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SPECIFICATION OF THE INDIVIDUAL'S DEMAND FUNCTION: THE TREATMENT OF TIME

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CHAPTER 5

SPECIFICATION OF THE INDIVIDUAL'S DEMAND FUNCTION: THE TREATMENT OF TIME*

Economists, especially those working in the area of recreational demand, have long recognized that time spent in consuming a commodity may, in some cases, be an important determinant of the demand for that commodity. It remains true, however, that even though the potential importance of time has been discussed at some length in the literature it is only relatively recently, and in a fairly small set of papers, that the problem of explicitly incorporating time into the behavioral framework of the consumer has been addressed.

This chapter provides a discussion of the ways in which researchers have traditionally incorporated time costs into recreational demand models and attempts to develop a more complete and general model. Improvements in both specification and estimation of the model draw on recent labor supply literature. An explanation of the nature of a decision model subject to two constraints is offered.

The treatment of time is one of the thorniest issues in the estimation of recreational benefits. A number of approaches to valuing time are currently in vogue, but no method is dominant and researchers often improvise as they see fit. Unfortunately, the benefit estimates associated with changes in public

* This Chapter is the work of Ivar E. Strand and Nancy E. Bockstael, Agricultural and Resource Economics, U. of Maryland, and W. Michael Hanemann, Agricultural and Resource Economics, U. of California, Berkeley.

recreation policy are extremely sensitive to these improvisations. Frank Cesario (1976), for example, found that annual benefits from park visits nearly doubled depending on whether time was valued at some function of the wage rate or treated independently in a manner suggested by Frank Cesario and Jack Knetsch (1970). More recently, Richard Bishop and Thomas Heberlein (1980) presented travel cost estimates of hunting permit values which differed four-fold when time was valued at one-half the median income and when time was omitted altogether from the model.

Recreational economists (e.g. Kenneth McConnell, 1975) have understood the applicability of the classical labor-leisure trade-off to this problem. In his 1975 article McConnell was the first to discuss the one vs. two constraint model. Recognizing that time remaining for recreation may be traded off for work time or it may be fixed, he shows how the nature of the decision problem is addressed by the nature of the time constraint. This chapter begins with the work of McConnell and develops a careful and endogenous general framework for incorporating time. After discussing the wide range of complex labor constraints which the model can handle, we turn to making the model operational. The approach developed below not only incorporates a defensible method for treating the value of time but also permits sample selection bias (Chapter 4) to be addressed and exact measures of welfare (Chapter 3) to be derived.

Time in Recreational Decisions

Despite the general acceptance that time plays an important role in recreational decisons (e.g. V. Kerry Smith, et al., 1983), no universally accepted method for incorporating time into recreational demand analysis has emerged and methods for "valuing" time in recreational demand models are numerous. While many have been developed from assumptions based on utility maximizing behavior, there is no consensus as to which is the "correct" method. In actual applications, researchers have often been forced to take a relatively ad hoc view of the problem by incorporating travel time in an arbitrary fashion as an adjustment in a demand function or, alternatively, by asking people what they would be willing to pay to reduce travel time.

Ad hoc econometric specifications or general willingness-to-pay questions are particularly problematic with respect to time valuation because time is such a complex concept. Time, like money, is a scarce resource, for which there is a constraint. Anything which uses time as an input consumes a resource for which there are utility-generating alternatives. While time is an input into virtually every consumption experience, some commodities take especially large amounts of time. These have frequently been modeled in a household production framework to reflect the individual's need to combine input purchases with household time to "produce" a commodity for consumption. Because time is an essential input into the production of any commodity which we might call an "activity", time is frequently used as a measure of that activity. Thus, while time is formally an input

into the production of the commodity, it may also serve as the unit of measure of the output.

The complexity of time's role in household decisions has implications for both travel and on-site recreational time. Both represent uses of a scarce resource and thus have positive opportunity costs. However, on-site time, and sometimes travel time, are used as units of measure of the utility generating activities themselves. Economists often measure the recreational good in terms of time, i.e. in hours or days spent at the site. Travel time may also be a measure of a utility generating activity, if the travel is through scenic areas or if it involves other activities such as visiting with travelling companions. Hence, direct questioning or poorly conceived econometric estimation may yield confusing results because the distinction between time as a scarce resource and time as a measure of the utility generating activity is not carefully made.

Does time belong in the utility function? Viewed as a scarce resource, time by itself does not belong in the utilty function. What does enter the utility function is a properly conceived measure (perhaps in units of time) of the quantity and quality of the recreational activity. This does not present major problems when the commodity is defined in terms of fixed units of on-site time and when travel does not in itself influence utility levels. Even when on site time is endogenous, however, it must be appropriately accounted for as a use of the scarce resource. Its exclusion from the model will bias results since the same on-site time cost will be valued differently by

people with different values of time. When time per trip is a decison variable, an appropriate and tractable measure is not easily conceived. This Chapter focuses solely on time as a scarce resource.

Time as a Component of Recreational Demand: A Review

The fact that time costs could influence the demand for recreation was recognized in the earliest travel cost literature (Marion Clawson, 1959; Marion Clawson and Jack Knetsch, 1966), although no attempt was made to explicitly model the role of time in consumer behavior.

The problems which arise when time is left out of the demand for recreation were first discussed by Clawson and Knetsch (1966). Cesario and Knetsch (1970) later argued that the estimation of a demand curve which ignored time costs would overstate the effect of price change and thus understate the consumer surplus associated with a price increase.

In practical application, both travel cost and travel time variables have usually been calculated as functions of distance. As a result, including time as a separate variable in the demand function tended to lead to multicollinearity. William Brown and Faras Nawas (1973) and Russell Gum and William Martin (1975) attempted to deal with the multicollinearity issue by suggesting the use of individual trip observations rather than zonal averages. In contrast, Cesario and Knetsch (1976) proposed combining all time costs and travel costs into one cost variable to eliminate the problem of multicollinearity. These papers had a primarily empirical focus, with emphasis given to getting

estimates. Demand functions were specified in an ad hoc way, with no particular utility theoretic underpinnings.

Bruce Johnson (1966) and Kenneth McConnell (1975) were among the first to consider the role of time in the context of the recreationalist's utlity maximization problem (although others had considered it in other consumer decision problems). McConnell specified the problem in the framework of the classical labor-leisure decision. The individual maximizes utility subject to a constraint on income and time. The income constraint is defined by his wage rate, w, such that

$$F(t_w) = px + \Sigma c_j r_j \tag{1a}$$

where t_w is work time, $F(t_w)$ is wage income, p is the price of good x, r_j, is the quantity of recreational activity j and c_j is the money cost for one unit of r_j. His time constraint is

$$T = \Sigma a_j r_j + t_w \tag{1b}$$

$$T^* = \Sigma a_j r_j, \tag{1c}$$

where a_j is the money cost of a unit of r_j and the choice between (1b) and (1c) depends on whether or not recreational time is fixed. Solving (1b) for t_w and substituting into (1a) yields the maximization problem

$$\max U(x,r) - \lambda(px + \Sigma c_j r_j - F(T - \Sigma a_j r_j)),$$

so that time cost is transformed into a money cost at the implicit wage rate if recreation time is not fixed.

McConnell (1975) also noted that if individuals were unable to choose the number of hours worked, the direct substitution of (1a) into (1b) is not possible. He suggests that in this case one would still need to value time in terms of money before incorporating it in the demand function. This is conceptually possible, since at any given solution there would be an amount of money which the individual would be just willing to exchange for an extra unit of time so as to keep his utility level constant. Unfortunately, this rate of trade-off between money and time, unlike the wage rate, is neither observable nor fixed. It is itself a product of the individual's utility maximizing decision. McConnell also suggested questions which might be asked in surveys to determine whether (1b) or (1c) was the appropriate constraint.

