Evaluating Yield Models for Crop Insurance Rating

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Generated crop insurance rates depend critically on the distributional assumptions of the underlying crop yield loss model. Using farm level corn yield data from 1972-2008, we revisit the problem of examining in-sample goodness-of-fit measures across a set of flexible parametric, semi-parametric, and non-parametric distributions. Simulations are also conducted to investigate the out-of-sample efficiency properties of several competing distributions. The results indicate that more parameterized distributional forms fit the data better in-sample due to the fact that they have more parameters, but are generally less efficient out-of-sample–and in some cases more biased–than more parsimonious forms which also fit the data adequately, such as the Weibull. The results highlight the relative advantages of alternative distributions in terms of the bias-efficiency tradeoff in both in- and out-of-sample frameworks.

Keywords: Yield distributions, Crop Insurance, Weibull Distribution, Beta Distribution, Mixture Distribution, Out-of-Sample Efficiency, Goodness-of-Fit, Insurance Rating Efficiency

Introduction

Crop insurance provides many options for farmers to manage yield and revenue risks emanating from uncertain weather, demand uncertainty and other perils. The pricing of crop insurance not only affects participation, but also relative indemnities paid by crop insurance companies. If crop insurance rates are too high, farmers may choose other methods to manage risk, such as hedging in the futures market; conversely, rates which are too low encourage adverse selection and contribute to other problems endemic to the overprovision of insurance (e.g., land degradation).

Generated crop insurance rates depend critically on the distributions assumptions of the underlying crop yield loss model. Many studies have focused on how best to establish parametric yield distributions in rating applications (e.g., Sherrick et al., 2004; Ramirez et al., 2003), while others have advanced the use of non-parametric (e.g., Ker and Goodwin, 2000) and semi-parametric methods (e.g., Wang and Zhang, 2002). Yet, no single family of distributions or method of selection for non-parametric models is widely accepted for rating insurance. While there is an extensively developed literature on goodness-of-fit measures of fitted distributions, crop insurance applications are more concerned with bias and efficiency of the rates resulting from alternative distributions. Moreover, past studies have been principally concerned with evaluating bias in rates generated from alternative distributions relative to (unknown) the "true" distribution approximated by empirical distributions, and tend to ignore the efficiency or precision aspect. Also, given that insurance rating is fundamentally a forecasting exercise, Norwood et al. (2004) urge for out-of-sample approaches to distribution assessment when evaluating yield models. The out-of-sample efficiency question is of critical importance in insurance applications since typically only a short history of data is available for any farm or county when making rates.

The purpose of this study is two-fold. First, using farm level corn yield data from the Illinois Farm Bureau Farm Management Association (FBFM) from 1972-2008, we revisit the problem of examining alternative in-sample goodness-of-fit measures and rates across a set of flexible two and four parameter distributions, as well as a semi-parametric and a non-parametric distribution. Next, to illustrate out-of-sample efficiency across alternative distributions, simulations are conducted whereby small yield samples are drawn from a known parametric distribution, and then fit to several candidate distributions; the estimated distributions are then used to estimate insurance rates under each candidate distribution; this process is repeated in order to estimate the rate sampling distribution of each candidate distribution. This approach to out-of-sample evaluation is similar in spirit to that articulated by Norwood et al. (2004), but differs somewhat from earlier approaches in that we assess the underlying *rate distribution* directly–an arguably more relevant objective in insurance contexts.

Literature Review

The choice of the "best" distribution has been a long debated question, and indeed remains contentious. Among the parametric family of distributions, many studies (Day, 1965; Ramirez, 1997; Atwood et al., 2003; Ramirez et al., 2003) reject normality as the "correct" distributional form of crop yields because of negative skewness and excess kurtosis. In contrast, Just and Weninger (1999) argue that the rejection of the normal distribution in preceding empirical research is an incorrect assumption due to methodological problems in typical yield distribution analyses. Other works (Nelson and Preckel, 1989; Nelson, 1990; Hennessy et al., 1997) use a beta distribution to depict crop yields. The beta distribution is arguably the most highly examined parametric form along with the normal distribution in empirical crop yield modeling literature. The beta distribution is flexible enough to take on varying forms of skewness and kurtosis, as well as being bounded at zero and a maximum value. Still other works attempt to examine alternative parameterizations of crop yield distributions, Gallagher (1986) and Pope and Ziemer (1984) with the gamma distribution and Sherrick et al. (2004) with the Weibull distribution. The gamma and Weibull distributions are similar to the beta distribution and its need for relatively few parameters to capture varying degrees of skewness–positive and negative–and variances.

Recently non- and semi-parametric distributional forms have received more attention in empirical literature (Goodwin and Ker, 1998; C.G. and Zhao, 1999; Ker and Goodwin, 2000; Norwood et al., 2004; Wang and Zhang, 2002; Ker and Coble, 2003) because of their increased flexibility in modeling in-sample crop yields. The increased flexibility allows the distributional form to cover a broader set of skewness and kurtosis values, but forecasting may suffer from efficiency problems due to their tendency to over-fit sample data. While some studies have suggested that semi-parametric methods may outperform parametric methods in similar applications in terms of out-of-sample efficiency (see e.g., Norwood, Roberts, and Lusk, 2004), the evidence to date is very weak.

Empirical Parametric Goodness-of-Fit and In-Sample Rate Analyses

The first section of this study focuses on in-sample analyses. Two in-sample analyses are conducted. First, goodness-of-fit tests are conducted comparing several parametric distributions. Second, in-sample rate analyses are conducted to evaluate the performance of several parametric and non-parametric distributions for insurance rating. The next section outlines the data used in these analyses, followed by an overview of the procedures.

Data

This study utilizes a high quality, extensive farm-level corn yield dataset from the Illinois FBFM from 1972 to 2008. FBFM, in cooperation with the University of Illinois, Department of Agricultural and Consumer Economics, is a cooperative educational-service program that assists farmers with management decision-making, and provides financial and production business analysis reports. Over 6,000 grain farms participate in the FBFM program each year, providing dependable and extensive yield histories. This dataset is unique in the United States for its long panel of certified corn yield data, which captures a uniquely representative cross-section of farms. The highly reliable data are representative of commercial scale farms with validated and commonly accounted yields.

The focus of this study is on commercially viable farms with long and complete yield histories. Thus, farms were selected out of the FBFM database that met the following criteria: at least twenty years of yields, more than eighty acres, and less than two consecutive years of missing data. This resulted in 2,088 corn farms, of which 768 have thirty or more years of data.

In order to accurately model yield distributions in the context of rating crop insurance products, the deterministic components of yields over time, namely the effects of improvements in farm technology, are removed to allow yields from early years to be compared with yields from more recent years. Empirical crop insurance rating studies use many techniques to detrend yield data. The most common approach is to use OLS with a linear trend, and is adopted here (Zanini, 2001). We investigated four different data aggregation levels when detrending: state, district, county and individual farm.

The optimal choice of aggregation when detrending is highly debated. Similar to previous work (see e.g., Atwood, Shaik, and Watts, 2003) our research (not reported) suggested that farm level detrending may result in highly inefficient trend estimates, while the state level trends may exhibit unacceptably high degrees of bias. For example, detrending at the individual farm-level resulted in highly volatile (and sometime negative) trend estimates across farms, even within a county. While the overall implications of the study were not sensitive to level of aggregation when detrending, following several other related studies we use district level trends when detrending farm yields. Summary statistics for the original and detrended yields are provided in Table 1.

