On the optimal design of income support and agri-environmental regulation

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Abstract

In this paper, we develop a model of regulation for a set of heterogenous farmers whose production yields to environmental externalities. The goal of the regulator is first to offer some income support depending on collective preferences towards income redistribution and second to internalize externalities. The optimal policy is constrained by the information available. We first consider the second best where the regulator is able to observe all individuals decisions in terms of inputs and individual profit, but not the individual farming labor supply. We characterized the generalized transfer in function of the desire to redistribute and the underlying characteristics of the production process. In a second step, we assume that the regulator has only information on aggregate consumption of inputs and hence can only tax/subsidy linearly inputs and output. However, because the accounting profit remains observable, a non linear transfer of profit is still part of the optimal policy. In the last part of the paper, we endogenize the market price of land and examine how the optimal policy should be modified.

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1   Introduction

Government intervention remains pervasive in agriculture, at least for two reasons. One
is to provide some income support to farmers and the second one is to promote positive
externalities and/or to reduce negative externalities arising from agricultural production. It
appears that agricultural policies in developed countries are often characterized by apparently
countervailing provisions. Indeed, income support motivated subsidies may have undesired
environmental consequences. As noted by Bourgeon and Chambers (2000), production sub-
sidies are granted but at the same time farmers are paid to reduce their acreage. Finally, the
different ways governments intervene to achieve several objectives are not equivalent as some
measures are less production and trade distorting than others. These issues and therefore the
design of efficient governmental intervention in agriculture have raised a considerable concern
in the literature.

For instance, Guyomard et al. (2004) (see also Leathers (1992) for an earlier reference)
compare four agricultural income support programs (output subsidy, land subsidy, a decou-
pled payment with or without mandatory production) according to achieve four goals (income
support, reduction of negative externalities, maintenance of a maximum number of farmers
and effects on trade). It is shown that no program uniformly dominates others. While these
kind of results are of importance, it might be needed to go a little further in order to bet-
ter understand the determinants of optimal governmental intervention and in particular to
understand how to combine efficiently the different available instruments.

This is precisely such an approach that Bourgeon and Chambers (2000) have used to
study the optimal design of income support in agriculture (as well as public investment) in
a context of imperfect discrimination due to asymmetric information between farmers and
the regulator.[1] There, in order to insure all farmers a minimum parity income, there is a
need to transfer money to high-cost and low income farmers. But these transfers can be

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[1]See also Lewis, Ware and Feenstra (1989), Chambers (1992) and Hueth (2000).

claimed by high income farmers as well and hence an optimal policy deters these claims by tying income support programs to production or acreage limits that are more costly to the high-income farmers. Innes (2003) pursues this line of research by showing that it is also important to incorporate the effects of the policy on all market prices and in particular on farmland prices. In particular, it may be optimal to implement some compensated acreage limitations for high-cost farmers together with low-cost farmers cultivating more acreage than they otherwise would.

Besides, the optimal design of environmental regulation in agriculture has been extensively studied. For instance, Bontems, Turpin and Rotillon (2005, 2007) study an output regulation aimed at reducing negative externalities and that takes into account the political power of farmers and the pre-intervention distribution of farming incomes. More recently, Sheriff (2008) analyzes the optimal design of environmental regulation taking into account the need to support income and the existence of price uncertainty while Feng (2007) looks at a model of optimal green payments for conservation and income support goals, where farmers are heterogeneous along two dimensions (farm size and conservation efficiency).[2]

Despite the interesting results gathered by the literature, it is fair to observe that many of these normative models typically rely on a rather crude modelling of farmer's behavior with often only one decision to be taken (production or land cultivated). Hence, the regulation is optimally designed on a exogenously and very limited set of variables in order to achieve several goals simultaneously. The purpose of the paper is to theoretically explore the optimal design of both income support and agri-environmental regulation in a more general model where farmer's decisions cover several variable inputs such as fertilizer, land cultivated and labor devoted to production. Given the existing policies, another important decision for a farmer is whether to stay as an active farmer or to give up production and lease out all his land endowment and allocating his labour to the next best alternative in terms of wages. In other words, the size of the agricultural sector is endogenous in the analysis. The goal of

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[2]See also Wu and Babcock (1995) for an earlier analysis.

the government is to redistribute income among a population of heterogenous farmers taking into account the potential negative externalities of production and its budget constraint. The intensity of redistribution depends on the social preferences towards redistribution through the degree of social aversion to inequality. The amount of damage caused by production depends on all polluting inputs used and also of the size of land cultivated. For instance, if environmental damage is primarily driven by intensification then increasing the land used reduces damages holding the level of polluting inputs constant.

Importantly, the policies that the government can implement are constrained by the information available. First, we assume that farmers are heterogenous according to their ability in the production process which is private information. In addition, the effort (or labour) devoted to production constitutes a private decision of farmers and hence is non observable to the regulator. However, we assume that the regulator is able to observe the accounting profit (profit gross of the disutility of labor) at the individual level. In addition, the status of agent (active farmer or not) is observable so that the regulator can implement a poll subsidy/tax on all non active farmers that lease out their lands.

We derive the optimal regulation policy in two different settings. First, we assume in addition that a very powerful regulator is able to gather observations of all relevant farmer's individual decisions (production, inputs). We show that unless some separability conditions hold for the production function, it is generally optimal to distort the taxation of polluting inputs like fertilizers from the traditional pigovian rule for redistributive purposes. We also study the shape of the optimal transfer and its progressive/regressive feature depending on the social preferences towards redistribution and the respective political weights of different types of farmers. Second, we consider a more realistic setting where the regulator has only access to aggregate decisions and hence cannot do better than employing linear tax/subsidy when regulating the output or the variable inputs. In this setting, the optimal policy is a combination of linear tax/subsidy on output and inputs and a non linear transfer based on the observation of accounting profit. Here, the Principle of Targeting breaks down as the income

4

subsidy based on the observed accounting profit is influenced by the negative externality.

We hence obtain results that are related to the ones obtained by Cremer et al. (1998) in the context of income non linear taxation and commodities taxes for the consumer case. Our model differs in that it is first in a context of production, also because the political or social weights of individuals appear in the analysis and finally because individuals may opt to quit the production sector making the size of the agricultural sector endogenous. Last, another difference lies into the fact that we endogenize the price of some good (namely the farmland price) which then depends on the policy implemented. There, the objective of the government now also takes into account the opportunity cost of land devoted to agriculture and the rents for landowners. We study the influence of income support and environmental policies on the equilibrium price of farmland.

The paper is organized as follows. The next section is devoted to assumptions and notations. Sections 3 and 4 are devoted to benchmark cases, the laissez-faire equilibrium and the first best. We analyze the second best regulation in section 5 and the optimal regulation under observable aggregate variables in section 6. Section 7 is devoted to the endogeneization of farmland price. Section 8 concludes.

2   Assumptions and notations

Consider a farmer with the following production technology:

$$q = f(l, z, e, \theta)$$

where q denotes the agricultural production, l is the land used, z is a variable marketed input (say chemical fertilizers, pesticides, energy...), e is the production-enhancing effort supplied by the farmer and θ is a one-dimensional productivity parameter.[3] We assume that land l is essential to production, i.e. $f(0, z, e, \theta) = 0$. We assume that f is smooth and is increasing in all its arguments, i.e. $f_i > 0$, $\forall i = l, z, e, \theta$.[4] Parameter θ can be interpreted as a value

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[3]The extension of the model to more than two inputs is straightforward.

[4]We denote $f_x = \frac{\partial f}{\partial x}$ as the partial derivate of f with respect to x.

characterizing the farmer himself in terms of ability to produce or some fixed characteristics of the production process. Here, a larger θ means a more efficient production process. We assume that θ belongs to a compact set $\Theta = [\underline{\theta}, \overline{\theta}]$ with distribution $K(\theta)$ and a positive density $k(\theta)$ on Θ.

The effort e can be interpreted as the quantity of effective labor devoted to production or even as the (continuous) choice of technology intensity employed on the farm. More effort which is costly in time or a more intensive technology allows to increase production.

In addition, we also assume that f is supermodular in (l, z, e, θ), that is, $f_{ij} \geq 0$, $\forall i \neq j$ with $(i, j) \in \{l, z, e, \theta\}$. This amounts to suppose that the technology is normal in all inputs in the sense of Rader (1968) and hence exhibits some complementarity between the variables.

The agricultural accounting profit absent any governmental intervention is given by the restricted profit function π defined as follows:

$$\pi(e; p, r, w, \theta, l^{\circ}) = \max_{l,z} \ \left\{ pq - r(l - l^{\circ}) - wz \ \text{ s.t. } q \leq f(l, z, e, \theta), \ l \geq 0, \ z \geq 0 \right\}$$

where l° is the initial endowment in land and r the market price of land. Also, p and w are respectively the market price of output and polluting input which are assumed to be constant. We denote $z(\theta)$ and $l(\theta)$ the optimal (interior) allocation of polluting input and land from the perspective of a type-θ farmer.

Each farmer has a utility function $U(I, e)$ where I is the net income. We assume that U takes the following quasi-linear form $U = I - \psi(e)$ where ψ is the (monetary) cost of effort with $\psi' > 0$ and $\psi'' > 0$.

The farmer can quit the agricultural sector and obtain an outside wage v together with the returns from renting his land endowment. Hence, in the laissez-faire situation, a farmer is actually active and produces if and only if

$$\pi(e; p, r, w; \theta, l^{\circ}) - \psi(e) \geq v + rl^{\circ}. \tag{1}$$

Note that this free entry/exit condition (1) does not depend on l° as the optimal allocation $z(\theta)$ and $l(\theta)$ do not depend on l° as well. But the distribution of income obviously depends on

the distribution of land endowment. We assume that the outside wage is constant (otherwise, we would have to model a second sector of production where workers are employed).

We assume that producing entails an environmental damage x which is related to the intensity of input usage

$$x = x(z, l)$$

and we assume that $x(., .)$ is increasing in the first argument z. Concerning the land use, if the environmental damage is primarily caused by the intensification of production process then it is natural to assume that $x(., .)$ is decreasing in l. In that case, obviously the intervention of the regulator would call a for land subsidy. Conversely, if damage is instead driven by excessive use of marginal land, then it would be natural to assume that $x(., .)$ is increasing in l.