Much of the recent recreation demand literature follows this line of reasoning and relates the opportunity cost of time in some way to the wage rate. Of the many models of this sort, the one offered by Kenneth McConnell and Ivar Strand (1981) is one of the most recent in this vein (see also Frank Cesario, 1976; Smith and Kavanaugh; Nichols et al.). Their work demonstrates a methodology from which a factor of proportionality between the wage rate and the unit cost of time can be estimated within the traditonal travel cost model.

More recently, V. Kerry Smith, William Desvousges and Matthew McGivney (1983) have attempted to modify the traditional recreational demand model so that more general constraints on individual use of time are imposed. They consider two time

constraints, one for work/non-recreational goods and another for recreational goods. The available recreation time cannot be traded for work time. The implications of their model suggest that when time and income constraints cannot be reduced to one constraint, the marginal effect of travel and on-site time on recreational demand is related to the wage rate only through the income effect and in the most indirect manner. Unfortunately, their model "does not suggest an empirically feasible approach for treating these time costs" (p. 264). For estimation, they confine themselves to an ad hoc modification of a traditonal demand specification.

Researchers are thus left with considerable confusion about the role of the wage rate in specifying an individual's value of time. But there is an important body of economics literature, somewhat better developed, which has attempted to deal with similar issues. Just as the early literature on the labor-leisure decision provided initial insights into the modelling of time in recreational demand, more recent literature on labor supply behavior provides further refinement.

Labor Supply Literature: A Review

The first generation of labor supply models resembled the traditional recreational demand literature in a number of ways. These models treated work time as a continuous choice variable. A budget constraint such as that depicted in Figure 5.1 was assumed for each individual, suggesting the potential for a continuous trade-off between money and leisure time at the wage rate, w. In this graph, E is non-wage income and T is total

available hours. Participants in the labor force were assumed to be at points in the open interval (BC) on the budget line, equating their marginal rates of substitution between leisure and goods to the wage rate. Those who did not participate were found at the corner solution B.

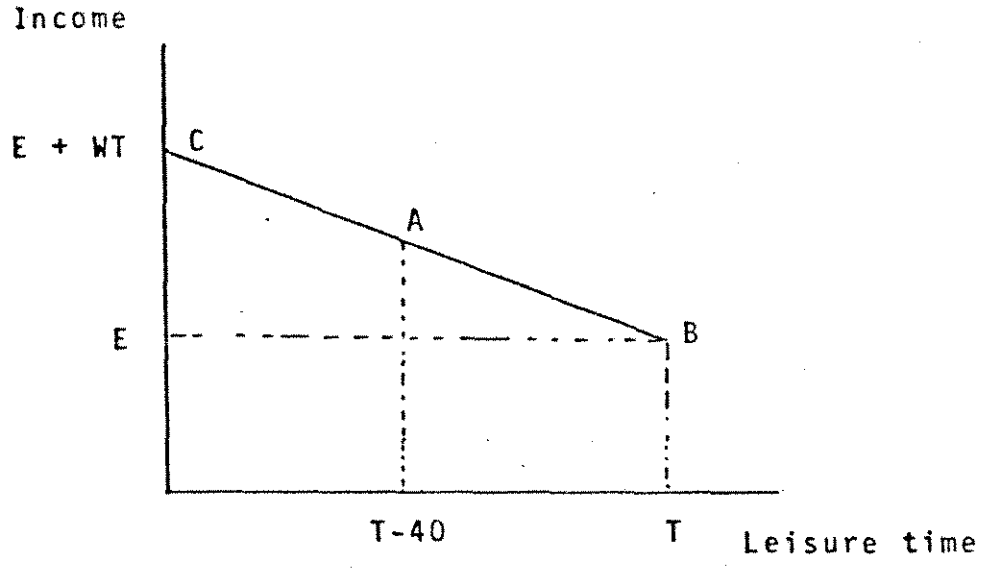

Figure 5.1: The first generation budget constraint

Other researchers argued that work time may not be a choice variable. Individuals might be "rationed" with respect to labor supply in a "take-it-or-leave-it" fashion, that is they may be forced to choose between a given number of work hours (say 40 hours/week) or none at all (Perlman, 1966; Mossin and Bronfenbrenner, 1967). In this context, there is no opportunity for marginally adjusting work hours, and all individuals are found at one of two corner solutions (A or B in Figure 5.1).

While useful in characterizing the general nature of a time allocation problem, first generation labor supply models were criticized on both theoretical and econometric grounds. These concerns fostered a second generation of labor supply research which made improvements in modelling of constraints and in estimating parameters as well as making models more consistent with utility maximizing assumptions (see Mark Killingsworth, 1983, p. 130-1). Each of these areas of development have implications for the recreation problem.

The second generation of labor supply literature (see for example O. Ashenfelter, 1980; J. C. Ham, 1982; Gary Burtless and Jerry Hausman, 1978) generalized the budget line to reflect more realistic assumptions about employment opportunities. As Killingsworth states in his survey, "...the budget line may not be a straight line: Its slope may change (for example, the wage a moonlighter gets when he moonlights may differ from the wage he gets at his 'first' job), and it may also have 'holes' (for example, it may not be possible to work between zero and four hours)".

To appreciate this point, consider an example: an individual whose primary job requires T_p hours per week within a total time constraint of T hours per week. The relevant wage rate at this primary job is w_p and is depicted in Figure 5.2 as the slope of the implied line segment between A and B. This individual can earn more wage income only by moonlighting at a job with a lower wage rate (depicted by the slope of the segment between A and C). His relevant budget line is segment AC and point B. Depending on his preference for goods and leisure, he may choose not to work and be at B; he may work a fixed work week at A; or he may take a second job and be along the segment BC. Consideration of more realistic employment constraints such as these have implications for model specification. Only those individuals who choose to work jobs with flexible work hours (such as second jobs or part-time jobs) can adjust their marginal rates of substitution of goods for leisure to the wage rate. All others can be found at corner solutions where no such equimarginal conditions hold.

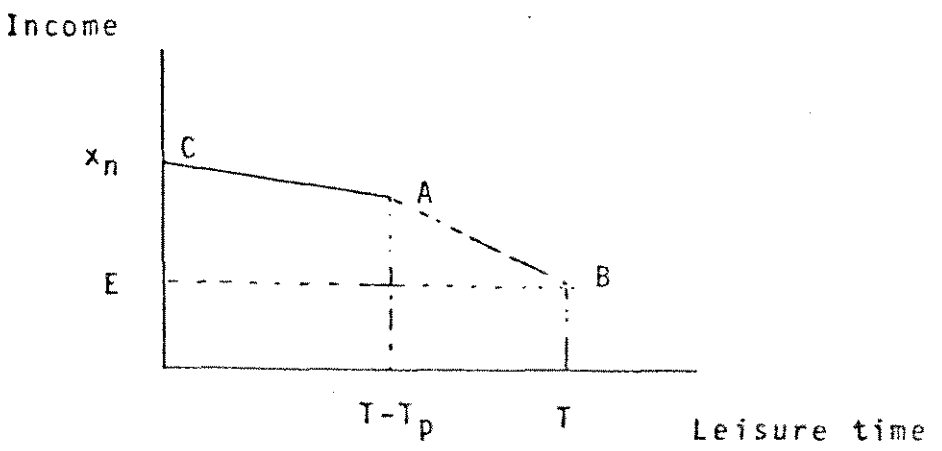

Figure 5.2: Second generation budget constraints

Two other aspects of the second generation labor supply models are noteworthy. The first generation studies estimated functions which were specified in a relatively ad hoc manner. By contrast, second generation models have tended to be utility-theoretic. This has been accomplished by deriving specific labor supply functions from direct or indirect utility functions (James Heckman, Mark Killingsworth, and T. MacCurdy, 1981; Gary Burtless and Jerry Hausman, 1978; Wales and Woodland, 1976, 1977). Such utlity-theoretic models have particular appeal for recreational benefit estimation because they allow estimation of exact welfare measures. Additionally, first generation research was concerned either with the discrete work/non-work decision or with the continuous hours-of-work decision. Second generation empirical studies recognized the potential bias and inefficiency of estimating the two problems independently and employed estimation techniques to correct for this.

