Valuing Monitoring Networks for Invasive Species: The Case of Soybean Rust

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In 2004, soybean rust (*Phakopsora pachyrhizi*) arrived in the United States, likely carried by the winds of Hurricane Ivan (Isard et al., 2005). Soybean rust is a fungal plant pathogen that can cause large yield losses on commercial soybean farms. Soybean rust spores are transported by the wind, and are capable of traveling hundreds of miles. Soybean rust is different from other invasive species in that it cannot overwinter in most of the United States. Thus, soybean rust does not slowly spread across the landscape like other invasive species. Instead, the probability that a particular county is infected in a particular year depends on the prevailing winds, the weather that year, and the county's proximity to the Gulf Coast (where soybean rust can over winter).

When soybean rust first arrived in the United States, there were concerns that the economic costs of its presence (control costs + damages) could be large. A study by the United States Department of Agriculture's Economic Research Service estimated that the losses in the first year of infestation to be between $640 million and $1.3 billion (Livingston et al. 2004). In 1984, a different study estimated that the economic consequences of soybean rust, once fully established, could be as high $7.1 billion per year (Kuchler et al., 1984). In response to the arrival of this invasive species, the USDA and the United Central Soybean Board created a national monitoring system for the disease.

The soybean rust monitoring network consists of several components. A key component is the sentinel plot. Sentinel plots are an area of soybeans grown specifically to detect rust. The sentinel plots are managed through each state's extension service. Leaf samples from the sentinel plots are sent to processing labs at each state's land grant university. These processing labs are important because soybean rust is hard to detect early in its life cycle before significant yield loss has occurred. A microscope or magnifying hand lens is required to reliably diagnose the pathogen while it is in a treatable phase of its lifecycle. In 2007, there were more than 700 sentinel plots in operation across the

US. The sentinel plot data is made publically available via the integrated pest management pest information platform for extension and education's website (IPM PIPE). This website displays a map of all the confirmed cases of rust to the general public. Farmers can also signup to receive email alerts if a case of rust is detected in their region. In addition to providing detection information, the IPM PIPE website provides predictions of spore depositions using an aerobiology model developed by Isard, Russo and Ariatti (2007). Spore deposition is necessary, but not sufficient, for soybean rust infections to occur. Sentinel plot data and atmospheric data are used to developing these forecasts. Because the forecasting model relies on wind and weather forecasts, spore deposition predictions are limited to approximately one week in advance.

While initial estimates of the magnitude of potential losses due to soybean rust were large, the problem so far has been less severe than anticipated. There have not been any infections of economic consequence in the corn-belt region of the country (which accounts for the majority of US soybean production) since soybean rust came to the US. In states where rust infections have occurred, extension agents report that only 1 to 10% of the soybean acreage had yield losses due to infection. Because of this, the USDA and United Central Soybean Board want to reevaluate their investments in the soybean rust monitoring network. Specifically, they want to consider the costs and benefits of changing the extent and intensity of the monitoring effort.

The purpose of this paper is to evaluate the benefits of the soybean rust monitoring network to producers and to determine how this value changes as producers become more aware of the true risk of infection. To accomplish this objective, we build upon the work of Roberts et al. (2007). Their paper also sought to value the US soybean rust sentinel plot system using a static farm level model of decision making. We expand upon their work in two ways. First we develop a dynamic model so we can consider how producers accumulating knowledge about their risk of rust affects the value of the monitoring network over time. Second, we assemble the county level data necessary to optimize the spatial arrangement of sentinel plots.

The contributions of this paper are twofold. First, the analytical model we develop is applicable to a variety of situations where there is an opportunity for publically provided information to help producers learn about random processes that affect their success. Second, the results provide useful information for guiding future investments in the current soybean rust network.

Model

First, we review the strategies available to farmers to manage soybean rust. Following Roberts *et al.* (2006), we focus on three strategies: applying a preventative fungicide at the beginning of the susceptible period, scouting for rust and applying a curative fungicide only if rust is found and, lastly, taking no action. In addition to the three strategies discussed by Roberts *et al*., we also consider the possibility of farmer's conditioning their strategy choice (preventative or no action) on the infection status of the sentinel plot nearest to them. We use the cost estimates from Roberts *et al.* for following the preventative, curative and no action strategy which are shown in Table 1. Table 1 has two columns, the left shows the cost of the strategy if rust does not come and the right column denotes the cost if it does. If the probability of rust arriving each year is known the expected cost of each strategy can be calculated. The within season signal strategy is a composite of the preventative and no action strategy. The payoff of this strategy will depend on the degree of correlation between the infection status of the sentinel plot and the farmer's field. We choose to represent this with a single signal accuracy parameter, *S*.

Table 1: Cost of following management strategies (dollars/acre)

Management Strategy	No Infection	Infection
Preventative	$25.63	$25.63+1% yield loss
Curative	$6.71	$20.52+7% yield loss
No Action	$0.00	25% yield loss
Follow the within season signal with accuracy, *S*	S*$0+(1-S)$25.63	S*($25.63+1% yield loss)+ (1-S)(25% yield loss)

If the price of soybeans and yield information is known, then it is easy to identify the expected cost minimizing strategy for managing soybean rust. As an example, if the price of soybeans were $8.00 per bushel and the yield was 37 bushels/acre then farmers should choose the preventative strategy if the probability of a rust infection is greater than 0.59. If the probability is between 0.17-0.59 then the curative strategy is best and if p is less than 0.17 then no action is the best. When it is best to utilize the within season signal is a function of both the probability of infection and the signal quality.

