Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

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MULTIPLE SITE DEMAND MODELS. PART I: SOME PRELIMINARY CONSIDERATIONS

W. Michael Hanemann

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CHAPTER 8

MULTIPLE SITE DEMAND MODELS:
SOME CONSIDERATIONS

This chapter focuses on some of the conceptual and empirical issues which arise when one sets out to model consumer demand for a set of recreation sites, as opposed to a single site. Many of these issues also arise when a single demand equation is being estimated, but they tend to assume added force in the context of multiple site demand systems.

Integrability Considerations

An example   is the question of how one goes about generating specifications for entire demand systems. One approach consists of specifying a direct or indirect utility function explicitly and then deriving the ordinary demand functions either by maximizing the direct utility function or by applying Roy's Identity to the indirect utility function. The second, and simpler, approach consists of specifying the ordinary demand functions directly. Here, however, there is an important distinction between what is possible when dealing with a demand equation for a single good and demand systems for multiple goods.

Suppose that we wish to estimate the demand for a single good, x, and we are willing to treat all other goods as a Hicksian composite commodity, z, so that the direct utility function is u(x,z). Recognizing the homogeneity of demand functions in prices and income, we write the demand function of interest as:

$$(1) \qquad x = h^x(p_x,\ p_z,\ y) = \bar{h}\ (\frac{p_x}{p_z}\ ,\ \frac{y}{p_z}),$$

where it is understood that the implied demand function for z is

$$(2) \qquad z = h^z\ (p_x,\ p_z,\ y) = [y - \bar{h}\ (\frac{p_x}{p_z},\ \frac{y}{p_z})]/p_z .$$

We do <u>not</u> intend to estimate (2), and we would usually set $p_z = 1$ in (1), but it is convenient here to retain p_z as an explicit variable. We could generate the formula for $\bar{h}(\cdot)$ by specifying some quasiconcave, increasing direct utility function u(x,z) or some quasiconvex indirect utility function $v(p_x,\ p_z,\ y)$, increasing in y and decreasing in (p_x, p_z). Or we could simply write down an arbitrary formula for the function $\bar{h}(\cdot)$. But, if we do the latter, we must ensure that our function satisfies the integrability conditions.

(3) $$\frac{\partial h^x}{\partial p_x} + x \frac{\partial h^x}{\partial y} \leq 0$$

(4) $$\frac{\partial h^x}{\partial p_z} + z \frac{\partial h^x}{\partial y} = \frac{\partial h^z}{\partial p_x} + x \frac{\partial h^z}{\partial y}.$$

Now, (3) is a substantive restriction which, however, it is not too difficult to satisfy. By contrast, (4) is a trivial restriction which is always satisfied: as long as it is understood that $\bar{h}^z(\cdot)$ is given by (2), any bivariate function $\bar{h}(\cdot)$, automatically satisfies (4) (for a demonstration, see Katzner, 1970, p. 68).

We are not so fortunate when dealing with demand functions for two or more goods (in addition to the numeraire). In the general case with $u(x_1,\ldots,x_N)$, where $N > 2$, and ordinary demand functions $x_i = h^i(p_1,\ldots,p_N, y)$, $i = 1,\ldots N$, satisfying $\sum p_i h^i(p,y) = y$, the integrability conditions are

(5) $$\frac{\partial h^i}{\partial p_i} + x_i \frac{\partial h^i}{\partial y} \leq 0$$

(6) $$\frac{\partial h^i}{\partial p_j} + x_j \frac{\partial h^i}{\partial y} = \frac{\partial h^j}{\partial p_i} + x_i \frac{\partial h^j}{\partial y} \quad \text{all } i, j.$$

The symmetry conditions (6) are now a non-trivial restriction, since they are obviously not satisfied by an arbitrary set of homogeneous functions $h^1(\cdot),\ldots,h^N(\cdot)$. [1] For this reason it is likely to be considerably more convenient to specify multiple site demand systems by starting from some explicit direct or indirect utility function.

