Managing Flounder Openings for Maximum Revenue[*]

by

Wei Chen, Dan Kauffman, Daniel B. Taylor and Everett B. Peterson[**]

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[**] Wei Chen (chenw@vt.edu) is a Ph.D. candidate, Daniel B. Taylor (taylord@vt.edu) is a Professor, and Everett B. Peterson (petrsone@vt.edu) is an Associate Professor in the Department of Agricultural and Applied Economics (0401), Virginia Tech, Blacksburg, VA 24061. Dan Kauffman (dkauffma@vt.edu) is a Seafood Business Specialist, Virginia Agricultural Research and Extension Center, 102 South King Street, Hampton, VA 23072.

Introduction

Because of its light delicate taste, summer flounder (Paralichthys dentatus) is an object of desire for many seafood consumers. As evidenced by its dockside price, it is among the most esteemed of the American flounders. However, on an anecdotal basis some industry participants believe that summer flounder, also called fluke, is more in desired in March than November. This question of seasonal demand is important since the money commercial fishermen receive for their catch may depend, in part, on the time of the year the Federal Government allows them to fish. The average monthly real prices from 1991 to 2005 are contained in Figure 1. The average monthly real prices were slightly above $1.20/pound from March to August. In the other months, the average monthly prices were less than $1.05/pound and the lowest average monthly real price was $0.88/pound in January.

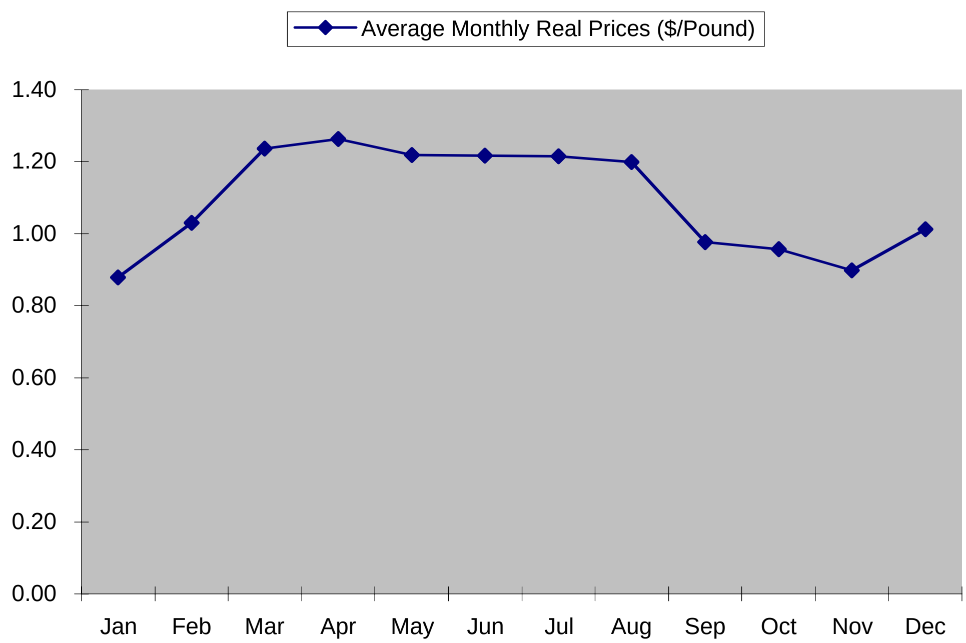

Figure 1. Average Monthly Real Price of Summer Flounder from 1991 to 2005 (Source: Office of Science and Technology, National Marine Fisheries Service; Base Year of Price: 1982-1984

Before 1988, when the catch was only lightly regulated, seasonal demand wasn't as important because then fishermen mostly fished when they could catch fish. Now however they can fish only when the government says it is allowed. With the quota system that is now in place, the time of year fishermen are allowed to fish assumes more importance because it mostly likely affects revenue.

Before strict government controls, what Mother Nature gave was more or less taken. In the 1980's, commercial landings were between 21 and 38 million pounds but by 1990 the catch fell to nine million pounds. This precipitous decline in the catch led to increasingly stringent governmental regulations so that now nature and the Federal Government determine the fluke catch jointly. In 1993 an east coast wide quota was implemented and the flounder catching states began receiving a percentage of the Total Allowable Catch (TAL), which was set by Federal Government under the terms of the Magnuson-Stevens Fishery Act.

Each year the National Marine Fisheries Service (NMFS) assesses current biomass stock levels and compares that estimate to the 2013 target stock biomass level required under the terms of reauthorized Magnuson-Stevens Act of 2006. Using those two numbers the agency then suggests the TAL for the year. In this process NMFS consults with the Mid-Atlantic Fisheries Management Council and the Atlantic States Marine Fisheries Commission. After the councils approve the TAL, each flounder catching state is then allocated its historical percentage of the TAL with instructions that 60 percent of the TAL goes to commercial and 40 percent to recreational fisherman.

The reauthorized Magnuson-Stevens Act of 2006 required much more stringent Federal controls than previously. Figure 2 shows how the commercial quota was cut almost in half between 2005 and 2008 under the terms of the reauthorized act. In 2005 the quota was 18.2 million pounds and in 2008 it dropped to 9.5 million pounds.

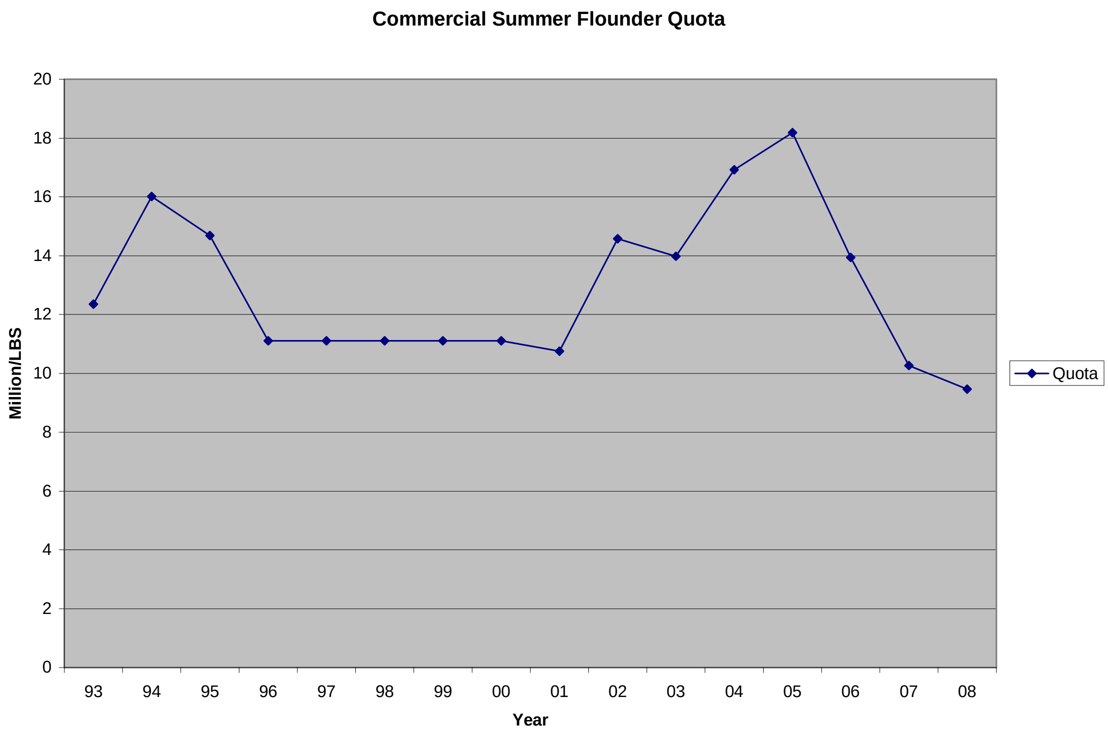

Figure 2. Annual Federal Summer Flounder Quota (Source: Personal communication from Jessica Coakley, Mid-Atlantic Fishery Management Council, July 2007.)

