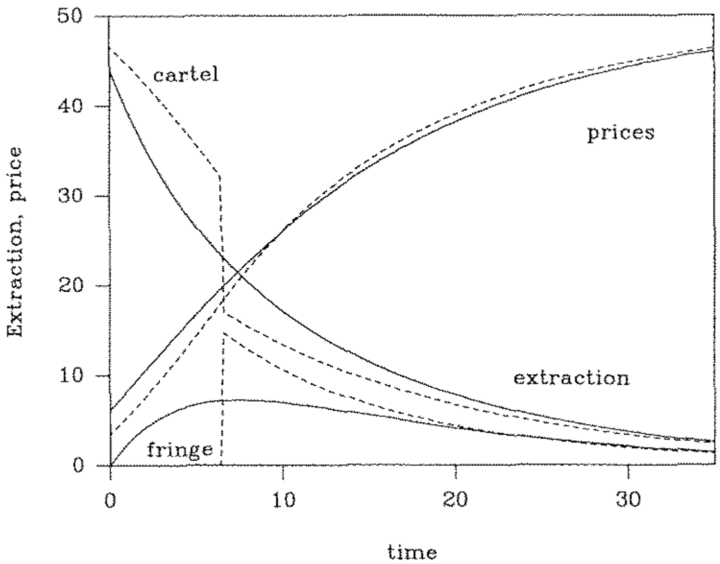

Figure 4. OLSE and MPSE time paths
and OLSE switch (c)–(b)
Solid lines: MPSE
Dotted lines: OLSE

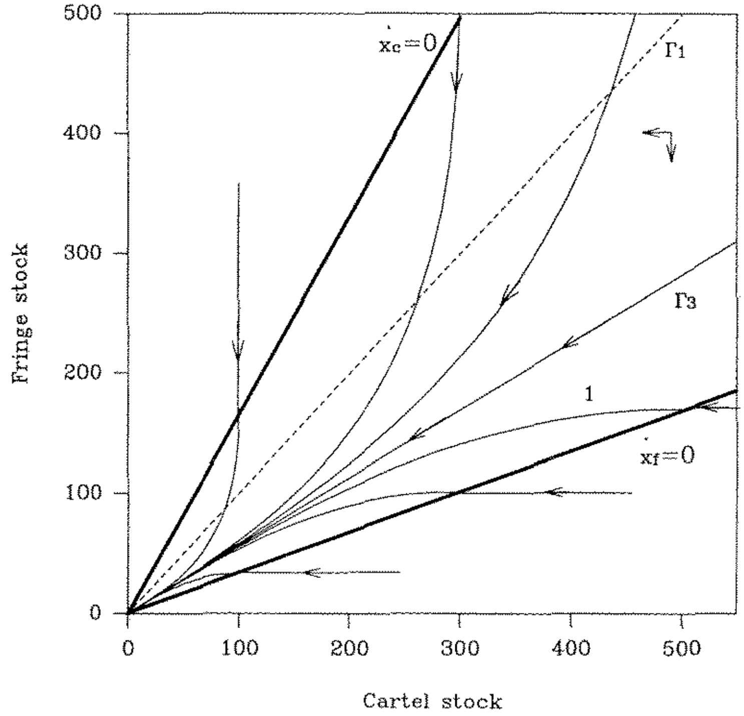

Figure 5. MPSE in state space.