Incomplete Demand Systems

The discussion above raises another issue which is often of practical importance when modelling demands for multiple goods, namely what to do about the demand functions for the goods that are not of interest. In the recreation context, for example, one has data on the costs, perhaps quality attributes, and rates of visitation of a set of recreation sites by a sample of households but, typically, no data on the households' consumption and/or prices of other, nonrecreation goods. How then is one to proceed?

Writing on the subject of demand functions for multiple recreation sites Cicchetti, Fisher and Smith (1976, fn. 12) address this very question and conclude that, if one has data on only a subset of consumption activities, it is not appropriate to employ a system of demand equations that is consistent with the hypothesis of utility maximization. This is not always an acceptable conclusion.[2] If one wishes to employ the fitted demand system merely for the empirical prediction of demand responses to changes in prices, the employment of a system of demand functions that violates homogeneity or the integrability conditions may be satisfactory. But if, as is often the case, one intends to derive welfare evaluations from the fitted demand equations (e.g. estimates of the value of a particular site or the benefits from some quality enhancement program), it is unclear to us how this can ever be conducted on the basis of demand functions which violate the very postulates that are the foundation of welfare analysis.[3] Moreover, the conclusion reached by Cicchetti et al. is unduly pessimistic since several strategies exist for handling, albeit imperfectly, data on a subset of commodities in a manner consistent with utility theory.

One strategy has already been illustrated, namely to assume that the prices of the goods that are <u>not</u> of concern move in proportion, so that they can be aggregated into a Hicksian composite commodity. Thus we take the utility function to be $u(x_1,\ldots,x_N,z)$ where z, a scalar, is the composite commodity, and the x_i's are the goods whose demands we wish to estimate; these demand functions take the form $x_i = h^i(p_1,\ldots,p_N,q,y)$, $i = 1, \ldots, N$, where q, the price of z, would be set at unity.

Another strategy is to assume that the utility function is separable with respect to the commodities that are of concern. Before discussing this approach, however, it is instructive to examine the consequences when we adopt <u>neither</u> strategy - we assume neither aggregation to a composite commodity nor separability. In this case the situation is as follows: The utility function is $u(x_1,\ldots,x_N,z_1,\ldots,z_M)$ where the x_i's are the goods of concern, and the z_j's are all the other goods consumed by the individual. The respective price vectors are $p_1,\ldots,p_N$ and $q_1,\ldots,q_M$. The individual maximizes $u(x,z)$ subject to the budget constraint $\sum p_i x_i + \sum q_j z_j = y$, which generates two sets of ordinary demand functions

$$(7) \qquad x_i = h_i^x\left(\frac{p_1}{q_1},\cdots,\frac{p_N}{q_1},\frac{q_2}{q_1},\cdots,\frac{q_M}{q_1},\frac{y}{q_1}\right) \quad i = 1,\ldots,N$$

$$(8a) \qquad z_j = h_j^z\left(\frac{p_1}{q_1},\cdots,\frac{p_N}{q_1},\frac{q_2}{q_1},\cdots,\frac{q_M}{q_1},\frac{y}{q_1}\right) \quad j = 2,\ldots, M$$

$$(8b) \qquad z_1 = \left[y - \sum_{i=1}^{N} p_i\, h_i^x(\cdot) - \sum_{j=2}^{M} q_j\, h_j^z(\cdot)\right]/q_1$$

and an indirect utility function $v(p,q,y)$, where z_1 has been taken as the numeraire. In practice, therefore, we would set q_1 = 1 in (7) and (8). The demand functions in (7) and (8) taken together are known as a "complete" demand system. However, we only care about (7), which is known as an "incomplete" demand system. What can we do with this?