Although flounder stocks are at the highest levels they have been in 25 years they are still at only about half the level that government regulations require by 2013. The estimated spawning stock biomass is just above 100 million pounds now and Magnuson-Stevens requires almost 200 million pounds in 2013 (Terceiro, 2006). This is the biomass that biologists think would support maximum sustainable yield. Some fishermen believe this target biomass is unobtainable and are trying to marshal evidence that might convince congress to change the law. Regardless, the current stringent quotas make it even more desirable that maximum revenue is gained from the summer flounder that is allowed to be caught.

Each state's allocation is based on the percent of total catch each state had historically before 1992. The percentage does not change from year to year but quota does. It is the state's responsibility to manage the catch so that its percentage of the commercial quota is not exceeded. Each state allocates its quota amongst the months as best it can.

Four states have the lion's share of the quota. North Carolina has the largest quota, followed by Virginia, New Jersey and Rhode Island. New York, Massachusetts, Connecticut and Maryland have much smaller shares. Table One shows how the quota was allocated to the states in 2007 and how the states, in turn, allocated the quota among months.

Table 1. Summer Flounder Quota Allocation in Mid-Atlantic States: Percentage of State Level Quota Allocated per Month and Pounds of Flounder in 2007.

<table>
<tr><th></th><th colspan="8">State</th><th></th></tr>
<tr><th>Month</th><th>CN</th><th>MD</th><th>MA</th><th>NJ</th><th>NY</th><th>NC</th><th>RI</th><th>VA</th><th>Total</th></tr>
<tr><td>January</td><td rowspan="12">100.0%
231,737</td><td rowspan="12">100.0%
209,356</td><td rowspan="4">15.0%
105,039
(to Apr. 22)</td><td rowspan="2">28.0%
469,869</td><td rowspan="2">20.0%
157,025</td><td rowspan="4">80.0%
2,254,313</td><td rowspan="4">54.0%
869,491</td><td rowspan="3">64.3%
1,375,260</td><td rowspan="12"></td></tr>
<tr><td>February</td></tr>
<tr><td>March</td><td rowspan="2">11.0%
184,591</td><td rowspan="2">16.0%,
125,620</td></tr>
<tr><td>April</td><td rowspan="3">6.4%
136,884</td></tr>
<tr><td>May</td><td rowspan="6">70%
490,184
(from Apr.23)</td><td rowspan="2">10.5%
176,201</td><td rowspan="3">27.0%
211,983</td><td rowspan="6">0%</td><td rowspan="4">35.0%
563,559</td></tr>
<tr><td>June</td></tr>
<tr><td>July</td><td rowspan="2">10.5%
176,201</td><td rowspan="3">0%</td></tr>
<tr><td>August</td><td rowspan="3">27%
211,983</td></tr>
<tr><td>September</td><td rowspan="2">29.0%
486,650</td><td rowspan="4">11.0%
177,118</td></tr>
<tr><td>October</td><td rowspan="3">29.3%
626,674</td></tr>
<tr><td>November</td><td rowspan="2">15.0%
105,039</td><td rowspan="2">11.0%
184,591</td><td rowspan="2">10.0%
78,512</td><td rowspan="2">20.0%
563,578</td></tr>
<tr><td>December</td></tr>
<tr><td>Total (lbs)</td><td>231,737</td><td>209,356</td><td>700,262</td><td>1,678,103</td><td>785,123</td><td>2,817,891</td><td>1,610,168</td><td>2,138,818</td><td>10,171,458</td></tr>
<tr><td>Percent of Total Quota</td><td>2.3</td><td>2.0</td><td>6.9</td><td>16.5</td><td>7.7</td><td>27.7</td><td>15.8</td><td>21.0</td><td>100.0</td></tr>
</table>

Source: Electronic Code for Federal Regulations

The allocation situation is further complicated because fluke are apparently moving to new areas. Fishermen say that for some reason, possibly warmer water temperatures, fish are moving 50 or so miles further north each year than they were the previous year at the corresponding time. Thus, fisherman report they are now often catching fish off New Jersey and Virginia that were formerly caught off North Carolina. This means that they may be catching fish off of New Jersey but steaming back to North Carolina to unload them.

States would like to allocate the quota so as to maximize revenue from the summer flounder catch. But without an analysis of seasonality of demand, regulators can not factor economics into their quota opening decisions except on an anecdotal basis. In these yearly arguments between the states, one of the regulators notes, “There is never any agreement because no one can prove anything (Personal communication from Jack Travelstead, Deputy Commissioner, Virginia Marine Resource Commission, July 2004).”

The three states with the largest quota usually cooperate so they don’t flood the market with multiple openings at the same time but occasionally they will make almost unannounced unilateral openings if they think it to their advantage. This can lead to very uneven market situations where there is a glut of flounder and then there is no flounder. Some market participants believe this makes it difficult to build a strong market. This flounder/no flounder cycle even stronger because of the current relatively large bio-mass by historical standards and the small quotas. Now when quota is opened in a month, fishermen can sometimes fill it in just several days. Fishermen like this because it means they use less fuel but further exacerbates the boom bust cycle and mitigates against regulators ability to smooth the production cycle.

Those in charge of managing the quota allocation in North Carolina, Virginia and New Jersey usually talk to each other about when they will allow fisherman to land fish at their ports and fill the quota. However, in the end, each state opens its season more or less independently based on past experiences. Summer flounder migration patterns are, of course, considered. Seasonal demand, however, enters into the decision only on an ad hoc basis because it has not been systematically analyzed.

The objectives of this research are: to analyze demand seasonality, and to determine if altered seasonal quota allocations can lead to improved revenues for the sector. In order to accomplish these objectives, an econometric model of flounder demand is estimated. This econometric model is then used in a non-linear mathematical programming model to optimize revenue from the sector.

The Demand Model

The quantity of fish landed usually determines wholesale prices received by fishermen. For this reason, an inverse demand function, where price is the dependent variable, is the typical model specification for fish demand. Barten and Bettendorf (1989) argue this approach is justified because of fish's perishability. Eales (1997), in a study of Japanese fish demand, found that inverse demand systems dominate ordinary demand systems in forecasting performance. For these reasons, we also used an inverse demand system to estimate summer flounder demand. (See Barten and Bettendorf (1989) and Matsuda (2005) for a discussion of inverse demand systems.) Because this demand function will be used in a second stage of this research that maximizes producer revenues from summer flounder, a single equation structure model for summer flounder that has a flexible functional form that allows for second-order effects (Greene, 2003; and Berndt and Christensen, 1973) is employed.[1]

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[1] Autoregressive and moving average (ARIMA) and vector autoregressive (VAR) time series models were also considered. Because the structural inverse demand model had better forecast performance than the time series models, the results for the time series models are not presented in this paper.

