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ECONOMETRIC MODELS OF THE DEMAND FOR QUALITY-DIFFERENTIATED GOODS

by

W. Michael Hanemann

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
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W. Michael Hanemann

University of California, Berkeley, CA 94720, USA

Abstract

This paper presents several models of the demand for quality-differentiated goods in which the consumer decides which brand of product to select as well as how many units to buy. The models cover a variety of preference structures and can readily be estimated using standard techniques for switching regressions. From the fitted demand equations, one can calculate monetary measures of the welfare effects of changes in the price, quality, or variety of the brands. The models are then applied to data on households' demands for recreation sites in the Boston area, and the values of the sites are calculated.

Address: Professor W. Michael Hanemann
207 Giannini Hall
University of California
Berkeley, CA 94720

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1. Introduction

Recently there has been an explosion of interest in the study of markets with differentiated products. Some developments in this literature are surveyed in the symposia edited by Gould et al. (1980) and Phlips and Thisse (1982). For the most part, these theoretical studies have focused on a comparative static analysis based on the first-order conditions for equilibrium under various market structures. The empirical application of such models, however, has lagged behind due, in part, to the difficulty of specifying parametric utility and production functions which are both realistic in their degree of detail and sufficiently tractable to permit the derivation of closed-form expressions for the demand and supply equations. The present paper may be of some assistance in this regard since it provides some parametric utility models for a consumer's choice among differentiated products which cover a variety of preference structures and yield tractable estimating equations. It is somewhat less ambitious than many of the theoretical treatments since it concentrates exclusively on the demand side of markets for differentiated products, taking their supply as given. The perspective is thus similar to that adopted by Novshek and Sonnenschein (1979), although the demand models developed here differ somewhat in their structure from the theoretical models of Novshek and Sonnenschein. However, unlike some of the general equilibrium treatments such as Mussa and Rosen (1978), Gabszewicz and Thisse (1980), and Shaked and Sutton (1982), the utility models presented here do not impose the assumption that the consumer buys only one unit of the differentiated product.[1]

Thus, in these models a consumer has to decide how many units to buy as well as which brand of product to select. Both decisions--the discrete (quality) choice and the continuous (quantity) choice--are determined simultaneously as the solution to a single utility maximization problem. The resulting demand equations can be cast in the form of a switching regression model, and the statistical techniques developed by Heckman (1979) and Lee and Trost (1978) can be employed to estimate them. Moreover, because the discrete and continuous choices both flow from the same underlying utility function, there are additional restrictions on the coefficients and disturbance terms appearing in the discrete and continuous equations of the switching regression model which can be exploited in the estimation process.

Since the fitted demand equations provide information about the underlying utility function, they permit one not only to predict the consumer's response to exogenous changes in the price, quality, or variety of the brands available to him but, also, to compute monetary measures of the effect of these changes on his welfare. These calculations, which are illustrated below, represent an extension of the welfare-analytic methodology initiated by Small and Rosen (1981) to the case of mixed discrete/continuous choices.

The paper is organized as follows. In section 2, I present the utility models, derive their discrete and continuous choice equations, and show how they can be estimated. In section 3, as an illustration, these models are applied to data on household recreation behavior in the Boston area. The choice among different recreation sites with exogenously given quality characteristics is here taken as an example of consumer choice among differentiated products. In section 4, I describe the procedure for computing the welfare measures for changes in the set of prices and qualities available to the consumer and apply it to the fitted recr[illegible] models to derive estimates of the value of each site (i.e., t[illegible] surplus lost if the site were shut down). In the concluding sec[illegible]cate some directions for further research.

2. Model specification and estimation

2.1. General Structure

The theoretical set-up is as follows. There are N different brands of commodity; the consumption of the <u>j</u>th brand is denoted by x_j. The brands may differ with respect to their prices and quality characteristics, which the consumer takes as exogenous. The prices are denoted by $p = (p_1, \ldots, p_N)$. I assume that there are K different dimensions of quality. Let $b = (b_1, \ldots, b_N)$ and $b_j = (b_{j1}, \ldots, b_{jK})$, where b_{jk} is the amount of the <u>k</u>th quality characteristic associated with a unit of consumption of brand j. The consumer's utility depends on his consumption of the various brands, their quality characteristics, and his consumption of other, nonbranded goods represented by the composite commodity z, which I take as the numeraire. The consumer's preferences may also be influenced by his own observable characteristics (age, education, etc.), but I will ignore these variables for now.

In addition, I assume that, although the consumer's utility function is deterministic for <u>him</u>, it contains some components which are unobservable to the econometric investigator and are treated by the investigator as random variables. These random elements could be unobservable characteristics of the consumer and/or attributes of the brands. They will be denoted by the vector ε, and the utility function will be written compactly as $u(x, b, z; \varepsilon)$.

Different types of discrete/continuous choice model can be generated, depending on how one specifies the interaction of ε with the other arguments of the utility function. Here I assume that $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_N)$ and $u(x, b, z; \varepsilon) = u[x, \psi_1(b_1; \varepsilon_1), \ldots, \psi_N(b_N; \varepsilon_N), z]$. The function ψ_j may be interpreted as an index of the quality of the _j_th brand; my assumption is that it is here, in the consumer's overall perception of each brand's quality, that the random component is located.

The consumer chooses (x, z) so as to maximize $u(x, \psi, z)$ subject to the budget constraint $\Sigma p_j x_j + z = y$ and the nonnegativity conditions $x_j \geq 0$, $j = 1, \ldots, N$ and $z \geq 0$. Moreover, I assume that this decision leads to a corner solution in which z and <u>only one</u> of the x_j's is positive. This is because the consumer considers the different brands as substitutes for one another and prefers to consume only one of them at a time. Thus, his decision simultaneously involves a discrete and a continuous choice: the discrete choice is which brand to select (which one of the x_j's is nonzero), and the continuous choice is how much of it to buy (the magnitude of the nonzero x_j). If the consumer has this preference, his indifference curves for all pairs of x_j's must be linear or concave. A general family of utility functions with this property is

$$u(x, \psi, z) = u(\Sigma x_j, \Sigma \psi_j x_j, z). \tag{2.1}$$

Some specific examples of (2.1) are presented below.

Before describing these utility models in more detail, it is useful to summarize the general procedure by which the discrete and continuous demand functions are derived from them.[2] Suppose, for the moment, that the consumer has chosen the _j_th brand. Conditional on this choice, his utility is

$\bar{u}_j \equiv u(0, \ldots, 0, x_j, 0, \ldots, 0, \psi_1, \ldots, \psi_N, z)$. Observe that the family of utility models (2.1) has the property that

$$x_j = 0 \rightarrow \frac{\partial u}{\partial \psi_j} = 0 \qquad j = 1, \ldots, N, \tag{2.2}$$

i.e., a brand's quality does not matter to the consumer unless that brand is actually consumed. Therefore, his utility conditional on selecting the jth brand can be written as $\bar{u}_j = \bar{u}_j(x_j, \psi_j, z)$; I refer to this as the conditional direct utility function. In order to decide how much of the brand to buy, the consumer maximizes $\bar{u}_j$ subject to the conditional budget constraint, $p_j\, x_j + z = y$. Assuming that $\bar{u}_j$ is strictly quasiconcave in x_j and z, and x_j is essential with respect to $\bar{u}_j$ (i.e., none of the indifference curves intersect the z-axis), this leads to an interior solution with $\bar{x}_j > 0$. The resulting conditional ordinary demand functions will be denoted $\bar{x}_j(p_j, \psi_j, y)$ and $\bar{z}(p_j, \psi_j, y)$, and the conditional indirect utility function is $\bar{v}_j(p_j, \psi_j, y) \equiv \bar{u}_j[\bar{x}_j(p_j, \psi_j, y), \psi_j, \bar{z}(p_j, \psi_j, y)]$.

