Multiproduct Optimal Hedging by

Time-Varying Correlations

in a State Dependent model of Regime-Switching

Hernan A. Tejeda

Graduate Research Assistant
North Carolina State University
email: hatejeda@ncsu.edu

Barry K. Goodwin

William Neal Reynolds Distinguished Professor
North Carolina State University
email: barry_goodwin@ncsu.edu

Selected Paper prepared for presentation at the Agricultural & Applied Economics Association 2010 AAEA, CAES &WAEA Joint Annual Meeting, Denver, Colorado, July 25-27, 2010

Multiproduct Optimal Hedging by

Time-Varying Correlations

in a State Dependent model of Regime-Switching

Abstract

We determine time-varying hedge ratios in a multiproduct setting using a multivariate state dependent model of regime switching dynamic correlations. The model enables one to depict the time-varying correlations for multiple series of cash and future prices in two or more different regimes (i.e. the conditional correlation is not constant in this multivariate model). This provides an improved characterization of the multiproduct dynamic hedging process as it captures the evolution of the cash/futures correlation matrix when the model switches from a regime of low correlation to one of higher correlation or vice-versa. The model switches regimes according to a Markov chain process and does not have a dimensionality problem for larger numbers of series, as does the more conventional BEKK model. In addition, we introduce fundamental, economically related factors in the regime switching process to assess their effect. These are (weakly) exogenous variables with respect to the markets being considered.

Results show that these explicit weakly exogenous variables may have an impact on the dynamic process. We determine the optimal hedge ratios for the soybean complex by specifically introducing the stocks-to-use ratio of soybeans as a variable in determining the probability of switching correlation regimes, and compare to the case of constant transition probability between regimes. The stocks-to-use ratio contains specific, up-to-date information on the supply and demand conditions relevant to the soybean markets, and hence has a direct role in determining the price of the commodities. By introducing this variable, our model achieves a relative improvement in the characterization of the process over the case of constant transition probabilities between regimes. More importantly, there is an improvement in our estimated hedge ratios over simple and naïve hedge ratio estimations. Additionally, shocks to these related variables may permit us to identify the effect on the hedging ratios and comparison to simpler hedging estimation procedures. The model applied is from Tejeda et al. (2009).

Hernan A. Tejeda is a Graduate Research Assistant in the Departments of Economics and Agricultural and Resource Economics at North Carolina State University.

Barry K. Goodwin is William Neal Reynolds Distinguished Professor in the Departments of Economics and Agricultural and Resource Economics at North Carolina State University.

Introduction

Certain production settings consider the use of inputs and outputs in futures markets, enabling the potential use of a multiproduct hedging strategy for risk management purposes. Multiproduct hedging considers a multivariate portfolio approach with the potential advantage of production-related commodities decreasing the price risk faced over the case of singular commodity hedge. The usual optimal hedging strategies rely on a mean variance (M-V) structure, as per Anderson and Danthine (1980), Fackler and McNew (1993), Noussinov and Leuthold (1999) among others. Agents seek to maximize their expected returns and minimize the variance of their returns. Within the setting of a production process, these returns incorporate the difference between the cash prices of the inputs and the output, and also for changes of futures prices at different periods for each of the inputs and also for the output. Details of these returns in the soybean complex process are in Garcia et al. (1995).

It is noted by Fackler and McNew (1993) that the optimal futures position can be partitioned into a pure speculative and a pure hedging component. By assuming unbiased futures markets - through the condition of expected futures price differences being equal to zero, only the pure hedging component remains. This hedging component - otherwise known as the minimum variance hedge, denotes the proportion of the cash position to hold in futures in order to minimize price risk. This minimum variance hedge may be expressed by the following ratio *z/q*:

$$\frac{z}{q} = \frac{Cov(F,S)}{Var(F)}$$

Where z and q represent the futures and cash positions, respectively; such that $z > 0$ is a long (i.e. buy) position and $z < 0$ is a short (i.e. sell) position. Also F and S represent the futures and spot or cash prices, respectively.

Anderson and Danthine (1980) lay the theoretical ground for a static scenario, where hedging between multiple contracts in an efficient market responds to this covariance between the future and cash prices and the variance of the future prices. However, ensuing studies by Myers and Thompson (1989) and Baillie and Myers (1991) determined that the condition for the proper covariance between cash and futures prices required incorporating information up to the date the hedge is made. (i.e. minimum variance hedging should consider conditional covariance instead of unconditional covariance). Estimation of the conditional covariance became available with the use of ARCH and GARCH models, and Baillie and Myers (1991) estimate optimal time-varying hedge ratios by using a bivariate GARCH model with diagonal vech parametrization. Another study by Garcia et al. (1995) used constant correlation within a MGARCH model to estimate the optimal hedging ratio of a soybean complex.