3 Laissez-faire equilibrium with free entry/exit

In the absence of governmental support, the farmer has to take four decisions: first whether to stay active or to leave the sector, if active how much land, fertilizer and effort to put in the production process. Note that the entry condition only depends on θ, hence it follows that a farmer with type $\hat{\theta}$ decides to stay then any farmer with type $\theta > \hat{\theta}$ stays active as well. This implies that the set of active farmers is only determined by the ability θ and is $[\theta_s, \overline{\theta}]$.

The equilibrium is characterized by the following conditions:

$$
\begin{aligned}
p f_l(l, z, e, \theta) &= r \\
p f_z(l, z, e, \theta) &= w \\
p f_e(l, z, e, \theta) &= \psi'
\end{aligned}
$$

which gives the equilibrium allocation for an active θ-type farmer, $l(\theta)$, $z(\theta)$ and $e(\theta)$. Also, the (interior) marginal farmer θ_s who is indifferent between producing and leaving the agri-

cultural sector is such that:

$$U(\theta_s) = pf(l(\theta_s), z(\theta_s), e(\theta_s), \theta_s) - wz(\theta_s) - r(l(\theta_s) - l^\circ) - \psi(e(\theta_s)) = v + rl^\circ$$

or equivalently

$$pf(l(\theta_s), z(\theta_s), e(\theta_s), \theta_s) - wz(\theta_s) - rl(\theta_s) - \psi(e(\theta_s)) = v.$$

This condition means that the return of production must be at least superior to the next best alternative v. Note that this condition is independent of the land endowment l°.

4 The First Best

We now examine the benchmark situation where the regulator intervenes without informational constraints. Indeed, we assume that the regulator can observe costlessly the status of agents (farmer or not), the individual decisions over l, z, q and the accounting profit π. In addition, we assume in this section that the regulator is able to observe the effort e and the type θ. The objective of the regulator is to maximize a weighted sum of social value of utilities. The social value of utility U is denoted $\mathcal{W}(U)$ where $\mathcal{W}(.)$ is increasing, concave, reflecting the desire to redistribute income from the richer to the poorer farmers. The social (or political) weight in the welfare function is denoted $\alpha(\theta)$ for a type-θ farmer. If this weight function is increasing in θ then the basic desire to redistribute ($\mathcal{W}(U)$ is concave) is counterbalanced by the fact that richer farmers have also a higher weight in the welfare function. A transfer $T(\theta)$ is paid to any type-θ active farmer and a transfer τ is paid to any non active farmer. The budget devoted to the agricultural sector is denoted B while the maximal environmental damage which is sustainable is given by X.

The program of the regulator can thus be written as follows:

$$\max \int_{\theta_s}^{\overline{\theta}} \alpha(\theta) \mathcal{W}(U(\theta)) dK(\theta) + \int_{\underline{\theta}}^{\theta_s} \alpha(\theta) \mathcal{W}(v + rl^\circ + \tau) dK(\theta)$$

s.t.

$$\int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) + \int_{\theta_s}^{\overline{\theta}} T(\theta) dK(\theta) \leq B \tag{2}$$

$$\int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta)) dK(\theta) \leq X \tag{3}$$

$$U(\theta) \geq v + rl^\circ + \tau \text{ for any } \theta \geq \theta_s$$

$$U(\theta) = \pi(\theta) + T(\theta) - \psi(e(\theta))$$

where $\pi(\theta) = pf(l(\theta), z(\theta), e(\theta), \theta) - r(l(\theta) - l^\circ) - wz(\theta)$. Also $\alpha(\theta)$ is a positive function of θ with the normalization $\int_{\underline{\theta}}^{\overline{\theta}} \alpha(\theta) d\theta = 1$ which represents as indicated above the social (or political) weight of type-θ farmers in the welfare function.

Let us denote by μ the Lagrange multiplier of the environmental constraint (3) and by ν the Lagrange multiplier of the budget constraint (2). Solving the regulator's program, we obtain the following result.

Proposition 1 *At the first best, the optimal allocation devoted to a type-θ farmer is such that*

$$pf_l(l, z, e, \theta) = r + \frac{\mu}{\nu} x_l(z, l)$$
$$pf_z(l, z, e, \theta) = w + \frac{\mu}{\nu} x_z(z, l)$$
$$pf_e(l, z, e, \theta) = \psi'(e)$$

and the (interior) marginal farmer is such that

$$pf(l(\theta_s), z(\theta_s), e(\theta_s), \theta_s) - rl(\theta_s) - wz(\theta_s) - \psi(e(\theta_s)) - \frac{\mu}{\nu} x(z(\theta_s), l(\theta_s)) = v$$

Last, the first best is characterized by the equality between marginal social value of utility across types:

$$\alpha(\theta) \mathcal{W}'(U(\theta)) = \alpha_0 \mathcal{W}'(v + rl^\circ + \tau) = \nu.$$

9

for any $\theta \geq \theta_s$ and where $\alpha_0 = \frac{1}{K(\theta_s)} \int_{\underline{\theta}}^{\theta_s} \alpha(\theta) dK(\theta)$.

Proof: See appendix A. ∎

As can be seen from the Proposition, the First Best allocation entails a Pigovian tax/subsidy on both z and l. More precisely, if the optimal allocation is $l^*(\theta)$ and $z^*(\theta)$, then the regulator can decentralize the first best allocation by implementing a personalized tax per unit of land equal to $\frac{\mu}{\nu} x_l(z^*(\theta), l^*(\theta))$ and a personalized tax per unit of fertilizer equal to $\frac{\mu}{\nu} x_z(z^*(\theta), l^*(\theta))$.[5] Note that in the case of land, it can be a subsidy if damage is primarily driven by intensification ($x_l < 0$). In addition, the optimal distribution of incomes is obtained through personalized transfers $T(\theta)$ for active farmers and a uniform transfer τ for inactive agents. Note also that there is no need to regulate the effort once z and l are driven to their efficient levels.

Last, the identity of the marginal farmer is such that the return of production from the marginal farmer net of the social damage weighted by the shadow price of public funds should be equal to the outside wage. It follows that the intervention tends to reduce the size of the agricultural sector by taking into account the social cost of production in terms of environmental damages.

5 The second best with observable individual decisions

We now suppose that the regulator can always observe the accounting profit π and the status of any agent (being an active farmer or not). In addition, we will assume that the regulator may observe individual decisions like the land used, the quantity of fertilizer used and the production level. However, the effort level and the productivity parameter are unobservable to the regulator.

Given its information set, the regulator is able to consider a general policy of the form $\{\tau, \hat{t}(\pi, z, l, q)\}$ where τ is the transfer paid to any farmer stopping his activity and t is

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[5]The tax would have been uniform if we had instead assumed that the aggregate pollution level is a function $\tilde{X}(Z, L)$ where Z is the aggregate consumption of polluting input z and L the aggregate use of farmland. The constraint would then write $\tilde{X}(Z, L) \leq X$.

the transfer paid to each active farmer as a function of observable variables. Note that actually a transfer $t(\pi, z, l)$ is equivalent to the transfer $\hat{t}(\pi, z, l, q)$, because we have $q = (\pi + wz + r(l - l^\circ))/p$ and (z, l, l°) are observable.

5.1 Analysis

From the Revelation Principle, any mechanism $\{\tau, t(\pi, z, l)\}$ is equivalent to a direct revelation mechanism $\{\tau, \pi(\theta), z(\theta), l(\theta), T(\theta)\}$ where $T(.)$ is the transfer paid by the regulator to a producing agent and in which truthtelling is an optimal strategy for each farmer. For simplicity, we assume the differentiability of the policy.[6] The program of the regulator can be written as follows:

$$\max_{e, l, z, \theta_s, \tau, U} \int_{\theta_s}^{\overline{\theta}} \alpha(\theta) \mathcal{W}(U(\theta)) dK(\theta) + \int_{\underline{\theta}}^{\theta_s} \alpha(\theta) \mathcal{W}(v + rl^\circ + \tau) dK(\theta)$$

s.t.

$$\int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) + \int_{\theta_s}^{\overline{\theta}} T(\theta) dK(\theta) \leq B$$

$$\int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta)) dK(\theta) \leq X$$

$$U(\theta_s) = v + rl^\circ + \tau$$

$$U(\theta) \geq U(\theta, \tilde{\theta}) \text{ for any } \theta, \tilde{\theta}$$

$$U(\theta) = \pi(\theta, l(\theta), z(\theta), e(\theta)) + T(\theta) - \psi(e(\theta))$$

$$\pi(\theta, l(\theta), z(\theta), e(\theta)) = pf(l(\theta), z(\theta), e(\theta), \theta) - wz(\theta) - r(l(\theta) - l^\circ)$$

Given that the regulator observes π together with l and z, it is interesting to denote the effort $E(\theta, l, z, \pi)$ needed to generate a profit π using l and z for a type-θ farmer. As $f_e > 0$, the equation $\pi = \pi(\theta, l, z, e) = pf(l, z, e, \theta) - wz - r(l - l^\circ)$ defines implicitly the function $E(\theta, l, z, \pi)$. Note that we easily get:

$$E_\theta = -\frac{f_\theta}{f_e} < 0, \ E_\pi = \frac{1}{pf_e} > 0, \ E_l = -\frac{pf_l - r}{pf_e}, \ E_z = -\frac{pf_z - w}{pf_e}.$$

Hence, the effort needed decreases with ability and increase with the profit goal. Whether E

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[6] Standard arguments allow to prove the differentiability almost everywhere.

increases or not with l or z depends on whether the optimum involves under-use (compared to the private optimum) of land or fertilizer or not.