Choosing the expected cost minimizing strategy, however, requires knowledge of the probability of infection. Because soybean rust is an invasive species, the probability of infection was unknown when it first arrived in the US. Compounding the problem, producers face a dilemma in choosing a management strategy because the strategy they choose affects how much they learn about the probability of infection. Specifically, if a farmer applies a preventative fungicide at the beginning of the susceptible period then there is a significant risk that they might observe a false negative; inoculum that lands on the fields may not develop because of the presence of the fungicide. To capture this tradeoff, we assume that farmers do not get information to update their prior beliefs about infection probability if they apply a preventative fungicide. Due to the nature of soybean rust, false positives are less of a problem because infected leaves have a fairly distinct appearance. Alternatively, using a curative fungicide controls an infection after it occurs and provides an opportunity for the producer to learn how often an infection is likely to occur. The downside to this strategy is that the farmer incurs scouting costs when using this strategy.

We conceptualize the soybean rust sentinel plot system as providing two major benefits. The first is that it provides a within season signal that farmers can condition their management choice upon. Because soybean rust can be difficult to detect early in its life cycle we associate an accuracy parameter, S, with the within season signal. This accuracy parameter can be thought of as a function of how spatially auto-correlated infections are and the probability of identifying the disease in time to treat with fungicides. Applying fungicides after the infection is sufficiently established has no economic benefit, so timing is very important. The second benefit that the sentinel plot system provides is that it allows producers to learn about their risk of rust infection even if they apply a preventative fungicide.

Because of the tradeoffs between current returns and future knowledge, we frame the producer's problem in terms of the two-armed bandit problem first discussed in the economics literature by Rothschild (1974). A two-armed bandit problem is a situation in which an agent is in a casino and forced to repeatedly play one of two slot machines (one-armed bandits) each timer period for eternity. Each machine yields a payoff, R, with probability, P, and no payoff with probability (1 - P). The probability of a payoff can be different between the two machines. The agent is assumed to have complete information about one machine but have no information about the other. The agent's problem is to choose which machine to play each time period to maximize his returns or, more likely, minimize his losses. Rothschild solved this problem using dynamic programming and Bayesian updating and found that, when there is a positive discount rate, that the optimal solution takes the form of a stopping rule; a series of sufficiently disappointing outcomes from the unknown slot machine will lead the agent to play the known slot machine for eternity. Rothschild also showed that there is a positive probability that the agent observes such a series of disappointing outcomes from the unknown slot machine and stops playing it even if it yields an objectively higher return.

This type of problem is a useful metaphor for the soybean rust problem. On the one hand, producers can apply a preventative fungicide and get a certain return, but learn nothing about the risk of soybean rust. On the other hand, they can apply a curative fungicide only if they observe a rust infection and learn about their risk. The expected cost minimizing choice depends on the probability that rust will infect their farm in a given year. In this problem the preventative fungicide strategy is equivalent to the agent playing the slot machine with the known return and the curative and no action strategies are like playing the slot machine with an unknown return. In this context, one benefit of the soybean rust monitoring network is that it allows an agent to apply the preventative fungicide but still receive information about what his or her return would have been had they chosen to apply a curative fungicide if soybean rust were observed. The monitoring network reduces the probability that agents will make poor management decisions because it allows for greater learning and, while it is being provided, eliminates the tradeoff between current returns and future learning.

To better join the problem of soybean rust and the two-armed bandit model, we assign a constant payoff to the preventative strategy and allowing for an explicit strategy of following a within season signal. If the payoffs to the preventative strategy were different when rust came and when it didn’t, the farmer could deduce the outcome of that year and update his or her prior beliefs. Field trials by Livingston et al. (2004) found an average of 1% yield loss if rust spores infect a field treated with a preventative fungicide. Because there are many phenomena that can cause a 1% yield loss, it would be difficult for a farmer to know whether rust spores arrived at their farm in years when they apply a preventative fungicide just from their yield.

The farmer's problem can be formulated as a dynamic programming problem where the state variable is the farmer's beliefs about the probability of a rust infection *p*. To make this problem tractable, it is important to find a parsimonious way of representing the farmer's beliefs. To do this, we assume that the probability that a farmer will observe *X* infections in *N* years will follow a binomial distribution with parameter *p*. When we make this assumption, the beta distribution is an ideal way to describe a famer's beliefs about the value of *p*. Using a beta distribution allows a farmer's beliefs to be completely described by two parameters, *m* and *n*. The probability density function of the beta distribution

1) $$\frac{(1-p)^{n-1} * p^{m-1}}{\mathrm{Beta}[m,n]} : 0 \leq p \leq 1$$

where beta(m,n) refers to a beta distribution and Beta[m,n] is the Euler beta function.