A Proposed Recreational Demand Model

It is clear that the nature of an individual's labor supply decision determines whether his wage rate will yield information about the marginal value of his time. In the recreational literature, researchers have conventionally viewed only two polar cases: either individuals are assumed to face perfect substitutability between work and leisure time or work time is assumed fixed. The choice between these two cases is less than appealing. Few people can be considered to have absolutely fixed work time, since part-time secondary jobs are always possible. On the other hand, only some professions allow free choice of

work hours at a constant wage rate. Additionally no sample of individuals is likely to be homogeneous with respect to these labor market alternatives. A workable recreation demand model must reflect the implications which labor decisions have on time valuation and allow these decisions to vary over individuals.

In developing a behavioral model that includes time as an input it is useful to broaden the description of the nature of the decision problem beyond the simple travel cost framework. The more general household production model depicts the individual maximizing utility by choosing a flow of recreational services, x_R, and a vector of other commodities, x_N. A vector of goods, S_R, is combined with recreation time, T_R, to produce x_R. Both time, T_N, and purchased inputs, S_N, may be required to produce x_N.

The individual's constrained utility maximizing problem can be represented as

(2)
$$\max_{S,T} U(x_R, x_N)$$

subject to
$$x_R = f(S_R, T_R),$$
$$x_N = g(S_N, T_N),$$
$$E + W = v_N{}'S_N + v_R{}'S_R,$$

and

$$T = T_W - T_R - T_N,$$

where U(...) is a quasi-concave, twice-differentiable utility function, f(...) and g(...) are vectors of quasi-convex, twice-differentiable production functions, E + W is the sum of the individual's non-wage and wage income, v_R and v_N are the price

vectors associated with the vectors of recreational and non-recreational inputs respectively, T_W is labor time supplied, and T is the total time available.

We reduce the problem by assuming (as do Oscar Burt and Thomas Brewer, 1971, and others before us) a Leontief, fixed-proportions technology. This is equivalent to assuming that the commodities, i.e. the x's, have fixed time and money costs per unit given by t and p, respectively. For the recreation good, x_R, it implies that a unit of x_R (e.g. a visit) has a constant marginal cost (p_R) and fixed travel and on-site time requirements (t_R). All other commodities are subject to unit money or time costs and the general problem becomes

$$\text{(3)} \qquad \max_{x_R, x_N} \quad U(x_R, x_N)$$

subject to $\quad E + W - p_R' x_R - p_N' x_N = 0,$

and $\quad T - T_W - t_R' x_R - t_N' x_N = 0,$

were p and t are the unit money and time prices of the x's.

In order to characterize an individual's solution to the problem posed in (3), it is necessary to know the nature of the labor market constraints. We consider two possibilities. First, an interior solution may be achieved, such as along line segment AC in Figure 5.2. He can adjust work time such that his marginal rate of substitution between leisure and goods equals his effective (marginal) wage rate. As Killingsworth points out, this is most likely to be true for individuals who work overtime or secondary jobs, but may also be true for those with part-time jobs and those (e.g. the self-employed) with discretion over their work time. An individual may, alternatively, be at a

corner solution such as point A or B in Figure 5.2. Point B is associated with unemployment, while an individual at point A works some fixed work week at wage w_p and has the opportunity to work more hours only at a lower wage. In neither case is there a relationship between the wage rate the individual faces and his valuation of time.[1]

Strictly speaking, the problem in (3) requires the simultaneous choice of T_w and all of the x's. In order to avoid the complexity of estimating both the labor supply and recreation demand decisions, an individual's decision as to his location on the labor constraint is treated as an initial condition and the implications of this decision for recreational demand analysis is examined.

The problem as posed in (3) is restated and the first-order conditions provided, given alternative solutions to the labor supply problem. For individuals at corner solutions (such as B or A in Figure 5.2), the problem becomes

$$\text{(4)} \qquad \text{Max } U(x) + \lambda(Y - \Sigma p_i x_i) + \mu(T - \Sigma t_i x_i)$$

where Y is effective income (including the individual's wage income if he works and nonwage income which may include the individual's share of the earnings of other household members). The variable T is time available (after work) for household production of commodities, including recreation.

First order conditions are

$$\partial u/\partial x_i - \lambda p_i - \mu t_i = 0$$

(4a) $$Y - \Sigma p_i x_i = 0,$$

$$T - \Sigma t_i x_i = 0.$$

Note that since work time cannot be adjusted marginally, the two constraints are not collapsible. Solving (4a) for the demand for x_R yields a demand functon of the general form

(4b) $$x_i = h^c(p_i, t_i, p^o, t^o, Y, T) + \epsilon$$

where p^o and t^o are the vectors of money and time costs of all other goods and ϵ is the random element in the model. (The properties of this demand function are detailed in the Appendix to this Chapter.)

For an interior solution in the labor market, however, at least some component of work time is discretionary and time can be traded for money at the margin. Thus, the time constraint can be substituted into the income constraint, yielding

$$Y + w_D T - \Sigma(p_i + w_D t_i)x_i = 0$$

where w_D is the wage rate applicable to discretionary employment, and Y is income from non-discretionary sources, i.e. the individual's nonwage income plus any income earned at jobs with fixed time requirements. The variable T is discretionary time, i.e. time available for discretionary work and household production.

The maximization problem conditioned on an interior solution to the labor supply decision is

$$\text{(5)} \qquad \underset{x}{\text{Max}}\ U(x) + \delta(Y + w_D T - \Sigma(p_i + w_D t_i)x_i).$$

First order conditions are

$$\text{(5a)} \qquad \partial u/\partial x_i - \delta(p_i + w_D t_i) = 0$$

$$Y + w_D T - \Sigma(p_i + w_D t_i)x_i = 0.$$

Solving for the general form of recreational demand yields

$$\text{(5b)} \qquad X_i = h^I(p_i + w_D t_i,\ p^o + w_D t^o, Y + w_D T) + \varepsilon.$$

Note that, for empirical purposes, the term $Y + w_D T$ can be re-expressed as $Y + w_D t_D + w_D(T - t_D)$ where t_D is discretionary work time, Y is total income, and $T-t_D$ is the time available for household production (or total time minus all hours worked).

Consideration of demand functions (4b) and (5b) suggests that the data requirements of estimation are not overly complex or substantial. In addition to the usual questions about income, time and money costs, one need only ask whether or not the individual has discretion over any part of his work time. If he does, his discretionary wage must be elicited.