All of the foregoing is conditional on the consumer's selecting brand j. The discrete choice of which brand to select can be represented by a set of binary valued indices, $\delta_1, \ldots, \delta_N$, where $\delta_j = 1$ if $x_j > 0$ and $\delta_j = 0$ if $x_j = 0$. The consumer selects the brand which gives the highest utility; that is,

$$\delta_j(p, \psi, y) = \begin{cases} 1 \text{ if } \bar{v}_j(p_j, \psi_j, y) \geq \bar{v}_i(p_i, \psi_i, y), \text{ all } i \\ 0 \text{ otherwise.} \end{cases} \tag{2.3}$$

Now consider the original, unconditional problem of maximizing $u(x, \psi, z)$ subject to $\Sigma p_j\, x_j + z = y$. The unconditional ordinary demand functions associated with this problem will be denoted by $x_j(p, \psi, y)$, $j = 1, \ldots, N$ and $z(p, b, y)$. The resulting unconditional indirect utility function is $v(p, \psi, y) \equiv u[x(p, \psi, y), \psi, z(p, \psi, y)]$. These are related to the corresponding conditional functions by

$$x_j(p, \psi, y) = \delta_j(p, \psi, y)\, \bar{x}_j(p_j, \psi_j, y) \tag{2.4}$$

$$v(p, \psi, y) = \max\, [\bar{v}_1(p_1, \psi_1, y), \ldots, \bar{v}_N(p_N, \psi_N, y)]. \tag{2.5}$$

For the consumer, the quantities $\bar{x}_j$, $\bar{z}$, $\bar{v}_j$, δ_j, x_j, z, and v are known numbers but, because his preferences are incompletely observed, they are random variables from the point of view of the econometric investigator. For example, the discrete choice indices are Bernoulli random variables with a mean, $E\{\delta_j\} \equiv \pi_j$, given by

$$\pi_j = \Pr\{\bar{v}_j(p_j, \psi_j, y) \geq \bar{v}_i(p_i, \psi_i, y), \text{ all } i\}. \tag{2.6}$$

This probability can be evaluated by manipulating the joint density of the ε_j's, $f_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N)$. Define the sets $A_j \equiv \{\varepsilon | \bar{v}_j(p_j, \psi_j, y) \geq \bar{v}_i(p_i, \psi_i, y)$, all $i\}$, $j = 1, \ldots, N$. From f_ε one can construct $f_{\varepsilon_j | \varepsilon \epsilon A_j}$, the conditional marginal density of ε_j given that $\varepsilon \epsilon A_j$, i.e., given that brand j is selected. Then, the probability density of $\bar{x}_j$, $f_{x_j | \varepsilon \epsilon A_j}(x) = \Pr\{x_j = x | \varepsilon \epsilon A_j\}$, can be derived from $f_{\varepsilon_j | \varepsilon \epsilon A_j}$ by an appropriate change of variables. Finally, the probability density of x_j, $f_{x_j}(x) = \Pr\{x_j = x\}$, takes the form

$$f_{x_j}(x) = \begin{cases} 1 - \pi_j & x = 0 \\ \pi_j\, f_{x_j | \varepsilon \epsilon A_j}(x) & x > 0 \end{cases} \tag{2.7}$$

Accordingly, suppose these are observations on a sample of T consumers, each of whom selects one brand of the commodity. Let the subscript t denote the individual consumer, let j^* be the index of the brand which he selects, and let x_t^* be the particular quantity which he is observed to consume. From (2.7), the likelihood function for the sample is

$$\mathcal{L} = \prod_{t=1}^{T} \left\{ \pi_{j^*t} \, f_{x_{j^*t} | \epsilon \epsilon A_{j^*t}} (x_t^*) \right\}. \tag{2.8}$$

In principle, the unknown parameters of the model can be estimated by full information maximum likelihood. As an alternative, one can employ the two-stage estimation procedure suggested by Heckman (1979) and Lee and Trost (1978). Before discussing this, I will describe the specific utility models to which it is applied.

2.2. *The Blackburn model*

The first model was originally developed by Blackburn (1970) for the analysis of aggregate travel demand. The utility function is[3]

$$u(x, \psi, z) = \Sigma x_j (1 + \ln \theta - \ln \Sigma x_j) + hz + \Sigma \psi_j x_j \qquad \theta > 0, \; h > 0 \tag{2.9}$$

$$\psi_j(b_j; \epsilon_j) = \alpha_j + \sum_k \gamma_k b_{jk} + \epsilon_j, \; j = 1, \ldots, N \tag{2.10}$$

where θ is a constant or, more generally, a function of observable characteristics of the individual consumer. Furthermore, the random terms ϵ_j are i.i.d. according to the extreme value (EV) distribution with scale parameter $\mu > 0$. Thus, their joint c.d.f. is

$$F_\varepsilon(\varepsilon_1, \ldots, \varepsilon_N) = \exp \text{ [illegible]} \tag{2.11}$$

The maximization of (2.9) subject to the budget constraint clearly leads to a corner solution in which, except on a set of measure zero, only one brand is selected.

Suppose that the consumer selects brand j; his conditional direct utility is $\bar{u}_j(x_j, \psi_j, z) = x_j(1 + \ln \theta - x_j) + hz + \psi_j x_j$, which is strictly quasi-concave in x_j and z. Maximization of $\bar{u}_j$ yields the conditional demand and indirect utility functions

$$\bar{x}_j(p_j, \psi_j, y) = \theta e^{\lambda_j + \varepsilon_j} \tag{2.12}$$

$$\bar{v}_j(p_j, \psi_j, y) = hy + \theta e^{\lambda_j + \varepsilon_j} \tag{2.13}$$

where

$$\lambda_j \equiv \alpha_j + \Sigma\gamma_k b_{jk} - hp_j.$$

It follows from (2.13) that the single brand selected by the consumer is that for which $\lambda_j + \varepsilon_j = \psi_j - hp_j$ is highest. Thus, the discrete choice probabilities (2.6) take the form

$$\pi_j = \Pr\{\lambda_j + \varepsilon_j \geq \lambda_i + \varepsilon_i, \text{ all } i\}. \tag{2.14}$$

With the EV distribution (2.11), one obtains

$$\pi_j = \frac{\exp(\bar{\alpha}_j + \Sigma\bar{\gamma}_k b_{jk} - \bar{h}p_j)}{\Sigma \exp(\bar{\alpha}_i + \Sigma\bar{\gamma}_k b_{ik} - \bar{h}p_i)} \tag{2.15}$$

where

$$\overline{\alpha}_j \equiv \frac{\alpha_j}{\mu}$$

$$\overline{\gamma}_k \equiv \frac{\gamma_k}{\mu}$$

and

$$h \equiv \frac{\overline{h}}{\mu}.$$

This is similar to the standard multinominal logit formula [see McFadden (1974)] except that the latter usually sets the scale parameter μ equal to unity. Indeed, this normalization would be unavoidable if one were estimating purely discrete choices. It can be avoided here because I am also estimating the continuous choices, which serve to identify μ.[4]

Given the structure of the discrete choice probabilities (2.14), $A_j = \{\varepsilon | \varepsilon_j + \lambda_j \geq \varepsilon_i + \lambda_i, \text{ all } i\}$ and the conditional marginal density $f_{\varepsilon_j | \varepsilon\epsilon A_j}$ has the form

$$f_{\varepsilon_j | \varepsilon\epsilon A_j}(t) = \pi_j^{-1} F_\varepsilon^j(t + \lambda_j - \lambda_1, \ . \ . \ ., \ t + \lambda_j - \lambda_N)$$

where F_ε^j is the derivative of F_ε with respect to its jth argument. With the EV distribution (2.11) one obtains

$$f_{\varepsilon_j | \varepsilon\epsilon A_j}(t) = \mu^{-1} \beta_j \ e^{-t/\mu} \exp\left[-\beta_j \ e^{-t/\mu}\right] \tag{2.16}$$

where

$$\beta_j \equiv e^{-\lambda_j/\mu} \cdot \sum_{i=1}^{N} e^{\lambda_i/\mu} = \pi_j^{-1}.$$