An advantage of formulating beliefs in this way is that it makes the application of Bayes' rule straightforward. The beta distribution is a conjugate prior for Bayes' rule when the focus is on the parameter of a binomial distribution. This means that if a farmer has a prior described by a beta distribution then their posterior beliefs will also be described by a beta distribution.[1] Specifically, given a prior belief described by a beta distribution with parameters {*m, n}*, if a famer observes an infection in the current period then their posterior belief from applying Bayes rule is given by beta distribution with parameters {m+1, n}. Conversely, if the farmer observes no infection the posterior belief is described by the parameters {m, n+1}.

[1] See Appendix for proof

One more statistical preliminary needs to be discussed before presenting the formal model. To write out the model, it is necessary to know what the farmer's expectations are regarding the probability of observing or not observing an infection next period. In essence, this is a farmer's expectations over what they expect to learn next period if they choose a strategy that allows them to gather evidence. This is calculated by taking the expectation of the beta distribution:

2) $E[beta(m,n)] = \frac{m}{m+n}$

The Producer's Problem when no Monitoring Network Exists

Now we can formulate the farmer's decision problem when there is no monitoring network as a dynamic program with three strategies.

3)

$$V[M,N] = Max[\pi_{N1} * \frac{M}{M+N} + \pi_{N0} * \left(1 - \frac{M}{M+N}\right) + \delta\left(V[M+1,N] * \frac{M}{M+N} + V[M,N+1] * \left(1 - \frac{M}{M+N}\right)\right),$$

$$\pi_{C1} * \frac{M}{M+N} + \pi_{C0} * \left(1 - \frac{M}{M+N}\right) + \delta\left(V[M+1,N] * \frac{M}{M+N} + V[M,N+1] * \left(1 - \frac{M}{M+N}\right)\right),$$

$$\pi_P + \delta * V[M,N]]$$

where π_{N1} and π_{N0} represent the one period return from following the no action strategy with and without a rust infection; π_{C1} and π_{C0} represents the one period returns from following the curative strategy with and without a rust infection; π_P represents the return from following the preventative strategy; and δ is the discount rate. A reasonable initial prior for a farmer with no information is to assume that p is uniformly distributed over the interval [0,1]. This is equivalent to a beta distribution with m=n=1. The important part of this formulation is to note that choosing the preventative strategy prohibits the farmer from gaining additional information about their risk of infection. Once the preventative strategy is optimal, it will be chosen for all remaining time periods.

The Producer's Problem when a Monitoring Network does Exist

Next, we turn our attention to how the existence of the monitoring network would change the farmer's problem. From an informational standpoint, the soybean rust monitoring network allows farmers to gather evidence about their risk of infection regardless of the management strategy they choose. The monitoring network also provides a within season signal upon which farmers can condition their actions. In the context of the two armed bandit problem, the monitoring network allows a producer to see what return they would have earned had they played the unknown arm. To place a value on the monitoring network farmers need to know when and how long the monitoring network will be in existence. For instance, a monitoring network that exists during years one to three will be more valuable than a network that only exists during years one and two. Likewise, because of discounting, a monitoring network that exists during years one to three will be more valuable than one that exists during years four to six. Because of this, the dynamic program representing the farmer's problem with information will require more notation than the farmer's problem without information. To simplify this notation, we assume that the monitoring network always begins in year one and lasts a finite number of periods given by the parameter *T*. To measure the value of a monitoring network that exists for a specified number of periods, we have to re-solve the dynamic programming problem allowing farmers to observe evidence when choosing the preventative strategy only during periods when the monitoring network exists. After solving this modified problem, the value of the information can be found by comparing the two different value functions. We represent the farmer's value function without information as $V_t[M,N]$ to mean the value of arriving in year *t* with the prior described by *M* and *N*. We represent the farmer's value function with information as $V1_t[M,N,T,S]$ to mean the value of arriving in year *t* with priors *M* and *N*, a monitoring network that exists during years 1-*T* and a within season signal quality of *S*.

An efficient way (in the computational sense of the word) of calculating the value function with the monitoring network is to make use of the previously calculated value functions from the farmer's original decision problem. The key insight is that the value function for the decision problem with information becomes the same as the original problem once the monitoring network is discontinued. Mathematically, the functional equation can be written as follows:

4)

$V1_t$[M,N,T,S]= IF t<T,

Max[

$$\pi_{N1} * \frac{m}{m+n} + \pi_{N0} * \left(1 - \frac{m}{m+n}\right) + \delta\left(V1_{t+1}[\mathrm{M}+1,\mathrm{N},\mathrm{T}] * \frac{m}{m+n} + V1_{t+1}[\mathrm{M},\mathrm{N}+1,\mathrm{T}] * \left(1 - \frac{m}{m+n}\right)\right), \pi_{C1} * \frac{m}{m+n} + \pi_{C0} * \left(1 - \frac{m}{m+n}\right) + \delta\left(V1_{t+1}[\mathrm{M}+1,\mathrm{N},\mathrm{T}] * \frac{m}{m+n} + V1_{t+1}[\mathrm{M},\mathrm{N}+1,\mathrm{T}] * \left(1 - \frac{m}{m+n}\right)\right), \pi_P + \delta\left(V1_{t+1}[\mathrm{M}+1,\mathrm{N},\mathrm{T}] * \frac{m}{m+n} + V1_{t+1}[\mathrm{M},\mathrm{N}+1,\mathrm{T}] * \left(1 - \frac{m}{m+n}\right)\right), \frac{m}{m+n}(\mathrm{S} * \pi_P + (1-\mathrm{S})\pi_{N1}) + \left(1 - \frac{m}{m+n}\right)(\mathrm{S} * \pi_{N0} + (1-\mathrm{S})\pi_P) + \delta\left(V1_{t+1}[\mathrm{M}+1,\mathrm{N},\mathrm{T}] * \frac{m}{m+n} + V1_{t+1}[\mathrm{M},\mathrm{N}+1,\mathrm{T}] * \left(1 - \frac{m}{m+n}\right)\right)]$$