In problem (5) the recreational demand function is conditioned on the individual having chosen an interior solution in the labor market. The wage rate (w_D) reflects the individual's value of time because work and leisure can be traded-off marginally. However, when this is not the case as in problem (4), the marginal value of the individual's time in other uses is not

equal to the wage rate he faces. As a result we have no observable evidence as to the value of his time. However no such evidence is necessary for recreation demand estimation since, from (4a-b) we see that the time price enters the demand function directly.

Considerations for Estimating Recreational Benefits

In order to estimate recreational demand functions and thus derive benefit estimates, it is necessary to define a specific form for the demand equation and to postulate an error structure. In previous chapters, the nature of this choice, as well as the means by which exact welfare measures can be obtained, has been discussed.

The method of integrating back to a utility function from a demand function has been shown only for the case where the demand function derives from utility maximization subject to a budget constraint. The nature of the labor market decision complicates the constraint set facing the individual. When the individual is at an interior solution the time and income constraints collapse, and the problem is formally analogous to the classical, one constraint problem. However, when the individual is at a corner solution, he faces two noncollapsible constraints.

The comparative statics and general duality results of utility maximazation in the context of two constraints are developed in the Appendix to this Chapter. There, it is demonstrated rigorously that maximization under two linear constraints yields a demand function with properties analogous to the one constraint case. The demand function is still

homogeneous of degree zero, but in a larger list of arguments - money prices, time prices, income and time endowments. It also satisfies usual aggregation conditions. In addition, two duals are shown to exist - one which minimizes money costs subject to utility and time constraints and the other which minimizes time costs subject to utility and income constraints. Associated with each dual is an expenditure function and a compensated demand. Both income and time compensated demands are own price downward sloping and possess symmetric, negative semidefinite substitution matrices.

Integrating back from a demand function to an indirect utility function is not so straightforward with two constraints. Consequently it is useful to begin with a direct utility function and solve for recreational demand functions by maximizing utility subject to the appropriate constraint set. The form of the demand functions and the indirect utility function will depend on which constraint set is relevant. Rather than deal with the general model, we prefer showing a specific case which has been developed in Chapter 3.

The utility function chosen for illustration is

$$U(x) = \frac{(\gamma_1 + \gamma_2)x_1 + \beta}{(\gamma_1 + \gamma_2)^2} \quad \exp \quad \frac{(\gamma_1 + \gamma_2)(\alpha + \gamma_1 x_2 + \gamma_2 x_3 - x_1 + \varepsilon)}{(\gamma_1 + \gamma_2)x_1 + \beta}. \tag{7}$$

In the above expression α, β, γ_1, and γ_2 are parameters common to all individuals and ε represents a random element reflecting the distribution of preferences over the population. The random variable, ε, is assumed to be distributed normally with mean zero and constant variance, σ^2.

The recreational good is designated as x_1. We partition the set of other goods such that x_2 is a bundle of goods with money but no significant time costs. The bundle, x_2, is a numeraire such that the money price of recreation is normalized with respect to p_2. Hicksian bundle x_3 is a bundle of goods with time but not significant money costs and serves as a numeraire such that time prices are normalized with respect to t_3. Thus the general constraint set is

$$Y - p_1x_1 - p_2x_2 = 0$$

and

$$T - t_1x_1 - t_3x_3 = 0$$

where p_2 and t_3 are assumed to be equal to one forthwith.

Solving the system for the optimum value of x_1 and denoting $\beta/(\gamma_1 + \gamma_2)$ as β', yields ordinary recreational demand functions, conditioned on each labor supply decision, of the form

$$x_1 = \alpha + \gamma_1 Y + \gamma_2 T + \beta'\gamma_1 p_1 + \beta'\gamma_2 t_1 + \varepsilon \tag{8}$$

for individuals at corner solutions in the labor market, and

$$x_1 = \alpha + \gamma_1(Y + w_D T) + \beta_1(p_1 + w_D t_1) + \varepsilon \tag{9}$$

for individuals at interior solutions in the labor market.[2] These estimating equations are useful if they allow us to recover estimates of the parameters of our welfare measure.

We choose compensating variation of a price change which drives x_1 to zero as our money measure of welfare. The compensating variation expression found in Table 3.1 is directly applicable to the one constraint case, yielding

$$CV_1 = \frac{\beta_1}{\gamma_1^2} - \exp[\gamma_1(p_1 - p_1^o)] \frac{x^o}{\gamma_1} + \frac{\beta_1}{\gamma_1^2}$$

for the interior solution. Compensating variation for the two constraint case can be specified by first substituting demand functions into (7) to obtain the indirect utility function

$$V(p,t,Y,T) = \exp(-\gamma_1 p_1 - \gamma_2 t_1) \frac{\alpha + \gamma_1 Y + \gamma_2 T + \beta'\gamma_1 p_1 + \beta'\gamma_2 t_1 + \beta'}{\gamma_1 + \gamma_2}$$

and inverting to obtain the money expenditure function

$$m(p,t,V,T) = \frac{\gamma_1 + \gamma_2}{\gamma_1} U^o \exp(\gamma_1 p_1 + \gamma_2 t_1) - \frac{1}{\gamma_1}(\alpha + \gamma_2 T + \beta'\gamma_1 p_1 + \beta'\gamma_2 t_1 + \beta').$$

The compensating variation for a loss of the recreation good is then

$$CV_c = \frac{\beta'}{\gamma_1} - \exp[\gamma_1(p_1 - p_1^o)] \left(\frac{x^o}{\gamma_1} + \frac{\beta'}{\gamma_1}\right).$$

Estimating the Model: The Likelihood Function

As discussed in Chapter 4 a random sample of the population will produce a significant portion of non participants. We employ the Tobit model (discussed in that chapter) which implies that the jth individual will be observed to take some positive number of recreational trips, x, if and only if the cost of the trip, p, is less than his reservation price p_j, where the reservation price is a function of other factors influencing the individual. Thus

$$x_j = h_j(\cdot) + \varepsilon_j \quad \text{if and only if } h_j(\cdot) + \varepsilon_j > 0$$

$$x_j = 0 \quad \text{otherwise}$$

where $h_j(.)$ is the systematic portion of the appropriate demand function evaluated for individual j (eq. 4b or 5b).

If the sample of persons is divided so that the first m individuals recreate and the last n - m do not, then the likelihood function for this sample is

$$L_1 = \prod_{j=1}^{m} f(\varepsilon_j/\sigma)/\sigma \prod_{j=m+1}^{n} F(-h_j(\cdot)/\sigma).$$

This general form of the likelihood function will be true for each labor-market group. However, account must be given to the difference in the demand functions for each group. Thus, for our entire system of persons with interior and corner solutions in the labor market, the likelihood function is

$$(12) \qquad L^* = \prod_{j=1}^{m_c} f(\varepsilon_j/\sigma)/\sigma \prod_{j=m_c+1}^{n_c} F(-h_j(\cdot)/\sigma) \prod_{j=1}^{m_I} f(\varepsilon_j/\sigma)/\sigma \prod_{j=m_I+1}^{n_I} F(-h_j(\cdot)/\sigma)$$

where the subscripts c and I refer to numbers of individuals with corner and interior solutions respectively.

Should we only possess observations on participants, we can still avoid sample selection bias, by specifying the appropriate likelihood (Amemiya, 1973). The conditional probability of an individual j taking x_j visits given that x_j is positive is given by

$$(13) \qquad L^c = \prod_{j=1}^{m_c} \frac{f(\varepsilon_j/\sigma)/\sigma}{F(h_j(\cdot)/\sigma)} \prod_{j=1}^{m_I} \frac{f(\varepsilon_j/\sigma)/\sigma}{F(h_j(\cdot)/\sigma)}.$$

An Illustration

The purpose of this section is to demonstrate the application of our proposed approach for estimating recreational demand functions and for calculating recreational losses associated with elimination of the recreational site. We compare our results to those generated by traditional approaches. The exercise gives an example of how the traditional approaches can produce biased parameter estimates and inaccurate benefit measures.