Structural Model

The structural inverse demand model posits that the price of summer flounder is a function of the landings of summer flounder, landings of other flounder species, such as Atlantic, winter, yellowtail and imports of fresh and frozen flounder. To account for any scale effects, a flounder quantity index, which is a function of all flounder species, is included in the model. A second-order trans-log functional form is posited for all continuous variables in the model. Because the model is estimated using monthly landings and prices for the years 1991 through 2004 (cite NMFS data here), seasonal and annual variations in demand are accounted for by introducing monthly and annual binary variables. The inverse demand function is specified as:

$$\begin{aligned} lfsp = {} & \alpha_0 + \xi_1 lfsq + \xi_2 lfaq + \xi_3 lfwq + \xi_4 lfyq + \xi_5 lfeq + \xi_6 lfoq + \xi_7 QI + \frac{1}{2}\Big[\delta_{11}\left(lfsq\right)^2 + \\ & 2\delta_{12} lfsq \times lfaq + 2\delta_{13} lfsq \times lfwq + 2\delta_{14} lfsq \times lfyq + 2\delta_{15} lfsq \times lfeq + 2\delta_{16} lfsq \times lfoq + \\ & 2\delta_{17} lfsq \times QI + \delta_{22}\left(lfaq\right)^2 + 2\delta_{23} lfaq \times lfwq + 2\delta_{24} lfaq \times lfyq + 2\delta_{25} lfaq \times lfeq + \\ & 2\delta_{26} lfaq \times lfoq + 2\delta_{27} lfaq \times QI + \delta_{33}\left(lfwq\right)^2 + 2\delta_{34} lfwq \times lfyq + 2\delta_{35} lfwq \times lfeq + \\ & 2\delta_{36} lfwq \times lfoq + 2\delta_{37} lfwq \times QI + \delta_{44}\left(lfyq\right)^2 + 2\delta_{45} lfyq \times lfeq + 2\delta_{46} lfyq \times lfoq + \\ & 2\delta_{47} lfyq \times QI + \delta_{55}\left(lfeq\right)^2 + 2\delta_{56} lfeq \times lfoq + 2\delta_{57} lfeq \times QI + \\ & \delta_{66}\left(lfoq\right)^2 + 2\delta_{67} lfoq \times QI + \delta_{77}\left(\left(QI\right)^2\right)\Big] + \sum_{j=2}^{12} \beta_j M_j + \sum_{k=92}^{04} \gamma_k Y_k \end{aligned} \tag{1}$$

where: $lfsp$ is the logarithm of monthly real prices of summer flounder, $lfsq$ is the logarithm of monthly landing quantities of summer flounder, $lfaq$ is the logarithm of monthly landing quantities of Atlantic flounder, $lfwq$ is the logarithm of monthly landing quantities of winter flounder, $lfyq$ is the logarithm of monthly landing quantities of yellowtail flounder, $lfeq$ is the logarithm of monthly imports of fresh flounder, $lfoq$ is the logarithm of monthly imports of frozen flounder, QI is the quantity index, M_i are monthly dummy variables, and Y_j are annual dummy variables. The quantity index is computed by the following equation:

$$QI = lfsq \times w_{fs} + lfaq \times w_{fa} + lfwq \times w_{fw} + lfyq \times w_{fy} + lfeq \times w_{fe} + lfoq \times w_{fo} \qquad (2)$$

where $w_i = \dfrac{m_i}{\sum_{i \in I} m_i}$, $i \in I = \{ fs, fa, fw, fy, fe, fo \}$, and m_i is the monthly ex-dock value and import value in dollars of landed and imported fish. In equation (2), the subscript fs represents summer flounder, fa represents Atlantic flounder, fw represents winter flounder, fy represents yellowtail flounder, fe represents imported fresh flounder, and fo represents imported frozen flounder.

Data

The authors use 1991 to 2005 data for summer flounder and its substitutes from the website of the Office of Science and Technology, National Marine Fisheries Services (NMFS). Real prices are used to estimate demand. Nominal prices were transformed to their corresponding real ones by using the Consumer Price Index (CPI) from the U.S. Department of Labor. The specific index used was the CPI - All Urban Consumers U.S. all items, 1982-84=100. The average monthly prices of summer flounder are plotted in Figure 1 above. Average monthly landings of summer flounder are plotted in Figure 3. The highest average monthly landings were in January (2.99 million pounds), February (2.10 million pounds) and October (2.02 million pounds). In March the average landings drop to 1.35 million pounds. The five lowest average monthly landings are April through August at just under 1 million pounds. In September and October, the average landings increase to 1.64 and 2.02 million pounds respectively. In December, the average landing decreased to 1.24 million pounds.

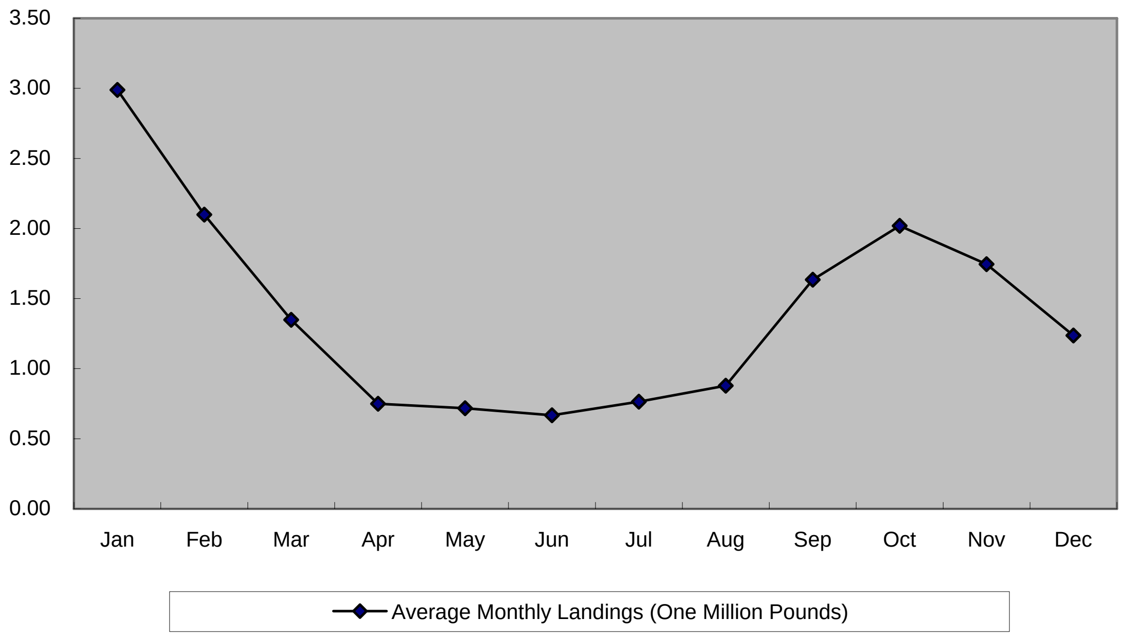

Figure 3. Averages of Monthly Summer Flounder Landings from 1991 to 2005 (Source: Office of Science and Technology, National Marine Fisheries Service)

Model Estimation

In examining the full model, with all possible substitutes included, multicollinearity is detected. In order to find combinations of explanatory variables that are not multicollinear, a condition index is used. To a matrix X, the condition index of $X'X$ is the ratio of the square root of the largest characteristic root of $X'X$ to the smallest. If the condition index is greater than 20, then the multicollinearity problem is serious. After scanning the condition indices of all combinations of explanatory variables, there are five combinations without multicollinearity problems. These five combinations of variables and their condition indices are contained in Table 2. The five variable groups are sequentially put into the structural model to estimate the monthly real price of summer flounder in order to determine which group gives the best prediction of summer flounder price.

Table 2. Condition Indices of Different Variable Groups

Variable Group	**Variable Group**	**Condition Index**
1	*lfsq, lfaq, lfwq, lfyq*	4.06
2	*lfsq, lfaq, lfwq, lfoq*	2.93
3	*lfsq, lfaq, lfyq, lfeq*	4.21
4	*lfsq, lfaq, lfwq, lfyq, lfeq*	4.22
5	*lfsq, lfaq, lfwq, lfeq, lfoq*	3.54

In estimation of the models with variables from group one to five, monthly data from January 1991 to December 2004 are used. The monthly real prices in 2005 are then forecast by using the model parameter estimates and the 2005 data on landings. The model with the best forecasting performance is used to maximize revenue in the mathematical programming model.