As mentioned previously, certain production settings – such as cattle feeder production or a soybean complex process consider the use of inputs and outputs in futures markets. This setting enables the use of a multiproduct hedging strategy for risk management purposes. Multiproduct hedging considers a multivariate portfolio approach, with the potential advantage of production-related commodities decreasing the price risk faced over the case of singular commodity hedge. Early studies by Peterson and Leuthold (1987), Tzang and Leuthold (1990), Fackler and McNew (1993) and Garcia et al. (1995) determined empirical estimates of multiproduct optimal hedges with relative advantages over single commodity hedging strategies. These latter three studies incorporated the conditional covariance requirement posed by Myers and Thompson (1989) and Baillie and Myers (1991).

Subsequent papers by Noussinov and Leuthold (1999), Manfredo et al. (2000) and Haigh and Holt (2000 and 2002) analyzing multiple hedging estimations within a time-varying context, have arrived at favorable results for periods of higher volatility. Recent studies for time-varying minimum hedge ratios in univariate settings have been made by Lee et al. (2006), Lee and Yoder (2007a and 2007b) with some improvements over previous dynamic models. Our model considers hedging estimation within a multiproduct setting.

Specifically, our model determines optimal hedge ratios for a soybean complex process by considering time-varying conditional correlations, in a two correlation regime setting. The switch between one regime of correlation and the other is governed by a Markov chain. The optimal hedge ratios for the soybean complex are estimated by specifically introducing the stocks-to-use ratio of soybeans as a variable in determining the probability of switching between correlation regimes. Yet similar results are also obtained by considering constant transition probabilities between the two regimes.

We next discuss the multiproduct hedging method in a soybean complex, followed by a presentation of the econometric model used which includes time varying correlations in a state dependent model of regime switching. A brief description of the estimation procedure is presented afterwards, along with results for a multiproduct optimal hedging strategy for a soybean complex and also with a comparison to a simple hedging strategy which just considers time-varying correlations. Discussions and Conclusions follow.

Multiproduct Hedging in a Soybean Complex

A soybean processor operation requires soybeans as input and results in soybean meal and soybean oil as output. Hence the return or margin from the soybean process is the difference between the sale prices of soybean meal and soybean oil and the cost prices of soybeans. This margin varies according to the variability of these prices, and soybean processors may hedge these three prices in the cash markets, forward cash markets, and the futures and options markets. This study considers hedging with futures instruments (i.e. options are not included in this study).

The processor's crushing margin depends on the ratio of input/output soybean crushing technology employed. It is assumed here that 48 pounds of soybean meal and 11 pounds of soybean oil are produced from each bushel of soybeans (i.e. 59 lbs.), neglecting any loss for simplicity.

A framework in line with Tzang and Leuthold (1990), Garcia et al. (1995), and Manfredo et al. (2000) for soybean processing, is established considering two stages in a total of three periods or weeks in this case. The first stage involves two weeks in just production planning (i.e. previous to the actual purchase of soybeans). Here futures hedges include concurrently going long (i.e. buy) in soybeans[1] ($F_{b,t-3}$) and short (i.e. sell) in both soybean meal ($F_{m,t-3}$) and soybean oil ($F_{o,t-3}$). The second stage involves the operation, which includes one week in actually buying the soybeans in the cash market ($S_{b,t-1}$) and concurrently placing a short ($F_{b,t-1}$) in the futures market, thus liquidating previous soybeans long position. Subsequently, after a week following a period

[1] Soybean is denoted by subscript "b"; Soybean meal is denoted by subscript "m"; Soybean oil is denoted by subscript "o".

of crushing, the producer sells the soybean meal ($S_{m,t}$) and soybean oil ($S_{o,t}$) in the cash market and concurrently places a long in the futures markets for both these outputs ($F_{m,t}$ and $F_{o,t}$), and thus liquidates previous shorts of soybean meal and soybean oil.