This may be recognized as the density of a univariate EV r.v. with scale parameter μ and location parameter $(\mu \ln \beta_j)$. Its mean and moment-generating function are

$$E\{\varepsilon_j | \varepsilon \in A_j\} = \mu(\ln \beta_j + 0.57722) \tag{2.17}$$

$$E\{e^{t\varepsilon_j} | \varepsilon \in A_j\} = \beta_j^{\mu t}\, \Gamma(1 - \mu t). \tag{2.18}$$

The conditional density $f_{x_j|\varepsilon\in A_j}$, derived from $f_{\varepsilon_j|\varepsilon\in A_j}$ by a change of variable based on (2.12), is

$$f_{x_j|\varepsilon\in A_j}(x) = \mu^{-1}\, \theta^{1/\mu}\, x^{-(1+\mu)/\mu}\, \Sigma e^{\lambda_i/\mu} \exp\left(-\theta^{1/\mu}\, x^{-1/\mu}\, \Sigma e^{\lambda_i/\mu}\right). \tag{2.19}$$

The conditional mean quality of brand j demanded can be obtained by integrating (2.19) or, more simply, from (2.12) and (2.18)

$$\begin{aligned} E\{x_j | \varepsilon \in A_j\} &= \theta e^{\lambda_j}\, E\{e^{\varepsilon_j} | \varepsilon \in A_j\} \\ &= \theta \left(\Sigma e^{\lambda_i/\mu}\right)^{\mu} \Gamma(1 - \mu). \end{aligned} \tag{2.20}$$

Note that this conditional mean exists only if $\mu < 1$. For estimation purposes, it is more convenient to work with the mean of the conditional distribution of $\ln x_j$, which is, from (2.17),

$$\begin{aligned} E\{\ln x_j | \varepsilon \in A_j\} &= \ln \theta + \lambda_j + E\{\varepsilon_j | \varepsilon \in A_j\} \\ &= \ln \theta + \mu\left\{\ln\left(\Sigma e^{\lambda_i/\mu}\right) + 0.57722\right\}. \end{aligned} \tag{2.21}$$

The estimation of this demand model on the basis of (2.15) and (2.. ` : (2.21) is discussed in section 2.4. Before proceeding, however, two points about the model should be mentioned. First, I have implicitly assumed that the conditional utility maximization yields an interior solution for x_j. In fact, it can be shown that, for the utility model (2.9), x_j is essential with respect to $\bar{u}_j$ so that this assumption is justified. However, it is not true that z is essential with respect to $\bar{u}_j$--it can be seen from (2.12) that there is a nonzero probability of obtaining $z < 0$, which is economically meaningless.[5] In the empirical application below, this probability turns out to be negligibly small so that it can reasonably be ignored. Second, the demand function (2.12) implies a zero income elasticity of demand for all the brands, which may be unduly restrictive. This restriction is removed in the next group of models, which offer considerable flexibility in modeling the shape of the Engel curves for the branded goods.

2.3. Perfect substitute models

Consider the following utility function in which the different brands are perfect substitutes

$$u(x, \psi, z) = u^*(\Sigma\psi_j x_j, z) \tag{2.22}$$

where u^* is a conventional bivariate utility function. Clearly, the maximization of (2.22) subject to a budget constraint leads to a corner solution in which only one brand is selected. Given that the consumer has chosen brand j, his conditional direct utility is $\bar{u}_j(x_j, \psi_j, z) = u^*(\psi_j x_j, z)$. Regard $u^*(\cdot, \cdot)$ as a function of two arguments, say, w_I and w_{II}. If u^* is strictly quasiconcave in w_I and w_{II}, then $\bar{u}_j$ is strictly quasiconcave in x_j and z.

Similarly, if w_I and w_{II} are essential with respect to u^*, then x_j and z are essential with respect to $\bar{u}_j$. Let $w_I^*(p_I, y)$ and $w_{II}^*(p_I, y)$ be the ordinary demand functions arising when $u^*(w_I, w_{II})$ is maximized subject to the budget constraint $p_I w_I + w_{II} = y$, and let $v^*(p_I, y) \equiv u^*[w_I^*(p_I, y), w_{II}^*(p_I, y)]$ be the corresponding indirect utility function. It can be shown that the conditional ordinary demand functions and indirect utility function associated with $\bar{u}_j$ have the form[6]

$$\bar{x}_j(p_j, c_j, y) = \psi_j^{-1} w_I^*\left(\frac{p_j}{\psi_j}, y\right) \tag{2.23a}$$

$$\bar{z}_j(p_j, \psi_j, y) = w_{II}^*\left(\frac{p_j}{\psi_j}, y\right) \tag{2.23b}$$

$$\bar{v}_j(p_j, \psi_j, y) = v^*\left(\frac{p_j}{\psi_j}, y\right) \tag{2.23c}$$

Since v^* is decreasing in its first argument, it follows from (2.23c) that the single brand selected by the consumer is that for which p_j/ψ_j is lowest.

Instead of (2.10) it is convenient here to adopt the following specification for the ψ_j's.

$$\psi(b_j; \varepsilon_j) = \exp(\alpha_j + \Sigma\gamma_k\, b_{jk} + \varepsilon_j) \tag{2.24}$$

where, as before, the ε_j's have the EV distribution (2.11). Applying (2.6), the discrete choice probabilities may be written

$$\pi_j = \Pr\{\ell n\, \psi_j - \ell n\, p_j \geq \ell n\, \psi_i + \ell n\, p_i, \text{ all } i\},$$

which has the same general form as (2.14) but with λ_j now defined as $\lambda_j \equiv \alpha_j + \Sigma\gamma_k b_{jk} - \ln p_j$. With the EV distribution, these probabilities are given by

$$\pi_j = \frac{\exp\left(\bar{\alpha}_j + \Sigma\bar{\gamma}_k b_{jk} - \frac{1}{\mu} \ln p_j\right)}{\Sigma \exp\left(\bar{\alpha}_i + \Sigma\bar{\gamma}_k b_{ik} - \frac{1}{\mu} \ln p_i\right)} \tag{2.25}$$

where, as before,

$$\bar{\alpha}_j \equiv \frac{\alpha_j}{\mu}$$

and

$$\bar{\gamma}_k \equiv \frac{\gamma_k}{\mu}.$$

This is similar to corresponding formula for the Blackburn model (2.15) except for the appearance of the term $(1/\mu)\ln p_j$ in place of $\bar{h}p_j$. Because of this difference, the scale parameter of the EVD, μ, can be identified directly from the discrete choices.