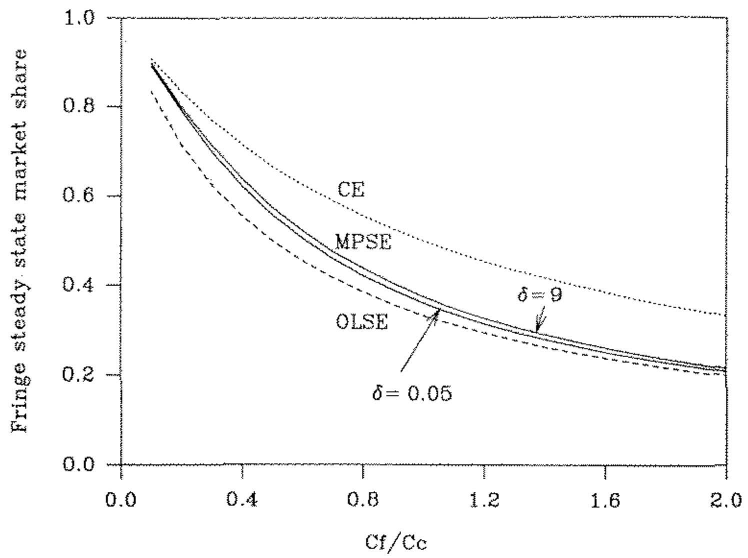

Figure 6. Comparison of steady state market shares

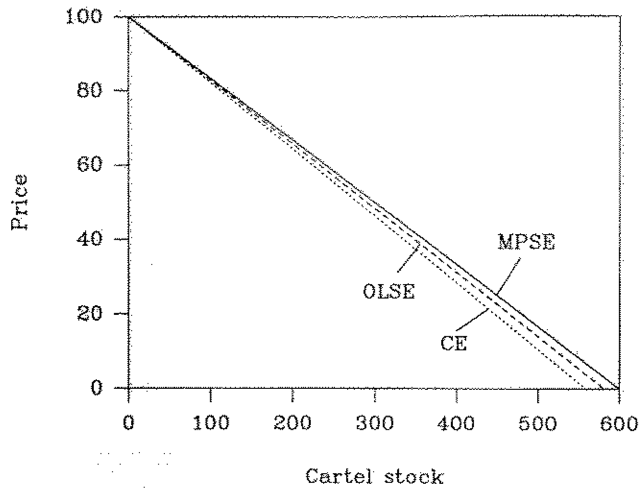

Figure 7. Comparison of prices along Γ_1

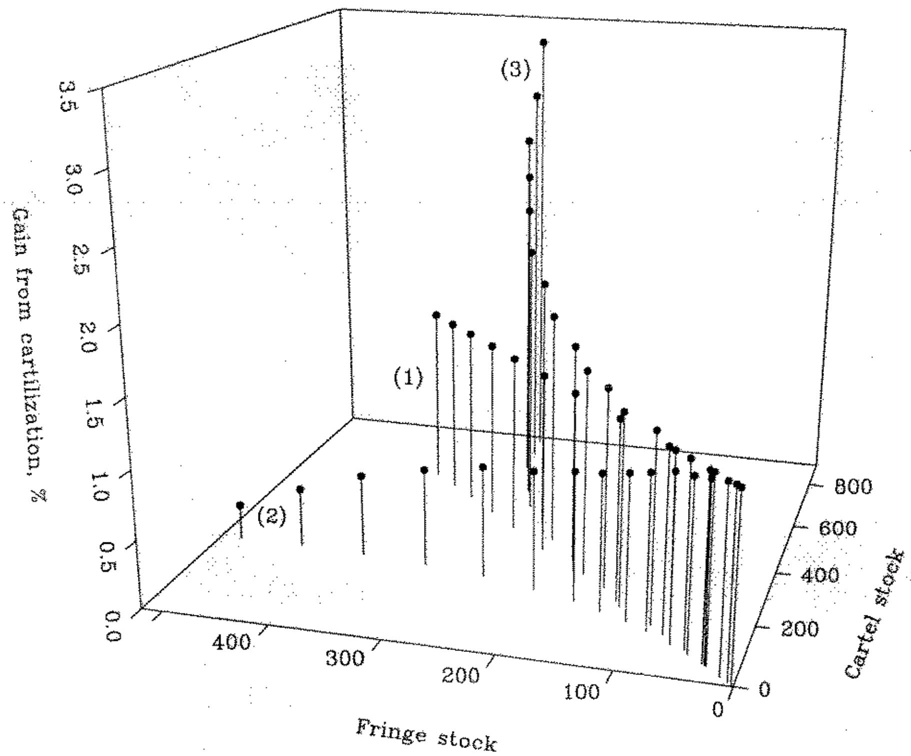

Figure 8. Gain from cartelization along three MPSE solutions