In-Sample Goodness-of-Fit Analysis

This analysis examines the in-sample fitting capability of alternative parametric distributional forms with three empirically popular goodness-of-fit tests-Anderson-Darling (A-D), Kolmogorov-Smirnov (K-S) and Chi-Squared (χ^2). The parametric distributions are comprised of the conditional beta, gamma, inverse Gaussian, normal and Weibull. First, the distributions are fit to the farm-level yields using maximum likelihood estimation for each farm individually. Next, the three goodness-of-fit tests and a weighted average of the three test ranks are used to rank each of the distributions in terms of their respective in-sample fitting performance. The goodness-of-fit rankings for the parametric distributional forms are summarized and tabulated for each of the nine Illinois National Agricultural Statistics Service (NASS) Crop Reporting Districts–Northwest, Northeast, West, Central, East, West Southwest, East Southeast, Southwest and Southeast.

In some cases, the estimation of the conditional beta distribution did not converge, and as a result 132 farms were dropped from the sample, and the remaining results did not appear to be sensitive to dropping these observations. The test statistics from the goodness-of-fit tests for the remaining farms are then calculated, and the distributions are then ordered by the number of times they rank in first through fifth place. All of the goodness-of-fit tests are generally accepted but differ somewhat in their focus. The A-D test puts more weight on the fit in the tails of the distribution, while the K-S test is more sensitive to the center of the distribution, and the χ^2 test tends to put emphasis on a combination of the two. A weighted goodness-of-fit measure is also constructed which equals the average of the K-S, A-D, and χ^2 ranks.

Results of In-Sample Goodness-of-Fit Analysis

The results for the parametric goodness-of-fit application are aggregated by crop reporting district and tabulated in Table 2. The number of FBFM farms in each region and the total FBFM farms in Illinois for this application are: Northwest - 379, Northeast - 154, West - 88, Central - 492, East - 280, West Southwest - 140, East Southeast - 236, Southwest - 111, Southeast - 76 and state total - 1,956. Table 2 is formatted as follows: the far-left column contains the three goodness-of-fit tests and the weighted average of the three tests; the next column has the parametric distributions being examined. The values represent the percentage of times that a distribution is ranked between one and five; a ranking of one means that the distribution fit the best according to the specified test statistic. For example, the K-S test ranks Weibull best 44% in the Northwest district, 40% second best and 4% third best.

With respect to the A-D test, the Weibull distribution fits seven of the nine districts best and comes in second in the other two districts. The Weibull also performs best across all 1,956 farms, with 46% of the first place finishes. The conditional beta is the next best fitting parametric distributional form, coming in first place 28% of the time and second place 27% of time for the weighted state level results. The normal distribution has the highest percentage of third place finishes and is consistently the next best after the conditional beta. The gamma and inverse Gaussian distributions are in fourth and fifth place in every category.

The results for the K-S, χ^2 and weighted tests are similar with the Weibull performing best overall. The conditional beta and normal distributions consistently come in second and third place, respectively, across all categories, while the gamma and inverse Gaussian distributions appear in fourth and fifth place. From the results of all four goodness-of-fit tests, the Weibull distribution is the overwhelming favorite for fitting in-sample farm-level yields in this sample.

Empirical In-Sample Rating Analysis

The next analysis examines the bias and efficiency of rates generated under alternative fitted distributions for each farm, and thus focuses on the left-tail of alternative distributional forms which is critical in insurance rating. We investigate four parametric distributions (conditional beta, gamma, normal, and Weibull), one semi-parametric (two-component mixture-of-normals) and one non-parametric (Gaussian kernel density).

First, all distributions are fitted to each farm. The resulting distributions are then used to calculate yield insurance rate estimates at coverage levels of 85%, 75% and 65%. The expected yield insurance rate is expressed for each farm and candidate distribution as:

$$InsRate_{i,d} = \int_{0}^{k_i} Max\{0, k_i - Y_i\} f_{d,i}(Y_i|\theta_{i,d})\mathrm{d}Y_i$$

where $k_i = Cover \times E(Y_i)$, $E(Y_i)$ is the expected yield of farm i, $Cover$ is the coverage level, Y_i is the yield for farm i and $f_{d,i}(Y_i|\theta_i)$ is the probability density function d and estimated parameter set $\theta_{i,d}$ for farm i and candidate distribution d. The rate estimates are then compared to the empirical (or "burn") rates from the underlying farm-level yields to evaluate in-sample bias and efficiency/precision across all farms.

The bias statistic is calculated as:

$$Bias_d = \sum_i [InsRate_{i,d} - BurnRate_i]$$

where $BurnRate_i$ is the empirical rate for each farm. The bias of the distributional forms has been studied extensively in previous literature, but the efficiency, or precision, of distributional forms has not been examined as extensively. The in-sample precision/efficiency is measured using the root mean squared error (RMSE) across farms as:

$$Efficiency_d = \sqrt{\frac{1}{m}\sum (EstRate_{i,d} - BurnRate_i)^2}$$

where m is the number of farms in the sample being evaluated.

Results of In-Sample Rating Analysis

Referring to Table 3, the average of the empirical burn rates ranges from a high of 3.70 bu./acre in the East district, to a low of 2.25 bushels/acre in the West-Southwest district at an 85% coverage level. The averages of the empirical rates for each FBFM corn farm across all districts and coverage levels are 2.89 bushels/acre, 1.18 bushels/acre and 0.41 bushels/acre. Focusing on the bias test statistic, the two-component mixture-of-normals fits best in 10 of the 27 coverage level/districts combinations ($9 \times 3 = 27$), and also for the State as a whole. The Weibull and conditional beta fit best in terms of bias in 9 and 8 of the 27 combinations, respectively. The gamma, normal and inverse Gaussian distributions do not produce the best fitting rates in any of the districts. The estimated rates from the conditional beta and kernel density estimator are similar with respect to the fact that they both have rates that consistently are greater than the empirical burn rate. A surprising characteristic of the differences between the empirical rates and the estimated rates from the distributional forms is the fact that the kernel density estimator performs the worst in terms of bias.

In contrast, the kernel density estimator is generally the most efficient distribution in-sample as measured by the RMSE statistic. The kernel density estimator is the most efficient in 14 of the 27 coverage level/district combinations, while the mixture-of-normals is the most efficient in 13. The Weibull and beta distributions are the least efficient.

Overall, the non- and semi-parametric distributional forms are shown to fit in-sample yields quite well in terms of in-sample precision, while the parametric distributions perform less well. Nevertheless, in both the goodness-of-fit examination and the empirical insurance application, the conditional beta and Weibull distributions outperform all other parametric distributional forms and are still capable of representing a relatively large range of skewness and kurtosis values. The normal distribution does not tend to perform as well in either the goodness-of-fit examination or the empirical insurance rating application, most likely due to its inability to capture any variance in skewness or kurtosis.

Out-of-Sample Insurance Simulation Analysis

Although the underlying distributional form of an individual farm is unknown in empirical applications, the empirical distribution is used as a proxy for its representation earlier in this study and used to assess in-sample bias and efficiency. In this section we approach the issue from a slightly different angle in order to assess out-of-sample efficiency and bias. Specifically, starting with known parametric distributions, we iteratively draw small samples from a known theoretical distribution, and then fit the resulting sample to three candidate distributions (Weibull, beta, and mixture-of-normals). The fitted distributions are then used to estimate rates. This process is repeated 5,000 times to generate a sampling distribution for each method relative to the known true rate. Thus, this exercise allows for out-of-sample assessment of the performance of these distributions in terms of rates generated relative to known underlying distributions.