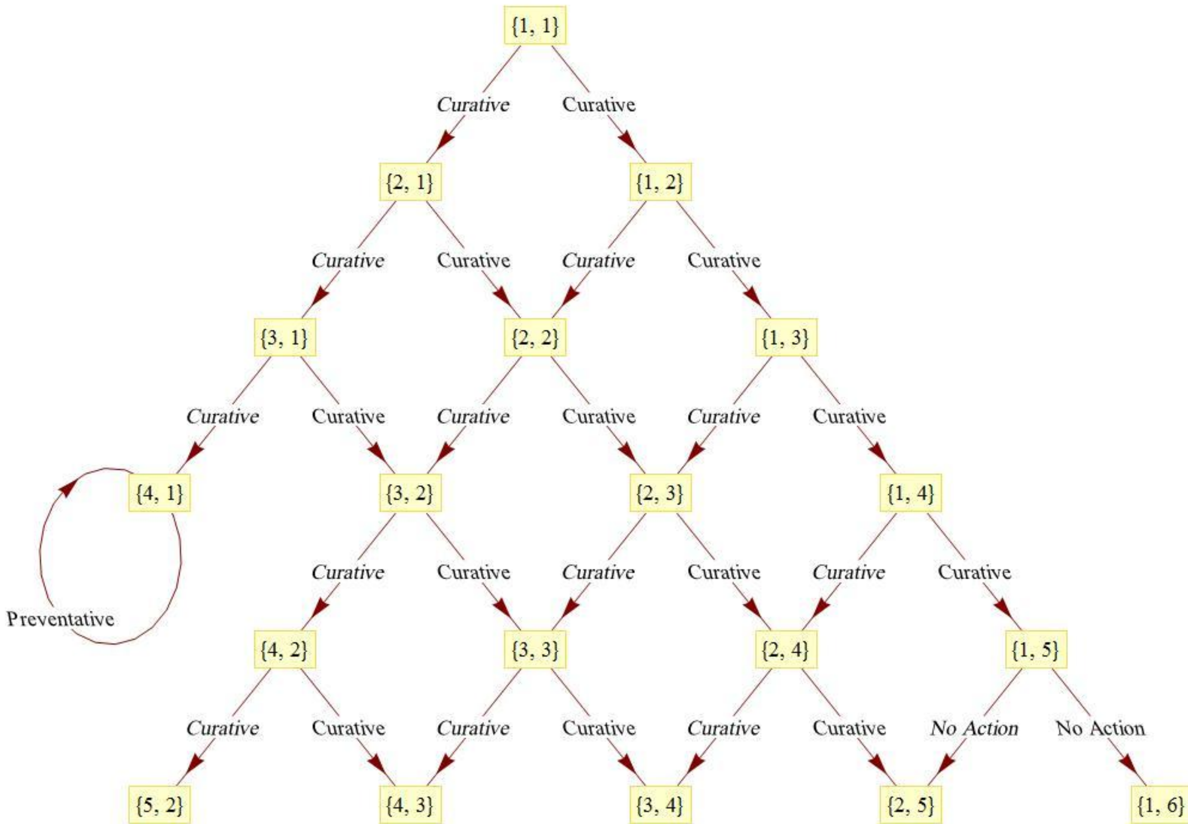
ELSE

$V1_t$[M,N,T]= $V_t[\mathrm{M},\mathrm{N}]$

Having these two models of decision making allows us to calculate the value of information generated by the soybean rust sentinel plot system. Conceptually, the value of information is the expected cost of managing soybean rust with information minus the cost without information. Mathematically, this is equivalent to V1[M,N,T]-V[M,N]. A nice feature of this type of model is that it allows us to value the sentinel plot system for counties with heterogeneous risks of soybean infection described by the parameters *{M,N}*, as well as looking at the marginal value of extending the life of the network.