In answering this question it is important to distinguish between problems of estimation and welfare evaluation. Starting with the latter, suppose for a moment that we had estimates of all the coefficients in the incomplete demand system (7). Suppose, too, that these demand functions satisfy the local integrability conditions for incomplete demand systems which, as shown by Epstein (1982), involve the symmetry of the Slutsky terms with respect to the p_i's,

$$\frac{\partial h_i^x}{\partial p_k} + x_k \frac{\partial h_i^x}{\partial y} = \frac{\partial h_k^x}{\partial p_i} + x_i \frac{\partial h_k^x}{\partial y} \quad \text{all } i,\ k = 1,\ldots,N$$

and the negative definiteness (<u>not</u> semi-definiteness) of the Slutsky matrix,

$$\frac{\partial h_i^x}{\partial p_i} + x_i \frac{\partial h_i^x}{\partial y} < 0 \quad i = 1,\ldots, N.$$

Given these assumptions, LaFrance and Hanemann (1984) show that the incomplete demand system (7) contains sufficient information to allow us to calculate correctly money welfare measures for changes in the p_i's. Specifically, they show that the system (7) can be integrated, treating the q_j's as fixed parameters, to obtain what is called a "quasi-indirect utility function", $\theta(p,y;q)$, with the property that, if one uses it to calculate measures of compensating or equivalent variation for some price change from p' to p''

$$\theta(p'', y-C; q) = \theta(p', y; q) \tag{9a}$$

$$\theta(p'', y; q) = \theta(p', y+E; q), \tag{9b}$$

one obtains exactly the same results as if one knew true indirect utility function $v(p,q,y)$ obtained by integrating the complete system (7) and (8), i.e., the quantities C and E defined by (9a, b) also satisfy

$$v(p'', q, y-C) = v(p', q, y) \tag{10a}$$

$$v(p'', q, y) = v(p', q, y+E). \tag{10b}$$

Thus, having access only to the incomplete demand system for the x's, (7), does not cause problems for welfare evaluations with respect to changes in the prices of the x's.[4]

The problems arising when one works with incomplete demand systems, therefore, have to do with estimation rather than welfare evaluation. Here it is useful to distinguish between the case where one has data on the q_j's as well as the p_i's and x_i's but not the z_j's, and the case where one has data on neither the q_j's nor the z_j's. This distinction is important because data on the prices of other goods (i.e. non-recreation activities) may be readily available from some published source. However, the data on the z_j's would have to be obtained from a household survey, but one may be forced to confine the interview to questions about recreation activities. Under this scenario where one knows the q_j's but not the z_j's, there is in principle no problem in estimating the incomplete demand system (7). But if one has no data on either the q_j's or the z_j's then there is a problem in estimating (7). An exception is when one can assume that the q_j's do not vary across the sample so that they can be taken as being subsumed in the coefficients of the demand functions in (7). In the latter case the estimated demand equations can be integrated to obtain $\theta(p,y;\, q)$, from which welfare can safely be constructed. Otherwise, the estimation of (7) omitting $q_2,\ldots,q_M$ is a form of specification error which may produce biased estimates of the coefficients pertaining to the p_i's, the nature of the bias depending on the functional form of the demand equations and the degree of correlation between the included p_i's and the omitted q_j's.

Partial Demand Systems

By contrast, the strategy of assuming a separable utility function avoids many of the estimation problems, but it raises some conceptual problems for welfare evaluation. In this case one assumes that $u(x_1,\ldots,x_N,z_1,\ldots,z_M) = f[\bar{u}(x_1,\ldots,x_N),z_1,\ldots,z_M]$ for some functions $f(\cdot)$ and $\bar{u}(\cdot)$, where f is an increasing function of M + 1 arguments and quasiconcave in z, and $\bar{u}$ is increasing and quasiconcave in x. Thus the marginal rate of substitution between any pair of x_i's is independent of each z_j. Ignoring the selection of a numeraire, the ordinary demand functions for the x_i's and z_j's resulting from the maximization of $u(x,z)$ subject to $\sum p_i x_i + \sum q_j z_j = y$ are of the form