The forecasts for 2005 monthly real prices of summer flounder for the five variable groups are contained in Table 3. The plots of these forecasts are contained in Figures 4 and 5. Groups 1, 2 and 5 have one missed turning point in October, less than other groups. Three statistics, Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE) and the Theil Inequality Coefficient (TIC), are calculated to evaluate the different group's forecasts. The results are contained in Table 3. Group Two's forecast has the smallest values for all three statistics. The original full model of Group Two has forty-five parameter estimates. Many of the parameters are not significant and can be removed. In the original model, any variables whose parameter estimates have p – values higher than 15% are removed and the model is estimated again. Then the process to remove variables whose coefficients had p – values higher than 15% is repeated two more times and the coefficients of all the variables left in the model have p – values less than 5%. These variables and their coefficients are labeled as the Final Model in Table 4 and are used in the mathematical programming model.

Table 3. Predictions of Real Monthly Price of Summer Flounder in 2005, Group One to Group Five

Month	Real Price	Group One	Group Two	Group Three	Group Four	Group Five
January	0.71	0.82	0.74	0.91	0.83	0.86
February	0.64	0.80	0.75	0.83	0.83	0.84
March	0.87	0.91	0.85	0.99	0.94	0.99
April	1.08	0.96	0.98	1.13	0.97	1.03
May	1.23	1.11	1.24	1.16	1.13	1.21
June	1.10	1.03	1.06	1.05	1.03	1.00
July	1.12	1.11	1.08	1.16	1.11	1.07
August	1.02	1.04	1.04	1.14	1.11	1.01
September	0.94	0.97	1.00	1.15	1.02	0.99
October	0.88	1.00	1.04	1.15	1.05	1.04
November	0.81	0.80	0.78	0.89	0.82	0.82
December	0.90	0.92	0.92	0.98	0.94	1.02
RMSE		0.0072	0.0046	0.0204	0.0105	0.0108
MAPE		7.880	6.068	14.354	10.182	10.152
TIC		0.0626	0.0503	0.1056	0.0758	0.0768
Turning Point Errors		1	1	2	2	1

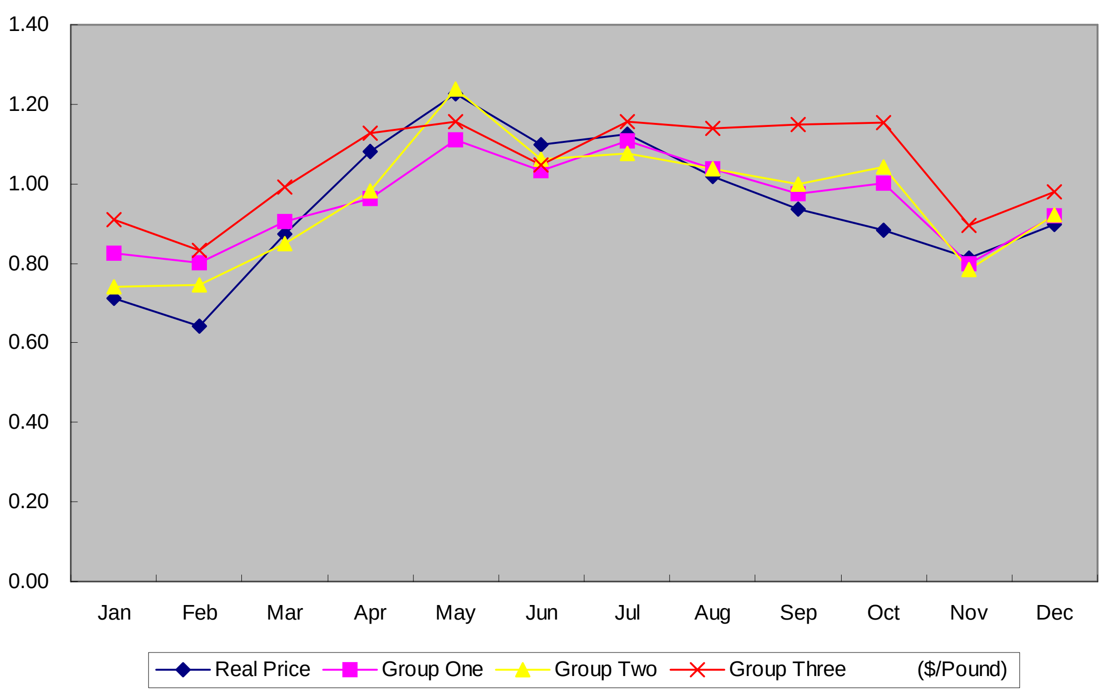

Figure 4. Predictions of Real Monthly Price of Summer Flounder in 2005, Group One to Three and the Actual Historical Prices with Log Transformed Data.

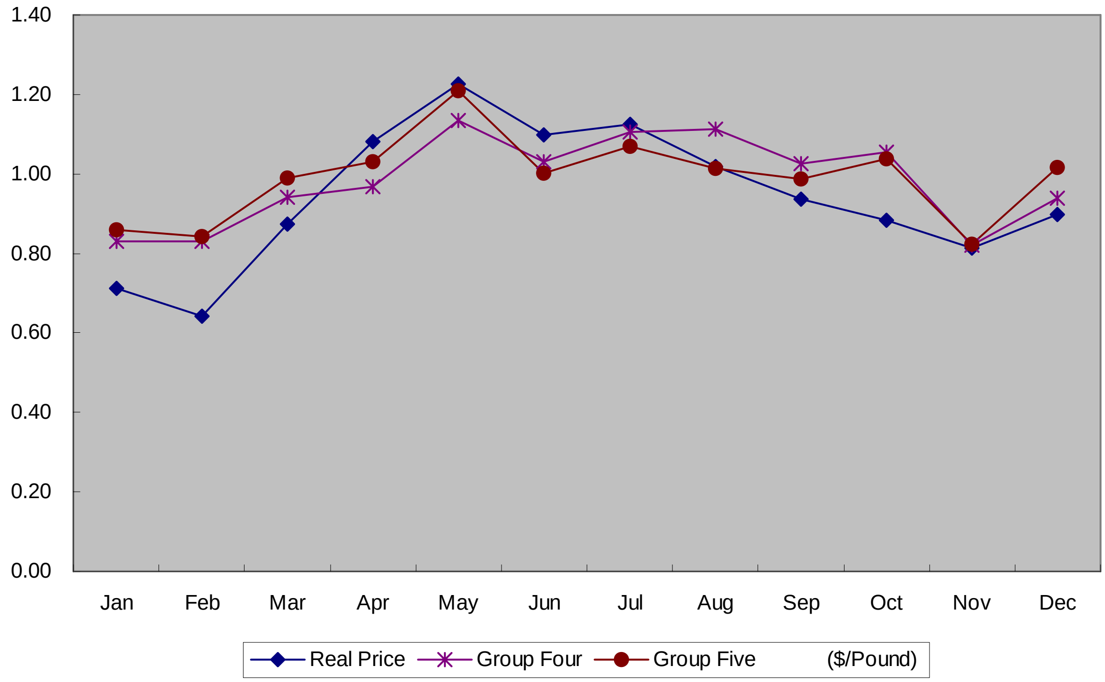

Figure 5. Predictions of Real Monthly Price of Summer Flounder in 2005, Group Four to Five and the Actual Historical Prices with Log Transformed Data.