To obtain the continuous choice probabilities, one must specify a parametric bivariate utility model. Here I consider three models which lead to reasonably tractable formulas for the continuous choice probabilities. Expressed in dual form, the utility models are

$$v^*(p_I, y) = \frac{\theta}{\rho - 1} p_I^{1-\rho} + \frac{1}{1-\eta} y^{1-\eta} \qquad \theta > 0, \eta \neq 1 \tag{2.26a}$$

$$v^*(p_I, y) = \frac{\theta}{\rho - 1} p_I^{1-\rho} - \frac{e^{-\eta y}}{\eta} \qquad \theta > 0 \tag{2.27a}$$

$$v^*(p_I, y) = (\ln y - \theta \ln p_I)p_I^{-\eta} \qquad \theta < \theta < 1. \tag{2.28a}$$

In each case, θ is a constant or, more generally, a function of the characteristics of the individual consumer. The ordinary demand functions associated with these utility models are

$$w_I^*(p_I, y) = \theta p_I^{-\rho} y^{\eta} \tag{2.26b}$$

$$w_I^*(p_I, y) = \theta p_I^{-\rho} e^{\eta y} \tag{2.27b}$$

$$w_I^*(p_I, y) = \frac{\theta y}{p_I} + \frac{\theta y}{p_I} (\ell n\ y - \theta\ \ell n\ p_I). \tag{2.28b}$$

Thus, the utility model (2.26) leads to a constant income elasticity of demand, η, while the utility model (2.27) leads to an income elasticity of demand, ηy, which varies with the consumer's income. I will refer to these as the LOG-LOG and SEMI-LOG models, respectively. In both models a necessary and sufficient condition for w_I to be an essential good is that $\rho < 1$; however, w_{II} is not essential. The utility model (2.28) is a bivariate version of a special case of Muellbauer's (1976) PIGLOG model; given that $\theta < 1$, both goods are essential. It should be noted that none of these indirect utility functions is quasiconvex over the entire price-income space. Therefore, one must check for quasiconvexity at the sample data points.

To show how these conventional bivariate utility functions can be combined with the random utility model (2.22), I will focus on the LOG-LOG model (2.26); the analysis of the other models proceeds in a similar manner. From (2.23a) the conditional ordinary demand functions associated with the random utility model, which consists of (2.22), (2.24), and the direct utility function dual to (2.26), are[7]

$$\bar{x}_j(p_j, \psi_j, y) = \theta\left(\frac{p_j}{\psi_j}\right)^{-\rho} y^{\eta} \psi_j^{-1}$$

$$= \theta p_j^{-\rho} y^{\eta} \psi_j^{\rho-1}.$$

Substituting (2.24) and simplifying yields

$$\bar{x}_j(p_j, \psi_j, y) = \theta p_j^{-1} y^{\eta} e^{(\rho-1)\lambda_j} e^{(\rho-1)\varepsilon_j}. \tag{2.29}$$

Using (2.29) to make a change of variable in (2.19) from ε_j to x_j, one obtains the conditional density[8]

$$f_{x_j|\varepsilon\epsilon A_j}(x) = \left(\frac{\theta}{p_j}\right)^{1/\mu(\rho-1)} y^{\eta/\mu(\rho-1)} x^{-[\mu(\rho-1)+1]/\mu(\rho-1)} \Sigma e^{\lambda_i/\mu}$$

$$* \exp\left[-\left(\frac{\theta}{p_j}\right)^{1/\mu(\rho-1)} y^{\eta/\mu(\rho-1)} x^{-1/\mu(\rho-1)} \Sigma e^{\lambda_i/\mu}\right] (\mu|\rho - 1|)^{-1}. \tag{2.30}$$

For the purpose of estimating the model, it is convenient to work with the mean of the conditional distribution of $\ell n(p_j\, x_j)$ which is, from (2.29) and (2.17),

$$E\{\ell n\, p_j\, x_j | \varepsilon \epsilon A_j\} = \ell n\, \theta + \eta\, \ell n\, y + (\rho - 1)\, \lambda_j + (\rho - 1)\, E\{\varepsilon_j | \varepsilon \epsilon A_j\}$$

$$= \ell n\, \theta + \eta\, \ell n\, y + (\rho - 1)\left[\mu\, \ell n\left(\Sigma e^{\lambda_i/\mu}\right) + 0.57722\, \mu\right]. \tag{2.31}$$

To save space, the formulas corresponding to (2.30) and (2.31) for the other two utility models are presented in the Appendix.

2.4. Estimation

The demand models presented above can all be estimated in a similar manner. In each case, given observations on a sample of consumers, the likelihood function of the sample can be cast in the form of (2.8), where π_{j*t} is given by (2.15) or (2.25), and $f_{x_{j*t}|\epsilon\epsilon A_{j*t}}(x_t^*)$ is given by (2.19), (2.30), or the formulas in the Appendix. As noted above, the unknowns in the model could be estimated by full information maximum likelihood. It is simpler, however, to employ a two-stage estimation procedure along the lines originally suggested by Heckman (1979) and Lee and Trost (1978) for the switching regression model with normal errors. I will describe this estimation procedure in the context of the Blackburn model; the details vary slightly for the other demand models.

The first step is maximum likelihood estimation of the logit model for the discrete choice probabilities alone (2.15). This yields estimates of $\bar{\alpha}$, $\bar{\gamma}$, and $\bar{h}$ which are consistent but not efficient since they ignore the information contained in the data on continuous choices. With these estimates one can form consistent estimates of $(\lambda_i/\mu)_t$. The next step is a regression analysis of the data on the continuous choices. By virtue of (2.21) one can set up the following regression model for these data.

$$\ell n\ x_t^* = \ell n\ \theta_t + \mu\left\{\ell n\ \Sigma e^{(\lambda_i/\mu)_t} + 0.57722\right\} + \nu_t \qquad t = 1, \ldots, T \qquad (2.32)$$

where the ν_t's are i.i.d as EV $(\mu, -0.57722\mu)$. Hence, $E\{\nu_t\} = 0$ and $\text{var}\{\nu_t\} = \pi^2\mu^2/6$. The estimated values of $(\lambda_i/\mu)_t$ are used to form the regressor variable in braces in (2.32), and the equation is fitted by least squares--by OLS or nonlinear least squares depending on the form of $\ell n\ \theta_t$. Since this is a regression with nonnormal but homoscedastic and finite-variance disturbances, the resulting estimates of μ and the coefficients in $\ln \theta_t$ are consistent.[9]

At this point, therefore, one has consistent estimates of all the unknowns in the model. The final step is to use these as initial estimates for the maximization of the likelihood function (2.8). Since they are consistent, a single Newton-Raphson iteration will provide estimates with the same asymptotic distribution as the global MLE. Thus, these two-step maximum likelihood (2SML) estimates are consistent and asymptotically normal and efficient, and their covariance matrix is consistently estimated by the information matrix. The computations can be further simplified by following the suggestion of Berndt et al. (1974) and substituting the covariance matrix of the gradient of the log-likelihood function for the information matrix when performing the Newton-Raphson iteration and computing the covariance matrix of the parameter estimates.[10] This procedure is employed in the empirical application described in the next section.

3. An application to recreation demand

3.1. *The data*

In this section, the demand models developed in the preceding section are applied to some data on households' visitation of water-based recreation sites in the Boston area. The data come from two surveys, both conducted in 1974 and described in more detail in Binkley and Hanemann (1978): a survey in their homes of a stratified random sample of households in the Boston SMSA to ascertain which sites they had visited during the summer of 1974 for swimming and beach recreation and the frequency of their visits; and a survey of the major recreation sites in the Boston area to inventory their facilities and sample their water quality. From this sample, 83 households who each visited only one site during the summer of 1974 form the basis for the application of the discrete/continuous demand models. My maintained hypothesis is that these households freely chose to visit only one site because of their particular recreation preferences. Between them, they visited some 20 different sites which cover all the main sites in the area. Each household is conceived of as selecting one of these 20 sites and choosing to make some number of visits to the site over the summer; in the sample, the number of visits by a household ranged from 1 to 100, with a median of about 5.