(11) $$x_i = h_i^x(p,q,y) = \bar{h}_i^x[p,H^x(p,q,y)] \quad i = 1,\ldots N$$

(12) $$z_j = h_j^z(p,q,y) \quad j = 1,\ldots,M.$$

Here, the functions

(13) $$x_i = \bar{h}_i^x(p,y_x) \quad i = 1,\ldots,N$$

may also be obtained by solving the maximization problem

(14) maximize $\bar{u}(x_1, \ldots, x_N)$ s.t. $\sum p_i x_i = y_x$,

and the function

(15) $$y_x = H^x(p,q,y),$$

as well as the functions $h_j^z(\cdot)$ may be obtained by solving the maximization problem

$$\underset{y_x,z}{\text{maximize}}\ f[\bar{v}(p,y_x),z] \quad \text{s.t.} \quad y_x + \sum q_j z_j = y$$

where $\bar{v}(p,y_x)$ is the indirect utility function arising from the maximization problem (14).[5]

If one has data on the x_i's, p_i's and q_j's but not the z_j's, rather than estimating the demand functions (11), he may as well relax the separability assumption and estimate the incomplete demand system (7). It is when one has no data on either the q's or the z's that the separability assumption comes into its own, since the estimation of (7) becomes problematic but the estimation of the demand functions in (13) remains perfectly feasible. (Note that y_x can be calculated from the p_i's and x_i's).

Equations (13) were termed "partial demand functions" by Pollak (1971);

we will also refer to them as "micro-allocation" functions and to (15) as the "macro-allocation" function . The reason for our terminology is that the functions in (13) indicate how a fixed total recreation budget, y_x, should optimally be allocated in terms of visits to individual recreation sites as a function of their costs, while the function in (15) indicates how the overall recreation budget should optimally be determined. Since they are derived from conventional (sub-) utility functions, the functions in (13) possess the standard properties of ordinary demand functions - homogeneity, summability, and symmetry and negative semi-definiteness of the Slutsky terms, $[\partial \bar{h}_i^x / \partial p_j + \bar{h}_j^x (\partial \bar{h}_i^x / \partial y_x)]$. In particular, while some of them may exhibit zero income effects, i.e., $\partial \bar{h}_i^x / \partial y_x = 0$, it is <u>not</u> possible for <u>all</u> of the demand functions in (13) to have zero income effects; nor is it possible for all of them to have negative income effects, $\partial \bar{h}_i^x / \partial y_x < 0$.[6] This is one of the ways in which they differ from incomplete demand systems, since it is entirely possible for all of the demand functions in (7) to have zero or negative derivatives with respect to total income.

Although partial demand functions for the x_i's can be estimated under circumstances in which it is impossible to estimate incomplete demand functions, this advantage is not obtained without some cost. First, the partial demand functions can only be used to predict site visitation patterns conditional on a given total recreation budget. If prices change, say from p' to p'', y_x also will change from $y_x' = H^x(p',q,y)$ to $y_x'' = H^x(p'',q,y)$, and this change cannot be ascertained from the partial demand functions for the x's. In practice one might try to circumvent this problem by estimating some sort of ad hoc macro-allocation function relating y_x to p and y and perhaps some general price index as a crude approximation to (15); this approximation would be used to predict the change from y_x' to y_x''.

The second problem, the construction of welfare measures, is more stubborn. Knowledge of the partial demand functions provides insufficient information about the individual's preferences to permit exact calculations of the welfare effects of the price change from p' to p''. By integrating (13) one can obtain the partial indirect utility function $\bar{v}(p,y_x)$, but not the full indirect utility function v(p,q,y). The true welfare measures for the price change should be based on the latter; they are the quantities C and E defined in (10a and b). Suppose that, in addition to knowing $\bar{v}(p,y_x)$, one knows y_x' and y_x'', or can estimate them from some crude, non-utility-theoretic macro-allocation function. The best that one can hope to do is calculate the quantities $\bar{C}$ and $\bar{E}$, where

(16a) $$\bar{v}(p'', y_x' - \bar{C}) = \bar{v}(p', y_x')$$

(16b) $$\bar{v}(p'', y_x'') = \bar{v}(p', y_x'' + \bar{E}).$$

Hanemann (1982b) showed that these are in general different from the true welfare measures, but they at least provide bounds on them:

(17) $$\bar{C} \leq C \text{ and } E \leq \bar{E}.$$

The empirical adequacy of these bounds, however, remains an open question.[7]