To have a standard by which we can compare results, we begin with a direct utility function of the form in (7), choose parameter values (see Table 5.1, true model), and generate ten samples of individual observations. Each sample or replication of composed of 240 drawings, one third of which are consistent with each of the following situations: a) an interior solution in the labor market, b) a fixed work week solution, and c) unemployment. Two hundred forty values for wage income, non-wage income, secondary wage rate, travel cost and travel time are randomly drawn from five rectangular distributions R($0,$25,000), R($0,$1000), R($2.5, $5.0), R($0, $60) and R(0,4), respectively, and these values for the exogenous variables are repeated in each replication. The replications are different in that independent error terms are drawn from a normal distribution, N(0,25), for each of the 2400 individual observations. Total recreational time is taken to be the sum of travel and on-site time. While we assume on-site time to be exogenous, fixed at six hours per trip for all individuals, it is still necessary to include this fixed

amount since in the collapsible time model it will be valued differently by individuals with different time values.

The true demand models have three forms, conditioned on the labor supply choice:

(14a) $$x = \underset{(\alpha+\gamma_2 T)}{-3.22} - \underset{(\beta'\gamma_1)}{.06}\, p + \underset{(\gamma_1)}{.0005}\, Y - \underset{(\beta'\gamma_2)}{.04}\, t + \varepsilon_h \quad \text{(fixed work week)}$$

(14b) $$x = \underset{(\alpha+\gamma_2 T)}{-2.56} - \underset{(\beta'\gamma_1)}{.06}\, p + \underset{(\gamma_1)}{.0005}\, Y - \underset{(\beta'\gamma_2)}{.04}\, t + \varepsilon_h \quad \text{(unemployment)}$$

(14c) $$x = \underset{(\alpha)}{-4.00} - \underset{(\beta'\gamma_1)}{.06}\,(p + w_D t) + \underset{(\gamma_1)}{.0005}\,(Y + w_D T) + \varepsilon_h \quad \text{(discretionary work time)}$$

where the terms in parentheses under coefficients indicate how the coefficient is related to the utility model (equations 7, 8 and 9). The available time is assumed constant over all individuals in the sample. The Y denotes the relevant income level depending on the labor market choice. Each replication of 240 observations generated between 100 and 120 participants (i.e. observations for which $x > 0$).

Estimates for the parameters α, β', γ_1, and γ_2 are obtained using six procedures for each of the ten replications. The first two procedures (OLS-I and OLS-C) approach the problem in the traditional manner: all individuals are treated identically with respect to time valuation and only participants are included in the sample. Ordinary least squares estimates of parameters are obtained for both models. The two models differ in the way time is incorporated in the model. In the OLS-I model, everyone is assumed to value time at his wage rate. In OLS-C, time and money costs are introduced as separate variables for all individuals. To distinguish the biases which may arise

due to model specification from those attributable to sample selection bias, a second set of estimates are obtained from a maximum likelihood formulation (ML) which corrects for the truncated sample problem. However, all individuals are incorrectly presumed to be at interior labor market solutions in ML-I, and all individuals are incorrectly presumed to be at corner solutions in ML-C. The third pair of estimations represent the "correct" approach in that both the truncated sample problem and the specification problem are addressed. ML* and CML* correspond to the maximization of the likelihood functions in (12) and (13). Thus ML* includes both participants and non-participants, while CML* offers a solution for the conditional likelihood and examines only participants. Both allow individual's recreational choices to be conditioned on their labor supply decisions.

Means of the parameter estimates, estimated bias, and mean-square errors (MSE) are presented in Table 5.1 for the various procedures. The results of the proposed approach (ML*) are superior, both on the basis of unbiasedness and MSE. Two of the procedures which used only participants, OLS-I and CML*, produced similar RMSE, with OLS-I having smaller coefficient variance and CML* having smaller bias. The OLS-C model yielded the poorest results. The mean-square errors for the sample of estimates produced by ML-I and ML-C do not differ inordinantly from those produced by OLS-I and OLS-C and are larger than those of the proposed model, ML*.

Table 5.1

Estimated Preference Parameters, Standard Errors, and Mean-Square Errors for Ten Replications of 240 Random Drawings

Model	Parameters (Standard Errors)					Root-mean-square Errors				
	α	β'	γ_1 1/	γ_2 1/	σ	α	β'	γ_1 1/	γ_2 1/	σ
True	-4.00	-120.48	.50	.33	5.00	.....	.....	.....	.....	.....
OLS-I	3.66	-104.68	.38	....	3.88	60.26	2244.	.018	.....	1.30
	1.26	(44.66)	(.06)	....	(.21)					
ML-I	-6.45	-166.30	.60	....	5.38	34.95	5741.	.033	.....	.69
	(5.38)	(60.34)	.15)	....	(.74)					
OLS-C	5.04	-196.03	.22	2.05	3.78	94.47	17975.	.082	7.16	4.62
	(3.57)	(110.76)	(.06)	2.05	(1.77)					
ML-C	-.56	-204.28	.53	.06	5.15	21.38	16404.	.027	1.79	.56
	(3.09)	(96.86)	(.16)	(1.31)	(.73)					
ML*	-4.72	-113.65	.52	.43	4.65	4.56	999.	.003	.56	.23
	(2.01)	(30.87)	(.05)	(.74)	(.33)					
CML*	-6.11	-118.22	.57	.77	4.95	36.60	2999.	.027	1.02	.49
	5.67	(54.72)	(.15)	(.91)	(.70)					

1/ Because of scaling differences, estimated values for γ_1 and γ_2 are one one thousandth of the values shown in the table.

In addition to estimating parameters for each procedure, we estimate welfare measures for hypothetical price increases sufficient to eliminate the recreational activity (Table 5.2). The standard to which estimated measures are compared is the mean sample compensating variation based on the true values of the parameters. For our ten replications, the mean sample compensating variation is $429.09, where the relevant sample is all participants. For the ML*, the relevant standard is $524.03, which is the mean compensating variation for the sample of participants used to generate the ML* estimates. The ML* procedure used the same number of observations, but included both participants and non-participants. The subset of participants had a slightly higher average compensating variation.

The largest estimate of lost benefits ($1176) was associated with the OLS-C procedure when time and trip costs were considered separately and sample selection bias was ignored. This estimate is over 2 1/2 times the true value. The results are consistent with the expectation that ignoring the truncated sample problem will bias welfare measures upward. The ML* procedure produced a compensating variation only 12% higher than the true value for the sample. The error produced by CML* procedure was also relatively small, suggesting potential for using only on-site interviews providing the estimation routine is properly executed. The compensating variation estimates generated by ML-I and ML-C are particularly interesting in that they are the only underestimates of the true value. The elimination of the sample selection problem considerably reduces the bias for these estimates.