We investigate this using both Weibull and beta starting distributions over a range of parameterizations and draw sizes. The parameters of the Weibull and beta are selected to reflect parameterizations consistent with typical farms in the Central Illinois region. Specifically, Weibull and beta distributions are constructed to represent candidate pseudo-farms defined by their mean and standard deviation; two mean levels (160 and 180 bu./acre) and three standard deviation levels (20, 30 and 40 bu./acre) are investigated. In fitting the starting distributions, a modified method-of-moments based approach is used to estimate the parameters of the Weibull and beta distributions. These fitted distributions are then employed as the starting distributions from which sampling is conducted.

The Weibull distribution has two parameters, a scale parameter, α , and a shape parameter β. The method-of-moments approximation follows that of Garcia (1981), and expresses the shape parameter:

$$1/\beta = z\left(1 + (1-z)^2 \sum_{i=0}^{n} k_i z^i\right)$$

where $z = \sigma/\mu$ and the k_i and n coefficients are $k_0 = -0.220009910$, $k_1 = -0.001946641, k_2 = 0.153109251, k_3 = -0.083543480, k_4 = 0, k_5 = 0.007454537$. The scale parameter is estimated given the shape parameter and the mean, μ, as:

$$a = [\Gamma(1 + 1/\beta)/\mu]^{\beta}$$

where Γ is the *gamma* function.

The conditional beta distribution has two shape parameters, α and β, as well as an upper and lower limit. The lower limit is bounded at zero. The method-of-moments approximation of the two shape parameters and the function for the upper limit are expressed as:

$$\alpha = \left(\frac{\mu}{h}\right)\left(\frac{\left(\frac{\mu}{h}\right)\left(1-\left(\frac{\mu}{h}\right)\right)}{\frac{v}{h^2}} - 1\right),$$

$$\beta = \left(1-\frac{\mu}{h}\right)\left(\frac{\left(\frac{\mu}{h}\right)\left(1-\left(\frac{\mu}{h}\right)\right)}{\frac{v}{h^2}} - 1\right),$$

$$h = \mu + 3 * \sigma$$

where σ and μ are the sample standard deviation and mean, respectively.

Results of Out-of-Sample Insurance Simulation Analysis

The results are found in Tables 4–9 and are arranged as follows. Each table-set is comprised of three individual tables, one for each of the three varying sample sizes, grouped by mean, standard deviation and data generating distributional form. The table-sets contain the comparison statistics – average, bias and efficiency – in the far-left column and the three distributional forms being compared in the next column. The bias and efficiency (RMSE) are presented in terms of percentages relative to the known theoretical "true" rate. The results are presented by coverage level and distribution. The first line of the table-set contains the true empirical rate for the given data generating distributional form, mean and standard deviation.

Referring to Table 4, which presents results for the known starting distribution with an expected yield of 160 bu./acre and standard deviation of 20, the true rates from both starting/underlying distributional (Weibull and beta) are quite similar at all coverage levels. In fact, this was true regardless of mean and standard deviation level. With respect to the bias statistic, the fitted beta consistently overestimated rates, ranging from 55%-90% bias for small N=15 at the 85% coverage level, and increasing at lower coverage levels. For example, at 65% and N=15, the bias was typically several hundred percent. These findings were true regardless of sample size drawn, N. In contrast, the Weibull and mixture-of-normals performed quite well in terms of bias regardless of the mean, standard deviation, and type of the underlying distribution, and sample size drawn. The exception was that the Weibull performed poorly in some cases for low risk levels, but always outperformed the beta. For example, at the 85% coverage level, the Weibull and mixture-of-normals typically resulted in rate bias of less than 5% when standard deviation of the underlying was 40 bu./acre and N=15; and at 65% coverage level the bias was always less than 10%. In general, the bias decreased as the sample size increased as well, and increased at lower coverage levels.

Turning attention to the out-of-sample efficiency results, the RMSE of the estimated conditional beta rates is, on average, 32.2% greater than the RMSE of the Weibull estimated rates at a coverage level of 85%; 110.6% greater at a 75% coverage level. Compared to the mixture-of-normals, the Weibull typically performed better in terms of out-of-sample efficiency, although in a few cases they performed similarly. The efficiency appeared to be lower at lower coverage levels, and naturally also increased as the sample size increased. At a sample size of N=15, the estimated rates of the Weibull are more efficient than the estimated rates of the beta by 51.53%, 179.12% and 745.36% at coverage levels of 85%, 75% and 65%, respectively. As the sample size increases to N=30, the differences between the conditional beta and Weibull efficiencies shrinks to 14.21%, 45.92% and 174.68% at coverage levels of 85%, 75% and 65%, respectively.

Overall, we find that the parsimonious two-parameter Weibull tended to consistently outperform both the mixture-of-normals and beta in terms of out-of-sample efficiency, and was also comparable in performance to the mixture-of-normals in terms of bias.

Conclusions

Issues surrounding the choice of distribution for modeling yields, as well as the manner in which one should go about evaluating and comparing them, are and remain contentious issues. We shed light on these issues using a comprehensive dataset from the Illinois FBFM using matched commercial scale corn yields from 1972-2008. Using three standard goodness-of-fit tests, we first examine the in-sample fitting performance of five parametric distributional forms. We then develop an application to examine in-sample rate bias and efficiency of several alternative parametric and non-parametric distributions. Last, a simulation approach is used to compare the out-of-sample bias and efficiency of the beta, Weibull and two-component mixture-of-normal distributions.

The results from the first section show that the Weibull, conditional beta and normal distributions perform better than the gamma and inverse Gaussian distributions at representing yield samples across virtually all conditions represented in these farms. While the results from the second section demonstrate that the two-component mixture-of-normals and kernel density estimators are the most efficient in-sample, the results from the out-of-sample analysis suggest that the more parsimonious Weibull distribution outperforms both the conditional beta and two-component mixture-of-normals on the basis of out-of-sample efficiency, and that this is particularly true in small samples.

This finding is somewhat in contrast to the findings of Norwood, Roberts, and Lusk (2004), who find that the mixture-of-normals was superior to other distributions investigated in that study, and calls into question generalization of the “best” distribution (whether in-sample or out-of-sample) for any particular application. The results of the simulations illustrate the bias-efficiency tradeoff when evaluating distributions with different levels of parameterization, and also add insight to the in-sample versus out-of-sample question as it relates to crop insurance rating and distribution selection.

The scope of this study does not include analysis with greater than thirty years of yields due to the small number of actual farms that contain near-perfect yield histories in our data. Nevertheless, insurers typically group large numbers of farms together with like characteristics when making rates. Thus, further research is needed in order to assess the sampling distribution questions addressed here in more realistic and comprehensive frameworks when several like risk farms are combined to estimate rates. Also, our out-of-sample analysis is based on simulated pseudo-data from known and restrictive parametric distributions, and thus the out-of-sample results found here may not always carry over to cases representing actual data for any particular application (e.g., if the data have larger tails than the fitted beta and Weibull distributions used here). Thus, frameworks need to be developed which can effectively assess out-of-sample rate performance using actual yield data.