Incorporating Quality

Another issue in modelling demands for multiple sites is the incorporation of quality differences among the sites. Much of the early literature on recreation tended to ignore site quality both in the specification of demand functions and in their application for welfare evaluations. A common practice, whether using visitation data for a single site or several sites, was to estimate a demand function relating the number of visits to a site (by individual households or, say, residents of a county) to the cost of visiting the site, income, and perhaps other socio-economic variables. Denote this demand function by $x = f(p,y)$ where x and p are scalars. When it was necessary to evaluate the benefits from opening up a new recreation site, this was based on a criterion of price dominance. Let p' be the cost to an individual of visiting the cheapest existing site, and p'' the cost of visiting the new site. Using the Marshallian measure of consumer's surplus, the benefit from the opening of the new site was computed as

(18a) $$\text{Benefit} = \begin{cases} \int_{p''}^{p'} f(p,y)dp & \text{if } p' > p'' \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that N sites are available to the individual, including the new site, with prices $p_1,\ldots,p_N$. The implicit ordinary demand function for any site, say the i th site, is

$$x_i = h^i(p_1,\ldots,p_N,y) = \begin{cases} f(p,y) & \text{if } p_i < p_j \text{ all } j \neq i \\ 0 & \text{otherwise.} \end{cases} \tag{18b}$$

The only utility model which would generate this type of demand function is

$$u(x_1,\ldots,x_N,z) = \bar{u}\left(\sum_{i=1}^{N} x_i,\ z\right) \tag{18c}$$

i.e., the various sites are perfect substitutes and either they offer exactly the same quality of recreation experience or else consumers are entirely indifferent to the nonprice aspects of recreation experience.

None of these assumptions seems very plausible. It is an empirical fact that many individuals who participate in waterbased recreation visit more than one site, and even those who visit only one site rarely visit their cheapest site. Moreover, in most regions there is some variation in the quality of recreation experience afforded by different sites, and casual evidence suggests that recreationists care about at least some dimensions of site quality and trade off price and quality in making their recreation decisions. Therefore, there is a strong case for introducing site quality into models of recreation behavior.

This can be done in two ways, and the choice among them depends largely on the ease with which one can obtain reasonable measures of site quality; as indicated in Chapter 7, this is not always a simple task. If quality is not easily measured explicitly, one can introduce it implicitly by treating the various sites as different commodities and estimating separate demand functions for them. Thus, utility is some function $u(x_1,\ldots,x_N,z)$, but not $\tilde{u}(\sum x_i, z)$ as in (18c): the individual has different preferences for different sites, and therefore does not regard them as perfect substitutes for one another, but the reasons for the differences in his preferences (presumably differences in site quality) are not made explicit. Consequently, the resulting demand system for recreation sites, $h^1(p_1,\ldots,p_N,q,y),\ldots, h^N(p_1,\ldots,p_N,q,y)$, tells one how the demand for sites differs - e.g. for sites 1 and 2, even if $p_1 = p_2$, $h^1(\cdot) > h^2(\cdot)$ - but not why the demand differs.[8a] This causes some problems when one wants to predict the consequences of changing the quality of an existing site or the opening of a new site. In practice, one would have to assume that the new site is a perfect substitute for one of the existing sites and apply the price dominance criterion implicit in (18a,b).[9]

The alternative approach is to introduce some measures of site quality - ordinal or cardinal measures - explicitly into the utility function and the demand functions. Some procedures for doing this were discussed in Chapter 7 in the context of a demand function for a single site, and they carry over directly to demand systems for multiple sites. Let b_{ik} represent the level of the k^{th} quality characteristic associated with a visit to site i, $k = 1,\ldots,K$ and $i = 1,\ldots,N$ and let $b_i = (b_{i1},\ldots,b_{iK})$. The utility function is assumed to be $u(x_1,\ldots,x_N,b_1,\ldots,b_N,z)$ and the resulting ordinary demand functions are $x_i = h^i(p_1,\ldots,p_N,b_1,\ldots,b_N,q,y)$, $i=1,\ldots,N$.[10] One implication should immediately be noted: the demand for any site depends in principle not only on its own quality characteristics but also on those of all other sites. As noted in Chapter 7, this may cause some problems where one employs subjective rather than objective measures of site quality and when individuals do not visit all available sites, because it is often difficult in practice to elicit subjective ratings of site quality for sites that people do not visit (Hanemann, 1984b; Hanemann, 1978, Ch 6).[11]