Hence the hedged soybean returns or margin, considering the two previous stages (with two periods/weeks for planning and one period/week for operation), is as follows:

$$R_t = S_{m,t} + S_{o,t} – S_{b,t-1} + b_{b,t-3} (F_{b,t-1} – F_{b,t-3}) – b_{m,t-3} (F_{m,t} – F_{m,t-3}) – b_{o,t-3} (F_{o,t} – F_{o,t-3}) – c$$

where b_b, b_m, b_o are respectively soybeans, soybean meal and soybean oil bushels of futures contracts on a per bushel soybean basis at the first time period t-3, and c is a processing cost which is assumed constant. These bushels of futures contracts will determine the respective hedge ratio obtained below, by providing the optimal number of futures contracts from minimizing the variation of the returns as mentioned previously.

That is, by using the mean variance framework described in the introduction under the condition of unbiased futures markets, (i.e. expected futures price differences being equal to zero), we are able to determine the minimum hedge ratios from the variance of the returns[2] presented below, as per Garcia et al. (1995) and Manfredo et al. (2000).:

$$V(R) = V(S_b) + V(S_m) + V(S_o) + b_b^2 V(F_b) + b_o^2 V(F_o) + b_m^2 V(F_m) – 2cov(S_o, S_b) – 2cov(S_m, S_b) +$$

$$+ 2cov(S_m, S_o) – 2b_b cov(F_b, Sb) + 2b_b cov(F_b, S_o) + 2b_b cov(F_b, S_m) + 2b_o cov(F_o,S_b) –$$

$$- 2b_o cov(F_o,S_o) – 2b_o cov(F_o, S_m) – 2b_o b_b cov(F_o, F_b) + 2b_m cov(F_m, S_b) – 2b_m cov(F_m, S_o) –$$

$$- 2b_m cov(F_m, S_m) – 2b_m b_b cov(F_m, F_b) + 2b_m b_o cov(F_m, F_o).$$

The minimum variance hedge ratios are obtained by partially differentiating the previous variance with respect to b_b, b_m, b_o and equating each to zero, and then solving for each b_b, b_m, $b_{o,}$ which is calculated with Cramer's rule for simplicity. Equations for the computation of these optimal hedge ratios are in Appendix 1. These time-varying hedge ratios are computed by concurrently estimating the time-varying variances and covariance terms.

[2] The time scripts are omitted for simplicity.

Econometric Methods

The conditional mean and covariance of market prices must be defined in order to estimate the conditional time-varying covariance matrix. For this purpose, the conditional returns of the respective spot and futures prices are identified and computed (i.e. in order for the covariance matrix to be estimated). In line with Manfredo et al. (2000), the soybean cash and futures prices consider the timing between planning and production period, resulting in the following conditional returns:

$$R_{b,t} \mid I_{t-3} = 100*ln(P_{b,t-1}/P_{b,\ t-3})$$

or $$R_{b,t} = 100*ln(P_{b,t-1}/P_{b,\ t-3}) + u_{b,t} \qquad (1.1)$$

with information available at the planning stage, (i.e. at *t-3*), and P being Spot or Futures prices.

Once again, by incorporating the timing between planning and production for soybean meal and soybean oil, respectively, the following conditional returns are obtained:

$$R_{m,t} \mid I_{t-3} = 100*ln(P_{m,t}/P_{m,\ t-3}) \quad \text{or}$$

$$R_{m,t} = 100*ln(P_{m,\ t}/P_{m,\ t-3}) + u_{m,t} \qquad (1.2)$$

and

$$R_{o,t} \mid I_{t-3} = 100*ln(P_{o,\ t}/P_{o,\ t-3}) \quad \text{or}$$

$$R_{o,t} = 100*ln(P_{o,\ t}/P_{o,\ t-3}) + u_{o,t} \qquad (1.3)$$

The prediction errors are specified as the time-varying covariance matrix:

$$H_t = E(\varepsilon_t \varepsilon_t' \mid I_{t-3}) \qquad (1.4)$$

Estimation of the time-varying variances and covariances of cash and futures price changes is made with a Regime Switching Dynamic Correlation (RSDC) model and a State Dependent Regime Switching Dynamic Correlation model, per Pelletier (2006) and Tejeda et al. (2009), respectively.