For the site characteristics, I employ five variables. Two are measures of water quality, chemical oxygen demand (COD) and total phosphorus content (PHOS), both measured as mg/l. Nonwater aspects of site quality are captured by a dummy variable, NUISANCE. Higher values of these three variables signify poorer site quality. The fourth variable, SITE TYPE, a dummy variable for freshwater (= 1) as opposed to ocean sites (= 0), is included because there may be distinct preferences for the two types of site. The fifth variable, MINORITY ATT, is intended to allow for racial segmentation in the selection of recreation sites which is a significant phenomenon in the Boston area. This is a dummy variable which takes the value 1 if the household is from a minority group and the site is one of those identified as being especially accessible to minorities and 0 otherwise. These five variables constitute the b_{jk}'s in (2.10) and (2.24). Following the custom in recreation demand studies, the price variable, p_j, is taken to be the travel cost defined as the estimated road distance from the household's home to the recreation site multiplied by an estimated travel cost of 7 cents per mile (in 1974 prices).

Finally, in formulating the model I adopt an entirely "generic" specification of site demand by setting $\alpha_1 = \alpha_2 = \ldots = \alpha_N = 0$ in (2.10) and (2.24).

The observable household characteristics which enter into the term θ in (2.9) and (2.26) through (2.28) are an index of highest household education, EDUC; the number of persons aged 18 and older in the household, ADULTS; and the number of persons under 18 in the household, CHILDREN. In addition, there is a dummy variable, SWIMPOOL, which takes the value 1 if the household has access to a private swimming pool and 0 otherwise. The specific formula for θ is

$$\theta = \theta_{10}\ \mathrm{EDUC}^{\theta_{11}} \exp[\theta_{12}\ \mathrm{SWIMPOOL} + \theta_{13}\ \mathrm{ADULTS} + \theta_{14}\ \mathrm{CHILDREN}] \qquad (3.1)$$

where θ_{10} is a positive constant. Finally, y is annual household income in 1974 dollars.

3.2. Results

The four demand models described above were estimated by the method described in section 2.4. The results are presented in tables 1 and 2. In the first stage, a logit model of the discrete choice was estimated using (2.15) for the Blackburn model and (2.25) for the other models. In the case of the Blackburn model, this yielded estimates of $\bar{\gamma}_1, \ldots, \bar{\gamma}_5$ and $\bar{h}$. The next step was the estimation of the regression model (2.32) by OLS. This provided consistent estimates of μ and $\theta_{11}, \ldots, \theta_{14}$, which are shown in the first column of table 1, as well as of $\ell n\ \theta_{10}$. The implied estimates of θ_{10}, $\gamma_k = \bar{\gamma}_k \cdot \mu$ and $h = \bar{h} \cdot \mu$, which are also consistent, are shown in the table. These estimates were used as the starting values for a Newton-Raphson iteration of the normal equations for the log-likelihood function. The resulting 2SML estimates are shown in the first column of table 2.

TABLE 1

CONSISTENT TWO-STAGE ESTIMATES OF DEMAND MODELS

COEFFICIENT/ VARIABLE	DEMAND MODEL			
	BLACKBURN	LOG-LOG	SEMI-LOG	PIGLOG[b]
γ_1 COD	-0.0164		-.0161	
γ_2 PHOS	-9.4114		-9.7673	
γ_3 NUISANCE	-0.2979		-0.363	
γ_4 SITE TYPE	-1.1965		-1.2883	
γ_5 MINORITY ATT	0.8773		0.3418	
h	0.7675		N/A	
μ	0.5607		0.465	
ρ	N/A	0.0941	0.1139	N/A
η	N/A	-0.1252	$2.9 * 10^{-6}$	$-9.61 * 10^{-4}$
θ_{10}	1.4024	1.5959	0.5335	0.0012
θ_{11} EDUC	0.2804	0.7038	0.6421	0.3812
θ_{12} SWIMPOOL	-0.7303	-0.8131	-0.8478	-0.1518
θ_{13} ADULTS	0.4185	0.4733	0.4623	0.1327
θ_{14} CHILDREN	0.0826	-0.0114	-0.0105	0.0291
$\ell n\ L$[c]	-199.68		-167.86	
R^2[d]	0.263	0.356	0.354	0.268

[a]The sample is 83 households. The estimates of $\gamma_1, \ldots, \gamma_5$ and μ are the same for the LOG-LOG, SEMI-LOG, and PIGLOG demand models.

[b]Income measured in thousands of dollars.

[c]Value of log-likelihood function for discrete choice equation.

[d]R^2 for least squares regression of continuous choices; note that different models have different dependent variables.

TABLE 2

2SML ESTIMATES OF DEMAND MODELS [a]

COEFFICIENT	DEMAND MODEL		
	BLACKBURN	LOG-LOG	SEMI-LOG
γ_1	-0.0202 (3.74)	-0.0177 (6.18)	-0.0166 (6.07)
γ_2	-9.4137 (3.89)	-9.2811 (4.89)	-9.2118 (5.11)
γ_3	-0.3855 (1.77)	-0.3297 (2.33)	-0.313 (2.32)
γ_4	-1.4106 (4.39)	-1.2593 (7.58)	-1.0967 (7.95)
γ_5	0.8314 (4.70)	0.3388 (2.50)	0.3316 (2.36)
h	0.8309 (8.79)	N/A	N/A
μ	0.6009 (22.01)	0.4939 (36.26)	0.4702 (38.57)
ρ	N/A	0.101 (54.00)	0.1156 (55.26)
η	N/A	-0.1125 (9.53)	$4.22 * 10^{-6}$ (3.28)
θ_{10}	1.5165 (2.89)	1.5149 (7.44)	0.581 (9.70)
θ_{11}	0.3265 (3.05)	0.6611 (20.48)	0.6167 (17.82)
θ_{12}	-0.8612 (5.56)	-0.905 (25.08)	-0.8772 (22.44)
θ_{13}	0.4418 (14.83)	0.472 (41.73)	0.4601 (45.71)
θ_{14}	0.0945 (3.51)	-0.0157 (1.89)	-0.0108 (1.37)
$\ell n\ L$	-506.15	-961.46	-1,077.73

[a] The sample is 83 households. The starting values of the coefficients for the Newton-Raphson iteration are those given in Table I. The numbers in brackets are the absolute values of the asymptotic t-statistics.

In the case of the other demand models, the logit model of the discrete choices yielded estimates of $\bar{\gamma}_1, \ldots, \bar{\gamma}_5$ and $(1/\mu)$ from which estimates of μ and $\gamma_1, \ldots, \gamma_5$ were obtained; these are shown in the upper portion of the third column of table 1. Next, the continuous choices were estimated using (2.31) for the LOG-LOG model, (A.2) for the SEMI-LOG model, and (A.9) for the PIGLOG model. The first two of these regressions were estimated by OLS and the last by nonlinear least squares. The resulting coefficient estimates are shown in the lower portion of table 1. These were then employed as the starting values in a Newton-Raphson iteration. The resulting 2SML estimates for the LOG-LOG and SEMI-LOG models are shown in table 2; because of numerical difficulties, the 2SML estimates could not be obtained for the PIGLOG model.[11]

It should be noted that the coefficient estimates for the LOG-LOG, SEMI-LOG, and PIGLOG models all satisfy the conditions required for x_j to be an essential good: the estimates of ρ in the LOG-LOG and SEMI-LOG models are each less than unity, and the implied estimates of θ in the PIGLOG model satisfy $0 < \theta < 1$ for every household. Moreover, the coefficient estimates ensure that each of these three indirect utility functions is quasiconvex at the sample values of the variables.[12]

In order to get a feel for the implications of these coefficient estimates and their differences across the various demand models, it is useful to focus on the implied own price and income elasticities of demand. Using (2.7), I will define the own price elasticity as

$$p_j E\{x_j\}^{-1} \frac{\partial E\{x_j\}}{\partial p_j} = p_j E\{x_j | \varepsilon \in A_j\}^{-1} \frac{\partial E\{x_j | \varepsilon \in A_j\}}{\partial p_j} + p_j \pi_j^{-1} \frac{\partial \pi_j}{\partial p_j} . \qquad (3.2)$$

The first term on the right-hand side of (3.2) is the price elasticity of demand given that brand j has been selected, while the second term is the price elasticity of the selection probability. I will refer to these as the continuous and discrete choice elasticities, respectively. The specific formulas for these elasticities corresponding to the various demand models are presented in table 3. The income elasticity of demand is defined similarly except that, from (2.15) and (2.25), $\partial\pi_j/\partial y = 0$. Thus,

$$yE\{x_j\}^{-1} \frac{\partial E\{x_j\}}{\partial y} = yE\{x_j | \varepsilon \in A_j\}^{-1} \frac{\partial E\{x_j | \varepsilon \in A_j\}}{\partial y} . \tag{3.3}$$

The specific formulas for the income elasticity are shown in the last column of table 3. Point estimates of these elasticities were calculated for each household in the sample, using the coefficient estimates of table 2 for the Blackburn, LOG-LOG and SEMI-LOG models and the coefficient estimates of table 1 for the PIGLOG model. The averages of these point estimates are shown in table 3.