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Table 1: Illinois District Sample Characteristics from Filtered FBFM Corn Farms

		Sample Summary Statistics									
Region	**Data**	**Mean***	**St Dev***	**CV***	**Skew***	**Kurt***	**Max***	**Min***	**Farm Count**	**Yield Count**	**Avg % of Tot Acreage**
NW	**Original**	135.97	29.00	0.21	-0.42	0.72	242.00	15.50	395	10,959	19.50%
	Detrended - β= 2.07	173.81	23.97	0.14	-0.83	1.35	252.34	44.27			
NE	**Original**	141.39	28.68	0.20	-0.20	0.30	250.00	24.34	176	4,662	8.30%
	Detrended - β= 1.86	172.69	24.46	0.14	-0.51	0.36	265.10	46.03			
West	**Original**	136.35	32.75	0.24	-0.48	0.63	265.00	17.00	94	2,647	5.06%
	Detrended - β= 2.12	175.58	28.17	0.16	-0.60	0.60	275.27	48.37			
Central	**Original**	146.71	30.95	0.21	-0.63	0.80	263.00	17.35	519	14,292	25.75%
	Detrended - β= 2.03	181.82	26.23	0.14	-0.75	0.93	281.36	59.10			
East	**Original**	139.86	32.17	0.23	-0.75	0.64	239.00	18.96	296	7,849	13.90%
	Detrended - β= 1.84	171.49	28.37	0.17	-0.86	0.81	253.68	57.58			
WSW	**Original**	141.07	28.06	0.20	-0.39	0.33	238.00	27.02	151	4,106	6.93%
	Detrended - β= 1.87	177.59	24.45	0.14	-0.58	0.43	257.84	62.08			
ESE	**Original**	126.85	28.20	0.22	-0.35	0.27	218.00	15.00	253	6,682	12.17%
	Detrended - β= 1.61	155.54	26.73	0.17	-0.37	-0.09	239.09	44.24			
SW	**Original**	106.05	28.80	0.27	-0.08	0.05	220.00	16.66	119	3,426	3.41%
	Detrended - β= 1.71	136.81	26.02	0.19	-0.30	-0.11	228.57	29.00			
SE	**Original**	112.05	26.62	0.24	-0.18	0.20	206.64	16.84	85	2,277	4.97%
	Detrended - β= 1.68	142.55	24.77	0.17	-0.33	-0.01	235.20	37.76			
Total	**Original**	136.25	29.80	0.22	-0.46	0.55	265.00	15.00	2088	56,900	100%

* in bushels/acre; for years 1972 to 2008

Table 2: Goodness-of-Fit Results: Illinois Districts for Corn Farms

		Illinois Districts																								
		NW					NE					West					Central					East				
		Goodness-of-Fit Rankings																								
Test	**Distribution**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
A-D[a]	**Beta**	36%	**43%**	2%	2%	17%	**44%**	36%	5%	2%	14%	39%	**42%**	6%	3%	10%	34%	**45%**	4%	1%	15%	43%	30%	3%	1%	22%
	gamma	1%	3%	20%	**75%**	0%	1%	4%	19%	**76%**	0%	3%	3%	15%	**78%**	0%	1%	2%	20%	**77%**	0%	0%	2%	23%	**74%**	0%
	invGauss	1%	1%	3%	19%	**77%**	1%	1%	1%	18%	**79%**	1%	1%	2%	12%	**83%**	1%	1%	1%	18%	**80%**	0%	0%	2%	22%	**75%**
	Weibull	**52%**	34%	5%	3%	6%	42%	**38%**	8%	4%	8%	**43%**	38%	8%	5%	7%	**52%**	34%	5%	4%	5%	**50%**	**43%**	2%	2%	2%
	normal	11%	19%	**70%**	0%	0%	12%	20%	**67%**	1%	0%	14%	16%	**69%**	1%	0%	11%	18%	**70%**	0%	0%	7%	24%	**70%**	0%	0%
K-S[b]	**Beta**	41%	35%	8%	5%	11%	40%	34%	8%	8%	8%	38%	**42%**	9%	5%	7%	35%	**41%**	10%	7%	8%	45%	41%	5%	4%	5%
	gamma	4%	5%	9%	**80%**	1%	1%	9%	13%	**77%**	0%	5%	9%	8%	**78%**	0%	3%	7%	10%	**79%**	1%	1%	2%	8%	**88%**	1%
	invGauss	3%	4%	3%	8%	**83%**	3%	1%	5%	10%	**82%**	3%	5%	1%	7%	**84%**	2%	1%	5%	10%	**82%**	1%	1%	1%	7%	**91%**
	Weibull	**44%**	**40%**	4%	5%	6%	**41%**	**37%**	8%	5%	10%	**40%**	40%	3%	8%	9%	**49%**	33%	5%	3%	9%	**46%**	**43%**	6%	1%	3%
	normal	8%	15%	**75%**	1%	0%	15%	18%	**66%**	1%	0%	15%	5%	**78%**	2%	0%	10%	18%	**70%**	1%	0%	6%	14%	**80%**	0%	0%
χ^{2}[c]	**Beta**	37%	23%	16%	11%	12%	42%	**26%**	12%	7%	14%	**38%**	20%	17%	15%	10%	33%	**28%**	12%	13%	14%	38%	25%	13%	10%	14%
	gamma	15%	18%	23%	**39%**	5%	16%	18%	26%	**31%**	9%	17%	18%	18%	**39%**	8%	18%	18%	23%	**34%**	6%	15%	14%	25%	**41%**	6%
	invGauss	13%	11%	17%	23%	**36%**	16%	10%	20%	29%	**25%**	14%	12%	19%	30%	**25%**	13%	13%	16%	25%	**32%**	10%	11%	15%	23%	**41%**
	Weibull	**45%**	25%	8%	12%	10%	**48%**	25%	12%	4%	10%	32%	18%	**23%**	14%	14%	**43%**	21%	12%	12%	12%	**46%**	**29%**	9%	8%	8%
	normal	25%	**25%**	**40%**	6%	5%	21%	24%	**42%**	5%	7%	**38%**	**22%**	19%	10%	11%	27%	21%	**36%**	9%	7%	29%	20%	**35%**	10%	7%
Weighted[d]	**Beta**	27%	**28%**	12%	9%	24%	32%	27%	11%	10%	20%	28%	24%	17%	15%	16%	27%	**29%**	13%	9%	22%	28%	25%	12%	5%	29%
	gamma	6%	13%	25%	**50%**	7%	5%	12%	24%	**51%**	8%	7%	23%	18%	**43%**	9%	8%	14%	23%	**48%**	7%	6%	9%	31%	**49%**	6%
	invGauss	3%	7%	12%	24%	**54%**	5%	4%	10%	26%	**55%**	6%	5%	**24%**	19%	**47%**	3%	7%	12%	24%	**53%**	2%	6%	7%	31%	**54%**
	Weibull	**44%**	27%	7%	12%	10%	**41%**	**31%**	12%	8%	9%	**34%**	20%	17%	10%	18%	**41%**	25%	12%	10%	11%	**46%**	**31%**	11%	6%	6%
	normal	20%	25%	**44%**	5%	5%	17%	27%	**44%**	5%	8%	25%	**28%**	**24%**	12%	10%	20%	24%	**40%**	9%	6%	18%	29%	**39%**	9%	6%

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Table 2 – continued from previous page