Table 4. Parameter Estimates for Fluke and Summer Flounder Price Forecasting Model

Full Model			**Final Model**		
Variable	**Estimate**	***p*-value**	**Variable**	**Estimate**	***p*-value**
Intercept	0.12039	0.0374	Intercept	0.141867	<.0001
Monthly Binary			*Monthly Binary*		
February	0.03858	0.289	March	0.060426	0.0081
March	0.07623	0.043	November	-0.11625	<.0001
April	-0.016	0.758	December	-0.05824	0.0162
May	-0.057	0.381	*Yearly Binary*		
June	-0.0816	0.221	1994	0.085059	0.0005
July	-0.0524	0.393	1995	0.159146	<.0001
August	-0.0019	0.975	1997	0.098475	<.0001
September	-0.0207	0.694	2001	-0.13164	<.0001
October	-0.0014	0.978	2002	-0.20382	<.0001
November	-0.1191	0.014	2003	-0.16895	<.0001
December	-0.0676	0.147	2004	-0.17652	<.0001
Yearly Binary			*lfsq*	-0.24208	<.0001
1992	0.06077	0.117	$lfsq^2$	-0.1081	<.0001
1993	0.05607	0.162	*lfsq*lfaq*	-0.12919	<.0001
1994	0.16051	0.001	*lfsq*lfoq*	-0.28858	<.0001
1995	0.21179	<.0001	*lfsq*QI*	0.291496	<.0001
1996	0.07923	0.077	*lfaq*lfoq*	-0.19049	0.0001

Full Model			Final Model		
Variable	**Estimate**	***p*-value**	**Variable**	**Estimate**	***p*-value**
1997	0.14583	6E-04	*lfaq*QI*	0.266235	0.0003
1998	0.0614	0.137			
1999	0.02537	0.552			
2000	0.01021	0.803			
2001	-0.0963	0.014			
2002	-0.1596	<.0001			
2003	-0.1351	6E-04			
2004	-0.1484	6E-04			
lfsq	-0.2046	0.015			
lfaq	0.03299	0.608			
lfwq	0.09505	0.124			
lfoq	0.05856	0.553			
QI	-0.2788	0.293			
$lfsq^2$	-0.1128	0.007			
*lfsq*lfaq*	-0.158	0.011			
*lfsq*lfwq*	-0.0527	0.511			
*lfsq*lfoq*	-0.4423	2E-04			
*lfsq*QI*	0.40701	0.042			
$lfaq^2$	0.03243	0.299			
*lfaq*lfwq*	-0.0094	0.839			
*lfaq*lfoq*	-0.2262	0.017			
*lfaq*QI*	0.30735	0.043			
$lfwq^2$	0.01143	0.728			
*lfwq*lfoq*	-0.0726	0.365			
*lfwq*QI*	-0.0582	0.684			
$lfoq^2$	-0.0546	0.648			
*lfoq*QI*	0.35403	0.19			
QI^2	-0.0534	0.846			

The substitutes for summer flounder in the final model are Atlantic flounder, winter flounder and imported frozen flounder. Winter flounder doesn’t show up in the final model directly as a substitute, but it is used to calculate the quantity index QI. As to monthly effects, March, November and December are significant in estimating the logarithm of summer flounder prices. In terms of annual effects, summer flounder price was significantly impacted in seven years: 1994, 1995, 1997, 2001, 2002, 2003 and 2004.

Forecasting and Model Evaluation

The final model is also evaluated by several tests related to the price forecasts and a set of misspecification tests. Sanders and Manfredo (2003) developed three different tests for the optimality of a forecast using the residuals of the out-of-sample forecast: a bias test, delta efficiency test of whether the residuals contain any information on the forecasted variables, and a gamma efficiency test on whether the residuals of the forecast are autocorrelated. Table 5 presents the results for these tests using the residuals for our forecasts for the year 2005. The null hypothesis is rejected for all three tests. Sanders and Manfredo also presented a test whether the accuracy of a forecast improves or worsens over time. As shown in the last column of table 5, the null hypothesis that the accuracy of our forecast changes over time is rejected.

Table 5. Optimality Test and Time Improvement Test for Out-of-Sample Residuals for the Final Model

	Test for Optimality			
	Forecast Bias Test	**Delta Efficiency Test**	**Gamma Efficiency**	**Time Improvement or Worsening Test**
Estimate	-0.0142	0.0441	0.1017	0.0003
Std Error	0.0201	0.1435	0.3149	0.0038
***p*-value**	0.4944	0.7650	0.7533	0.9480

The goodness-of-fit and other diagnostic tests for the final model are listed in Table 6. The R^2 and adjusted R^2 of the estimation are 0.8813 and 0.8679 respectively. The diagnostic tests are calculated from in-sample residuals. The Durbin-Watson statistic is 1.9901 indicating no autocorrelation in residuals. Both White's and Breusch-Pagan tests are tests for homoskedasticity. The results in Table 6 imply that the residuals are homoskedastic and do not vary through time. The Augmented Dickey-Fuller test indicates that residuals are stationary and do not contain a unit root. Lastly, the Shapiro-Wilk test shows that the hypothesis that the residuals are standard normally distributed can't be rejected. Therefore, based on the diagnostic test results, the model is very well specified.

Table 6. Goodness-of-Fit and Diagnostic Tests for Final Model

Goodness-of-Fit		**Diagnostic Tests**				
R^2	Adj. R^2	Durbin-Watson	White's Test	Breusch-Pagan Test	Augmented Dickey-Fuller	Shapiro-Wilk Test
0.8813	0.8679	1.9901	0.4637	0.5474	No unit root	0.4484

Note: Durbin-Watson: tests for autocorrelation in residuals, a number close to 2 indicates no autocorrelation; White's Test: the null hypothesis is that the residual are homoscedastic; Breusch-Pagan Test: the null hypothesis is that the residuals are homoscedastic; Augmented Dickey-Fuller Test: a test for a unit root in the residuals; Shapiro-Wilk Test: the null hypothesis is that the residuals are normally distributed.

Mathematical Programming

Numerous scenarios were run and sensitivity analyses conducted as part of this study. These scenarios are solved by the CONOPT Solver of GAMS 21.5 (Brook et al. 1998). The General Algebraic Modeling System (GAMS) has been widely applied in the fishery industry. Lee et al. (2000) used the nonlinear solver of GAMS to maximize the net present value of returns to the U.S. Atlantic pelagic longline fleet. Using different assumptions about production technology, Anderson et al. (2003) set up models to allocate the 2001 catch optimally among vessels in the Danish fishery management system. He used GAMS to find optimal solutions for these models. Bisack and Sutinen (2006) used GAMS to calculate optimal profit levels and measure the efficiency losses due to an in-season stock externality in an actual fishery. Lee and Gates (2007) set up a Virtual Population Unit (VPU) problem as an iterative mathematical programming model and solved it by applying GAMS with nonlinear solver.

As with the econometrics section, in the interest of brevity, this paper focuses on only one model. A revenue maximizing model is developed with one constraint representing the total annual quota, but there is no constraint representing each state's quota illustrated in Table 1. As discussed above, the percentage of the total quota allocated to each state has not changed from year to year thus it is like an endowment and rarely does a state government transfer any part of its quota to other states. Also as discussed above, it is common for a fisherman, who catches summer flounder off the coast of one state, to land the catch at the port of another state whose quota is open. That is, fishermen fish where the concentration of fish is the greatest and then land the fish where they legally can, given the licenses that they have. Thus the main reason for focusing on this particular model is that it really does not matter to the fishermen what state has its quota open in terms of their fishing activities, as long as some quota is open. We then refine the model by adding some biological seasonal catch constraints that were estimated by experienced fishermen.