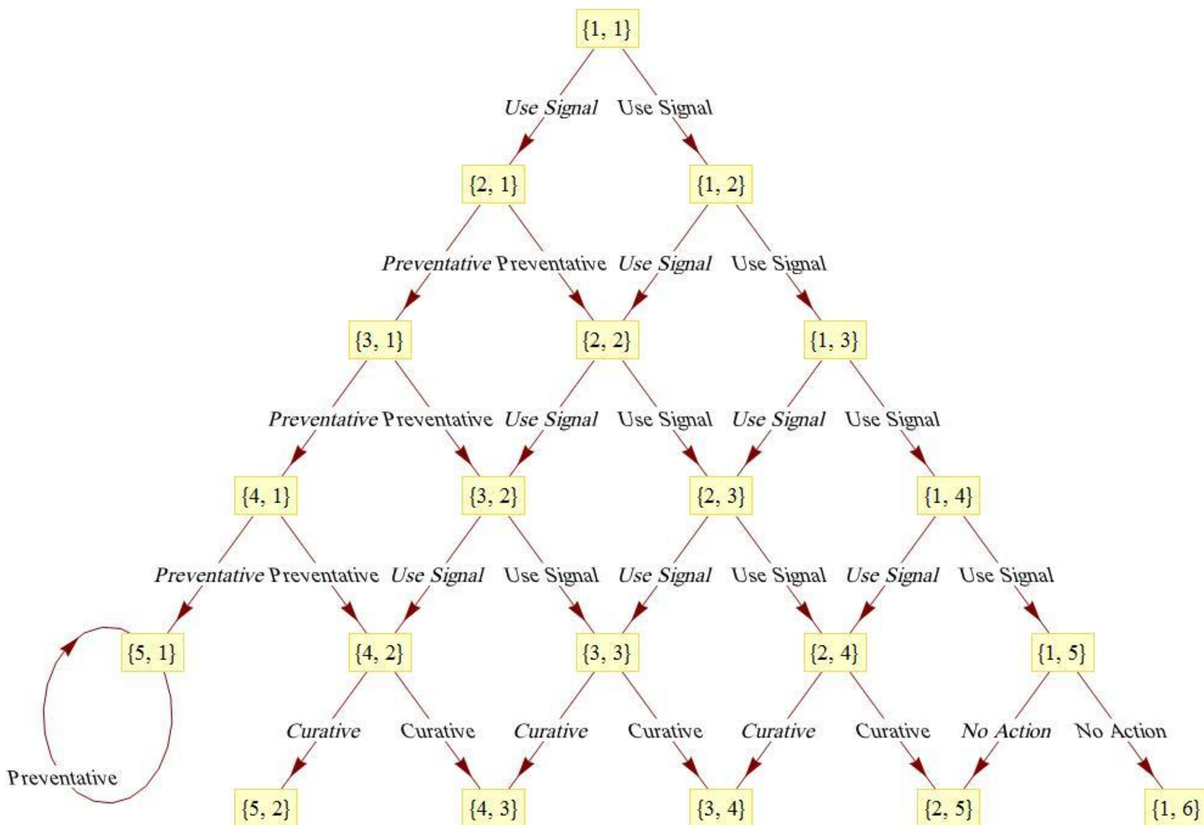
An interesting question is, when no monitoring network is present, at what points the model predicts farmers will choose to the preventative strategy and stop learning for all remaining timer periods. To solve the model to answer this question, we rely on the fact that at an interest rate of 6%, the costs and benefits more than 90 years in the future contribute very little to the overall value function. Because of this, we solve the model by making it into a 90 period finite horizon problem and using backwards induction. For the example below, we assumed a soybean price of $8.00/ bushel and a yield of 37 bushels/acre. Figure 1 shows a decision tree that denotes the optimal strategy at each time period given the farmer's beliefs over the probability of infection. This is only the top portion of the decision tree that would fit into a figure, the decision tree continues until the end of the time horizon. The vertices of the decision are labeled according to the state variables in the dynamic programming problem. For instance, the first vertex is labeled (1,1) which represent the parameter values of a beta distribution that equate to a uniform distribution. Branches to the left represent observed infections and branches to the right are non-infections. This figure shows that the first place where it is optimal for farmers to choose the preventative strategy is upon reaching a prior belief described by beta (4,1). This corresponds to the first 3 observations being infections. Alternatively, if a famer reaches a state space of (1,6), then it is optimal for him or her to choose the no action strategy. This corresponds to the first 5 observations being non-infections. For the majority of the states it is optimal for the farmer to choose the curative strategy. We feel that these results lend credibility to our model of producer behavior. In regions of the country where there have been 3 years of non-infections anecdotal information from extension agents indicates that the majority of farmers are not applying preventative fungicides for soybean rust and are spending little if any time scouting for soybean rust. Conversely, farmers in many areas in the Deep South are regularly applying a fungicide before soybean rust is specifically detected. To be fair though, there are also other fungal pathogens in the Deep South and fungicide spraying in those areas is control more than just soybean rust.

Figure 1: Optimal management decisions for a farmer with no monitoring network



Next, we turn our attention to the decision tree when a monitoring network is present. Figure 2 shows the optimal management decisions when a monitoring network will be present for 4 years and there is a signal with quality 0.75. At the start of this tree the optimal management decision is to use the within season signal. This is the case when the probability of infection is closer to 0.5 than either extreme and there is a sufficiently high signal quality. Using the within season signal allows farmers to outsource their scouting costs and obtain very good yield protection and is a very valuable part of the sentinel plot network. Looking at the vertex labeled (2,1), we see that the best management decision is to apply a preventative. Because the monitoring network is present for the first 4 years in this example, the farmer can continue to learn while doing so. Once the monitoring network ceases to exist, choosing the preventative strategy halts learning as can be seen at the vertex (5,1).