		Illinois Districts																								
		WSW					ESE					SW					SE					Total				
		Goodness-of-Fit Rankings																								
Test	**Distribution**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
A-D[a]	**Beta**	36%	31%	7%	6%	21%	31%	**33%**	10%	4%	22%	31%	**39%**	14%	4%	13%	**50%**	29%	8%	3%	11%	37%	**38%**	5%	2%	17%
	gamma	4%	4%	24%	**67%**	0%	3%	11%	25%	**62%**	0%	4%	5%	19%	**72%**	0%	4%	4%	17%	**74%**	1%	2%	4%	21%	**73%**	0%
	invGauss	1%	4%	2%	20%	**73%**	3%	1%	6%	21%	**69%**	2%	2%	3%	14%	**79%**	3%	3%	3%	16%	**76%**	1%	1%	2%	19%	**77%**
	Weibull	**43%**	**36%**	8%	6%	6%	**40%**	29%	10%	12%	8%	**38%**	37%	8%	9%	8%	24%	**50%**	9%	5%	12%	**46%**	36%	6%	5%	6%
	normal	16%	25%	**59%**	1%	0%	23%	26%	**50%**	0%	0%	26%	17%	**56%**	1%	0%	20%	14%	**63%**	3%	0%	14%	20%	**65%**	0%	0%
K-S[b]	**Beta**	36%	31%	9%	6%	16%	36%	24%	14%	10%	17%	28%	**39%**	14%	9%	11%	**43%**	30%	8%	7%	12%	38%	36%	9%	6%	10%
	gamma	5%	10%	16%	**68%**	1%	6%	11%	16%	**64%**	2%	10%	10%	5%	**75%**	1%	5%	12%	18%	**63%**	1%	4%	7%	11%	**77%**	1%
	invGauss	2%	4%	6%	19%	**70%**	3%	7%	6%	12%	**72%**	3%	3%	11%	8%	**76%**	4%	4%	5%	16%	**71%**	2%	3%	4%	10%	**81%**
	Weibull	**42%**	**32%**	7%	6%	12%	**36%**	**34%**	8%	12%	9%	**39%**	30%	14%	5%	13%	24%	**41%**	9%	11%	16%	**43%**	**37%**	6%	5%	8%
	normal	14%	23%	**62%**	1%	0%	19%	23%	**56%**	1%	0%	21%	19%	**58%**	3%	0%	24%	13%	**59%**	4%	0%	12%	17%	**69%**	1%	0%
χ^2[c]	**Beta**	36%	**23%**	11%	16%	14%	**42%**	20%	11%	12%	14%	**38%**	**26%**	17%	12%	7%	**45%**	9%	16%	12%	18%	37%	**24%**	13%	12%	13%
	gamma	20%	16%	26%	**36%**	2%	24%	20%	17%	**26%**	12%	20%	23%	21%	**31%**	5%	21%	18%	24%	**33%**	4%	18%	18%	23%	**35%**	7%
	invGauss	16%	14%	21%	22%	**27%**	21%	12%	18%	25%	**24%**	23%	10%	15%	21%	**32%**	21%	14%	14%	22%	**28%**	15%	12%	17%	24%	**32%**
	Weibull	**44%**	19%	14%	13%	11%	35%	17%	17%	17%	15%	**38%**	23%	12%	12%	15%	32%	**33%**	11%	14%	11%	**42%**	23%	12%	12%	11%
	normal	32%	17%	**34%**	12%	5%	29%	**23%**	**27%**	12%	8%	23%	16%	**36%**	15%	10%	28%	17%	**43%**	9%	3%	27%	22%	**35%**	9%	7%
Weighted[d]	**Beta**	29%	21%	11%	11%	28%	25%	**26%**	8%	11%	29%	23%	**40%**	8%	14%	16%	**37%**	11%	18%	16%	18%	28%	**27%**	12%	10%	24%
	gamma	6%	19%	27%	**41%**	7%	13%	20%	24%	**31%**	12%	11%	17%	23%	**41%**	8%	11%	16%	12%	**53%**	9%	8%	15%	24%	**46%**	8%
	invGauss	5%	7%	17%	21%	**50%**	8%	10%	15%	29%	**37%**	12%	9%	11%	19%	**50%**	9%	9%	12%	14%	**55%**	5%	7%	12%	25%	**51%**
	Weibull	**35%**	**27%**	14%	16%	9%	**31%**	19%	17%	17%	15%	**36%**	20%	16%	12%	16%	20%	**46%**	9%	12%	13%	**39%**	27%	12%	11%	11%
	normal	26%	26%	**31%**	11%	6%	22%	25%	**35%**	11%	7%	19%	14%	**41%**	15%	10%	24%	18%	**49%**	5%	4%	21%	25%	**39%**	9%	6%

[a] Anderson-Darling Test - (Stephens, 1974)
[b] Kolmogorov-Smirnov Test - (Chakravart et al., 1967)
[c] χ^2 Test - (Snedecor and Cochran, 1989)
[d] Weighted Test = (.334*A-D + .333*K-S +.333*χ^2)

Table 3: In-Sample Rate Statistics: Illinois by Regions for FBFM Corn Farms at Three Coverage Levels (in bu/acre).

		Illinois Districts														
		NW			**NE**			**West**			**Central**			**East**		
		Coverage Levels														
Statistics	**Distribution**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**
Average[a]	**Empirical**[b]	2.36	0.97	0.37	2.56	1.02	0.35	3.27	1.44	0.49	2.74	1.09	0.36	3.70	1.67	0.65
	Weibull	1.97	0.63	0.18	2.30	0.78	0.24	2.98	1.09	0.35	2.30	0.74	0.21	2.75	0.99	0.31
	gamma	1.80	0.40	0.07	2.02	0.50	0.10	2.68	0.69	0.13	2.05	0.44	0.07	2.92	0.82	0.17
	normal	1.84	0.48	0.11	2.12	0.62	0.16	2.80	0.86	0.23	2.13	0.55	0.12	2.91	0.94	0.26
	Beta	2.60	0.99	0.36	2.84	1.14	0.44	3.53	1.42	0.51	2.94	1.12	0.40	3.87	1.72	0.71
	Mix2Norm	2.12	0.73	0.22	2.54	0.94	0.31	3.24	1.28	0.39	2.67	0.96	0.29	3.71	1.59	0.57
	kernel	2.82	1.18	0.46	3.10	1.27	0.47	3.94	1.74	0.67	3.24	1.32	0.47	4.28	2.00	0.83
Bias[c]	**Weibull**	-0.38	-0.35	-0.19	-0.26	-0.23	-0.11	-0.28	-0.35	-0.14	-0.44	-0.35	-0.15	-0.95	-0.68	-0.34
	gamma	-0.56	-0.58	-0.30	-0.54	-0.51	-0.25	-0.58	-0.75	-0.36	-0.69	-0.65	-0.29	-0.78	-0.85	-0.48
	normal	-0.52	-0.50	-0.26	-0.44	-0.40	-0.18	-0.47	-0.58	-0.26	-0.61	-0.53	-0.24	-0.79	-0.73	-0.39
	Beta	0.25	**0.01**	**-0.01**	0.28	0.12	0.09	0.26	**-0.02**	**0.03**	0.20	**0.03**	**0.04**	0.17	**0.05**	**0.05**
	Mix2Norm	**-0.23**	-0.24	-0.15	**-0.03**	**-0.08**	**-0.03**	**-0.03**	-0.16	-0.09	**-0.07**	-0.13	-0.07	**0.01**	-0.08	-0.08
	kernel	0.46	0.20	0.09	0.54	0.25	0.13	0.67	0.30	0.18	0.49	0.23	0.11	0.59	0.34	0.18
Efficiency[d]	**Weibull**	0.95	0.78	0.51	0.90	0.72	0.43	1.01	0.80	0.45	1.00	0.76	0.44	1.39	1.10	0.68
	gamma	0.69	0.76	0.52	0.69	0.73	0.45	0.74	0.90	0.52	0.80	0.81	0.49	0.92	1.03	0.71
	normal	0.70	0.73	0.50	0.64	0.66	0.41	0.70	0.77	0.45	0.77	0.74	0.44	0.96	0.97	0.64
	Beta	0.75	0.62	0.45	0.74	0.60	0.44	0.66	0.46	0.31	0.73	0.61	0.40	0.79	0.71	0.51
	Mix2Norm	0.83	0.62	0.40	**0.43**	**0.32**	0.20	**0.58**	0.47	0.31	0.57	0.41	0.26	**0.55**	0.43	0.31
	kernel	**0.54**	**0.28**	**0.15**	0.60	0.33	**0.20**	0.73	**0.36**	**0.25**	**0.56**	**0.29**	**0.15**	0.66	**0.40**	**0.24**