Optimization Model

Since the inverse demand function is estimated as the logarithm of monthly real price, the objective function is changed to an exponential function of a sum of two log values (log prices and log quantities). The basic optimization problem is then set up as:

$$\max_{lfsq_i} \; R = \sum_{i=1}^{12} \exp\left(lfsp_i + lfsq_i\right) \tag{3}$$

subject to:

$$\sum_{i=1}^{12} \exp\left(lfsq_i\right) \leq Quota \tag{4}$$

where *lfsp* is determined by the inverse demand function.

Then the Lagrangian problem is:

$$L = \sum_{i=1}^{12} \exp\left(lfsp_i + lfsq_i\right) - \lambda\left(\sum_{i=1}^{12} \exp\left(lfsq_i\right) - Quota\right) \tag{5}$$

The first order conditions are:

$$\frac{\partial L}{\partial lfsq_i} = 0, \quad \frac{\partial L}{\partial \lambda} = 0. \tag{6}$$

The second order conditions require that the augmented Hessian matrix is negative semidefinite.

The solution of the problem is as follows, starting with the first order conditions:

$$\frac{\partial L}{\partial lfsq_i} = r_i g_i - \lambda \exp\left(lfsq_i\right) = r_i g_i - \lambda h_i = 0 \tag{7}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{12} \exp\left(lfsq_i\right) - Quota = 0 \tag{8}$$

where:

$$\begin{aligned} g_i &= \frac{\partial\left(lfsp_i + lfsq_i\right)}{\partial lfsq_i} \\ &= -0.2421 - 0.2162 \times lfsq_i - 0.1292 \times lfaq_i - 0.2886 \times lfoq_i + 0.2915 \times QI_i \\ &\quad +0.2915 \times lfsq_i \times w_{fs,i} + 0.2662 \times lfaq_i \times w_{fs,i} + 1 \\ &= 0.7579 - 0.2162 \times lfsq_i - 0.1292 \times lfaq_i - 0.2886 \times lfoq_i \\ &\quad + \left(0.2915 \times lfsq_i + 0.2662 \times lfaq_i\right) w_{fs,i} + 0.2915 \times QI_i \end{aligned} \tag{9}$$

$$r_i = \exp\left(lfsp_i + lfsq_i\right) = fsp_i \times fsq_i \tag{10}$$

and

$$h_i = \exp\left(lfsq_i\right) = fsq_i \tag{11}$$

Since the Hessian matrix is diagonal, only the signs of the diagonal elements need to be checked for the second order conditions as follows:

$$\begin{aligned} \frac{\partial^2 L}{\partial lfsq_i^{\,2}} &= r_i^2 g_i^2 + r_i(-0.2162 + 0.2915 \times w_{fs,i} + 0.2915 \times w_{fs,i}) - \lambda h_i \\ &= r_i^2 g_i^2 + r_i(-0.2162 + 0.5830 \times w_{fs,i}) - \lambda h_i \leq 0 \end{aligned} \tag{12}$$

Failure to satisfy equation (12) is illustrated below in Appendix A when annual rather than average market shares are used to calculate the quantity index and no feasible solutions are found in some years.

Biological Constraints

In addition to solving the optimization model with just the quota constraint, what the authors' refer to as "biological constraints" are added to the optimization model in order to reflect experts' opinions regarding the percentage of the total quota that could be caught in any given month. The twelve constraints are:

$$lfsq_i \le \rho_i \times Quota, \tag{13}$$

where: ρ_i is the monthly percentage of the total quota that can be caught, $i = \{1, 2, ..., 12\}$. The experts are three fishermen whom the authors know to be among the most experienced trawl fishermen on the east cost , with a combined total of more than 100 years fishing experience. The percentages are the averages of their estimates, to the nearest five per cent, of the per cent of the annual quota that could be caught in any one month. Table 7 contains these percentages.

Table 7. Fishermen Estimates of the Percentage of the Total Quota that can be caught in Each Month if Unfettered Fishing were allowed

Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
75%	75%	65%	40%	5%	15%	15%	5%	20%	25%	35%	50%

The fisherman's estimates do not constrain the model, except in May or August when fish are moving in or off shore. Migrating fish generally don't school making them difficult to catch in commercial quantities, which explains the reason for the fishermen's low estimates in those months.

Results of the Mathematical Programming

The CONPOT Solver of GAMS 21.5 is used to solve the revenue maximization problem. The constraint on landing summer flounder is 10,171,458 pounds, which is the 2007 federal quota. Starting values are specified for the decision making variables to solve this nonlinear problem to avoid infeasibility in the initial solution where all variables are set to zero. Starting values for the decision making variables are 840,000 pounds, which is less than one twelfth of the total landing quota.

The monthly landings of the three substitutes, Atlantic, winter, and imported frozen flounder, are put into the model as exogenous variables. The most recent data available for the quantities of these three substitutes is 2005, which is used in the model. There are fifteen years of market share data available for the three substitutes to calculate the quantity index. The sensitivity of the solution to the choice of market share year used for quantity index calculation is examined by solving the model with each year's market share and an average of all 15 years. Feasible solutions were obtained for the average market share in all years except for 1999, 2004, and 2005. The results from models using sixteen sets (1991 to 2005 and their average) of market shares, with 2005 substitute quantities is contained in Table 8. The upper part of the table contains landing quantities in million pounds, and the lower part of the table contains monthly real prices of the fish in dollars per pound. The highest revenue is $12.87 million in 1997, and the lowest is $11.87 million in 1991. The average of these yearly revenues is $12.48 million, compared to the solution with the average of the market shares from 1991 to 2005 at $12.35 million. The optimization model is not very sensitive to the market shares from different years, so the 1991 to 2005 average is used in the remainder of this paper.

Table 8. Optimized Monthly Landings and Prices for Summer Flounder in 2007 using the Monthly Quantities in 2005 of All Substitutes, and the Market Shares from 1991 to 2005

Monthly Landings of Summer Flounder (Million Pounds)

Month	Year of Market Share								
	2005	2004	2003	2002	2001	2000	1999	1998	1997
January	.	.	0.578	0.518	0.451	0.442	.	0.251	0.191
February	.	.	0.407	0.699	0.811	0.562	.	0.783	0.959
March	.	.	0.791	1.114	1.254	1.227	.	1.285	1.12
April	.	.	1.441	1.215	1.338	1.353	.	1.314	1.221
May	.	.	0.996	1.073	1.019	1.035	.	1.018	0.989
June	.	.	0.798	0.757	0.804	0.762	.	0.826	0.716
July	.	.	0.954	0.935	0.906	0.825	.	0.985	0.905
August	.	.	0.868	0.804	0.813	0.855	.	0.924	0.874
September	.	.	0.982	0.813	0.699	0.816	.	0.833	0.791
October	.	.	1.064	0.941	0.808	0.953	.	0.856	0.727
November	.	.	0.581	0.513	0.549	0.592	.	0.482	0.675
December	.	.	0.712	0.789	0.719	0.750	.	0.615	1.005

Monthly Prices of Summer Flounder ($/Pound)

	2005	2004	2003	2002	2001	2000	1999	1998	1997
January	.	.	1.455	1.607	1.587	1.613	.	2.399	3.112
February	.	.	1.687	1.386	1.248	1.312	.	1.206	1.231
March	.	.	1.461	1.239	1.290	1.203	.	1.248	1.370
April	.	.	1.144	1.256	1.187	1.138	.	1.226	1.304
May	.	.	1.172	1.180	1.176	1.147	.	1.276	1.401
June	.	.	1.099	1.126	1.063	1.138	.	1.076	1.194
July	.	.	1.046	1.097	1.122	1.213	.	1.053	1.110
August	.	.	1.173	1.197	1.212	1.225	.	1.124	1.170
September	.	.	1.050	1.148	1.200	1.126	.	1.111	1.154
October	.	.	1.097	1.159	1.214	1.149	.	1.193	1.287
November	.	.	1.119	1.198	1.150	1.106	.	1.221	1.088
December	.	.	1.292	1.218	1.287	1.284	.	1.503	1.117
Revenue ($ Million)	.	.	12.16	12.41	12.37	12.20	.	12.48	12.87