Proposition 2 *At a separating optimum, the allocation devoted to a type-θ active farmer is characterized by:*

$$
\begin{aligned}
pf_l(l(\theta), z(\theta), e(\theta), \theta) &= r + \frac{\mu}{\nu}x_l(z(\theta), l(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)}\psi'(e(\theta))\frac{d(E_\theta)}{dl} \\
pf_z(l(\theta), z(\theta), e(\theta), \theta) &= w + \frac{\mu}{\nu}x_z(z(\theta), l(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)}\psi'(e(\theta))\frac{d(E_\theta)}{dz} \\
pf_e(l(\theta), z(\theta), e(\theta), \theta) &= \psi'(e(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)}\frac{d(\psi' E_\theta)}{de}
\end{aligned}
$$

where

$$
\lambda(\theta) = -\nu(1 - K(\theta)) + \int_{\underline{\theta}}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta)
$$

and

$$
\nu = \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta) + \alpha_0 \mathcal{W}'(v + rl^\circ + \tau)K(\theta_s) > 0.
$$

Also the marginal farmer has identity θ_s given by:

$$
pf(l(\theta_s), z(\theta_s), e(\theta_s), \theta_s) - rl(\theta_s) - wz(\theta_s) - \psi(e(\theta_s)) - \frac{\mu}{\nu}x(z(\theta_s), l(\theta_s)) - \frac{\lambda(\theta_s)}{\nu k(\theta_s)}\psi'(e(\theta_s))\left. E_\theta\right|_{\theta=\theta_s} = v
$$

Proof: See appendix B. ∎

A direct comparison between the first best and the second best suggests that the first-order conditions for l, z and e are now corrected by a new term due to incentive compatibility. Let us consider for instance the condition for land use:[7]

$$
pf_l(l(\theta), z(\theta), e(\theta), \theta) = r + \frac{\mu}{\nu}x_l(z(\theta), l(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)}\psi'(e(\theta))\frac{d(E_\theta)}{dl}
$$

This equation can be interpreted as follows: in the absence of asymmetric information on θ (first best) then $\lambda(\theta) = 0$ so that we recover the first-best rule described before. Under asymmetric information, there is a need to introduce an incentive distortion due to the fact that by distorting l it is possible to extract rents in order to redistribute income across

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[7]The interpretation of the condition for z is similar.

farmers. Note that we get the usual result of no distortion at the top as $\lambda(\bar{\theta}) = 0$ which means no distortion for the highest type of farmers. Actually, $\frac{d(E_\theta)}{dl}$ measures how much land use l affects the potential effort savings ($E_\theta < 0$) associated with an increase in efficiency. It also measures how the rent $U'(\theta)$ evolves with l. Note that we have:

$$\frac{d(E_\theta)}{dl} = \frac{\partial}{\partial l}\left(-\frac{f_\theta}{f_e}\right) = -\frac{f_{\theta l} f_e - f_\theta f_{el}}{(f_e)^2} > 0$$

if and only if

$$\frac{f_{el}}{f_e} > \frac{f_{\theta l}}{f_\theta}(> 0).$$

which means in terms of elasticity that the elasticity of marginal productivity of ability with respect to land use is lower than the elasticity of marginal productivity of effort with respect to land use. In other words, land use has a more (positive) influence on the marginal productivity of effort than on the marginal productivity of ability.

It follows that by increasing the land use of a type-θ farmer, one also decreases the (positive) rate of growth $U'(\theta) = -\psi' E_\theta$ of the rent devoted to this farmer and hence the rents to all more efficient farmers (between θ and $\bar{\theta}$). The social cost of this decrease is $\lambda(\theta)\frac{d(U'(\theta))}{dl}$. On the other hand, the distortion in l for type θ relative to the first best level $\left(pf_l - r - \frac{\mu}{\nu}x_l\right)$ has social cost $\nu\left(pf_l - r - \frac{\mu}{\nu}x_l\right)$ and occurs with probability $k(\theta)$. The trade-off between redistribution and efficiency thus yields to the condition in the Proposition. Finally, for the highest type, there is no more efficient farmers on which one would want to extract rents and this calls for efficiency at the top, i.e. $\lambda(\bar{\theta}) = 0$.

Now it is possible to obtain a result in the spirit of Laffont-Tirole (1991) on the dichotomy between the tasks of redistributing income among farmers and the tasks of correctly pricing land and fertilizer by taking into account externalities.

Proposition 3 *The tasks of distributing income support among farmers and the tasks of correctly pricing land and fertilizer by taking into account externalities are disconnected if* $f(l, z, e, \theta) = f(l, z, h(e, \theta))$. *Under this assumption, we obtain the first-best rules for land*

and fertilizer (for a given effort)

$$pf_l(l(\theta), z(\theta), e(\theta), \theta) = r + \frac{\mu}{\nu} x_l(z(\theta), l(\theta))$$

$$pf_z(l(\theta), z(\theta), e(\theta), \theta) = w + \frac{\mu}{\nu} x_z(z(\theta), l(\theta))$$

while the effort remains distorted for redistributive purposes

$$pf_e(l(\theta), z(\theta), e(\theta), \theta) = \psi'(e(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)} \frac{d(\psi' E_\theta)}{de}.$$

Proof: If $f(l, z, e, \theta) = f(l, z, h(e, \theta))$ (Aggregation theorem of Leontieff (1947)), then

$$\frac{f_{el}}{f_e} = \frac{f_{\theta l}}{f_\theta}$$

$$\frac{f_{ez}}{f_e} = \frac{f_{\theta z}}{f_\theta}$$

which implies that $\frac{d(E_\theta)}{dl} = \frac{d(E_\theta)}{dz} = 0$. ∎

This Proposition applies for instance if f is a Cobb-Douglas function, i.e. $f = \theta e^{\alpha_1} z^{\alpha_2} l^{\alpha_3}$. Proposition 3 implies that the second best can be decentralized through a pure pigouvian regulation of x and l together with a non linear subsidy t as a function of π alone for redistributive purposes.

Concerning the effort decision, we have

$$pf_e(l(\theta), z(\theta), e(\theta), \theta) = \psi'(e(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)} \frac{d(\psi' E_\theta)}{de} \tag{4}$$

with

$$
\begin{aligned}
\frac{d(\psi' E_\theta)}{de} &= \psi' \frac{d(E_\theta)}{de} + \psi'' E_\theta \\
&= \psi' \left[\frac{\partial(-f_\theta/f_e)}{\partial e} \right] - \psi'' \frac{f_\theta}{f_e} \\
&= -\psi' \left[\frac{f_{\theta e} f_e - f_{ee} f_\theta}{(f_e)^2} \right] - \psi'' \frac{f_\theta}{f_e} < 0
\end{aligned}
$$

as $f_{ee} \leq 0$ and $f_{\theta e} \geq 0$. This means that increasing the effort allows to increase the rate $U'(\theta)$ of growth of rents. This means that if the objective is to redistribute income towards the poorer farmers (i.e. if $\lambda(\theta) < 0$), it might be optimal to reduce the incentives to exert effort.

This is because increasing the effort amounts to finally get a more unequal distribution of incomes in the population. This expresses the conflict between the search for more equity between heterogenous farmers and efficiency. On the contrary, when $\lambda(\theta) > 0$, it is optimal to give incentives for effort in order to generate more incomes to the more efficient farmers.

It then appears that it is crucial to understand how the nature of objective (the desire to redistribute and the presence of social weights for different types of farmers) will generate the direction of the distortions under second best and this is precisely the role of $\lambda(\theta)$.

5.2   Implementation

Before looking at the sign of $\lambda(\theta)$, it is interesting to go back to the way the optimal policy can be implemented through the generalized transfer $t(\pi, z, l)$ intented for active farmers.[8] Facing such a transfer, the type-θ farmer maximizes his utility by choosing the land l, the fertilizer level z and the effort e (or equivalently the profit level π):

$$\max_{\pi, z, l} U = \pi + t(\pi, z, l) - \psi(E(\theta, l, z, \pi))$$

The corresponding first-order conditions for an interior solution are as follows:

$$
\begin{aligned}
\frac{\partial U}{\partial \pi} &= 1 + \frac{\partial t}{\partial \pi} - \psi' E_\pi = 0 \\
\frac{\partial U}{\partial z} &= \frac{\partial t}{\partial z} - \psi' E_z = 0 \\
\frac{\partial U}{\partial l} &= \frac{\partial t}{\partial l} - \psi' E_l = 0
\end{aligned}
$$

Recalling that $E_\pi = 1/pf_e$, we first get for the effort:

$$\frac{\partial t}{\partial \pi} = \frac{\psi' - pf_e}{pf_e} = -\frac{\lambda(\theta)}{\nu k(\theta) pf_e} \frac{d(\psi' E_\theta)}{de} \tag{5}$$

using (4). This means that whenever $\lambda(\theta) > (<)0$, then the transfer t increases (decreases) in the observed level of profit. Intuitively, when $\lambda(\theta) > 0$, such a pattern gives incentives to the farmer to exert more effort. Conversely, when $\lambda(\theta) < 0$, it is optimal to reduce the incentives to exert effort as this goes against the search for a more equal distribution of

---

[8]Recall that non active farmers receive the uniform transfer τ.

incomes. Furthermore, at the top ($\theta = \overline{\theta}$), the marginal rate of subsidy is $\frac{\partial t}{\partial \pi} = 0$ because $\lambda(\overline{\theta}) = 0$.

Concerning the fertilizer level, we have

$$
\begin{aligned}
\frac{\partial t}{\partial z} &= \psi' E_z = -\psi' \frac{pf_z - w}{pf_e} \\
&= -\frac{\psi'}{pf_e} \left[\frac{\mu}{\nu} x_z(z(\theta), l(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)} \psi'(e(\theta)) \frac{d(E_\theta)}{dz} \right]
\end{aligned}
$$

It follows that the transfer should decrease with the fertilizer use due to the damage impact but this implicit tax also depends on the redistributive concern through the incentive distortion. If

$$
(0 <) \frac{f_{ez}}{f_e} < \frac{f_{\theta z}}{f_\theta}
$$

which once again means in terms of elasticity that the elasticity of marginal productivity of ability with respect to fertilizer use is larger than the elasticity of marginal productivity of effort with respect to fertilizer use, then $\frac{d(E_\theta)}{dz} < 0$. This means that if $\lambda(\theta) < 0$ then it is optimal to increase the implicit taxation of fertilizer for redistributive issues.[9]

In other words, if $\frac{d(E_\theta)}{dz} < 0$ then ceteris paribus the taxation on fertilizer is heavier than under the first best. In that case, if one increases the fertilizer use of a type-θ farmer, one also increases the rent of all more efficient farmers which goes against inequality preferences. Hence, the tax on fertilizer cannot escape from redistribution considerations except under the pricing-dichotomy assumption. The intuition goes as follows: when $\frac{dU'(\theta)}{dz} = -\psi' \frac{d(f_\theta / f_e)}{dz} \geq 0$, then more fertilizer makes it easier for the farmers to convert a superior ability into less effort. A farmer who wants to mimic the profit of a lower ability farmer uses more fertilizer than the farmer being mimicked. Finally, there is no reason to overtax the more able because there is nobody more able than him, hence first best taxation rule occurs.[10]

---

[9] Obviously, we would have the same situation but for opposite reasons when $\lambda(\theta) > 0$ and $\frac{d(E_\theta)}{dz} > 0$.