The RSDC model considers a K - multivariate time process:

$$Y_t = H_t^{1/2} U_t \quad \text{with } U_t \sim i.i.d.(0, I_K) \tag{1.5}$$

Where Y_t are the previous price returns from (1.1) to (1.3)

The time varying covariance matrix H_t to be estimated is decomposed into standard deviations and correlations, with different correlation values switching between different regimes through a Markov chain.

$$H_t \equiv S_t \Gamma_t S_t \tag{1.6}$$

where S_t is a Diagonal matrix with standard deviations: $s_{k,t}$ $k = 1 \ldots..K$ and Γ_t is the correlations matrix

The standard deviations $s_{k,t}$ for each time series k - from the diagonal matrix S_t, are assumed to follow an ARMACH model, per Taylor (1986). In the ARMACH model, the conditional standard deviation follows:

$$s_t = \omega + \sum_{i=1}^{q} \tilde{\alpha}_i |y_{t-i}| + \sum_{j=1}^{p} \beta_j s_{t-j} \quad \text{with } \tilde{\alpha}_i = \alpha_i / E|\tilde{u}_t|\text{, for stationary purposes} \tag{1.7}$$

The correlation matrix Γ_t follows a Markov chain, with different values for different regimes, i.e. for particular t periods it may be in one regime with a certain set of correlation values, and for other t periods it may be in another regime, with a different set of correlation values. The time-varying correlation matrix Γ_t is defined as:

$$\Gamma_t = \sum_{n=1}^{N} \mathbf{1}_{\{\Delta_t = n\}} \Gamma_n \tag{1.8}$$

where Δ_t is an unobserved Markov chain process independent of U_t, taking N possible regimes or values ($\Delta_t = 1,2, \ldots N$). And **1** is an indicator function. In this study two different regimes are considered.

The parsimonious or restricted model that will be estimated for the time-varying correlation matrix Γ_t is similar to Pelletier (2006) and Tejeda (2009). That is:

$$\Gamma_t = \Gamma\lambda(\Delta_t) + I_K\big(1 - \lambda(\Delta_t)\big) \tag{1.9}$$

where Γ is a fixed KxK correlation matrix – for every state or regime considered. I_K is a KxK identity matrix. And $\lambda(\Delta_t) \epsilon [0,1]$ (for assurance of eliminating possibilities of non-PSD correlation matrix) is a univariate random process governed by the unobserved Markov chain process Δ_t that takes N possible values ($\Delta_t = 1,2 \ldots N$), and is independent of U_t. Hence, the correlation matrix at time t (i.e. Γ_t) is a weighted average of two extreme states or regimes – uncorrelated returns by $\lambda(\Delta_t) = 0$, or totally correlated returns at $\lambda(\Delta_t) = 1$. Changes among correlations of different regimes are strictly proportional to $\lambda(\Delta_t)$, allowing for regimes of higher or lower correlations with the diagonals (own-correlations) being left at one.

The 'probability law' governing the Markov chain process Δ_t is defined by its state dependent transition probability matrix Π_t with elements of row i and column j : $\pi_t^{i,j}$, which is a function of a weakly exogenous variable x_{t-1}

Such that:

$$\pi_t^{i,i} = P(\Delta_t = i \mid \Delta_{t-1} = i\ , x_{t-1}; \beta_i) = \frac{\exp(x'_{t-1}\beta_i)}{1+\exp(x'_{t-1}\beta_i)} \;; \quad \text{and}$$

$$\pi_t^{i,j} = P(\Delta_t = j \mid \Delta_{t-1} = i\ , x_{t-1}; \beta_i) = 1 - \frac{\exp(x'_{t-1}\beta_i)}{1+\exp(x'_{t-1}\beta_i)}$$

For the specific case of two regimes:

i.e. $\Delta_t = 1$ $\boldsymbol{or}$ $\Delta_t = 2$

with

$$P(\Delta_t = 1 \mid \Delta_{t-1} = 2\ , x_{t-1}; \beta_2) = \frac{exp(x'_{t-1}\beta_2)}{1+exp(x'_{t-1}\beta_2)} \;; \quad \text{and}$$

$$P(\Delta_t = 2 \mid \Delta_{t-1} = 2\ , x_{t-1}; \beta_2) = 1 - \frac{exp(x'_{t-1}\beta_2)}{1+exp(x'_{t-1}\beta_2)} \tag{1.10}$$

Probabilities for both regimes when being previously in regime 2 (i.e. $\Delta_{t-1}= 2$) are analogous. The transition probability matrix Π_t is in Appendix 2. For the case of constant transition probabilities, the weakly exogenous variable x_{t-1} is equal to zero.