Table 8. (Cont.) Optimized Monthly Landings and Prices for Summer Flounder in 2007 using the Monthly Quantities in 2005 of All Substitutes, and the Market Shares from 1991 to 2005

Monthly Landings of Summer Flounder (Million Pounds)							
Month	Year of Market Share						
	1996	1995	1994	1993	1992	1991	Average 91 to 05
January	0.383	0.359	0.597	0.565	0.571	0.520	0.443
February	0.294	0.744	0.566	0.737	0.712	0.588	0.641
March	1.327	1.025	0.996	0.869	0.960	1.045	1.069
April	1.448	1.327	1.300	1.327	1.123	1.258	1.285
May	1.011	1.049	1.022	0.963	0.999	1.064	1.026
June	0.774	0.834	0.823	0.797	0.839	0.799	0.803
July	1.012	1.063	1.093	1.032	1.078	1.016	0.981
August	0.785	0.844	0.888	0.893	0.926	0.893	0.875
September	0.580	0.831	0.748	0.835	0.882	0.898	0.832
October	0.734	0.761	0.66	0.715	0.737	0.819	0.843
November	0.778	0.596	0.624	0.613	0.628	0.586	0.597
December	1.045	0.739	0.854	0.826	0.717	0.684	0.777
Monthly Prices of Summer Flounder ($/Pound)							
	1996	1995	1994	1993	1992	1991	Average 91 to 05
January	1.750	1.879	1.445	1.480	1.499	1.492	1.669
February	1.981	1.184	1.417	1.339	1.329	1.407	1.336
March	1.230	1.272	1.285	1.571	1.486	1.336	1.309
April	1.237	1.118	1.234	1.304	1.414	1.193	1.204
May	1.293	1.311	1.439	1.448	1.472	1.085	1.247
June	1.154	1.145	1.156	1.086	1.053	1.035	1.102
July	1.092	1.087	1.090	1.037	1.019	1.003	1.071
August	1.278	1.229	1.179	1.078	1.097	1.063	1.154
September	1.391	1.143	1.190	1.077	1.081	1.032	1.119
October	1.342	1.337	1.443	1.281	1.259	1.192	1.211
November	1.062	1.136	1.151	1.108	1.093	1.109	1.124
December	1.178	1.199	1.199	1.228	1.236	1.275	1.237
Revenue ($ Million)	12.83	12.40	12.79	12.72	12.74	11.87	12.35

Sensitivity of the Model to Substitutes

The influence of the three substitutes, Atlantic, winter, and imported frozen flounder on the solution is examined. First, the optimization model is solved with all the three-substitute quantities changed from 1991 to 2005. Then one by one the quantity of one substitute is varied from 1991-1995 while the others are at 2005 levels. A summary of the sensitivity analysis of the revenue to the monthly quantities of substitutes is contained in Table 9. The average revenues for variations in the substitute quantities for winter and imported frozen flounder are very close at $11.87 million and $11.90 million. The standard deviation, $0.52 million, indicates that revenue is the most sensitive to the year of the substitute quantities for Atlantic flounder, and least sensitive to winter flounder substitute quantities with a standard deviation of $0.21 million. However, overall the model is not very sensitive to these substitutes, so the 2005 quantities are used in the rest of this analysis.

Table 9. Averages and Standard Deviations of Maximized Annual Revenues from Substitute Quantities Combinations

	All Three Substitutes 91 - 05	**Atlantic Flounder 91 - 05**	**Winter Flounder 91 – 05**	**Imported Frozen Flounder 91 - 05**
Average ($ Million)	10.75	11.05	11.87	11.90
Standard Deviation ($ Million)	0.49	0.52	0.21	0.35

Optimized Monthly Quota Allocations

The percentages of the total annual quota in 2007 selected by the optimization model are compared to the historical percentages of the average monthly landings from 1991 to 2005. The historical percentage of the average monthly landing is the highest at 18% in January and falls to the level around 4% to 5% in the months from April to August. Then it reaches another high level of 12% in October. The January percentage in the optimization model with only the quota constraint is 5%, but with the biological constraint it increases to 6%. The optimization model with the quota only landed 8% of the flounder in October and 9% when the biological constraint is also imposed. The biological constraint is only binding in May and August where the percentage landed with the biological constraint is closer to the historical landings than the optimization model with just the quota constraint. The historical percentage of the average monthly landing has one cycle over the year. Low levels of historical landing occur from April to August, when landing percentages are around 4% to 5%. The optimization model with just the quota constraint has an increase in landings from 5% in January to a peak of 11% in April, down to 8% in June with a rise to 10% in July followed by a decline to 6% and 7% in November and December. A similar pattern is observed when the biological constraints are added, except for an increase in landings to 10% in September. Thus the optimization models dampen the cycle of landings relative to historical practices.

Table 10. Monthly Percentages of the Total Annual Landing in Different Scenarios of Landing Summer Flounder

	Historical Average, 1991-2005	**Optimization Model With Quota Constraint**	**Optimization Model With Quota and Biological Constraints**
January	18%	5%	6%
February	12%	7%	8%
March	8%	10%	12%
April	4%	11%	13%
May	4%	10%	5%
June	4%	8%	9%
July	5%	10%	11%
August	5%	9%	5%
September	10%	8%	10%
October	12%	8%	9%
November	10%	6%	7%
December	7%	7%	6%

The average real prices of the historical data from 1991 to 2005 and the optimization models are listed in Table 10. The cyclical fluctuation of the price produced by the optimization models is mild as one would expect given its dampening of the landing cycle. The historical average monthly real price has a high period covering six months from March to August at around $1.20/Pound. The highest prices with the optimization model with only the quota constraint imposed are in January ($1.49/pound) through March ($1.21/pound). With the biological constraint relatively high prices are also observed from January through March, but unlike the model with only the quota constraint, high prices also occur in May ($1.37/pound) and August ($1.26 per pound) when the biological constraint is binding.

Table 11. Historical and Mathematical Programming Model Results for Monthly Real Price of Summer Flounder ($/Pound)

	Historical Average 1991-2005	**Optimization Model With Quota Constraint**	**Optimization Model With Quota and Biological Constraints**
January	0.88	1.49	1.39
February	1.03	1.25	1.17
March	1.24	1.21	1.16
April	1.26	1.13	1.09
May	1.22	1.14	1.37
June	1.22	1.06	1.03
July	1.22	1.05	1.02
August	1.20	1.09	1.26
September	0.98	1.09	1.05
October	0.96	1.15	1.10
November	0.90	1.08	1.04
December	1.01	1.16	0.78

Historical Annual Landings and the Optimization Model

To study how the optimization model affects annual revenue, the historical annual landing for each year from 1991 to 2005 is put into the model as federal quotas. Models to maximize the annual revenue given these quotas are solved (Table 12). The maximized revenues are compared with the annual revenue calculated from historical data. In the comparison, revenues with the optimization model are higher than those derived from the historical data for all years 1991 through 2005. The total increased revenue over the historical revenue earned by applying the optimization model with only the quota constraint is $46.60 million over the fifteen years. Adding biological constraints decreases the total revenue gains by only $1.87 million. Revenue gains, in real terms, with additional monthly constraints, still amount to $44.73 million.