Table 5.2

Mean Individual Welfare Loss Estimates and Bias

Model	True Value	Estimated Welfare Losses (Compensating Variation)	Bias
OLS-I	$429.09	$745.98	$316.89
IL-I	$429.09	$296.59	$-132.50
OLS-C	$429.09	$1176.62	$747.48
ML-C	$429.09	$317.37	$-111.72
ML*	$524.03	$584.96	$60.93
CML*	$429.09	$554.43	$115.35

FOOTNOTES TO CHAPTER 5

1 In fact, the wage rate may not even serve as an upper or lower bound on the individual's marginal valuation of time when labor time is institutionally restricted. That is, an individual who chooses to be unemployed may simply value his marginal leisure hour more than the wage rate, or he may value it less but not be better off accepting a job requiring 40 hours of work per week. If restricted to an all or nothing decision, 40 hours may be less desireable than 0. An individual at a point such as A, however, may value the marginal leisure hour at more than w_p but choose 40 rather than 0 hours. Alternatively he may value leisure time at less than w_p but more than the wage he could earn for additional hours by working a secondary job.

2 Strictly speaking, γ_1 and β_1 could be replaced by γ_2 and β_2 for some individuals. A problem arises with the particular functional form chosen in (7). Because there are potentially two constraints, the utility function must be modified to accommodate three goods. Only x_1 is of interest, however, and in order to avoid the more complex problem of systems of demand equations, a bivariate direct utility function was modified to accommodate the three good case. The way in which x_2 and x_3 enters (7) implies that they are perfect substitutes. When they are subject to different "budget" constraints this restriction causes no particular problems. However when they enter the same

budget constraints when the two constraints collapse into one this form implies that either x_2 or x_3 is chosen depending on $\gamma_1, \gamma_2, t_3, p_2$ and w. A more realistic model of the decision process requires development of systems of demand functions as described in Part C.

APPENDIX TO CHAPTER 5

A COMPARATIVE STATICS ANALYSIS OF THE TWO CONSTRAINT CASE*

The subject of this Appendix is the consumer choice problem with two constraints. As we saw in Chapter 5, labor market restrictions and labor-leisure preferences cause individuals to be either at interior or corner solutions in the labor market. Classic comparative statics and welfare evaluation is directly applicable to interior solutions as the time and income constraints collapse into one. However the comparative statics and duality results associated with the corner solution case (i.e. utility maximization subject to time and income constraints) have received little attention.[1]

The first treatment of the problem was by A. C. DeSerpa (1971). Suzanne Holt's (1984) paper is the only other which explicitly deals with comparative statics of the time and income constraint. Both Holt's approach and that of DeSerpa's involves inversion of the Hessian, a tedious and difficult task for problems with large dimensionality. The Slutsky equation derived from this approach includes cofactors of the Hessian and, as such, is a complex function of the decision variables in the system. In what follows, a more modern approach is employed based on the saddle point theorem, as proposed by Akira Takayama (1977). Making use of the envelope theorem, this approach is simple to apply and far more

*/This appendix is the work of Terrence P. Smith, Agricultural and Resource Economics Department, University of Maryland.

revealing. From it can be derived Slutsky equations containing elements with clear economic interpretations.

This Appendix goes beyond the previous work by examining duality results and demand function properties in the context of the two constraints. Several new time analogs to the well known results in traditional demand theory are presented. Specifically, we derive a time analog to Roy's Identity and two generalized Slutsky equations. These Slutsky equations which describe the effect of a change in a money price are similar to the traditional Slutsky equation but contain additional income (time) effect terms which describe how demand responds indirectly to income (time) changes through the trade-off between time and money in producing utility.

Utility Maximization with Two Linear Constraints

Consider the household who maximizes a utility function, $U(x)$, where x is a vector of activities that produce utility. These activities need not be actual market commodities. The link to the market is through a set of household production functions. Suppose that the household produces these activities, x, according to the production functions, $f_i(s_i,t_i)$ where s_i and t_i represent a vector of purchased goods and time inputs into the production of x_i. The problem, then is to

(1) max $U(x)$ subject to $x_i = f_i(s_i,t_i)$ for all i, and

$$Y = R + wT_w = \Sigma p_i s_i, \text{ and}$$
$$T = T_w + T_r + T_n,$$

where Y is total income, the sum of nonearned income, R and wage

income, wT_w, p_i are the money prices for activity i, and t_i are the time prices associated with activity i. To proceed to specific results a fixed coefficients Leontief technology is assumed, that is, a technology with no substitution possibilities between the purchased inputs and time. This assumption implies that the activities, x_i, have fixed money and time costs, representable as p_i and t_i and that our problem may be rewritten

(2) $\max U(x)$ subject to $Y = p'x$ and $T = t'x$.

U(x) is a twice continuously differentiable concave utility function with x an n-dimensional vector of commodities. The consumer behaves so as to maximize this utility function. There is a commodity, say x_k, which represents savings such that the income constraint is always satisfied, and there is another commodity, say x_i, which is uncommitted leisure time such that the time constraint is effective.

Since the objective function is differentiable and concave in x, the constraints differentiable and linear in x and b, where b=(p,t,Y,T), the constraint qualification and curvature conditions are met. This implies that, if a solution exists, then the quasi-saddle point (QSP) conditions of Takayama will be both necessary and sufficient. Also, note that, given the assumption of the existence of slack variables, savings and uncommitted leisure time, the constraints are effective, and if a solution exists it will be an interior one. Collectively, these conditions allow the application of the envelope theorem to our problem.

Note that if a solution to (2) exists, it will be of the form $x(b)$, $\theta(b)$, $\phi(b)$. Hence we may substitute these solutions into the original Lagrangian to obtain

$$(3) \qquad L(b) = U(x(b)) + \phi(b)\,[Y - px(b)] + \theta(b)\,[T - tx(b)].$$

Now $U(x(b))$ may be written as $V(p,t,Y,T)$ and interpreted in the usual way as the indirect utility function. Note that, in addition to the traditional parameters affecting indirect utility (prices, p, and income, Y), the time prices, t, and time endowment, T, are also relevant parameters. Applying the envelope theorem to the above we obtain

$$(4a) \qquad \partial V(p,t,Y,T)/\partial Y = \phi(p,t,Y,T)$$

$$(4b) \qquad \partial V(p,t.Y,T,)/\partial T = \theta(p,t,Y,T)$$

$$(4c) \qquad \partial V(p,t,Y,T)/\partial p_i = -\phi(p,t,Y,T)\; x_i(p,t,Y,T)$$

$$(4d) \qquad \partial V(p,t,Y,T)/\partial t_i = -\theta(p,t,Y,T)\; x_i(p,t,Y,T)$$

Combining (4a) and (4c) gives Roy's Identity, viz.,

$$(5) \qquad \frac{\partial V(p,t,Y,T)/\partial p_i}{\partial V(p,t,Y,T)/\partial Y} = x_i \text{ for all } i.$$

Likewise, combining (4b) and (4d) gives another identity, viz.,

$$(6) \qquad \frac{\partial V(p,t,Y,T)/\partial t_i}{\partial V(p,t,Y,T)/\partial T} = x_i \text{ for all } i.$$

Note that (6) gives an alternative way to recover the Marshallian demand from the indirect utility function. However, both differential equations may be required to be solved to recover the indirect utility function from the demand function, since we will see that there are two expenditure functions.

We may manipulate these envelope results in other ways to demonstrate time extensions to traditional demand analysis. For example, combining (4a) and (4b) with (4c) and (4d) we obtain

$$(7) \qquad \frac{\partial V(p,t,Y,T)/\partial T}{\partial V(p,t,Y,T)/\partial Y} = \frac{\theta(p,t,Y,T)}{\phi(p,t,Y,T)} = \frac{\partial V(p,t,Y,T)/\partial t_i}{\partial V(p,t,Y,T)\partial p_i}$$

which is McConnell's m_T or "the opportunity cost of scarce time measured in dollars of income." Multiplying (4c) by p_i, (4d) by t_i and summing over all i yields

$$(8) \qquad \sum p_i \partial V/\partial p_i + \sum t_i \partial V/\partial t_i = -\phi \sum p_i x_i - \theta \sum t_i x_i$$

which by (4a) and (4b) is equal to

$$(9) \qquad \sum p_i \partial V/\partial p_i + \sum t_i \partial V \partial t_i + Y \partial V/\partial Y + T \partial V/\partial T = 0,$$

implying that the indirect utility function $V(p,t,Y,T)$ is homogeneous of degree 0 in money and time prices, income, and time.