[10] A similar interpretation holds for the land use as well.

5.3 The direction of distortions

As suggested above, of utmost importance for the direction of the incentive distortions is the sign of the marginal cost of incentive compatibility $\lambda(\theta)$. For instance, from (4), we know that if $\lambda(\theta)$ is negative for all θ in $\left[\theta_s, \overline{\theta}\right]$ then it is optimal to distort downward the effort provided by all active farmers (except at the top). On the contrary, each times $\lambda(\theta)$ is positive this entails that distorting upwards the effort is optimal.

First, we know that there is no distortion at the top $(\lambda(\overline{\theta}) = 0)$. Second, we also know that

$$\lambda(\theta_s) = \left[\nu - \alpha_0 \mathcal{W}'(U(\theta_s))\right] K(\theta_s) \tag{6}$$

Third, derivating the expression of $\lambda(\theta)$, we get

$$\lambda'(\theta) = \left[\nu - \alpha(\theta)\mathcal{W}'(U(\theta))\right] k(\theta). \tag{7}$$

It follows that a priori there are many possibilities for the pattern of $\lambda(\theta)$ over the set $\left[\theta_s, \overline{\theta}\right]$ depending in particular on the social weight function $\alpha(\theta)$. The following lemma is useful to obtain further results concerning the sign of $\lambda(\theta)$.

Lemma 4 *The marginal weighted social utility of income $\alpha(\theta)\mathcal{W}'(U(\theta))$ is increasing in θ if and only if the elasticity $\chi(\theta) = \theta\alpha'(\theta)/\alpha(\theta)$ of the social weight function is greater than $\rho(\theta) = -\theta\mathcal{W}''(U(\theta))U'(\theta)/\mathcal{W}'(U(\theta))$, the absolute value of the elasticity of the marginal social utility function $\mathcal{W}'(U(\theta))$ with respect to θ.*

Proof: We have

$$\frac{d}{d\theta}\left[\alpha(\theta)\mathcal{W}'(U(\theta))\right] = \alpha'(\theta)\mathcal{W}'(U(\theta)) + \alpha(\theta)\mathcal{W}''(U(\theta))U'(\theta)$$

$$= \alpha(\theta)\mathcal{W}'(U(\theta))\left[\frac{\alpha'(\theta)}{\alpha(\theta)} + \frac{\mathcal{W}''(U(\theta))U'(\theta)}{\mathcal{W}'(U(\theta))}\right].$$

Let us denote $\chi(\theta) = \theta\alpha'(\theta)/\alpha(\theta)$ the elasticity of the political weight function $\alpha(.)$ w.r.t θ. Also let us define $\rho(\theta) = -\frac{\theta\mathcal{W}''(U(\theta))U'(\theta)}{\mathcal{W}'(U(\theta))}$ as the absolute value of the elasticity of $\mathcal{W}'(U(\theta))$

with respect to θ (recall that $\mathcal{W}$ is concave and that $U' > 0$). Hence, $\alpha(\theta)\mathcal{W}'(U(\theta)) > 0$ if and only if $\chi(\theta) \geq \rho(\theta)$. ∎

Hence the social weight function $\alpha(\theta)$ has to be sufficiently increasing in θ in order to counterbalance the desire to redistribute which is expressed by the concavity of $\mathcal{W}$. In the following, we describe the situations where the function $\alpha(\theta)\mathcal{W}'(U(\theta))$ is assumed to be monotone in θ, either increasing or decreasing.

We now introduce the following condition.

Condition 5 *The marginal weighted social utility of the income of the mean non active farmer, $\alpha_0 \mathcal{W}'(U(\theta_s))$, is greater or equal to the average marginal weighted social utility of income for all agents, ν.*

From (6), this condition is equivalent to $\lambda(\theta_s) < 0$. This essentially means that the social weight of the poorest agents is rather high compared to the total population of farmers. In particular, Condition 5 holds in the particular case of equal social weights ($\alpha(\theta) = 1$ for any θ). We are now able to establish the following Proposition.

Proposition 6 *Assume that the marginal weighted social utility of income $\alpha(\theta)\mathcal{W}'(U(\theta))$ is decreasing in θ or equivalently that $\chi(\theta) \leq \rho(\theta)$. Then,*

(i) *the transfer $t(\pi, l, z)$ decreases in π if and only if Condition 5 holds,*

(ii) *the transfer $t(\pi, l, z)$ is first increasing and second decreasing in π if and only if Condition 5 does not hold.*

Proof: See appendix C. ∎

The context described by Proposition 6 is one where the priority in terms of income support is directed towards the less efficient farmers, including those who are not active. This is because the marginal weighted social utility of income is decreasing in θ. We would then expect that the transfer should optimally decrease with the observed level of profit π as

this is the way to reduce incentives to exert effort which entails a more equal distribution of incomes in the population. This intuitive result holds but only under Condition 1. Actually, as suggested by part (ii), it is possible for the optimal policy to gives incentives for effort for the less efficient active farmers by making t an increasing function of observed π.

Consider for instance the particular case where $\alpha(\theta) = 1$ for any θ (equal social weight situation). Here the only task of the government is to redistribute from the rich to the poor as $\mathcal{W}$ is concave. One can check that $\lambda(\theta)$ is negative, first decreasing then increasing. This means that the point where there is the highest decrease in the transfer when π increases lies somewhere between θ_s and $\overline{\theta}$.

Note finally that because of the no distortion at the top result, the transfer is always locally convex in the neighborhood of $\pi(\overline{\theta})$ which means that the rate of decrease in the transfer is diminishing when approaching the highest level of profit.

Examining the opposite situation where $\alpha(\theta)\mathcal{W}'(U(\theta))$ is an increasing function of θ, we obtain the following proposition.

Proposition 7 *Assume that the marginal weighted social utility of income $\alpha(\theta)\mathcal{W}'(U(\theta))$ is increasing in θ or equivalently that $\chi(\theta) \geq \rho(\theta)$. Then, the transfer $t(\pi, l, z)$ is increasing in π.*

Proof: See appendix D. ∎

From Proposition 7, we deduce that it is optimal to make the transfer an increasing function of π in order to give incentives to exert effort. Nevertheless, from the no distortion at the top result, t is locally concave around $\pi(\overline{\theta})$. The highest marginal rate of subsidy is hence interior or in θ_s.

6   Observable aggregate variables

In this section, we discuss the optimal design of the income support and environmental policy when the regulator cannot observe the individual variables like the output level, the use of

variable input like fertilizers or the amount of land involved in production. Hence, the policy can only rely on the observation of the status of each agent and their accounting profit. However, as the regulator can observe the aggregate consumption of fertilizer or land and the aggregate level of output, linear taxes are available. The taxes on output, land and fertilizer are denoted respectively t^q, t^l and t^z. In that case, we have to compute the optimal reaction of a farmer to these linear taxes.

The problem of the regulator is now to find the optimal uniform transfer τ for non active farmers, the optimal non linear transfer t as a function of observed π together with the linear taxes t^q, t^l and t^z in order to maximize the social welfare subject to the budget constraint and the environmental constraint. Hence, the utility of the type-θ farmer who is active writes as follows:

$$\max_{z,l,e} U = (p + t^q)f(l, z, e, \theta) - (w + t^z)z - (r + t^l)(l - l^\circ) + t(\pi) - \psi(e)$$

where $\pi = (p + t^q)f(l, z, e, \theta) - (w + t^z)z - (r + t^l)(l - l^\circ)$. We denote z^*, l^* and e^* the optimal decisions for this farmer given the existing policy $\{t^q, t^l, t^z, t(\pi), \tau\}$.

Solving the regulator's problem, we establish the following Proposition.

Proposition 8 *At an optimal separating policy, the non linear transfer $t(\pi)$ depends on the presence of environmental externalities. The optimal effort of a type-θ farmer is given by*

$$pf_e = \psi' + \frac{\lambda(\theta)}{\nu k(\theta)}\frac{d(\psi'E_\theta)}{de} - (pf_l - r - \frac{\mu}{\nu}x_l)l_e^* - (pf_z - w - \frac{\mu}{\nu}x_z)z_e^* \qquad (8)$$

Proof: See appendix E. ∎

Comparing with Proposition 2, we now have two additional terms in determining the incentives to exert effort and thereby the way the non linear transfer $t(\pi)$ evolves. The last two terms of (8) correspond to the marginal impact of the effort on the profit net of the environmental damage. This is due to the fact that with only one non linear instrument, the regulator has to take into account the redistributive concern (through the term $\frac{\lambda(\theta)}{\nu k(\theta)}\frac{d(\psi'E_\theta)}{de}$) but also the influence of $t(\pi)$ on the decisions over land and fertilizer taken by the farmer. In other

words, the income support policy must now take into account the presence of externalities contrary to the preceding section.

Intuitively, suppose for instance that $pf_z - w - \frac{\mu}{\nu}x_z < 0$ which means that at the optimum the type-θ farmer over-uses fertilizer compared to the first best. If increasing the effort also contributes to increase the fertilizer use, then the last term is positive and this means that it is optimal to reduce the transfer $t(\pi)$ at the margin because of the negative externalities due to z.