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Table 3 – continued from previous page

		Illinois Districts														
		WSW			ESE			SW			SE			Total		
		Coverage Levels														
Statistics	**Distribution**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**	**85%**	**75%**	**65%**
Average[a]	**Empirical**[b]	2.25	0.78	0.21	3.20	1.22	0.40	3.57	1.50	0.54	3.02	1.19	0.41	2.89	1.18	0.41
	Weibull	2.13	0.69	0.20	3.21	1.29	0.46	3.57	1.55	0.60	3.02	1.22	0.44	2.53	0.90	0.29
	gamma	1.71	0.35	0.05	2.76	0.81	0.19	3.19	1.08	0.28	2.65	0.80	0.19	2.30	0.59	0.12
	normal	1.85	0.47	0.10	2.97	1.05	0.33	3.36	1.33	0.47	2.82	1.02	0.32	2.39	0.73	0.20
	Beta	2.67	1.00	0.35	3.78	1.70	0.71	3.88	1.78	0.75	3.40	1.48	0.60	3.18	1.31	0.50
	Mix2Norm	2.12	0.62	0.15	3.26	1.20	0.38	3.59	1.44	0.49	3.06	1.11	0.34	2.83	1.06	0.34
	kernel	2.77	1.01	0.31	3.98	1.68	0.63	4.33	1.99	0.81	3.75	1.61	0.62	3.46	1.48	0.56
Bias[c]	**Weibull**	**-0.13**	**-0.09**	**-0.01**	**0.01**	0.06	0.06	**0.00**	**0.05**	0.06	**0.00**	**0.04**	**0.03**	-0.36	-0.28	-0.13
	gamma	-0.54	-0.43	-0.15	-0.44	-0.41	-0.22	-0.38	-0.43	-0.26	-0.37	-0.39	-0.22	-0.59	-0.59	-0.30
	normal	-0.40	-0.31	-0.10	-0.23	-0.17	-0.07	-0.21	-0.17	-0.08	-0.19	-0.17	-0.08	-0.50	-0.45	-0.21
	Beta	0.42	0.21	0.14	0.58	0.47	0.31	0.31	0.27	0.20	0.38	0.30	0.19	0.29	0.13	0.09
	Mix2Norm	-0.13	-0.16	-0.05	0.06	**-0.02**	**-0.02**	0.02	-0.07	**-0.05**	0.04	-0.07	-0.06	**-0.06**	**-0.12**	**-0.08**
	kernel	0.52	0.23	0.11	0.78	0.45	0.23	0.77	0.49	0.27	0.74	0.43	0.21	0.57	0.29	0.15
Efficiency[d]	**Weibull**	0.85	0.62	0.37	0.84	0.65	0.41	0.74	0.64	0.47	0.78	0.61	0.42	1.00	0.79	0.49
	gamma	0.68	0.62	0.37	0.59	0.59	0.41	0.58	0.57	0.43	0.56	0.56	0.42	0.74	0.78	0.51
	normal	0.63	0.56	0.35	0.52	0.50	0.35	0.49	0.50	0.40	0.49	0.48	0.38	0.72	0.72	0.47
	Beta	0.85	0.60	0.40	0.99	0.87	0.62	0.72	0.67	0.51	0.71	0.65	0.47	0.78	0.66	0.47
	Mix2Norm	0.65	0.49	0.33	**0.37**	**0.26**	**0.19**	**0.35**	**0.30**	**0.26**	**0.39**	**0.33**	**0.25**	**0.59**	0.44	0.30
	kernel	**0.58**	**0.28**	**0.16**	0.84	0.53	0.32	0.83	0.55	0.33	0.80	0.50	0.29	0.65	**0.38**	**0.22**

[a] Expected value of yield insurance at all farms in specified region.
[b] Expected burn rate [max(0,Yield Guarantee-Estimated Rate)] for all farms in specified region.
[c] Bias is expected value of empirical rate - estimated distributional rate for all farms in specified region.
[d] Efficiency is expected value of RMSE of all farms in specified region.

Table 4: Out-of-Sample Rate Simulation Analysis: μ=160; σ=20

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	1.587	0.431	0.095	1.424	0.302	0.044
n=15							
Average	**Beta**	2.461	1.065	0.468	2.392	1.005	0.430
	Weibull	1.582	0.463	0.117	1.640	0.483	0.123
	Mix2Norm	1.483	0.354	0.074	1.396	0.281	0.047
Bias (%)	**Beta**	-55.0%	-147.2%	-391.3%	-68.0%	-232.8%	-886.2%
	Weibull	**0.3%**	**-7.4%**	-23.1%	-15.2%	-60.0%	-181.4%
	Mix2Norm	6.6%	17.9%	**22.1%**	**2.0%**	**6.9%**	**-7.7%**
RMSE (%)	**Beta**	156.1%	380.3%	1059.8%	166.0%	517.4%	2150.7%
	Weibull	**71.4%**	**103.8%**	**157.4%**	**79.2%**	**157.3%**	383.3%
	Mix2Norm	95.3%	148.7%	245.7%	94.2%	162.1%	**328.5%**
n=20							
Average	**Beta**	2.131	0.802	0.298	2.106	0.772	0.282
	Weibull	1.599	0.463	0.115	1.680	0.493	0.124
	Mix2Norm	1.553	0.389	0.085	1.452	0.301	0.050
Bias (%)	**Beta**	-34.3%	-86.0%	-213.3%	-47.8%	-155.6%	-546.4%
	Weibull	**-0.7%**	**-7.3%**	-20.3%	-17.9%	-63.3%	-183.3%
	Mix2Norm	2.1%	9.6%	**10.6%**	**-1.9%**	**0.4%**	**-13.7%**
RMSE (%)	**Beta**	111.9%	247.7%	620.6%	124.5%	360.7%	1388.6%
	Weibull	**62.7%**	**90.7%**	**135.7%**	**70.9%**	**142.1%**	341.5%
	Mix2Norm	85.8%	141.9%	241.1%	83.6%	147.9%	**290.6%**
n=30							
Average	**Beta**	1.836	0.590	0.177	1.735	0.514	0.140
	Weibull	1.584	0.448	0.107	1.684	0.486	0.118
	Mix2Norm	1.566	0.393	0.084	1.422	0.291	0.048
Bias (%)	**Beta**	-15.7%	-36.8%	-86.4%	-21.8%	-70.1%	-219.9%
	Weibull	**0.2%**	**-3.9%**	-12.3%	-18.2%	-60.8%	-170.2%
	Mix2Norm	1.3%	8.7%	**12.2%**	**0.2%**	**3.6%**	**-9.5%**
RMSE (%)	**Beta**	78.4%	151.8%	326.9%	75.4%	182.1%	566.7%
	Weibull	**51.5%**	**73.1%**	**105.7%**	**57.6%**	**116.2%**	274.4%
	Mix2Norm	70.9%	120.8%	209.7%	65.8%	118.3%	**245.5%**