To examine the necessary conditions for an adjustment in the parameters while still maintaining a reference level of utility, consider the total differential

$$(10) \quad dV(p,t,Y,T) = (\partial V/\partial Y)dY + (\partial V/\partial Y)dT + \Sigma(\partial V/\partial p_i)\, dp_i + \sum(\partial V/\partial t_i)dt_i = 0$$

Rewriting (10) as

(11a) $$dY + \frac{\theta}{\phi}\, dt + \sum x_i dp_i + \frac{\theta}{\phi} \sum x_i dt_i$$

or

(11b) $$dt + \frac{\phi}{\theta}\, dY = \frac{\phi}{\theta} \sum x_i dp_i + \sum x_i dt_i$$

makes more explicit the types of compensation mechanisms possible. Here, there are more possibilities for compensation beyond the traditional income compensation for a price change.

The Two Duals and the Two Slutsky Equations

In this section we explore the dual of the utility maximization problem. Since there are two constraints, there are two duals to the problem. The first is (money) cost minimization subject to constraints on time and utility; the second is time cost minimization subject to constraints on income and utility. This exploration yields two expenditure functions, an income compensated function and a time compensated function. The existence of two expenditure functions allows one to compute welfare changes either in the traditonal way as income compensation measures or, alternatively, as time compensation meansures.

In addition, these expenditure functions are combined with the envelope theorem to reveal two generalized Slutsky equations. The first of these describes how Marshallian demand responds to money price changes and the second how the ordinary demand changes with a change in time prices. The manner of proof is in the style of the "instant Slutsky equation" as first introduced by Philip Cook (1972).

The duals to the utility maximization problem (2) are

$$(12) \qquad \min_x px \quad \text{subject to } T = tx \text{ and } U^0 = U(x)$$

and

$$(13) \qquad \min_x tx \quad \text{subject to } Y = px \text{ and } U^0 = U(x)$$

where U^0 is some reference level of utility.

Notice that (12) and (13) can be cast in the notation of our original maximization problem, where the objective function, px and tx, are linear and hence concave in x and p or t, and the constraint functions are quasiconcave since the first constraint is linear (either $T - tx = 0$ or $Y - px = 0$) and the second, concave. It follows then, as in our earlier analysis of the primal problem, that if a solution exists, the QSP conditions will be both necessary and sufficient. Furthermore, maintaining the existence of the slack variables, savings and freely disposable time, and requiring that the reference level of utility be maintained ensures that the constraints are effective, that we have an interior solution, and hence, that the envelope theorem may be applied.

Consider, then, the two Lagrangians,

$$(14a) \qquad \min L_Y(p,t,T,U^0) = px + \theta(T - tx) + \phi(U^0 - U(x))$$

and

$$(14b) \qquad \min L_T(p,t,Y,U^0) = tx + \mu(Y - px) + \delta(U^0 - U(x)).$$

Solutions to these minimization problems, if they exist, are given by,

(15a) $x_Y(p,t,T,U^0)$ and

(15b) $x_T(p,t,Y,U^0)$.

The first of these is the "usual" Hicksian income compensated demand, while (15b) is an analogous time compensated Hicksian demand. Of course, both depend (in general) on all money (p) and time (t) prices.

Solutions (15a) and (15b), when substituted back into the objective functions, imply the existence of two expenditure functions. The first of these,

(16a) $$E_Y(p,t,T,U^0) = px_Y(p,t,T,U^0)$$

is the well known classical expenditure function with the exception that the time prices, t, and the time endowment, T, appear as arguments.

The second,

(16b) $$E_T(p,t,Y,U^0) = tx_T(p,t,Y,U^0),$$

is a time compensated measure of the minimum expenditure level necessary to maintain U^0. Either (16a) or (16b) may be used to measure welfare effects of a change in money or time prices or both. The novelty of using (16b) for welfare analysis is that it measures the amount of time compensation, rather than income compensation, necessary to maintain a reference utility level in the face of, say, a money price change for one of the commodities.

Again, making the same assumptions as in the utility maximization problem above (i.e., all income and time are spent), and given that the reference utility level is U^0, we know the interior solutions to (14a) and (14b) exist. We may therefore apply the envelope theorem to derive substitution relationships for the two constraint problem.

First, consider x_Y, as given in (15a), and the money expenditure function, $E_Y = px_Y$, (16a). Note that for a change in p from p' to p'' with t^0, T^0, and U^0 constant, it is necessarily true (from the QSP conditions and the definition of the minimization problem) that,

$$E_Y(p') - E_Y(p'') \geq x_Y(p',t^0,T^0,U^0)(p' - p''). \tag{17}$$

Likewise, for a change from p'' to p' it follows that

$$E_Y(p'') - E_Y(p') \geq x_Y(p'',t^0,T^0,U^0)(p'' - p'). \tag{18}$$

Summing (17) and (18) yields $\Delta x \Delta p \geq 0$,

$\Delta x \equiv x_y(p',t^0, T^0,U^0) - x_y(p'',t^0,T^0,U^0)$ and $\Delta p \equiv p' - p''$, or in differential terms, $dx \cdot dp \leq 0$. Therefore it follows that $\partial x_i/\partial p_i \leq 0$. Alternately, using (14b) and (16b), we obtain $\partial x_i/\partial t_i \leq 0$. We therefore conclude that slopes of the (compensated) own money price and own time price demands are necessarily non-positive. Note that by the envelope theorem

$$\partial E_Y/\partial p_i = x_{Yi}(p,t,T,U^0) \text{ and} \tag{19a}$$

$$\partial E_T/\partial t_i = x_{Ti}\ (p,t,Y,U^0), \tag{19b}$$

(Shepard's Lemma), and so, by Young's theorem,

$$\partial^2 E_Y/\partial p_i \partial p_j = \partial x_{Yi}/\partial p_j = \partial x_{Yj}/\partial p_i = \partial^2 E_Y/\partial p_j \partial p_i \tag{20a}$$

and

$$\partial^2 E_T/\partial t_i \partial t_j = \partial x_{Ti}/\partial t_j = \partial x_{Tj}/\partial t_i = \partial^2 E_T/\partial t_j \partial t_i \tag{20b}$$

which implies symmetry. Also, define $S_{ij} \equiv \partial x_i/\partial p_j = \partial^2 E_Y/\partial p_j \partial p_i$, as the money substituton effect for good i, given a price change for good j, and $T_{ij} \equiv \partial x_i/\partial t_j = \partial^2 E_T/\partial t_i \partial t_j$ as the time substitution. Then it follows that S and T are negative semidefinite and symmetric, and since x_Y and x_T are homogeneous of degree 0 in p and t, respectively, we have

$$\sum p_i \partial x_i/\partial p_j = 0 = \sum S_{ij} p_i \tag{21a}$$

and

$$\sum t_i \partial x_i/\partial t_j = 0 = \sum T_{ij} t_i. \tag{21b}$$

That is, the aggregation conditions hold.