The linear taxes are designed optimally by taking account all their effects on incentives. Consider for instance the case of the optimal linear tax on fertilizer. From the appendix E we have the following condition:

$$\nu \int_{\theta_s}^{\overline{\theta}} \left[(pf_l - r)\, l_{t^z}^* + (pf_z - w)\, z_{t^z}^* \right] dK(\theta) - \mu \int_{\theta_s}^{\overline{\theta}} \left[x_z z_{t^z}^* + x_l l_{t^z}^* \right] dK(\theta)$$
$$- \int_{\theta_s}^{\overline{\theta}} \lambda(\theta) \frac{d(\psi' E_\theta)}{dt^z} d\theta = 0.$$

Rearranging, we have

$$\int_{\theta_s}^{\overline{\theta}} \left[\left(pf_l - r - \frac{\mu}{\nu} x_l \right) l_{t^z}^* + \left(pf_z - w - \frac{\mu}{\nu} x_z \right) z_{t^z}^* \right] dK(\theta) = \int_{\theta_s}^{\overline{\theta}} \frac{\lambda(\theta)}{\nu} \frac{d(\psi' E_\theta)}{dt^z} d\theta$$

Hence, the tax t^z is set such that the total marginal impact on profit net of damage is equal to the total marginal impact on the informational rents left to all active farmers. In other words, when manipulating the tax rate t^z the regulator will modify the decisions taken with respect to fertilizer and also land use. This will impact the social surplus of production $\pi - \frac{\mu}{\nu}x$. The other impact is that this tax influences the distribution of incomes in the population (which is reflected by the term . We have a similar interpretation for the tax t^l on land and the subsidy t^q on production.

7 Introducing an endogenous market price for lands

In this section, we go back to the second best analysis of section 5 but we also introduce the possibility of having an endogenous market price for land. For simplicity, we consider only

the case where the social weights are equal ($\alpha(\theta) = 1$ for any θ).

Let L the total amount of land used in the agricultural sector and $V(L)$ the value (or opportunity cost) of land in other sectors, that is increasing and concave. The market value of land per acre is then given by

$$r(L) = -V'(L)$$

where $r'(L) > 0$ because of diminishing returns to land use in other sectors.

Following Innes (2003), suppose that we have the endowment for agricultural usage denoted $L^{\circ} = N \int_{\Theta} l^{\circ} dK(\theta)$ and we denote also L the total land used for agriculture. N represents the number of farmers that is hereafter normalized to equal one, $N = 1$. Similarly, for non agriculture usage, we have L°_{NA} and L_{NA}. We have

$$V(L) = \max_{L_T \geq L^{\circ} + L^{\circ}_{NA}} B(L_T - L) - c(L_T - L^{\circ} - L^{\circ}_{NA})$$

where $L_T = L + L_{NA}$ is the aggregate land use and $c(.)$ is the increasing, convex cost of developing new lands. We also have

$$\max_{L_{NA}} B(L_{NA}) - r L_{NA} \Leftrightarrow B'(L_{NA}) = r = -V'(L)$$

It follows that the associated supply function is $L^S(r)$ and it is given implicitly by $r = r(L - L^{\circ})$. And the equilibrium on the market of land is described by

$$L^S(r^*) + L^{\circ} = \int_{\Theta} l^*(r^*; \theta) dK(\theta)$$

where we denote the demand for land from a type-θ farmer by $l^*(r^*; \theta)$. The model encompasses the limit cases where the supply of land is either fixed (perfectly inelastic) as in Guyomard et al. (2005) or perfectly elastic (which means that r is constant) as in Bourgeon and Chambers (2000).

The objective of the regulator is written as the sum of social utility of profits plus the value of land use and the land rents of the landowners. However, because the social utility $\mathcal{W}$ is only defined up to an increasing transformation, we normalize the problem by aggregating

the certainty equivalent:

$$CE(U) = \mathcal{W}^{-1} \left(\int_{\underline{\theta}}^{\bar{\theta}} \mathcal{W}(U(\theta)) dK(\theta) \right)$$

with the opportunity cost of land use and the rents of landowners which can be written as

$$B(L_{NA}) - r(L_{NA} - L_{NA}^{\circ}) + r(L_{NA} + L - L^{\circ} - L_{NA}^{\circ}) - c(L_T - L^{\circ} - L_{NA}^{\circ})$$

$$= B(L_{NA}) + r(L - L^{\circ}) - c(L_T - L^{\circ} - L_{NA}^{\circ})$$

$$= V(L) + r(L - L^{\circ})$$

Hence, the objective of the regulator sums up to maximize

$$CE(U) + V(L) + r(L - L^{\circ})$$

under the budget constraint, the environmental constraint and the incentive compatibility constraints as written in section 5. Solving this program, we establish the following proposition.

Proposition 9 *Assuming a separating optimal policy, the optimal allocation of land use with endogenous market price for land is given by*

$$pf_l(l(\theta), z(\theta), e(\theta), \theta) = r + \frac{\delta}{\nu} + \frac{\mu}{\nu} x_l(z(\theta), l(\theta)) + \frac{\lambda(\theta)}{\nu k(\theta)} \psi'(e(\theta)) \frac{d(E_\theta)}{dl} \qquad (9)$$

where

$$\lambda(\theta) = -\nu(1 - K(\theta)) + \int_{\theta}^{\bar{\theta}} \frac{\mathcal{W}'(U(u))}{\mathcal{W}'(CE(U))} k(u) du$$

and

$$\delta = r'(L) \left[(\nu - 1) L - \nu K(\theta_s) l^{\circ} \right].$$

Proof: See appendix F. ∎

Proposition 9 suggests that when the market price is endogenous, the optimal policy now takes into account the shadow price δ of the aggregate land demand L from the agricultural sector. Obviously, if the market price r is constant ($r'(L) = 0$) then we are back to Proposition

23

5, with the only minor difference in the evolution of $\lambda(\theta)$ due to the normalization through the certainty equivalent $CE(U)$ in the objective in place of $\mathbb{E}_\theta \mathcal{W}(U)$.

The intuition of the expression of δ goes as follows. As $r'(L) > 0$, due to diminishing returns to land use in other sectors, each time we increase marginally the land use $l(\theta)$ for a type-θ farmer, this has also some consequences for the rest of the economy. Indeed, as the price r increases, this induces a marginal loss to all active farmers who have to pay a larger price for land and this needs a compensation in terms of income support which costs ν times L. But on the other hand, we also increase the rents of landowners marginally over L units. Last, we also increase the income of non active farmers (in proportion $K(\theta_s)$) who rent their endowment l° and hence are less needed to get socially costly income support.

Consequently, if the drawback for active farmers outweighs the advantage for landowners' rents and non active farmers' income, then at the optimum, we have $\delta > 0$. Hence, if we assume for simplicity the absence of asymmetric information ($\lambda(\theta) = 0$) and the absence of land impact on the environmental constraint ($x_l = 0$), then the optimal land use of a type-θ farmer should be set such that the private marginal return should be equal to the social price which is equal to r plus the *positive* shadow price δ weighted by the shadow cost ν of the budget constraint. In other words, it is optimal ceteris paribus to induce an under-use of land compared to the first best with constant market price given by $pf_l = r$.

8 Conclusions

In this paper, we have developed a model of regulation for a set of heterogenous farmers whose production yields to environmental externalities. The goal of the regulator is first to offer some income support depending on collective preferences towards income redistribution and second to internalize externalities. The optimal policy is constrained by the information available. We first considered the second best where the regulator is able to observe all individuals decisions in terms of inputs and individual profit, but not the individual farming labor supply. We characterized the generalized transfer in function of the desire to redistribute

and the underlying characteristics of the production process. In a second step, we assumed that the regulator has only information on aggregate consumption of inputs and hence can only tax/subsidy linearly inputs and output. However, because the accounting profit remains observable, a non linear transfer of profit is still part of the optimal policy. In the last part of the paper, we have endogenized the market price of land and examine how the optimal policy should be modified.

Obviously, the limit of such an approach of income support and agri-environmental regulation lies in its static character. Also a natural extension would consider a model where farmers are heterogenous along other dimensions, for instance the endowment in land l° or the disutility of effort. Finally, it would be interesting to introduce price and production uncertainty (see Sheriff 2008 for a first approach) to better understand the insurance role of regulation in agriculture. All these interesting extensions are devoted to further research.

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Appendix

A   Proof of proposition 1

The Lagrangean writes as follows:

$$
\begin{aligned}
\mathcal{L} \;=\; & \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}(\pi(\theta) + T(\theta) - \psi(e(\theta)))dK(\theta) + \int_{\underline{\theta}}^{\theta_s} \alpha(\theta)\mathcal{W}(v + rl^{\circ} + \tau)dK(\theta) \\
& + \nu\left(B - \int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) - \int_{\theta_s}^{\overline{\theta}} T(\theta)dK(\theta)\right) + \mu\left(X - \int_{\theta_s}^{\overline{\theta}} x(z(\theta),l(\theta))dK(\theta)\right)
\end{aligned}
$$

Pointwise maximization gives us the following first-order conditions:

$$
\alpha(\theta)\mathcal{W}'(U(\theta))\left[pf_l - r\right] \;=\; \mu x_l
$$

$$
\alpha(\theta)\mathcal{W}'(U(\theta))\left[pf_z - r\right] \;=\; \mu x_z
$$

$$
\alpha(\theta)\mathcal{W}'(U(\theta))\left[pf_e - \psi'\right] \;=\; 0
$$

together with

$$
\alpha(\theta)\mathcal{W}'(U(\theta)) = \nu
$$

for any $\theta \geq \theta_s$. Also, derivating with respect to τ, we get

$$
\alpha_0 \mathcal{W}'(v + rl^{\circ} + \tau) = \nu
$$

where we denote $\alpha_0 = \frac{1}{K(\theta_s)}\int_{\underline{\theta}}^{\theta_s} \alpha(\theta)dK(\theta)$. Finally we also have (assuming an interior solution)

$$
\frac{\partial \mathcal{L}}{\partial \theta_s} = -\mu x(z(\theta_s), l(\theta_s)) - \nu(\tau - T(\theta_s)) = 0
$$

which gives the identity of the marginal farmer. This concludes the proof.