Table 5: Out-of-Sample Rate Simulation Analysis: μ=160; σ=30

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	4.098	1.715	0.621	3.989	1.539	0.479
n=15							
Average	**Beta**	5.384	2.896	1.514	5.246	2.765	1.411
	Weibull	4.021	1.735	0.667	4.063	1.756	0.674
	Mix2Norm	4.153	1.708	0.605	4.034	1.533	0.475
Bias (%)	**Beta**	-31.4%	-68.9%	-143.8%	-31.5%	-79.7%	-194.5%
	Weibull	1.9%	-1.2%	-7.5%	-1.8%	-14.1%	-40.7%
	Mix2Norm	**-1.3%**	**0.4%**	**2.6%**	**-1.1%**	**0.4%**	**0.9%**
RMSE (%)	**Beta**	102.0%	184.9%	363.2%	97.8%	192.8%	437.3%
	Weibull	**58.9%**	**77.1%**	**102.8%**	**58.2%**	**82.5%**	**129.6%**
	Mix2Norm	72.9%	105.0%	150.1%	70.0%	101.3%	146.5%
n=20							
Average	**Beta**	4.925	2.432	1.134	4.793	2.320	1.059
	Weibull	4.049	1.735	0.658	4.065	1.740	0.657
	Mix2Norm	4.170	1.719	0.609	4.068	1.573	0.497
Bias (%)	**Beta**	-20.2%	-41.8%	-82.6%	-20.1%	-50.8%	-121.0%
	Weibull	**1.2%**	-1.1%	-6.0%	**-1.9%**	-13.1%	-37.2%
	Mix2Norm	-1.8%	**-0.2%**	**1.8%**	-2.0%	**-2.2%**	**-3.8%**
RMSE (%)	**Beta**	74.4%	126.0%	229.0%	70.9%	132.9%	284.5%
	Weibull	**50.8%**	**65.8%**	**86.3%**	**49.4%**	**70.1%**	**110.1%**
	Mix2Norm	61.9%	89.9%	130.2%	59.2%	88.5%	135.1%
n=30							
Average	**Beta**	4.482	2.022	0.830	4.390	1.936	0.771
	Weibull	4.066	1.728	0.645	4.117	1.754	0.655
	Mix2Norm	4.157	1.704	0.599	4.041	1.537	0.480
Bias (%)	**Beta**	-9.4%	-17.9%	-33.7%	-10.0%	-25.8%	-60.8%
	Weibull	**0.8%**	-0.7%	-3.9%	-3.2%	-14.0%	-36.8%
	Mix2Norm	-1.4%	**0.7%**	**3.5%**	**-1.3%**	**0.1%**	**-0.2%**
RMSE (%)	**Beta**	53.8%	84.7%	141.2%	50.8%	87.1%	168.1%
	Weibull	**41.0%**	**52.8%**	**68.1%**	**40.5%**	**57.9%**	**91.4%**
	Mix2Norm	49.5%	72.7%	105.4%	47.5%	70.8%	108.4%

Table 6: Out-of-Sample Rate Simulation Analysis: μ=160; σ=40

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	7.132	3.695	1.710	7.151	3.607	1.581
n=15							
Average	**Beta**	8.341	4.995	2.852	8.206	4.897	2.783
	Weibull	6.926	3.625	1.722	6.895	3.601	1.704
	Mix2Norm	7.220	3.736	1.702	7.223	3.638	1.591
Bias (%)	**Beta**	-17.0%	-35.2%	-66.7%	-14.8%	-35.8%	-76.1%
	Weibull	2.9%	1.9%	-0.7%	3.6%	**0.2%**	-7.8%
	Mix2Norm	**-1.2%**	**-1.1%**	**0.5%**	**-1.0%**	-0.9%	**-0.6%**
RMSE (%)	**Beta**	70.2%	107.6%	175.5%	65.9%	104.4%	181.6%
	Weibull	**50.5%**	**61.8%**	**76.3%**	**48.3%**	**60.2%**	**78.5%**
	Mix2Norm	58.4%	77.1%	102.4%	56.4%	75.7%	102.6%
n=20							
Average	**Beta**	7.972	4.559	2.443	7.959	4.543	2.428
	Weibull	6.940	3.622	1.711	6.949	3.623	1.706
	Mix2Norm	7.221	3.755	1.729	7.251	3.658	1.595
Bias (%)	**Beta**	-11.8%	-23.4%	-42.8%	-11.3%	-25.9%	-53.6%
	Weibull	2.7%	2.0%	**0.0%**	2.8%	**-0.4%**	-8.0%
	Mix2Norm	**-1.3%**	**-1.6%**	-1.1%	**-1.4%**	-1.4%	**-0.9%**
RMSE (%)	**Beta**	55.6%	81.9%	127.3%	53.5%	82.0%	136.8%
	Weibull	**44.0%**	**53.9%**	**66.3%**	**42.8%**	**53.5%**	**69.5%**
	Mix2Norm	50.0%	66.6%	89.7%	49.0%	65.7%	89.4%
n=30							
Average	**Beta**	7.555	4.085	2.019	7.553	4.097	2.032
	Weibull	7.012	3.651	1.713	7.011	3.651	1.712
	Mix2Norm	7.178	3.685	1.672	7.229	3.622	1.565
Bias (%)	**Beta**	-5.9%	-10.6%	-18.1%	-5.6%	-13.6%	-28.5%
	Weibull	1.7%	1.2%	**-0.2%**	2.0%	-1.2%	-8.3%
	Mix2Norm	**-0.7%**	**0.3%**	2.2%	**-1.1%**	**-0.4%**	**1.0%**
RMSE (%)	**Beta**	42.7%	59.7%	86.5%	40.7%	58.9%	91.9%
	Weibull	**36.3%**	**44.4%**	**54.4%**	**35.3%**	**44.0%**	**57.1%**
	Mix2Norm	40.8%	53.9%	71.6%	39.7%	52.8%	71.0%

Table 7: Out-of-Sample Rate Simulation Analysis: μ=180; σ=20

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	1.292	0.298	0.055	1.114	0.185	0.020
n=15							
Average	**Beta**	2.267	0.935	0.396	2.109	0.829	0.339
	Weibull	1.326	0.339	0.074	1.395	0.362	0.080
	Mix2Norm	1.216	0.254	0.050	1.123	0.184	0.025
Bias (%)	**Beta**	-75.4%	-213.8%	-622.8%	-89.2%	-347.0%	-1627.8%
	Weibull	**-2.6%**	**-13.6%**	-35.6%	-25.2%	-95.1%	-308.4%
	Mix2Norm	5.9%	14.8%	**8.9%**	**-0.8%**	**1.0%**	**-27.5%**
RMSE (%)	**Beta**	193.6%	527.8%	1677.8%	207.6%	784.5%	4282.3%
	Weibull	**78.0%**	**119.7%**	**193.9%**	**91.5%**	208.1%	595.3%
	Mix2Norm	109.3%	186.6%	366.7%	107.8%	**201.3%**	**468.6%**
n=20							
Average	**Beta**	1.856	0.651	0.233	1.776	0.587	0.198
	Weibull	1.308	0.327	0.069	1.388	0.352	0.075
	Mix2Norm	1.241	0.255	0.045	1.123	0.184	0.025
Bias (%)	**Beta**	-43.6%	-118.3%	-325.1%	-59.3%	-216.3%	-907.5%
	Weibull	**-1.2%**	**-9.6%**	-26.3%	-24.6%	-90.0%	-284.5%
	Mix2Norm	4.0%	14.5%	**17.3%**	**-0.8%**	**0.6%**	**-26.7%**
RMSE (%)	**Beta**	137.3%	348.1%	1041.8%	145.8%	498.9%	2434.5%
	Weibull	**67.4%**	**101.4%**	**159.5%**	**79.5%**	**180.9%**	507.0%
	Mix2Norm	94.5%	159.8%	281.6%	93.0%	183.7%	**468.7%**
n=30							
Average	**Beta**	1.509	0.422	0.112	1.461	0.390	0.101
	Weibull	1.295	0.314	0.064	1.395	0.350	0.073
	Mix2Norm	1.232	0.249	0.044	1.107	0.178	0.023
Bias (%)	**Beta**	-16.8%	-41.7%	-104.3%	-31.1%	-110.3%	-413.8%
	Weibull	**-0.2%**	**-5.5%**	**-16.1%**	-25.2%	-88.5%	-271.8%
	Mix2Norm	4.7%	16.5%	19.8%	**0.7%**	**3.9%**	**-15.5%**
RMSE (%)	**Beta**	83.7%	171.1%	397.6%	100.0%	307.3%	1393.3%
	Weibull	**53.7%**	**78.0%**	**115.2%**	**68.4%**	157.7%	435.4%
	Mix2Norm	76.4%	131.0%	230.9%	76.2%	**144.6%**	**313.8%**