Following the discussion in Chapter 7, the utility function $u(x,b,z)$ may be generated either by adding the quality variables, b, to some known utility function $u(x,z)$ in an ad hoc manner, or by employing the transformation method. For example, with scaling, the utility function is

$$(19a) \qquad u(x,b,z) = \bar{u}[\psi_1(b_1)x_1,\ldots,\psi_N(b_N)x_N,z]$$

where $\psi_i(b_i)$ may be interpreted as an overall index of site i's quality, and the resulting demand functions are

$$(19b) \qquad x_i = h^i(p,b,q,y) = \frac{1}{\psi_i(b_i)}\bar{h}^i\left[\frac{p_1}{\psi_1(b_1)},\ldots,\frac{p_N}{\psi_N(b_N)},q,y\right] \quad i = 1,\ldots,N,$$

while with the cross product repackaging transformation, the utility function is

(20a) $$u(x,b,z) = \bar{u}\,[x_1,\ldots,x_N,\ z + \sum\psi_i(b_i)x_i]$$

and the demand functions are

(20b) $$x_i = h^i(p,b,q,y) = \bar{h}^i[p_1 - q\ \psi_1(b_1),\ldots,p_N - q\ \psi_N(b_N),q,y],$$
$$i = 1,\ldots,N.$$

The unfortunate implication of scaling mentioned earlier carries over to (19a,b). That is, if the demand for a site is price inelastic, an increase in quality reduces its demand.

However it is generated, a utility function incorporating quality characteristics of the general form u(x,b,z) may appear, from one point of view, to be too much of a good thing. This is because, even after quality differences have been accounted for, it treats each of the goods as different commodities: even if all the sites had exactly the same characteristics (i.e., $b_{ik}=b_{jk}$ for every i, j and all k), the general formuation u(x,b,z) implies that they would have different demand functions, which may be implausible. This can be remedied, for example, by specializing (19a) or (20a) to

(21) $$u(x,b,z) = \bar{u}[\sum\psi_i(b_i)x_i,z]$$

(22) $$u(x,b,z) = \bar{u}\,[\sum x_i,z + \sum\ \psi_i(b_i)x_i].$$

In these formulations, if all sites had exactly the same characteristics, they would have exactly the same demand; if they had different characteristics then, unlike the situation in (18c), they would have different demands. However, (21) and (22) imply that, allowing for quality differences, the sites are all perfect substitutes and an individual would generally visit only one site, the selection of this site involving a trade off between price and quality.[12]

A less extreme approach would be to assume that recreation sites can be grouped into several classes, each class representing a different type of recreation experience (freshwater versus saltwater sites, isolated versus heavily urban sites, etc.) and, therefore, having a different demand function. For example, if there were three classes indexed by A, B, and C, instead of (22) the utility function would be

$$(23) \qquad u(x,b,z) = \bar{u}\,[\sum_{i\epsilon A} x_i, \sum_{i\epsilon B} x_i, \sum_{i\epsilon C} x_i, z + \sum_{i\epsilon A} \psi_i(b_i)x_i + \sum_{i\epsilon B} \psi_i(b_i)\, x_i + \sum_{i\epsilon C} \psi_i(b_i)\, x_i].$$

Here, even if all N sites had exactly the same measured characteristics, sites of type A would have different demand functions from sites of type B or type C, but within each class all sites would have a common demand function. Models of this sort will be explored further in the next Chapter.

Corner Solutions - The Treatment of Zero Visits

The above discussion also serves to illustrate how "corner solutions" in which an individual has zero consumption of some goods can arise from purely theoretical considerations. Yet, this is more than a theoretical phenomenon. In practice, whenever one works with data on individuals' consumption behavior and a fairly disaggregated commodity classification, he is likely to observe instances of corner solutions.