B   Proof of Proposition 2

A type-θ farmer solves the following program:

$$
\max_{e,\tilde{\theta}} \pi(\tilde{\theta}) + T(\tilde{\theta}) - \psi(e)
$$

$$
\text{s.t.}
$$

$$
\pi(\tilde{\theta}) = pf(l(\tilde{\theta}), z(\tilde{\theta}), e, \theta) - wz(\tilde{\theta}) - r(l(\tilde{\theta}) - l^{\circ})
$$

Hence the farmer's program can be rewritten as follows:

$$\max_{\tilde{\theta}} \pi(\tilde{\theta}) + T(\tilde{\theta}) - \psi(E(\theta, l(\tilde{\theta}), z(\tilde{\theta}), \pi(\tilde{\theta})))$$

and we denote $U(\theta, \tilde{\theta}) = \pi(\tilde{\theta}) + T(\tilde{\theta}) - \psi(E(\theta, l(\tilde{\theta}), z(\tilde{\theta}), \pi(\tilde{\theta})))$ the utility of a type-θ farmer announcing to be of type $\tilde{\theta}$. Incentive compatibility requires that:

$$\left.\frac{\partial U}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = 0 \text{ and } \left.\frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta}\right|_{\tilde{\theta}=\theta} \geq 0$$

that is

$$\left.\frac{\partial U}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = \pi'(\theta) + T'(\theta) - \psi' \left[E_l l'(\theta) + E_z z'(\theta) + E_\pi \pi'(\theta)\right] = 0$$

and

$$
\begin{aligned}
\left.\frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta}\right|_{\tilde{\theta}=\theta} &= -\psi'' E_\theta \left[E_l l'(\theta) + E_z z'(\theta) + E_\pi \pi'(\theta)\right] - \psi' \left[E_{l\theta} l'(\theta) + E_{z\theta} z'(\theta) + E_{\pi\theta} \pi'(\theta)\right] \\
&= l'(\theta) \left[\psi'' E_\theta E_l + \psi' E_{l\theta}\right] + z'(\theta) \left[\psi'' E_\theta E_z + \psi' E_{z\theta}\right] + \pi'(\theta) \left[\psi'' E_\theta E_\pi + \psi' E_{\pi\theta}\right] \leq 0
\end{aligned}
$$

which will be checked ex-post.

If we denote $U(\theta) = U(\theta, \theta)$ then we have

$$U'(\theta) = -\psi'(e) E_\theta(\theta, l, z, \pi(\theta, l, z, e)) > 0$$

so that the rent is increasing in θ.

Hence the program to be solved (ignoring the second order constraint) is

$$\max_{e,l,z,\theta_s,\tau,U} \int_{\theta_s}^{\overline{\theta}} \alpha(\theta) \mathcal{W}(U(\theta)) dK(\theta) + \int_{\underline{\theta}}^{\theta_s} \alpha(\theta) \mathcal{W}(v + rl^{\circ} + \tau) dK(\theta)$$

s.t.

$$\int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) + \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - \pi(\theta, l(\theta), z(\theta), e(\theta)) + \psi(e(\theta))\right] dK(\theta) \leq B$$

$$\int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta)) dK(\theta) \leq X$$

$$U(\theta_s) = v + rl^{\circ} + \tau$$

$$U'(\theta) = -\psi'(e(\theta)) E_\theta(\theta, l(\theta), z(\theta), \pi(\theta, l(\theta), z(\theta), e(\theta)))$$

$$\pi(\theta, l(\theta), z(\theta), e(\theta)) = pf(l(\theta), z(\theta), e(\theta), \theta) - wz(\theta) - r(l(\theta) - l^{\circ})$$

or equivalently

$$\max \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}(U(\theta))dK(\theta) + \alpha_0 \mathcal{W}(v + rl^{\circ} + \tau)K(\theta_s)$$

s.t.

$$\tau K(\theta_s) + \int_{\theta_s}^{\overline{\theta}} [U(\theta) - \pi(\theta, l(\theta), z(\theta), e(\theta)) + \psi(e(\theta))]\, dK(\theta) \leq B$$

$$\int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta))dK(\theta) \leq X$$

$$U(\theta_s) = v + rl^{\circ} + \tau$$

$$U'(\theta) = -\psi'(e(\theta))E_\theta(\theta, l(\theta), z(\theta), \pi(\theta, l(\theta), z(\theta), e(\theta)))$$

$$\pi(\theta, l(\theta), z(\theta), e(\theta)) = pf(l(\theta), z(\theta), e(\theta), \theta) - wz(\theta) - r(l(\theta) - l^{\circ})$$

The Lagrangean writes as follows:

$$
\begin{aligned}
\mathcal{L} \;=\; & \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}(U(\theta))dK(\theta) + \alpha_0 \mathcal{W}(v + rl^{\circ} + \tau)K(\theta_s) \\
& + \nu \left(B - \tau K(\theta_s) - \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - pf(l(\theta), z(\theta), e(\theta), \theta) + wz(\theta) + r(l(\theta) - l^{\circ}) + \psi(e(\theta)) \right] dK(\theta) \right) \\
& + \mu \left(X - \int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta))dK(\theta) \right) + \int_{\theta_s}^{\overline{\theta}} \lambda(\theta)\left(-\psi' E_\theta - U'(\theta) \right) d\theta
\end{aligned}
$$

Integrating by parts the last term containing $U'(\theta)$ we get

$$
\begin{aligned}
\int_{\theta_s}^{\overline{\theta}} \lambda(\theta)U'(\theta)d\theta \;=\; & [\lambda(\theta)U(\theta)]_{\theta_s}^{\overline{\theta}} - \int_{\theta_s}^{\overline{\theta}} \lambda'(\theta)U(\theta)d\theta \\
\;=\; & -\lambda(\theta_s)U(\theta_s) - \int_{\theta_s}^{\overline{\theta}} \lambda'(\theta)U(\theta)d\theta
\end{aligned}
$$

as $\lambda(\overline{\theta}) = 0$ because the value of U at $\overline{\theta}$ is free. Replacing in the Lagrangean (and recall that $U(\theta_s) = v + rl^{\circ} + \tau$), we obtain:

$$
\begin{aligned}
\mathcal{L} \;=\; & \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}(U(\theta))dK(\theta) + \alpha_0 \mathcal{W}(v + rl^{\circ} + \tau)K(\theta_s) \\
& + \nu \left(B - \tau K(\theta_s) - \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - pf(l(\theta), z(\theta), e(\theta), \theta) + wz(\theta) + r(l(\theta) - l^{\circ}) + \psi(e(\theta)) \right] dK(\theta) \right) \\
& + \mu \left(X - \int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta))dK(\theta) \right) + \lambda(\theta_s)\left[v + rl^{\circ} + \tau \right] + \int_{\theta_s}^{\overline{\theta}} \left\{ \lambda'(\theta)U(\theta) - \lambda(\theta)\psi' E_\theta \right\} d\theta
\end{aligned}
$$

Derivating, we get the following necessary conditions (for $\theta \in \left[\theta_s, \overline{\theta}\right]$):

$$\frac{\partial \mathcal{L}}{\partial \tau} = \alpha_0 \mathcal{W}'(v + rl^{\circ} + \tau)K(\theta_s) - \nu K(\theta_s) + \lambda(\theta_s) = 0 \qquad (10)$$

$$\frac{\partial \mathcal{L}}{\partial l(\theta)} = \nu(pf_l - r)k(\theta) - \mu x_l k(\theta) - \lambda(\theta)\psi'\frac{d(E_\theta)}{dl} = 0$$

$$\frac{\partial \mathcal{L}}{\partial z(\theta)} = \nu(pf_z - w)k(\theta) - \mu x_z k(\theta) - \lambda(\theta)\psi'\frac{d(E_\theta)}{dz} = 0$$

$$\frac{\partial \mathcal{L}}{\partial e(\theta)} = \nu(pf_e - \psi')k(\theta) - \lambda(\theta)\frac{d(\psi' E_\theta)}{de} = 0$$

$$\frac{\partial \mathcal{L}}{\partial U(\theta)} = \alpha(\theta)\mathcal{W}'(U(\theta))k(\theta) - \nu k(\theta) + \lambda'(\theta) = 0$$

and $\frac{\partial \mathcal{L}}{\partial \theta_s} = 0$ which implies:

$$pf(l(\theta_s), z(\theta_s), e(\theta_s), \theta_s) - rl(\theta_s) - wz(\theta_s) - \psi(e(\theta_s)) - \frac{\mu}{\nu}x(z(\theta_s), l(\theta_s)) - \frac{\lambda(\theta_s)}{\nu k(\theta_s)}\psi'(e(\theta_s))\, E_\theta|_{\theta=\theta_s} = v.$$

Integrating $\lambda'(\theta)$, we get

$$\int_{\theta}^{\overline{\theta}} \lambda'(\theta)d\theta = \int_{\theta}^{\overline{\theta}} \left\{\nu - \alpha(\theta)\mathcal{W}'(U(\theta))\right\}k(\theta)d\theta$$

$$\lambda(\theta) = -\nu(1 - K(\theta)) + \int_{\theta}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta)$$

so that the (positive) shadow cost of the budget constraint writes (using (10)):

$$\nu = \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta) + \alpha_0\mathcal{W}'(v + rl^{\circ} + \tau)K(\theta_s) > 0$$

which represents the sum of all marginal (weighted) social utilities. And we also obtain that

$$\lambda(\theta_s) = \left[\nu - \alpha_0\mathcal{W}'(v + rl^{\circ} + \tau)\right]K(\theta_s)$$

$$= K(\theta_s)\left[\int_{\theta_s}^{\overline{\theta}} \left\{\alpha(\theta)\mathcal{W}'(U(\theta)) - \alpha_0\mathcal{W}'(v + rl^{\circ} + \tau)\right\}dK(\theta)\right]$$

This concludes the proof.

C   Proof of Proposition 6

Assume that the function $\alpha(\theta)\mathcal{W}'(U(\theta))$ is decreasing in θ. Part (i): from equation (5), $\frac{\partial t}{\partial \pi} < 0$ everywhere if and only if $\lambda(\theta)$ is non positive everywhere. As the function $\alpha(\theta)\mathcal{W}'(U(\theta))$ is

decreasing in θ, there are two possible situations. Either $\alpha(\theta)\mathcal{W}'(U(\theta))$ is lower than ν for any θ and consequently $\lambda'(\theta) > 0$ everywhere.[11] As $\lambda(\theta)$ is increasing and because $\lambda(\overline{\theta}) = 0$, it must be that Condition 5 $\lambda(\theta_s) < 0$ holds. Or $\alpha(\theta)\mathcal{W}'(U(\theta))$ intersects once ν for an intermediate value of θ and consequently $\lambda'(\theta)$ is first negative then positive. Once again, for $\lambda(\theta)$ to be non positive everywhere, Condition 5 must hold. Conversely, if Condition 5 holds then $\lambda(\theta)$ is non positive everywhere. Otherwise, the assumption that $\alpha(\theta)\mathcal{W}'(U(\theta))$ is decreasing in θ would be violated.