Estimation:

From equations (1.5) and (1.6), the log-likelihood can be written as:

$$L = -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + \log(|H_t|) + Y_t' H_t^{-1} Y_t]$$

$$= -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + \log(|S_t \Gamma_t S_t|) + Y_t' S_t^{-1} \Gamma_t^{-1} S_t^{-1} Y_t]$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}[Klog(2\pi) + 2\log(|S_t|) + \log(|\Gamma_t|) + \widetilde{U}_t' \Gamma_t^{-1} \widetilde{U}_t] \qquad (2.1.)$$

where $\widetilde{U}_t = S_t^{-1} Y_t$ and $\widetilde{U}_t = [\tilde{u}_{1,t} \ldots \ldots \ldots \ldots . \tilde{u}_{K,t}]'$ is a zero mean process with covariance matrix Γ_t; also $|H_t| = \det(H_t)$.

Estimation of the model parameters is made in two steps, with the assurance that the variance/covariance matrix is PSD (positive semi-definite). First the standard deviations are obtained, and then the correlations are calculated. This involves calculating the filtered probabilities for the complete data log-likelihood conditional on data observed, and then obtaining back the smoothed probabilities. The second part is the maximization step, which considers the use of these smoothed probabilities in our expected complete-data log likelihood function and maximizes directly with respect to the parameters. The parsimonious model enables calculation of dynamic correlations in the context of time-varying transition probabilities without the need for expectation maximization as does the full model; since the parsimonious model requires less number of parameters to be estimated. In other words, through maximum likelihood and using a correlation targeting method described in detail per Pelletier (2006) and Tejeda et al. (2009), we are able to estimate dynamic correlations between regimes when considering state dependent transition probabilities.

The data consists of weekly spot and futures prices for soybeans, soybean meal and soybean oil. The futures data considers the price of each Wednesday of the week, and if missing, then the value for that week's Tuesday or Thursday is taken into account. The cash soybean prices are quotes from the Central Illinois elevator and the Soybean meal and soybean oil prices are quotes from Decatur, Illinois. The futures quotes are for the closing prices at the Chicago Board of Trade (CBOT). Data spans from the second week of January in 2001 until the first week of October 2008, consisting of 408 observations. The out of sample data consists of weekly prices from the second week of October 2008 till the last week of April 2009, being 27 observations. The stock and use data is obtained from the monthly World Agricultural Supply and Demand Estimates (WASDE) reports from the USDA. A cubic spline is applied to this data in order to transform it into weekly data.

Results

Tables 1 and 2 below present estimated correlation values among the cash and future prices of soybean, soybean meal and soybean oil for the two regimes considered. Estimation was made with the model considering (i.) constant transition probabilities between regimes, and (ii.) with the stock to use ratio of soybeans in the state dependent transition probability, as per (1.8).

Table 1.

Regime 1	*Soybean Cash*	*Soybean Meal Cash*	*Soybean Oil Cash*	*Soybean Futures*	*Soybean Meal Futures*	*Soybean Oil Futures*
Soybean Cash	1.0000					
	-					
Soybean Meal Cash	0.7273	1.0000				
	0.0314	-				
Soybean Oil Cash	0.5858	0.4575	1.0000			
	0.0324	0.0431	-			
Soybean Futures	0.9911	0.7130	0.5864	1.0000		
	0.0023	0.0330	0.0329	-		
Soybean Meal Futures	0.7449	0.9865	0.4810	0.7366	1.0000	
	0.0301	0.0034	0.0417	0.0309	-	
Soybean Oil Futures	0.5982	0.4636	0.9948	0.5992	0.4906	1.0000
	0.0332	0.0439	0.0013	0.0338	0.0419	-