In the recreation context, for example, although individuals may visit several sites over the course of the recreation season, it is unusual to find that they visit *all* possible sites.[13]

The ramifications of corner solutions, both statistical and utility-theoretic, have only recently begun to receive attention. From a statistical point of view, perhaps the most important implication is that there is a probability mass at $x_i = 0$ which needs to be incorporated into the estimation procedure, as in Tobit models. From the point of view of economic model formulation, an implication is that the ordinary demand functions must satisfy an additional restriction besides homogeneity, summability, and the symmetry and negative-semidefiniteness of the Slutsky matrix, namely that they assume only non-negative values; thus a function like

$$x_i = h^i(p,y) = \alpha_i + \sum_{j=1}^{N} \beta_{ij} p_j + \gamma_i y \quad i=1,\ldots,N \tag{24}$$

cannot in fact be a valid formula for an ordinary demand system without some further modification because its range extends to the negative orthant. Furthermore, there is a more subtle problem in dealing with corner solutions in a manner consistent with the hypothesis of utility maximization. Suppose that, at the current prices and income, an individual is consuming some positive amounts of goods 3 through N but nothing of goods 1 and 2. Then, *small* changes in the prices p_1 or p_2 will have *no effect* on his demands for goods 3,...,N.[14] Within some region of (p,y) space his demand functions $h^i(\cdot)$, i = 3,...,N, will be independent of p_1 and p_2 and will depend only on $p_3,\ldots,p_N$ and y. Thus, as one moves from corner to corner the arguments of the demand functions change; only at an interior solution (i.e., the individual visits every site) do the demand functions depend on the full set of prices, $p_1,\ldots,p_N$. The pathology of corner solutions is examined in some detail in Hanemann (1984d), and will be summarized in the next Chapter.

A further consequence of corner solutions concerns the manner in which a stochastic element is combined with an economic model in order to generate a statistical model suitable for estimation. By far the most common practice in demand analysis is to postulate some utility model devoid of stochastic elements, $u(x,z)$, derive the corresponding demand system, also devoid of stochastic elements, $h^i(p,q,y)$, $i = 1,\ldots,N$, and then, in the last minute as it were, add some error terms $\varepsilon_1,\ldots,\varepsilon_N$ to justify the application of statistical procedures to the stochastic equations $x_i = h^i(p,q,y) + \varepsilon_i$. An alternative procedure would be to introduce the random elements into the utility function at the very beginning, $u(x,z,\varepsilon)$, and then derive the demand system in the conventional manner allowing the random elements to carry over to the ordinary demand functions, $x_i=h^i(p,q,y,\varepsilon)$. Although this has occasionally been considered in the context of demand models corresponding to interior solutions, notably by Pollak and Wales (1969, footnote 13), it turns out to be of crucial importance in demand models for corner solutions, as will be shown in the next Chapter.

The four topics discussed so far - the use of demand systems consistent with the hypothesis of utility maximization, the treatment of "other" goods, the method by which quality attributes are introduced into demand systems, and the treatment of corner solutions - provide criteria by whch existing multiple site demand models can be evaluated. In the following Chapter we review some of these models, and we outline some new approaches to the modelling of demands for multiple sites.

FOOTNOTES TO CHAPTER 8

1. The preference-structural implications of imposing symmetry on ad hoc demand systems such as $x_i = \alpha_i + \sum_j \beta_{ij} p_j + \gamma_i y$ or

$\ln x_i = \alpha_i + \sum_j \beta_{ij} p_j + \gamma_i y$, $i = 1, \ldots, N$, are explored in LaFrance and Hanemann (1984).

2. We are assuming a demand system which applies to the behavior of individual consumers. The question of modelling aggregate demand functions for recreation sites is considerably more complex and will not be addressed here.

3. It should be emphasized that this has nothing to do with the use of Willig's (1976) or Vartia's (1983) approximations. These are procedures for approximating the indirect utility function $v(p,y)$ underlying the demand functions $h^1(\cdot), \ldots, h^N(\cdot)$, and they are clearly needed when $v(p,y)$ does not have a closed-form expression. But they produce nonsensical results if the demand functions violate the integrability conditions, homogeneity, or summability.