Figure 2: Optimal management decisions for a farmer with a 4-year monitoring network with signal quality of 0.75

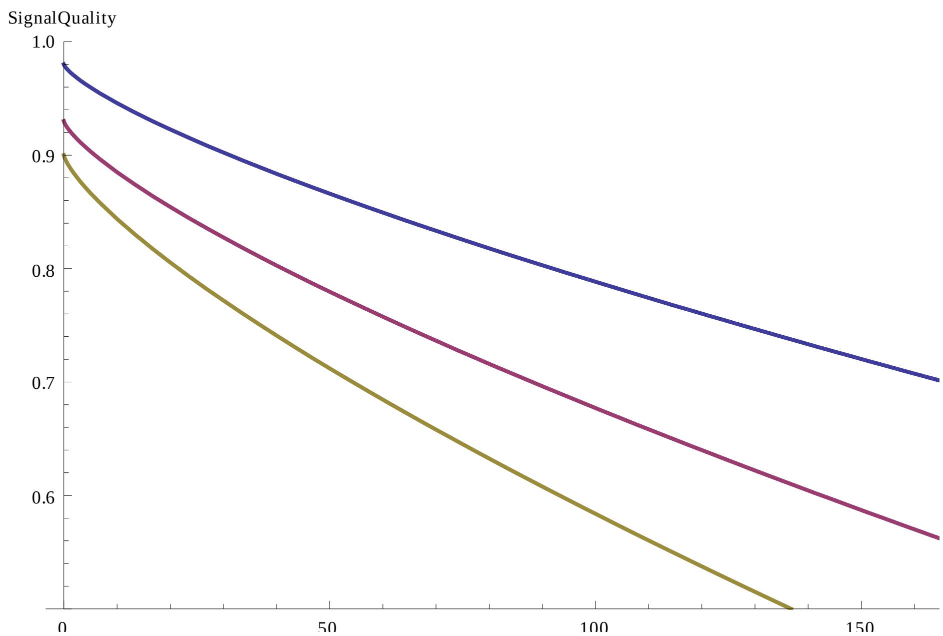


Having developed a dynamic model capable of valuing a monitoring network, we would like to apply this model to the problem of placing a limited number of sentinel plots to maximize the value of information. To accomplish this we assemble county level data on average soybean acreage and yield from 2005-2007. This data allows us to construct a representative farmer for each county that will form the basic unit of the model. We use county level estimates of the probability of soybean rust infection from a paper by Bekkerman, Goodwin and Piggott (2008) for farmers' prior beliefs on the likelihood of a rust infection in their county. Bekkerman, Goodwin and Piggott developed these estimates attempting to quantify soybean rust risk so that a soybean rust crop insurance product could be developed.

The last piece of information that needs to be discussed is how the signal quality that farmers receive depends on the spatial arrangement of sentinel plots. We choose to model signal quality as a function of the distance between the farmer and the sentinel plot nearest to them. The rate at which signal quality declines with distance should primarily be a function of the spatial autocorrelation of the disease spread. Given the wind borne nature of the disease, there is reason to expect that significant spatial autocorrelation exists. We investigated estimating a measure of spatial of autocorrelation from the infections from 2005-2008, however, this problem turned out to be beyond the computational resources available for the project[2]. Because of this we consider three scenarios (high, medium, and low) where signal quality decreases at different rates as a function of distance. These scenarios are shown in figure 3 where the top line (blue) denotes a high signal quality function, the middle line (red) denotes the medium signal quality function and the bottom line (yellow) denotes the low signal quality function.

[2] While estimating measures of spatial autocorrelation for continuous processes is well studied, methods for calculating autocorrelation for point processes have received less attention. One way to conceptualize the problem is to think of a spatially autocorrelated latent variable and when the latent variable is above a given threshold a county experiences an infection. The degree of spatial autocorrelation can be estimated using maximum likelihood estimation, but doing so requires the evaluation of high dimension integrals. In this case, the dimensionality was approximately 1600 and beyond the capabilities of a single desktop computer.

Figure 3: Signal Quality as a Function of Distance



Results

Using the county level data and model of farmer decision making described above, we ran a series of optimizations with the goal of placing a fixed number of sentinel plots to maximize the value of information generated from them. We only allowed one sentinel plot to be placed in a county and used the county centriods for calculating distances between farmers and the nearest sentinel plot. The value of extending the soybean rust sentinel plot system for one additional year using the optimal spatial arrangement is shown in Table 2 for a number of different signal quality and plot density combinations. Benefit estimates range from $52-106 million dollars. This is in the middle of the range estimated by Roberts et al., they estimated the benefits of a year of monitoring to be between $5-300 million dollars. It is important to point out that our benefits estimate is the *present value* of an additional year of monitoring. In our model of farmer decision making, knowledge is a capital good and can yield benefits many years after it is acquired.