The estimates of the own price elasticity for the different demand models are of a similar order of magnitude; the overall price elasticity ranges from -1.321 to -2.56. The estimates of the income elasticity display more variation which is not surprising because the underlying utility models involve very different Engel curves. As noted in section 2, the Blackburn model implies a zero income elasticity while the LOG-LOG model implies a constant income elasticity here estimated as -0.113. The other two models imply income elasticities which vary with the consumer's income or the budget share of the branded good. The mean-point estimate of the income elasticity is 0.054 for the SEMI-LOG model and -0.441 for the PIGLOG model.

TABLE 3

OWN PRICE AND INCOME ELASTICITIES OF DEMAND FOR RECREATION SITES[a]

MODEL	PRICE ELASTICITY					INCOME ELASTICITY	
	DISCRETE CHOICE		CONTINUOUS CHOICE		TOTAL		
Blackburn	$-p_j \frac{h}{\mu}(1-\pi_j)$	-1.245	$-p_j \pi_j h$	-0.076	-1.321	N/A	0
LOG-LOG	$-\frac{1}{\mu}(1-\pi_j)$	-1.481	$-[1+(\rho-1)\pi_j]$	-0.759	-2.239	η	-0.113
SEMI-LOG	"	-1.532	"	-0.753	-2.284	$y\eta$	-0.054
PIGLOG	"	-1.557	$-[1+\theta\eta\pi_j E\{w_j \mid \epsilon \in A_j\}^{-1}]$	-1.003	-2.56	$1+\eta E\{w_j \mid \epsilon \in A_j\}^{-1}$	-0.441

[a]Averages of point estimates over all households.

4. Welfare evaluations

Recall from (2.5) that $u = v(p, \psi, y)$ is the utility attained by the individual consumer when confronted with the choice set defined by (p, b, y). This is a known number for the consumer, but for the econometric investigator it is a random variable with a known probability distribution. In these circumstances it might be natural for the investigator to focus on the mean of this distribution, $E\{v\} \equiv V(p, b, y)$, in evaluating the consequences of a change in the choice set. Suppose that the available prices and qualities change from (p^0, b^0) to (p^1, b^1). By analogy with the Hicksian compensating variation of conventional utility theory, one could measure the effect of this change on the consumer's welfare by the quantity CV defined by

$$V(p^1, b^1, y - CV) = V(p^0, b^0, y). \tag{4.1}$$

CV is the amount of money that one would have to give the consumer after the change in order to render him as well off as he was before it where, because his preferences are partially unobservable, the welfare comparison is based on the investigator's expectation of his utility. An equivalent variation measure can be defined similarly.

In order to obtain $V(\cdot)$ it is convenient to first derive an expression for $E\{v_j | \varepsilon \in A_j\}$ and then calculate $E\{v\} = \Sigma\, \pi_j\, E\{v_j | \varepsilon \in A_j\}$. For the Blackburn model, for example, using (2.13) and (2.18) one obtains

$$E\{v\} = hy + \theta \left(\Sigma e^{\lambda_i/\mu} \right)^{\mu} \Gamma(1 - \mu), \tag{4.2}$$

while for the LOG-LOG model (2.26) using (2.23c) and (2.18) one obtains

$$E\{v\} = \frac{1}{1 - \eta}\, y^{1-\eta} + \frac{\theta}{\rho - 1} \left(\Sigma e^{\lambda_i/\mu} \right)^{\mu(\rho-1)} \Gamma[1 - \mu(\rho - 1)]. \tag{4.3}$$

Given the change in (p, b), λ_j, defined by (2.14), changes from λ_j^0 to λ_j^1. Let

$$Q_0 \equiv \Sigma e^{\lambda_i^0/\mu} \tag{4.4}$$

and

$$Q_1 \equiv \Sigma e^{\lambda_i^1/\mu}. \tag{4.5}$$

For the Blackburn model substitution of (4.2) into (4.1) yields

$$CV = \frac{1}{h}\,\theta\Gamma(1 - \mu)\ [Q_1^{\mu} - Q_0^{\mu}]. \tag{4.6}$$

For the LOG-LOG model substitution of (4.3) into (4.1) yields

$$\ell n(y - CV) = \frac{1}{1 - \eta}\,\ell n(y^{1-\eta} + \Delta) \tag{4.7}$$

where

$$\Delta \equiv \frac{1 - \eta}{\rho - 1}\ \theta\Gamma[1 - \mu(\rho - 1)]\left\{Q_0^{\mu(\rho-1)} - Q_1^{\mu(\rho-1)}\right\}.$$

This equation could be solved for CV by numerical techniques. Alternatively, one can write (4.7) as

$$\ell n\left(1 - \frac{CV}{y}\right) = \frac{1}{1 - \eta}\,\ell n\left(y^{1-\eta} + \Delta\right) - \ell n\ y$$

and then employ the approximation $\ell n(1 + x) \simeq x$ to obtain

$$CV \simeq y\ \ell n\ y - \frac{y}{1 - \eta}\,\ell n\left(y^{1-\eta} + \Delta\right). \tag{4.8}$$

The formulas for $E\{v\}$ and CV for the other two demand models, derived in a similar manner, are presented in the Appendix.

A related application of this welfare methodology is to analyze the effects of a change in the variety of brands available. Suppose that, in one case, there are N brands available with prices p^0 and qualities b^0 and, in another case, there are M (different) brands available with prices p^1 and qualities b^1. Assuming the "generic" specification of the ψ_j's in which $\alpha_j = 0$ all j, the effect of this change in variety on the consumer's welfare can be measured by the quantity CV given by the formulas (4.6) or (4.8) where Q_0 and Q_1 are defined by

$$Q_0 = \sum_{i=1}^{N} e^{\lambda_i^0/\mu} \text{ and } Q_1 = \sum_{i=1}^{M} e^{\lambda_i^1/\mu}.$$

Another application arises when one wishes to measure the value of the existence of a brand, i.e., the welfare loss sustained by the consumer if that brand were unavailable. With conventional, continuous demand models, this is approximated by the Marshallian triangle under the ordinary demand curve. The idea behind this calculation is that, if the brand were to become unavailable, this would be equivalent to its price rising from the current level to a level at which the demand for it fell to zero. In the present context with discrete/continuous choices, the relevant increase would be to a price of infinity since, from (2.15) and (2.25), this is required in order to drive the consumer's probability of selecting the brand to zero. Accordingly, suppose that one wishes to obtain the value of the jth brand. The CV measure is given by the formulas (4.6) or (4.8) where Q_0 is defined as in (4.4) while Q_1 is given by

$$Q_1 = \sum_{i \neq j} e^{\lambda_i^0/\mu} \tag{4.9}$$

since, when $p_j = \infty$, $e^{\lambda_j/\mu} = 0$.