Part (ii): from equation (5), $\frac{\partial t}{\partial \pi}$ is first positive then negative if and only if $\lambda(\theta)$ is first positive then negative. For $\lambda(\theta_s)$ to be positive, obviously Condition 5 must not hold. This amounts to

$$
\begin{aligned}
\lambda(\theta_s) &= \left[\nu - \alpha_0 \mathcal{W}'(U(\theta_s))\right] K(\theta_s) \\
&= \left[\int_{\theta_s}^{\overline{\theta}} \left[\alpha(\theta)\mathcal{W}'(U(\theta)) - \alpha_0 \mathcal{W}'(U(\theta_s))\right] dK(\theta)\right] K(\theta_s) > 0
\end{aligned}
$$

which is possible if $\alpha(\theta)$ increases sufficiently in θ.

Note also that $\lambda(\theta)$ cannot be positive for any θ. Indeed, for $\lambda(\theta)$ to be positive everywhere, we would have $\lambda'(\overline{\theta}) < 0$ or equivalently

$$
\begin{aligned}
\alpha(\overline{\theta})\mathcal{W}'(U(\overline{\theta})) &> \nu \\
\alpha(\overline{\theta})\mathcal{W}'(U(\overline{\theta})) &> \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta)) dK(\theta) + \alpha_0 \mathcal{W}'(U(\theta_s)) K(\theta_s)
\end{aligned}
$$

which is impossible as $\int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta)) dK(\theta) > \alpha(\overline{\theta})\mathcal{W}'(U(\overline{\theta}))$.

Conversely, if Conditions 5 does not hold then $\lambda(\theta_s)$ is positive and $\lambda(\theta)$ is necessarily negative in the neighborhood of $\overline{\theta}$. Hence, $\frac{\partial t}{\partial \pi}$ is first positive then negative. This concludes the proof.

---

[11]For this case to be possible, a sufficient condition is that $\alpha(\theta)$ decreases in θ. Indeed, $\alpha(\theta)\mathcal{W}'(U(\theta))$ is lower than ν everywhere if and only if $\alpha(\theta_s)\mathcal{W}'(U(\theta_s)) < \nu$ which is equivalent to

$$
\int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta)) dK(\theta) + \left[\alpha_0 K(\theta_s) - \alpha(\theta_s)\right] \mathcal{W}'(U(\theta_s)) > 0.
$$

A sufficient condition is $\alpha_0 K(\theta_s) > \alpha(\theta_s)$ which is guaranteed when $\alpha(\theta)$ decreases in θ.

D   Proof of Proposition 7

Assume that the function $\alpha(\theta)\mathcal{W}'(U(\theta))$ is increasing in θ. We have several possibilities for the pattern of $\alpha(\theta)\mathcal{W}'(U(\theta))$ compared to the constant ν. First consider the case where $\alpha(\theta)\mathcal{W}'(U(\theta))$ is greater than ν for any θ. This implies that $\lambda'(\theta) < 0$ and hence it must be that $\lambda(\theta)$ is positive everywhere so that t is increasing in π. Second consider the situation where $\alpha(\theta)\mathcal{W}'(U(\theta))$ intersects once ν. It follows that $\lambda(\theta)$ is first increasing then decreasing. However, it is impossible to have $\lambda(\theta_s) < 0$. Indeed,

$$
\begin{aligned}
\lambda(\theta_s) &= \left[\nu - \alpha_0 \mathcal{W}'(U(\theta_s))\right] K(\theta_s) \\
&= \left[\int_{\theta_s}^{\bar{\theta}} \left[\alpha(\theta)\mathcal{W}'(U(\theta)) - \alpha_0 \mathcal{W}'(U(\theta_s))\right] dK(\theta)\right] K(\theta_s) > 0
\end{aligned}
$$

as $\alpha(\theta)\mathcal{W}'(U(\theta))$ is increasing in θ. It follows that $\lambda(\theta)$ is positive everywhere. Last, it is easy to check that the case where $\alpha(\theta)\mathcal{W}'(U(\theta))$ is lower than ν for any θ cannot appear. Indeed, in such a case, we would have $\lambda'(\theta) > 0$ and hence it must be that $\lambda(\theta) < 0$ everywhere which contradicts the fact that $\lambda(\theta_s) > 0$. This concludes the proof.

E   Proof of Proposition 8

A type-θ farmer solves the following program:

$$
\max_{e,l,z,\tilde{\theta}} \pi(\tilde{\theta}) + T(\tilde{\theta}) - \psi(e)
$$

$$
\text{s.t.}
$$

$$
\pi(\tilde{\theta}) = (p + t^q)f(l, z, e, \theta) - (w + t^z)z - (r + t^l)(l - l^\circ)
$$

Once again, by defining the effort function $E(\theta, l, z, \pi)$, we can transform the program as follows:

$$
\max_{l,z,\tilde{\theta}} \pi(\tilde{\theta}) + T(\tilde{\theta}) - \psi(E(\theta, l, z, \pi(\tilde{\theta}))).
$$

Given $\tilde{\theta}$, the optimal solution for l and z is defined by minimizing $\psi(E(\theta, l, z, \pi(\tilde{\theta})))$, i.e.:

$$
\begin{aligned}
(p + t^q)f_l(l, z, E(\theta, l, z, \pi(\tilde{\theta})), \theta) &= r + t^l \quad (\text{i.e. } E_l = 0) \\
(p + t^q)f_z(l, z, E(\theta, l, z, \pi(\tilde{\theta})), \theta) &= w + t^z \quad (\text{i.e. } E_z = 0).
\end{aligned}
$$

This system implicitly defines the functions $l^*(\theta, \tilde{\theta})$ and $z^*(\theta, \tilde{\theta})$. Then the farmer's program becomes:

$$\max_{\tilde{\theta}} U(\theta, \tilde{\theta}) = \pi(\tilde{\theta}) + t(\tilde{\theta}) - \psi(E(\theta, l^*(\theta, \tilde{\theta}), z^*(\theta, \tilde{\theta}), \pi(\tilde{\theta}))).$$

Incentive compatibility requires that:

$$\left. \frac{\partial U}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} = 0 \text{ and } \left. \frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta} \right|_{\tilde{\theta}=\theta} \geq 0$$

We have (using the envelop theorem, i.e. $E_l = E_z = 0$)

$$\left. \frac{\partial U}{\partial \tilde{\theta}} \right|_{\tilde{\theta}=\theta} = \pi'(\theta) + T'(\theta) - \psi' E_\pi \pi'(\theta) = 0$$

or equivalently

$$U'(\theta) = -\psi' E_\theta \geq 0$$

Also, we get:

$$\left. \frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta} \right|_{\tilde{\theta}=\theta} = -\psi'' E_\pi \pi'(\theta) \left[E_\theta + E_l l_\theta^* + E_z z_\theta^* \right] - \psi' \pi'(\theta) \left[E_{\theta\pi} + E_{\pi z} z_\theta^* + E_{\pi l} l_\theta^* \right] \geq 0$$

Applying again the envelop theorem, we get

$$\left. \frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta} \right|_{\tilde{\theta}=\theta} = \pi'(\theta) \left[-\psi'' E_\pi E_\theta - \psi' \left[E_{\theta\pi} + E_{\pi z} z_\theta^* + E_{\pi l} l_\theta^* \right] \right] \geq 0$$

As $E_\theta < 0$, $E_\pi = 1/f_e > 0$ and

$$\begin{aligned} \frac{\partial^2 E}{\partial \pi \partial z} &= \frac{\partial}{\partial z}\left(\frac{1}{f_e}\right) = -\frac{f_{ez}}{(f_e)^2} \leq 0 \\ \frac{\partial^2 E}{\partial \pi \partial \theta} &= \frac{\partial}{\partial \theta}\left(\frac{1}{f_e}\right) = -\frac{f_{e\theta}}{(f_e)^2} \leq 0 \\ \frac{\partial^2 E}{\partial \pi \partial l} &= \frac{\partial}{\partial l}\left(\frac{1}{f_e}\right) = -\frac{f_{el}}{(f_e)^2} \leq 0 \end{aligned}$$

together with $z_\theta > 0$ and $l_\theta > 0$, then

$$\left. \frac{\partial^2 U}{\partial \tilde{\theta} \partial \theta} \right|_{\tilde{\theta}=\theta} \geq 0 \Leftrightarrow \pi'(\theta) \geq 0$$

The program of the regulator is thus

$$\max \int_{\theta_s}^{\bar{\theta}} \alpha(\theta) \mathcal{W}(U(\theta)) dK(\theta) + \int_{\underline{\theta}}^{\theta_s} \alpha(\theta) \mathcal{W}(v + rl^\circ + \tau) dK(\theta)$$

34

under the budget constraint, the environmental constraint and the incentive compatibility

constraint (ignoring the second order conditions) $U'(\theta) = -\psi' E_\theta \geq 0$.