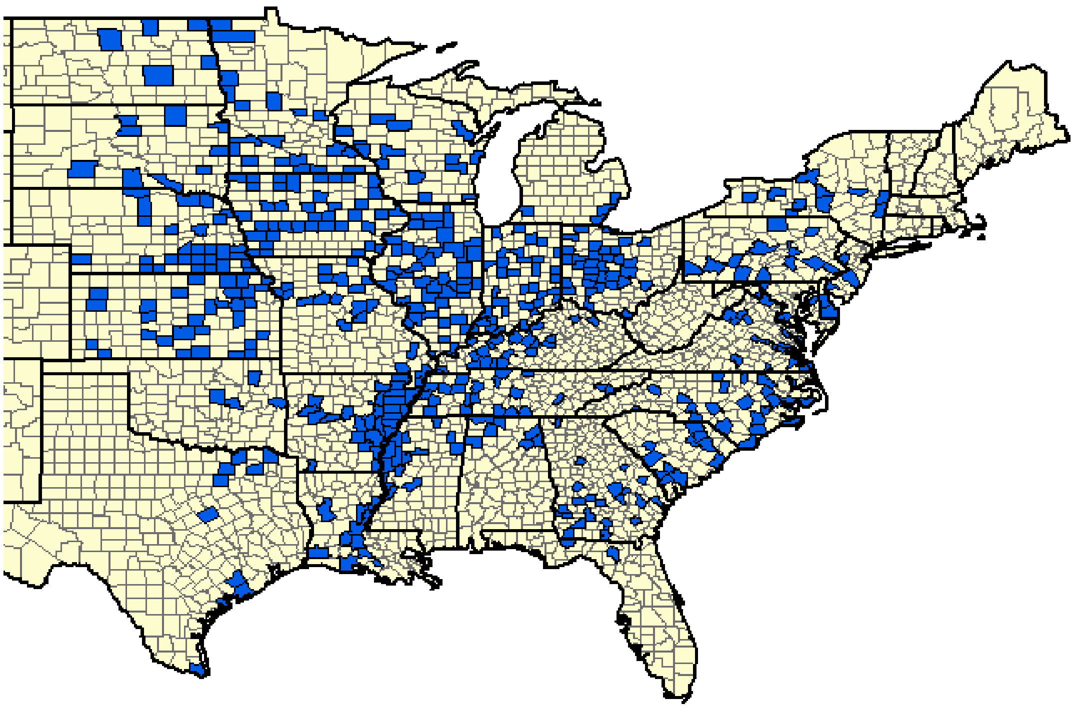
Table 2: Present Value of an Additional Year of Soybean Rust Monitoring

	High Quality Signal	Medium Signal Quality	Low Signal Quality
500 plots	$106,876,866	$74,217,175	$59,901,124
400 plots	$102,672,631	$72,055,069	$57,697,522
300 plots	$97,919,715	$69,110,845	$54,760,403
200 plots	$92,166,473	$65,149,613	$52,435,480

The optimal spatial arrangement of 500 sentinel plots under the high signal quality function scenario is shown in Figure 4. The counties were the model recommends a sentinel plot be place are highlighted. This corresponds closely which counties have the highest soybean production and is substantially different from allocation of sentinel plots planned for the 2010 growing season. The current plan is for all sentinel plots to be placed in the Southern US where the probability of infection is relatively high, but were only a small fraction of US soybean production occurs. In contrast, our model recommends placing substantial resources in the corn-belt where the probability of infection is low, but where production levels are higher. The optimal spatial arrangements of sentinel plots under different signal quality functions were qualitatively similar. The different signal quality scenarios seemed to affect primarily impact the value of the network, not the optimal arrangement.

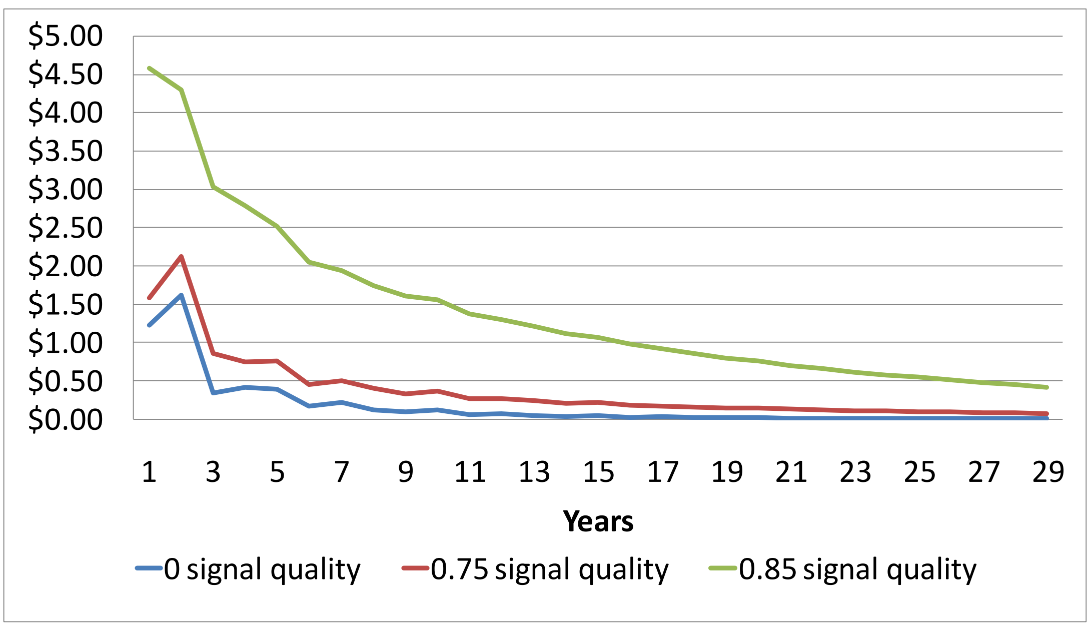
Figure 4: Optimal Placement of 500 Sentinel Plots with a High Signal Quality