As an illustration, I have applied this technique for measuring the value of the existence of a brand to the Boston area recreation sites, using the four demand models estimated in the preceding section. In the case of the Blackburn, LOG-LOG, and SEMI-LOG demand models, these values were calculated using the coefficient estimates in table 2 and the formulas (4.6), (4.8), and (A.5). In the case of the PIGLOG model, the values were calculated using the coefficient estimates in table 1 and the formula (A.13), together with the approximation $\Psi(1 - \mu\eta) \simeq \Psi(1) = -.57722$. Since p and θ are, in general, different for each household in the sample, I considered an average household and calculated the value of each of the 20 sites taken separately. To save space, only the estimates for the median site are presented in table 4.[13] These estimates are in 1974 prices and represent the median value of a site to an average household over the summer recreation season; in effect, they are annual values. It will be seen that the estimates vary across the different demand models, ranging from 7.3 cents per household with the SEMI-LOG model to 20.8 cents per household with Blackburn model.

5. Conclusions

In this paper I have presented several models of the demand for quality-differentiated goods which cover a variety of preference structures and yet are fairly simple to estimate. In these models consumers make a double choice--a discrete choice of which brand to select and a continuous choice

TABLE 4

ESTIMATES OF THE MEDIAN VALUE OF A RECREATION SITE
IN THE BOSTON AREA

DEMAND MODEL	MEDIAN VALUE OF A SITE
	cents
Blackburn	20.8
LOG-LOG	7.4
SEMI-LOG	7.3
PIGLOG	10.0

of how much of that brand to buy. Both choices flow from the same underlying utility maximization problem, and this provides restrictions on the coefficients and disturbance terms of the discrete and continuous choice equations which are incorporated into the estimation procedure. I have also shown how information from the fitted demand equations can be employed to construct monetary measures of the welfare effects of changes in the prices, qualities, or variety of the brands available to the consumer.

There are several directions for further research. In terms of the theoretical framework, one could develop alternative demand models by introducing the random element into the utility function in a different manner from that adopted here--in effect, telling a different economic story about the origin of the disturbance terms. One could also make a different assumption about their probability distribution. For example, the normal distribution could be employed in place of the EV distribution, yielding probit rather than logit models for the discrete choices.[14] A more substantial extension would be to endogenize the supply side, combining the demand models presented here with a discrete/continuous model of producer supply, in which firms decide which brands to manufacture and either how much to supply or what price to set. Such a development would generalize Bresnahan's (1981) model of markets for differentiated goods. Among the econometric issues, perhaps the most pressing is the problem of model discrimination. The various demand models presented here are not nested within one another. As the empirical application to recreation site choices shows, they can lead to quite different estimates of demand elasticities and welfare measures for changes in prices or qualities. It would be useful, therefore, to have some formal statistical criterion for discriminating among them. As an informal test, one can compare the values of the likelihood function, as suggested by Sargan (1964). If this is applied to the three demand models in table 2, the Blackburn model is ranked best and the SEMI-LOG model worst. However, it certainly would be desirable to extend Pesaran and Deaton's (1978) procedure for testing nonnested, nonlinear regression models to switching regressions of the type considered here.

Appendix

Here I record the formulas corresponding to (2.19), (2.21), (4.2), and (4.4) for the SEMI-LOG and PIGLOG demand models. For the SEMI-LOG model, (2.27), one obtains

$$f_{x_j|\varepsilon\epsilon A_j}(x) = (\theta/\rho_j)^{1/\mu(\rho-1)}\ e^{\eta y/\mu(\rho-1)}\ x^{-[\mu(\rho-1)+1]/\mu(\rho-1)}\ \Sigma e^{\lambda_i/\mu}$$

$$*\exp\left[-(\theta/\rho_j)^{1/\mu(\rho-1)}\ e^{\eta y/\mu(p-1)}\ x^{-1/\mu(p-1)}\ \Sigma e^{\lambda_i/\mu}\right](\mu|\rho - 1|)^{-1} \tag{A.1}$$

$$E\{\ell n\ p_j x_j|\varepsilon \in A_j\} = \ell n\ \theta + \eta y + (\rho - 1)\left[\mu\ \ell n\left(\Sigma\varepsilon^{\lambda_i/\mu}\right) + 0.57722\ \mu\right]. \tag{A.2}$$

The Slutsky matrix is negative semidefinite if $\eta p_j x_j \leq \rho$. Given that $\rho \leq 1$ x_j is essential, while

$$\Pr\{z > 0\} = 1 - \exp\left[-y^{-1/\mu(\rho-1)}\ e^{\eta y/\mu(\rho-1)}\ \theta^{1/\mu(\rho-1)}\ \Sigma e^{\lambda_i/\mu}\right]. \tag{A.3}$$

The formula for the expected unconditional indirect utility is

$$E\{v\} = -\frac{e^{-\eta y}}{\eta} + \left(\frac{\theta}{\rho - 1}\right)\left[\Sigma e^{\lambda_i/\mu}\right]^{\mu(\rho-1)}\ \Gamma[1 - \mu(\rho - 1)]. \tag{A.4}$$

Substitution into (4.1) yields a direct solution for CV

$$CV = \frac{1}{\eta}\ \ell n(1 + e^{\eta y}\ \Delta) \tag{A.5}$$

where

$$\Delta \equiv \frac{\eta\theta}{\rho - 1} \Gamma[1 - \mu(\rho - 1)]\{Q_1^{\mu(\rho-1)} - Q_0^{\mu(\rho-1)}\}.$$

For the PIGLOG demand model, (2.28), the conditional ordinary demand functions are

$$\begin{aligned}\bar{x}_j(p_j, \psi_j, y) &= \left[\frac{\theta y\psi_j}{p_j} + \frac{\eta y\psi_j}{p_j} \ln y - \frac{\eta y\theta\psi_j}{p_j} (\ln p_j - \ln \psi_j)\right] \psi_j^{-1} \\ &= \frac{\theta y}{p_j} + \frac{\eta y}{p_j} \ln y - \frac{\eta y\theta}{p_j} \ln p_j + \frac{\eta y\theta}{p_j} (\alpha_j + \Sigma\gamma_k b_{jk} + \varepsilon_j).\end{aligned} \tag{A.6}$$

For this demand model, it is more convenient to work with the budget share, $w_j \equiv p_j x_j / y$. In terms of this variable, (A.6) becomes

$$w_j = \theta + \eta \ln y + \eta\theta\lambda_j + \eta\theta\varepsilon_j. \tag{A.7}$$

Making a change of variable in (2.16) from ε_j to w_j, one obtains

$$f_{w_j|\varepsilon\epsilon A_j}(w) = y^{1/\theta\mu} e^{(\theta-w)/\eta\theta\mu} \Sigma e^{\lambda_i/\mu} \exp\left[-y^{1/\theta\mu} e^{(\theta-w)/\eta\theta\mu} \Sigma e^{\lambda_i/\mu}\right] (\theta\mu|\eta|)^{-1}. \tag{A.8}$$

The conditional mean of w_j is

$$E\{w_j|\varepsilon \epsilon A_j\} = \eta \ln y + \theta + \theta\eta\left\{\mu\left[\ln \Sigma e^{\lambda_i/\mu}\right] + 0.57722 \mu\right\}. \tag{A.9}$$

The Slutsky matrix is negative semidefinite if $w_j^2 + \eta w_j - w_j - \eta\theta \leq 0$; given that $0 < \theta < 1$, both x_j and z are essential.