The above serves to formalize the equivalence of several of the well known properties of Hicksian demands in the classical and two constraint systems. The Slutsky relations that follow from the present problem are now derived. Although our results show structural similarity to the classical equations, our derivation results in two Slutsky equations, each of which has a time effect as well as an income effect.2

Consider the solution to the primal problem posed in the preceding section. This solution is the set of Marshallian demands which may be written,

$$x^m = m(p, t^o, Y^o, T^o). \tag{22}$$

Now recall that the solution to our money minimization problem, Y^o, is just $p^o x_Y(p,t,T,U^o) = E_Y$, and likewise, the solution to the time minimization problem, T^o, is defined as $t^o x_T(p,t,Y,U^o) = E_T$, hence we may write,

$$x^o = f[p^o, t^o, E_y(p, t^o, T, U^o), E_T(p^o, t^o, Y, U^o)] \tag{23}$$

Note that (23) now defines the set of Hicksian demands. Differentiate (23) with respect to the jth price, p_j. We obtain,

$$\partial x_i/\partial p_j = \partial f/\partial p_j + (\partial f/\partial E_Y)(\partial E_Y/\partial p_j) + (\partial f/\partial E_T)(\partial E_T/\partial p_j) \tag{24a}$$

using the chain rule. Consider also how the demand for x_i changes with a change in one of the time prices, say t_j. Differentiating (23) with respect to t_j yields,

$$\partial x_i/\partial t_j = af/\partial t_j + (\partial f/\partial E)(\partial E_Y/\partial t_j) + (\partial f/\partial E_T)(\partial E_T/\partial t_j). \tag{24b}$$

These are two generalized Slutsky equations that result from our dual constraint problem. To cast them in more familiar terms use the envelope theorem applied to equations (14) to obtain,

$$\partial E_Y/\partial p_j = x_j \tag{25a}$$

$$\partial E_T/\partial t_j = x_j \tag{25b}$$

$$\partial E_Y/\partial T = \theta \tag{25c}$$

(25d) $$\partial E_T/\partial Y = \mu$$

(25e) $$\partial E_Y/\partial t_j = -\theta x_j$$

(25f) $$\partial E_T/\partial p_j = \mu x_j.$$

Substituting (25a) and (25f) into (24a) and rearranging, obtains the money price Slutsky equation,

(26a) $$\partial x_i^M/\partial p_j = \partial x_i^H/\partial p_j - x_j\,[\partial x_i^M/\partial Y - \mu\partial x_i^M/\partial T]$$

where x_i^m denotes Marshallian functions and x_i^H denotes Hicksian functions. This Slutsky equation is identical to the classical version with the exception of the additional term $\mu x_j \partial x_i^M/\partial T$, which is the indirect effect of income through time. If x_i is an income normal, time normal good then

$\partial x_i^M/\partial Y$ and $\partial x_i^M/\partial T$ are both positive, and since μ ,the Lagrangian multiplier on the income constraint in our time minimization problem, is necessarily non-positive, it follows that for a "normal-normal" good the "income" effect[3] is enhanced relative to the classic income effect.

Proceeding in exactly the same way, the time price Slutsky equation can be derived. Substituting (25b) and (25c) into the second Slutsky equation (24b) and rearranging, yields

(26b) $$\partial x_i^M/\partial t_j = \partial x_i^H/\partial t_j - x_j\,[\partial x_i^M/\partial T - \theta\,\partial x_i^M/\partial Y].$$

Notice that in addition to the "pure" time effect, $x_j \partial x_i/\partial T$, there is an additional indirect effect, $\theta x_j \partial x_i/\partial Y$, which, using the same argument as above, is an indirect time effect through income, converted to time by the marginal (time) cost of income.

Again the two terms will augment one another for a "normal-normal" good, and, of course, offset one another for a "normal-inferior" good, where "normal-inferior" is taken to represent a commodity which is income normal and time inferior or vice versa.

Utilizing the results that $\mu = \partial E_T/\partial Y$ and $\theta = \partial E_Y/\partial T$, an equivalent way of writing (26) is

$$\text{(27a)} \qquad \partial x_i^M/\partial p_j = \partial x_i^H/\partial p_j - x_j[\partial x_i^M/\partial Y - (\partial x_i/\partial T)\ (\partial E_T/\partial Y)]$$

$$\text{(27b)} \qquad \partial x_i^M/\partial t_j = \partial x_i^H/\partial t_j - x_j[\partial x_i/\partial T - (\partial x_i/\partial Y)\ (\partial E_Y/\partial T)].$$

This version makes clear the substitution between income and time in the two constraint model.

A Summary of Results

The "usual" properties of classical demand functions still hold when one solves the two constraint problem. The demand functions that solve our maximization problem are homogeneous of degree 0 in money and time prices, income and time, and satisfy the aggregation and integrability conditions. The compensated demands, be they income or time compensated, are own price (money or time) downward sloping. The "substitution" matrix is negative semidefinite, where the substitution matrix must be interpreted as the matrix which describes a response to a money (time) price change holding utility and the time (income) endowment constant. Finally, we can partition the ordinary demand response to a change in money (time) price as made up of two effects, a utility held constant effect, i.e. a movement along an indifference surface, and an income (time) effect, remembering the complication, however,

that this income (time) effect is made up of a "pure" income (time) effect and an indirect effect of time (income) converted to money (time) terms.

These new demand functions contain additional arguments relative to the "classic" demand function. That is, the ordinary demands are functions of not only money prices and income, but also of time prices and of the time endowment. Likewise, the money and time expenditure functions depend not only on money prices and utility, but also upon time prices, and the time endowment (for the money expenditure function) or income endowment (for the time expenditure function). Therefore, welfare analysis may be done in a straightforward way using these expenditure functions provided we account not only for money and income changes but also for time price and time endowment changes.

One final result is of particular interest. The Slutsky equations (27a) and (27b) indicate a two term income effect for the money price version and a two term time effect for the time price equation. Restating the Slutsky equations for money price changes,

$$\frac{\partial x_i^M}{\partial p_i} = \frac{\partial x_i}{\partial p_i} - x_i \frac{\partial x_i^M}{\partial Y} + x_i \frac{\partial x_i^M}{\partial T} \frac{\partial E_T}{\partial Y}.$$

The LHS is the Marshallian price slope. The first term on the right is the Hicksian price slope. The total income effect is made up of the usual income effect term $-x_i \partial x^M / \partial Y$ and the effect of income through the time effect $x_i (\partial x_i^M / \partial T)(\partial E_T / \partial Y)$ Both terms are negative if x_i is normal with respect to Y and T, because $\partial E_T / \partial Y$ is negative and represents the change in time costs necessary to

achieve a given level of utility if the individual is given more income.

From this expression we can see that the total (combined) income effect is greater in absolute value than the conventional (direct) effect. This has the interesting result of pushing compensated and ordinary demand functions farther away from each other. Note that the additional indirect income effect term causes the Marshallian demand function to be flatter and the Hicksian demand to be steeper, thus decreasing the consumer surplus measure but increasing the compensating variation measure.

FOOTNOTES TO APPENDIX OF CHAPTER 5

1 The solution to, and sensitivity analysis of, a more general problem, i.e. maximization of an objective function subject to multiple, possibly nonlinear, constraints has appeared in the mathematical economics literature.

2 The similarity can also be seen in the approach of DeSerpa and Holt. Unfortunately, that approach, which relies on the inverted Hessian, tends to obscure the detail of the time and income effects.

3 The interpretation of H is the marginal (money) cost of time, hence H converts the time effect into income units, and therefore the second term in brackets may be interpreted as an additional income effect.