Table 12. Maximized Annual Revenue from Landing Summer Flounder using the Monthly Quantities in 2005 of All Substitutes ($ Million)

Year	**Historical Average**	**Optimization Model With Quota Constraint**	**Optimization Model With Quota and Biological Constraints**
1991	17.06	20.86	20.74
1992	18.62	22.14	22.02
1993	17.01	20.47	20.34
1994	21.31	23.96	23.83
1995	23.02	26.20	26.06
1996	17.81	20.01	19.88
1997	15.00	18.63	18.49
1998	16.28	18.79	18.66
1999	14.50	17.53	17.40
2000	14.88	18.35	18.22
2001	13.07	16.03	15.91
2002	14.64	17.12	17.02
2003	14.66	16.99	16.88
2004	17.35	19.26	19.16
2005	16.60	22.05	21.93
Total Revenue	251.79	298.39	296.52
Increase in Revenue		46.60	44.73

Conclusions and Observations

An effective system for increasing revenues through improved season allocations was developed. In a perfect information world, the model could increase revenue $3.12 million per year 1991-2005 on average, in real terms, because of better quota allocation.

However, in order to keep the model current, the full econometric model would need to be re-estimated every year as new data become available. The re-estimated inverse demand equation would then need to be run in the optimization model to get optimal allocations for the year.

Even if the model is not re-estimated, regulators can use charts and graphs in the monograph to show them the directions in which they should be moving seasonal quotas.
In general the study calls for moving fishing effort away from January, in particular, towards other months; late spring and summer in particular. In general the study indicates that a smoother production cycle would enhance revenues when compared to the current peak and valley production cycle.

Tilapia imports are increasing exponentially. Industry participants often mention that they sometimes substitute tilapia when flounder is not available. Tilapia may well become a substitute in the future though the econometrics work in this study indicates it is not yet statistically significant.

Revenues only are considered in this study without referencing fishing costs. High fuel costs now make fishermen want to catch the quota quickly and in a concentrated fashion so they can reduce time at sea. Might higher fuel costs swamp revenue gains from better allocations in fishermen's eyes? Transferring slightly more of the quota to summer months might require new regulations with respect to allowing some additional inshore fishing. It might also engender additional soci-economic conflict amongst boats with different licenses. Such changes might also increase tensions between trawl and recreational fishermen because they might be fishing the same areas. Normally they do not see each other. Reducing quotas in the fall might engender protests from trawl flounder fishermen who have been without flounder income since the spring. These socio-economic considerations would require further study lest too many unintended consequences grow from changes in seasonal allocation.

Finally as summer flounder stocks have rebounded, fishermen have been able to fill the quota allocations extremely rapidly. It used to be that when regulators allocated part of a state's quota to a month, it would take fishermen several weeks to fill it if not the whole time period. Now because of the smaller quotas and larger quantities of fish to catch, an opening can be filled in just a few days. So the market alternates between having too many fish and not enough. This situation, of course, mitigates against regulators ability to smooth the curve and marketers ability to develop a market with at least some dependable supply.

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Appendix A

Results in Table 8 indicate that the revenue maximization can't be solved optimally for market shares from years 1999, 2004 and 2005. In order to find the season, the second order conditions are checked for all models with market shares from different years and their average. In checking it, since the Hessian matrix is diagonal in this model, only the signs of the diagonal elements need to be checked. In other words, check if the inequality (12) holds for all twelve months or not. The results are contained in Appendix Table A1.

Appendix Table A1. Values of the Second Order Condition Checks for the Optimization Problems with Market Shares from 1991 to 2005 and Their Average

	2005	2004	2003	2002	2001	2000	1999	1998
Jan	-5.1E+05	-4.3E+05	-4.5E+05	-4.1E+05	-3.7E+05	-3.6E+05	5.5E+11	-2.0E+05
Feb	5.8E+11	5.2E+11	-3.2E+05	-5.5E+05	-6.6E+05	-4.5E+05	-5.5E+05	-6.3E+05
Mar	-1.0E+06	-6.5E+05	-6.1E+05	-8.8E+05	-1.0E+06	-9.9E+05	-9.7E+05	-1.0E+06
Apr	-8.6E+05	-6.7E+04	-1.1E+06	-9.6E+05	-1.1E+06	-1.1E+06	-2.3E+04	-1.1E+06
May	-8.8E+05	-9.7E+05	-7.7E+05	-8.5E+05	-8.4E+05	-8.4E+05	-8.0E+05	-8.2E+05
Jun	-8.3E+05	-7.7E+05	-6.2E+05	-6.0E+05	-6.6E+05	-6.2E+05	-7.6E+05	-6.7E+05
Jul	-4.6E+04	-7.3E+05	-7.4E+05	-7.4E+05	-7.4E+05	-6.7E+05	-8.9E+05	-7.9E+05
Aug	-8.3E+05	-8.0E+05	-6.7E+05	-6.3E+05	-6.7E+05	-6.9E+05	-8.8E+05	-7.5E+05
Sep	-6.7E+05	-8.6E+05	-7.6E+05	-6.4E+05	-5.7E+05	-6.6E+05	-9.6E+05	-6.7E+05
Oct	-8.4E+02	-8.0E+05	-8.3E+05	-7.4E+05	-6.6E+05	-7.7E+05	-6.4E+05	-6.9E+05
Nov	-4.3E+05	-4.9E+05	-4.5E+05	-4.0E+05	-4.5E+05	-4.8E+05	-5.1E+05	-3.9E+05
Dec	-5.3E+05	-5.8E+05	-5.5E+05	-6.2E+05	-5.9E+05	-6.1E+05	-7.6E+05	-5.0E+05
	1997	1996	1995	1994	1993	1992	1991	Average 91 to 05
Jan	-1.5E+05	-3.1E+05	-2.8E+05	-4.6E+05	-4.7E+05	-4.7E+05	-4.4E+05	-4.6E+05
Feb	-7.8E+05	-2.3E+05	-5.8E+05	-4.3E+05	-6.1E+05	-5.8E+05	-5.0E+05	-3.2E+05
Mar	-9.1E+05	-1.1E+06	-8.0E+05	-7.6E+05	-7.2E+05	-7.9E+05	-8.9E+05	-6.3E+05
Apr	-9.9E+05	-1.2E+06	-1.0E+06	-1.0E+06	-1.1E+06	-9.2E+05	-1.1E+06	-1.1E+06
May	-8.0E+05	-8.1E+05	-8.2E+05	-7.8E+05	-8.0E+05	-8.2E+05	-9.0E+05	-7.9E+05
Jun	-5.8E+05	-6.2E+05	-6.5E+05	-6.3E+05	-6.6E+05	-6.9E+05	-6.8E+05	-6.3E+05
Jul	-7.3E+05	-8.1E+05	-8.3E+05	-8.4E+05	-8.6E+05	-8.8E+05	-8.6E+05	-7.6E+05
Aug	-7.1E+05	-6.3E+05	-6.6E+05	-6.8E+05	-7.4E+05	-7.6E+05	-7.6E+05	-6.9E+05
Sep	-6.4E+05	-4.6E+05	-6.5E+05	-5.7E+05	-6.9E+05	-7.2E+05	-7.6E+05	-7.8E+05
Oct	-5.9E+05	-5.9E+05	-5.9E+05	-5.1E+05	-5.9E+05	-6.0E+05	-6.9E+05	-8.4E+05
Nov	-5.5E+05	-6.2E+05	-4.6E+05	-4.8E+05	-5.1E+05	-5.1E+05	-5.0E+05	-4.6E+05
Dec	-8.2E+05	-8.3E+05	-5.7E+05	-6.5E+05	-6.9E+05	-5.9E+05	-5.8E+05	-5.7E+05

For models using 2005 and 2004 market shares, the inequality doesn’t hold in February and for 1999, it doesn’t hold in January. So, the three models have non-optimal solutions. As to models using all other year market shares and the average one of 1991 to 2005, the inequality holds for all twelve months.