From (2.23c) and (2.28a), the conditional indirect utility function is

$$\bar{v}_j(p_j, \psi_j, y) = (\ell n\ y - \theta\ \ell n\ p_j + \theta\ \ell n\ \psi_j)\ p_j^{-\eta}\psi_j^{\eta}$$

$$= (\ell n\ y + \theta\lambda_j)\ e^{\eta\lambda_j} e^{\eta\varepsilon_j} + \theta e^{\eta\lambda_j}\ \varepsilon_j\ e^{\eta\varepsilon_j}.$$

Its mean is

$$E\{v_j | \varepsilon \in A_j\} = (\ell n\ y + \theta\lambda_j)\ e^{\eta\lambda_j}\ E\left\{e^{\eta\varepsilon_j} \middle| \varepsilon \in A_j\right\} + \theta e^{\eta\lambda_j}\ E\left\{\varepsilon_j\ e^{\eta\varepsilon_j} \middle| \varepsilon \in A_j\right\}. \quad (A.10)$$

The first conditional mean on the right-hand side of (A.10) is given by (2.18); the second can be evaluated as follows

$$E\left\{\varepsilon_j\ \varepsilon^{\eta\varepsilon_j} \middle| \varepsilon \in A_j\right\} = \int_{-\infty}^{\infty} \varepsilon_j\ e^{\eta\varepsilon_j}\ f_{\varepsilon_j|\varepsilon\in A_j}(\varepsilon_j)d\varepsilon_j$$

$$= \int_{-\infty}^{\infty} \varepsilon e^{\eta\varepsilon}\ \frac{\beta_j}{\mu}\ e^{-\varepsilon/\mu}\ \exp(-\beta_j e^{-\varepsilon/\mu})d\varepsilon \quad (A.11)$$

$$= \mu\beta_j^{\mu\eta}\ \Gamma(1 - \mu\eta)[\ell n\ \beta_j - \Psi(1 - \mu\eta)]$$

where $\Psi(\cdot)$ is Euler's psi-function. Substituting (2.18) and (A.11) into (A.10) yields

$$E\{v\} = \left[\Sigma\varepsilon^{\lambda_i/\mu}\right]^{\mu\eta}\ \Gamma(1 - \mu\eta)\left\{\ell n\ y + \theta\mu\ \ell n\left[\Sigma e^{\lambda_i/\mu}\right] - \theta\mu\Psi(1 - \mu\eta)\right\}. \quad (A.12)$$

Applying (4.1), one finds that CV satisfies

$$\ell n(y - CV) = (Q_0/Q_1)^{\mu\eta}\ \{\ell n\ y + \theta\mu\ \ell n\ Q_0 - \theta\mu\ \Psi(1 - \mu\eta)\} - \theta\mu\ \ell n\ Q_1 + \theta\mu\ \Psi(1 - \mu\eta)$$

$$\equiv \Delta.$$

As with the LOG-LOG model, the solution for CV can be obtained numerically, or it can be approximated by

$$CV \simeq y \ln y - y \Delta. \tag{A.13}$$

Footnotes

[1]Bresnahan (1981) provides an empirical model of the supply and demand for differentiated products in the U. S. automobile market, but he imposes this restriction on consumer demands.

[2]A fuller account of the general structure of discrete/continuous demand models is presented in Hanemann (1982).

[3]Blackburn never explicitly presents this formula for the utility function, but it is implicit in his analysis.

[4]Some normalization is still required in order to estimate (2.15) since it is invariant to multiplication of both the numerator and the denominator by e^ρ, for an arbitrary constant ρ. An appropriate normalization would be to set $\alpha_i = 0$ for one index i.

[5]The c.d.f. corresponding to (2.19) is $F_{x_j|\varepsilon \epsilon A_j}(x) = \exp[-\theta^{1/\mu} x^{-1/\mu} \Sigma e^{\lambda_i/\mu}]$. Thus, $\Pr\{z > 0|\varepsilon \epsilon A_j\} = F_{x_j|\varepsilon \epsilon A_j}(y/p_j) = \exp[-(\theta p_j/y)^{1/\mu} \Sigma e^{\lambda_i/\mu}]$, and $\Pr\{z > 0\} = \Sigma \Pr\{z > 0|\varepsilon \epsilon A_j\}\pi_j$.

[6]See, for example, Muellbauer (1975).

[7]The quasiconvexity of the indirect utility function or, equivalently, the negative semidefiniteness of the Slutsky matrix is satisfied if $\eta p_j \bar{x}_j \leq \rho y$. In the empirical application below, this is tested by comparing the estimate of ρy with the estimate of $\eta p_j E\{x_j|\varepsilon \epsilon A_j\}$.

[8]From (2.30), $\Pr\{z > 0\} = 1 - \exp[-y^{(\eta-1)/\mu(\rho-1)} \theta^{1/\mu(\rho-1)} \Sigma e^{\lambda_i/\mu}]$.

[9]However, the usual formula for the covariance matrix of these coefficient estimates is incorrect [see Heckman (1979)].

[10]For details, see Lee and Trost (1978, pp. 368-69).

[11]When one fits the PIGLOG regression model (A.9) to data where the actual budget shares are very small (the median budget share in the sample is 0.14 percent), one obtains very small values for the estimates of η and θ. This can be seen from the estimates of η and θ_{10} in table 1. However, the likelihood function for the PIGLOG model, (A.8), involves the terms $y^{1/\theta\mu}$ and $e^{1/\eta\mu}$, and these terms explode when η and θ are small in absolute value.

[12]It was also noted above that the Blackburn, LOG-LOG, and SEMI-LOG models do not preclude the possibility that $z \leq 0$. However, when one uses the coefficients in table 2 to calculate $\Pr\{z > 0\}$ for each model, as indicate in footnotes 5 and 8 and (A.3), this probability exceeds 0.9999 for every household in the sample.

[13]Detailed results are available on request.

[14]The formulation of such normal models is discussed in Hanemann (1982).

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Appendix: Data

The recreation sites and their locations are:

Lynn Beach (Lynn), Nahant Beach (Nahant), Revere Beach (Revere), Constitution Beach/Orient Heights (Boston), Castle Island (Boston), City Point (Boston), L&M Street Beaches (Boston), Carson Beach (Boston), Malibu Beach/Savin Hill (Boston), Tenean Beach (Boston), Wollaston Beach (Quincy), Nantucket Beach (Hull), Wingaersheek Beach (Gloucester), Crane's Island (Ipswich), Plum Island (Newberry), Duxbury Beach (Duxbury), White Horse Beach (Plymouth), Wright's Pond (Medford), Walden Pond (Concord), and Cochituate State Park (Natick).

The last three are freshwater beaches; the others are all saltwater ocean beaches.

The variables are:

Dependent variable: x_{it} = the number of visits for swimming and beach recreation activities to site i by any members of household it during the period from Memorial Day to Labor Day, 1974, $i = 1, \ldots, 20$, $t = 1, \ldots 83$

Household characteristics:

$EDUC_t$ = highest level of educational attainment in household t (1 = elementary/junior high; 2 = some high school; 3 = completed high school; 4 = some college including junior college; 5 = vocational/technical school; 6 = completed college; 7 = postgraduate)

$SWIMPOOL_t$ = 1 if household t had access to a private swimming pool during Summer, 1974, 0 otherwise

$CHILDREN_t$ = number of persons under 18 in household t

$ADULTS_t$ = number of persons aged 18 and older in household t

y_t = total annual income after taxes of household t (\$)

Site characteristics:

COD_i = chemical oxygen demand of water at site i (mg/ℓ)

$PHOS_i$ = total phosphorus content of water at site i (mg/ℓ)

$SITE\ TYPE_i$ = 1 if freshwater site, 0 if saltwater site

$NUISANCE_i$ = 1 if site has heavily urban and/or noisy setting, 0 otherwise.

$MINORITY\ ATT_{it}$ = 1 if household t is from a minority group and site t is identified as having a special attractiveness to minorities, 0 otherwise.

p_{it} = cost (\$) of traveling by automobile from home of household t to site i.