Table 8: Out-of-Sample Rate Simulation Analysis: μ=180; σ=30

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	3.581	1.346	0.430	3.427	1.146	0.297
n=15							
Average	**Beta**	4.843	2.432	1.190	4.680	2.292	1.097
	Weibull	3.524	1.378	0.476	3.548	1.387	0.478
	Mix2Norm	3.571	1.302	0.406	3.414	1.110	0.294
Bias (%)	**Beta**	-35.2%	-80.7%	-176.4%	-36.6%	-99.9%	-268.7%
	Weibull	1.6%	**-2.4%**	-10.7%	-3.5%	-21.0%	-60.6%
	Mix2Norm	**0.3%**	3.3%	**5.8%**	**0.4%**	**3.2%**	**1.3%**
RMSE (%)	**Beta**	106.6%	205.0%	429.7%	106.7%	234.1%	607.4%
	Weibull	**61.4%**	**82.8%**	**114.3%**	**60.9%**	**93.3%**	**163.3%**
	Mix2Norm	78.0%	117.4%	177.2%	74.7%	114.0%	181.9%
n=20							
Average	**Beta**	4.408	2.024	0.888	4.348	1.969	0.856
	Weibull	3.565	1.383	0.470	3.617	1.406	0.478
	Mix2Norm	3.595	1.299	0.395	3.487	1.148	0.305
Bias (%)	**Beta**	-23.1%	-50.4%	-106.4%	-26.9%	-71.7%	-187.8%
	Weibull	0.4%	**-2.7%**	-9.1%	-5.6%	-22.7%	-60.6%
	Mix2Norm	**-0.4%**	3.5%	**8.3%**	**-1.8%**	**-0.1%**	**-2.5%**
RMSE (%)	**Beta**	83.2%	151.5%	299.5%	87.6%	186.2%	469.0%
	Weibull	**53.5%**	**71.4%**	**96.6%**	**53.2%**	**81.6%**	**142.3%**
	Mix2Norm	68.0%	101.9%	148.2%	64.2%	100.2%	161.6%
n=30							
Average	**Beta**	3.898	1.593	0.595	3.903	1.562	0.563
	Weibull	3.525	1.347	0.447	3.677	1.424	0.478
	Mix2Norm	3.562	1.277	0.389	3.507	1.148	0.302
Bias (%)	**Beta**	-8.8%	-18.4%	-38.2%	-13.9%	-36.3%	-89.4%
	Weibull	1.6%	**-0.1%**	**-3.8%**	-7.3%	-24.2%	-60.9%
	Mix2Norm	**0.5%**	5.1%	9.6%	**-2.3%**	**-0.1%**	**-1.6%**
RMSE (%)	**Beta**	58.7%	97.6%	174.6%	56.7%	106.4%	231.7%
	Weibull	**42.7%**	**56.0%**	**73.5%**	**44.7%**	**69.8%**	**122.9%**
	Mix2Norm	53.9%	82.6%	124.4%	51.7%	80.8%	131.1%

Table 9: Out-of-Sample Rate Simulation Analysis: μ=180; σ=40

Data Generating Process:		***Weibull***			***Beta***		
		Coverage Level			**Coverage Level**		
Statistic	**Distribution**	85%	75%	65%	85%	75%	65%
	True Empirical	6.458	3.090	1.305	6.408	2.931	1.134
n=15							
Average	**Beta**	7.820	4.479	2.459	7.863	4.503	2.471
	Weibull	6.235	3.027	1.324	6.310	3.073	1.346
	Mix2Norm	6.423	3.036	1.252	6.512	3.000	1.174
Bias (%)	**Beta**	-21.1%	-44.9%	-88.4%	-22.7%	-53.6%	-117.9%
	Weibull	3.5%	2.0%	**-1.4%**	**1.5%**	-4.9%	-18.7%
	Mix2Norm	**0.5%**	**1.8%**	4.0%	-1.6%	**-2.4%**	**-3.5%**
RMSE (%)	**Beta**	80.5%	131.2%	228.9%	80.4%	139.3%	268.7%
	Weibull	**52.9%**	**65.8%**	**83.0%**	**52.2%**	**68.3%**	**95.6%**
	Mix2Norm	62.7%	85.9%	117.1%	62.0%	86.1%	121.1%
n=20							
Average	**Beta**	7.432	4.012	2.034	7.264	3.892	1.955
	Weibull	6.330	3.062	1.327	6.278	3.036	1.315
	Mix2Norm	6.564	3.104	1.274	6.449	2.942	1.142
Bias (%)	**Beta**	-15.1%	-29.8%	-55.9%	-13.4%	-32.8%	-72.3%
	Weibull	2.0%	0.9%	**-1.7%**	2.0%	-3.6%	-15.9%
	Mix2Norm	**-1.6%**	**-0.4%**	2.3%	**-0.6%**	**-0.4%**	**-0.7%**
RMSE (%)	**Beta**	63.5%	99.3%	165.5%	60.4%	99.4%	181.6%
	Weibull	**46.0%**	**57.2%**	**71.5%**	**46.1%**	**60.0%**	**83.0%**
	Mix2Norm	54.0%	74.0%	101.2%	53.2%	74.0%	104.5%
n=30							
Average	**Beta**	6.975	3.525	1.621	6.876	3.459	1.580
	Weibull	6.422	3.107	1.342	6.370	3.078	1.326
	Mix2Norm	6.587	3.125	1.295	6.441	2.928	1.135
Bias (%)	**Beta**	-8.0%	-14.1%	-24.2%	-7.3%	-18.0%	-39.3%
	Weibull	**0.5%**	**-0.6%**	-2.9%	0.6%	-5.0%	-16.9%
	Mix2Norm	-2.0%	-1.1%	**0.8%**	**-0.5%**	**0.1%**	**0.0%**
RMSE (%)	**Beta**	46.5%	67.9%	102.8%	45.0%	69.2%	116.1%
	Weibull	**38.0%**	**47.6%**	**59.8%**	**37.9%**	**49.5%**	**68.7%**
	Mix2Norm	44.0%	61.2%	85.0%	43.1%	60.0%	85.1%