The budget constraint writes

$$\int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) + t^q \int_{\theta_s}^{\overline{\theta}} f(l^*, z^*, e, \theta) dK(\theta) - t^z \int_{\theta_s}^{\overline{\theta}} z^* dK(\theta)$$

$$-t^l \int_{\theta_s}^{\overline{\theta}} l^* dK(\theta) + \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - \pi(\theta, l^*, z^*, e) + \psi(e) \right] dK(\theta) \leq B$$

which simplifies into

$$\int_{\underline{\theta}}^{\theta_s} \tau dK(\theta) + \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - pf(l^*, z^*, e, \theta) + wz^* + r(l^* - l^\circ) + \psi(e) \right] dK(\theta) \leq B$$

The Lagrangean writes

$$
\begin{aligned}
\mathcal{L} \; = \; & \int_{\theta_s}^{\overline{\theta}} \alpha(\theta) \mathcal{W}(U(\theta)) dK(\theta) + \alpha_0 \mathcal{W}(v + rl^\circ + \tau) K(\theta_s) \\
& + \nu \left(B - \tau K(\theta_s) - \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - pf(l^*, z^*, e, \theta) + wz^* + r(l^* - l^\circ) + \psi(e) \right] dK(\theta) \right) \\
& + \mu \left(X - \int_{\theta_s}^{\overline{\theta}} x(z^*, l^*) dK(\theta) \right) + \lambda(\theta_s) \left[v + rl^\circ + \tau \right] + \int_{\theta_s}^{\overline{\theta}} \left\{ \lambda'(\theta) U(\theta) - \lambda(\theta) \psi' E_\theta \right\} d\theta
\end{aligned}
$$

Derivating, we get the following necessary conditions (for $\theta \in \left[\theta_s, \overline{\theta} \right]$):

$$\frac{\partial \mathcal{L}}{\partial \tau} = \alpha_0 \mathcal{W}'(U(\theta_s)) K(\theta_s) - \nu K(\theta_s) + \lambda(\theta_s) = 0 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial e} = \nu(pf_e - \psi') k(\theta) + \nu \left[p \left(f_l l_e^* + f_z z_e^* \right) - wz_e^* - rl_e^* \right] k(\theta)$$

$$-\mu \left[x_z z_e^* + x_l l_e^* \right] k(\theta) - \lambda(\theta) \frac{d(\psi' E_\theta)}{de} = 0$$

$$\frac{\partial \mathcal{L}}{\partial U(\theta)} = \alpha(\theta) \mathcal{W}'(U(\theta)) k(\theta) - \nu k(\theta) + \lambda'(\theta) = 0$$

$$\frac{\partial \mathcal{L}}{\partial t^q} = \nu \int_{\theta_s}^{\overline{\theta}} \left[(pf_l - r) l_{t^q}^* + (pf_z - w) z_{t^q}^* \right] dK(\theta) - \mu \int_{\theta_s}^{\overline{\theta}} \left[x_z z_{t^q}^* + x_l l_{t^q}^* \right] dK(\theta)$$

$$- \int_{\theta_s}^{\overline{\theta}} \lambda(\theta) \frac{d(\psi' E_\theta)}{dt^q} d\theta = 0$$

$$\frac{\partial \mathcal{L}}{\partial t^z} = \nu \int_{\theta_s}^{\overline{\theta}} \left[(pf_l - r) l_{t^z}^* + (pf_z - w) z_{t^z}^* \right] dK(\theta) - \mu \int_{\theta_s}^{\overline{\theta}} \left[x_z z_{t^z}^* + x_l l_{t^z}^* \right] dK(\theta)$$

$$- \int_{\theta_s}^{\overline{\theta}} \lambda(\theta) \frac{d(\psi' E_\theta)}{dt^z} d\theta = 0$$

$$\frac{\partial \mathcal{L}}{\partial t^l} = \nu \int_{\theta_s}^{\overline{\theta}} \left[(pf_l - r)\, l_{t^l}^* + (pf_z - w)\, z_{t^l}^* \right] dK(\theta)$$

$$-\mu \int_{\theta_s}^{\overline{\theta}} \left[x_z z_{t^l}^* + x_l l_{t^l}^* \right] dK(\theta) - \int_{\theta_s}^{\overline{\theta}} \lambda(\theta) \frac{d(\psi' E_\theta)}{dt^l} d\theta = 0$$

Once again, we obtain for $\lambda(\theta)$ and ν the following similar expressions:

$$\lambda(\theta) = -\nu(1 - K(\theta)) + \int_{\theta}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta)$$

and

$$\nu = \int_{\theta_s}^{\overline{\theta}} \alpha(\theta)\mathcal{W}'(U(\theta))dK(\theta) + \alpha_0 \mathcal{W}'(v + rl^\circ + \tau)K(\theta_s) > 0$$

This concludes the proof.

F   Proof of Proposition 9

The program of the regulator can be written as

$$\max CE(U) + V(L) + r(L)(L - L^\circ)$$

$$\text{s.t.}$$

$$CE(U) = \mathcal{W}^{-1}\left[\int_{\theta_s}^{\overline{\theta}} \mathcal{W}(U(\theta))dK(\theta) + \mathcal{W}(v + rl^\circ + \tau)K(\theta_s) \right]$$

$$\tau K(\theta_s) + \int_{\theta_s}^{\overline{\theta}} \left[U(\theta) - \pi(\theta, l(\theta), z(\theta), e(\theta)) + \psi(e(\theta)) \right] dK(\theta) \le B$$

$$\int_{\theta_s}^{\overline{\theta}} x(z(\theta), l(\theta))dK(\theta) \le X$$

$$U(\theta_s) = v + rl^\circ + \tau$$

$$U'(\theta) = -\psi'(e(\theta))E_\theta(\theta, l(\theta), z(\theta), \pi(\theta, l(\theta), z(\theta), e(\theta)))$$

$$\pi(\theta, l(\theta), z(\theta), e(\theta)) = pf(l(\theta), z(\theta), e(\theta), \theta) - wz(\theta) - r(l(\theta) - l^\circ)$$

$$L = \int_{\theta_s}^{\overline{\theta}} l(\theta)dK(\theta)$$

We denote by δ the multiplier of the last constraint determining the aggregate land use by the agricultural sector. Following the preceding analysis, we can write directly the Lagrangean

as follows

$$
\begin{aligned}
\mathcal{L} \;=\; & CE(U) + V(L) + r(L)(L - L^{\circ}) \\
& + \nu \left(B - \tau K(\theta_s) - \int_{\theta_s}^{\bar{\theta}} \left[U(\theta) - pf(l(\theta), z(\theta), e(\theta), \theta) + wz(\theta) + r(l(\theta) - l^{\circ}) + \psi(e(\theta)) \right] dK(\theta) \right) \\
& + \mu \left(X - \int_{\theta_s}^{\bar{\theta}} x(z(\theta), l(\theta)) dK(\theta) \right) + \lambda(\theta_s) \left[v + rl^{\circ} + \tau \right] \\
& + \int_{\theta_s}^{\bar{\theta}} \left\{ \lambda'(\theta) U(\theta) - \lambda(\theta) \psi' E_\theta \right\} d\theta + \delta \left(L - \int_{\theta_s}^{\bar{\theta}} l(\theta) dK(\theta) \right)
\end{aligned}
$$

The first-order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial l(\theta)} \;=\; & \nu(pf_l - r)k(\theta) - \mu x_l k(\theta) - \lambda(\theta)\psi' \frac{d(E_\theta)}{dl} - \delta k(\theta) = 0 \\
\frac{\partial \mathcal{L}}{\partial z(\theta)} \;=\; & \nu(pf_z - w)k(\theta) - \mu x_z k(\theta) - \lambda(\theta)\psi' \frac{d(E_\theta)}{dz} = 0 \\
\frac{\partial \mathcal{L}}{\partial e(\theta)} \;=\; & \nu(pf_e - \psi')k(\theta) - \lambda(\theta)\frac{d(\psi' E_\theta)}{de} = 0 \\
\frac{\partial \mathcal{L}}{\partial L} \;=\; & V'(L) + r'(L)L + r(L) - \nu r'(L)L + \delta + \lambda(\theta_s)r'(L)l^{\circ} + \frac{dCE(U)}{dr}r'(L) = 0 \quad (12) \\
\frac{\partial \mathcal{L}}{\partial \tau} \;=\; & \frac{dCE(U)}{d\tau} - \nu K(\theta_s) + \lambda(\theta_s) = 0 \quad\quad\quad\quad (13)
\end{aligned}
$$

Note that

$$
\begin{aligned}
\frac{dCE(U)}{dr} \;=\; & \frac{d}{dr} \left(\mathcal{W}^{-1} \left[\int_{\theta_s}^{\bar{\theta}} \mathcal{W}(U(\theta)) dK(\theta) + \mathcal{W}(v + rl^{\circ} + \tau) K(\theta_s) \right] \right) \\
\;=\; & \frac{\mathcal{W}'(U(\theta_s)) K(\theta_s)}{\mathcal{W}'(CE(U))} l^{\circ}
\end{aligned}
$$

and

$$
\frac{dCE(U)}{d\tau} = \frac{\mathcal{W}'(U(\theta_s)) K(\theta_s)}{\mathcal{W}'(CE(U))}
$$

Hence, from (12) and (13) we deduce that

$$
\begin{aligned}
r'(L)L - \nu r'(L)L + \delta + \left(\nu K(\theta_s) - \frac{dCE(U)}{d\tau} \right) r'(L)l^{\circ} + \frac{dCE(U)}{dr}r'(L) \;=\; & 0 \\
r'(L)L - \nu r'(L)L + \delta + \nu K(\theta_s)r'(L)l^{\circ} \;=\; & 0
\end{aligned}
$$

so that

$$
\delta = r'(L) \left[(\nu - 1) L - \nu K(\theta_s) l^{\circ} \right]
$$

With respect to $U(\theta)$, we have

$$\frac{\partial \mathcal{L}}{\partial U(\theta)} = \frac{dCE(U)}{dU(\theta)} - \nu k(\theta) + \lambda'(\theta) = 0$$

which is equivalent to

$$\frac{\mathcal{W}'(U(\theta))k(\theta)}{\mathcal{W}'(CE(U))} - \nu k(\theta) + \lambda'(\theta) = 0$$

Hence, we obtain that

$$\int_{\theta}^{\overline{\theta}} \lambda'(u)du = \int_{\theta}^{\overline{\theta}} \left\{ \nu - \frac{\mathcal{W}'(U(u))}{\mathcal{W}'(CE(U))} \right\} k(u)du$$

$$\lambda(\theta) = -\nu(1 - K(\theta)) + \int_{\theta}^{\overline{\theta}} \frac{\mathcal{W}'(U(u))}{\mathcal{W}'(CE(U))} k(u)du$$

As

$$\lambda(\theta_s) = \nu K(\theta_s) - \frac{dCE(U)}{d\tau}$$

$$= \nu K(\theta_s) - \frac{\mathcal{W}'(U(\theta_s))K(\theta_s)}{\mathcal{W}'(CE(U))}$$

we also have that

$$\nu = \int_{\theta_s}^{\overline{\theta}} \frac{\mathcal{W}'(U(\theta))}{\mathcal{W}'(CE(U))} dK(\theta) + \frac{\mathcal{W}'(U(\theta_s))K(\theta_s)}{\mathcal{W}'(CE(U))}.$$

This concludes the proof.