Function



Lastly, we investigate how the marginal value of extending the sentinel plot system changes as more knowledge is accumulated. Mathematically, the marginal benefit of extending the life of the network from T years to T+1 years is $V1_t[M,N,T+1,S]$- $V1_t[M,N,T,S]$. The marginal benefit of extending the life of the network for three different levels of signal accuracy is shown in figure 5. This figure was constructed assuming that farmer's had an initial prior described by a uniform distribution, \$8 per bushel soybean price and a yield of 37 bushels per acre. From this figure we see that the marginal benefit of extending the network depend critically on the accuracy of the within season signal. When the accuracy of the within season signal is 0, then the only benefit of extending the monitoring network is increased opportunity for learning while applying the preventative fungicide. If limited resources were available for processing leaf samples, the monitoring network might not be able to publish results in time for farmers to effectively condition their management strategies upon the results, but farmers could still update their prior beliefs from the results when they are eventually published if they choose to apply a preventative fungicide. In this case, the benefits of extending the network decline rapidly. When there is a within season signal, however, the marginal benefits of extending the network decline much more slowly.

Figure 5: Marginal Value of Extending Soybean Rust Sentinel Plot System



Conclusion

In this paper, we developed a dynamic model of the farmer's decision regarding the management of soybean rust. We used this model to value the US soybean rust sentinel plot system and to optimize the spatial arrangement of sentinel plots. We estimate that the value of extending the life of sentinel plot system for one year, given the optimal spatial arrangement of plots, to between $52-106 million dollars depending on the number of plots used and assumptions about the quality of the within-season signal. This is an order of magnitude greater than the cost of maintaining the system for another year. Also, the optimal spatial arrangement of sentinel plots indicated by our model is substantially different from the planned placement for the 2010 growing season. The current plan places no sentinel plots in the corn-belt region of the country, while our model indicates that some coverage is advantageous. Looking forward, we estimated the marginal value of extending the life of the sentinel plot system for multiple years. We find that the marginal value declines as more knowledge is accumulated, but the rate of decline is highly sensitive to the within-season signal quality parameter. When there is a low quality within-season signal, there are few benefits to having the network beyond the first decade.

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APPENDIX:

Proposition: If an agent's prior probability is described by a beta distribution, and the agent is estimating the probability of success for a binomial process, then the posterior belief will also follow a beta distribution.

Proof:

Suppose that the agent has a prior belief over the probability of success, *P*, that is described by a Beta distribution with parameters *m* and *n*. In the equation, Beta[m,n] refers the Euler beta function

$$Prob(P = x) = \frac{(1-x)^{n-1}x^{m-1}}{Beta[m,n]}$$

Suppose that the agent then observes *S* success and *F* failures and uses this data to update his beliefs. Bayes' Rule is where the prior is denoted by *A* and the observed evidence by *B* is

$$Prob(A|B) = \frac{Prob(B|A) * Prob(A)}{Prob(B)}$$

In the context of our problem we have:

$$Prob(B|A) = \frac{(S+F)!}{S!F!} * (1-x)^F x^S$$

$$Prob(A) = \frac{(1-x)^{n-1}x^{m-1}}{Beta[m,n]}$$

$$Prob(B) = \int_0^1 \frac{(S+F)!}{S!\,F!} * (1-x)^F x^S \frac{(1-x)^{n-1}x^{m-1}}{Beta[m,n]} dx$$

This Yields:

$$Prob(A|B) = \frac{\frac{(S+F)!}{S!F!} * (1-x)^F x^S \frac{(1-x)^{n-1} x^{m-1}}{Beta[m,n]}}{\int_0^1 \frac{(S+F)!}{S!F!} * (1-x)^F x^S \frac{(1-x)^{n-1} x^{m-1}}{Beta[m,n]} dx}$$

Integrating we get

$$Prob(A|B) = \frac{\frac{(S+F)!}{S!F!} * (1-x)^F x^S \frac{(1-x)^{n-1} x^{m-1}}{Beta[m,n]}}{\frac{(\mathrm{f}+\mathrm{s})!}{(\mathrm{Beta}[\mathrm{m},\mathrm{n}]\ \mathrm{Beta}[\mathrm{m}+\mathrm{s},\mathrm{f}+\mathrm{n}]\ \mathrm{f}!\ \mathrm{s}!)}}$$

Simplifying we get

$$Prob(A|B) = \frac{(1-x)^{n+f-1} x^{m+s-1}}{\mathrm{Beta}[\mathrm{m}+\mathrm{s},\mathrm{f}+\mathrm{n}]}$$

Which is the formula for a beta distribution with parameters m+s and n+f.