4. The same conclusion applies when the quality attributes of the x's affect preferences and therefore enter the demand functions (7) and (8). Welfare evaluations for changes in the quality of the x's can validly be based on the incomplete demand system (7), without knowing (8).

5. If the sub-utility function $\bar{u}(.)$ is homothetic, $\bar{v}(p, y_x) = \phi(p) y_x$ and (15) becomes $y_x = \tilde{H}^x(\phi(p), q, y)$, where ϕ is homogeneous of degree one.

6. It is possible, however, that $\partial y_x / \partial y = \partial H^x / \partial y \leq 0$.

7. It can be shown that the same argument applies to welfare measures for changes in the qualities of the x's, if these are arguments in $\bar{u}(.)$.

8. For examples of this procedure see Mansfield (1971), Ackerman et al. (1974) and Knetsch (1977).

8a. Here q is the price associated with z, and is a scalar or a vector depending on how one chooses to model the non-recreation activities

9. To be more specific, if the new site is labelled N + 1 and is determined to be similar to existing site 1, say, the assumed utility function for the old and new sites is

$$u(x_1, \ldots, x_N, x_{N+1}, z) = u(x_1 + x_{N+1}, x_2, \ldots, x_N, z),$$

the demand functions for sites 1 and N + 1 are

$$x_1 = \begin{cases} h^1(p_1, p_2, \ldots, p_N, q, y) & \text{if } p_1 < p_{N+1}, \\ 0 & \text{otherwise} \end{cases}$$

$$x_{N+1} = \begin{cases} 0 & \text{if } p_1 < p_{N+1} \\ h^1(p_{N+1}, p_2, \ldots, p_N, q, y) & \text{otherwise} \end{cases}$$

and the demand functions for all the other sites are

$$x_i = h^i[\min(p_1, p_{N+1}), p_2, \ldots, p_N, q, y] \qquad i = 1, \ldots, N.$$

This approach is followed by Burt and Brewer (1971) and Cicchetti et al. (1976).

10. This formulation is called a "Generalized Lancaster" model in Hanemann (1982a) which explains its relation to both Lancaster's (1966, 1971) quality model and the model employed by Houthakker (1951-52) and Theil (1951-52). The latter can be shown to underlie the recreation demand model of Brown and Mendelsohn (1983). The relative advantages of these two approaches - the Generalized Lancaster model and the Houthakker-Theil model - are discussed briefly on pp. 79-80 of Hanemann (1982a), and some related issues are raised in Chapter 7. A fuller comparison of Brown and Mendelsohn's work with our own will be forthcoming.

11. This raises a question about just what is the individual's effective choice set, which is discussed further below.

12. In the case of (21), the individual selects that site for which $p_i/\psi_i(b_i)$ is lowest; in the case of (22) he selects that site for which $p_i-\psi_i(b_i)$ is the lowest.

13. This raises the question already mentioned in footnote 11, of whether all the sites really enter into any person's choice set. It may well be that recreation behavior should be factored into two components: a mapping from all possible sites to an effective choice set, which may vary among individuals, and then a selection among the sites in the effective choice set, the latter following the utility maximization paradigm employed here. Modelling the formation of choice sets, however, raises both theoretical issues which

are not addressed by conventional economic theory and practical issues which are not addressed by conventional data sets. In some cases it may be obvious to the analyst what constitutes each individual's choice set; in the case of skiing or white water rafting, for example, where sites can readily be graded according to their difficulty and the recreationist's degree of skill or experience can be determined through a survey, it may be quite easy to construct effective choice sets for different individuals. In the case of swimming and beach recreation, by contrast, it may be more difficult to devise a suitable criterion for defining effective choice sets. When faced with exactly this problem, Caulkins et al (1984) assumed that the individual's choice set consists of only those sites which he actually visited during the course of the season, which appears overly restrictive to us.

All of this notwithstanding, it is still likely that corner solutions will be observed in selections among the effective choice set.

14. They will also have no effect on his (zero) consumption of goods 1 and 2.

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