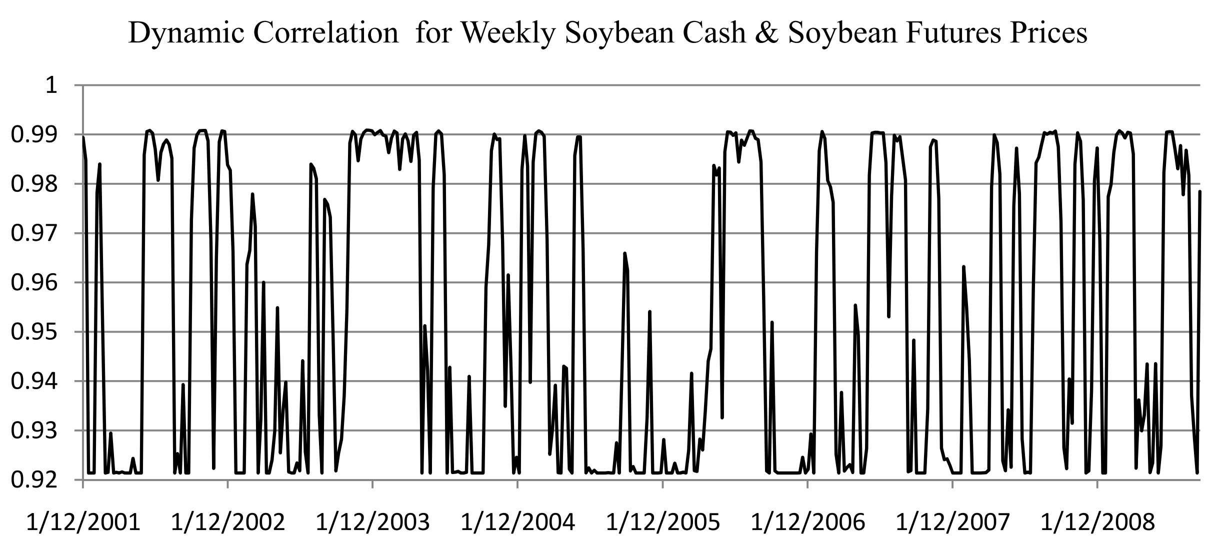
The correlation values obtained by each model for both regimes do not have a significant difference between them (i.e. these estimated correlation values are nearly the same for each model, differing a bit on their standard errors). Hence only the correlation values for each regime are presented.

Table 2.

Regime 2	*Soybean Cash*	*Soybean Meal Cash*	*Soybean Oil Cash*	*Soybean Futures*	*Soybean Meal Futures*	*Soybean Oil Futures*
Soybean Cash	1.0000 -					
Soybean Meal Cash	0.6874 0.0303	1.0000 -				
Soybean Oil Cash	0.5537 0.0310	0.4325 0.0409	1.0000 -			
Soybean Futures	**0.9367** 0.0086	0.6739 0.0318	0.5542 0.0315	1.0000 -		
Soybean Meal Futures	0.7041 0.0291	**0.9324** 0.0088	0.4547 0.0396	0.6962 0.0299	1.0000 -	
Soybean Oil Futures	0.5654 0.0317	0.4382 0.0417	**0.9402** 0.0084	0.5664 0.0324	0.4637 0.0398	1.0000 -

The chart in figure 1 below shows the correlation between the two regimes for Soybean Cash and Soybean futures.

Figure 1.



Regarding the different correlation regimes, it may be noted that each specific commodity has two distinct dynamic correlation regimes between their cash and futures prices. That is, soybeans, soybean cash and soybean meal each have two different correlation levels among their own cash and futures prices. These different correlation levels are quite similar for the three commodities, ranging from almost one at 0.99 to about 0.94. The correlation values for cash and futures prices between the three different commodities range from 0.745 for regime 1 and 0.704 for regime 2 considering soybean cash and soybean meal futures to 0.458 for regime 1 and 0.432 for regime 2 considering cash among soybean meal and soybean oil. Yet the difference in the magnitude of these values appears small when compared to the magnitude of their standard errors.

The ARMACH model results for each price are in table 3 below.

Table 3.

	Soybean		*Soybean Meal*		*Soybean Oil*	
	Cash	*Futures*	*Cash*	*Futures*	*Cash*	*Futures*
ω - omega	0.8197*	0.9763+	1.6248*	1.3673+	4.2507*	4.5348*
	0.3762	0.5012	0.7736	0.7240	1.0303	0.9748
α~ - alpha tilda	0.1828*	0.1688*	0.1677*	0.1274*	0.2649*	0.2702*
	0.0323	0.0364	0.0389	0.0286	0.0403	0.0451
β - beta	0.7012*	0.6804*	0.6563*	0.7128*	0.0608	0.0048
	0.0993	0.1292	0.1251	0.1196	0.1974	0.1835

*Significance at 5% level or less +Significance at 10% level or less

In general, the ARMACH parameters are significant for all price series, except those of soybean oil. For this latter case, the conditional volatility is only significantly dependent upon the previous observation or innovation, and not upon the previous volatility.

The average hedge ratios are computed considering each regime and compared to a simpler hedge ratio which only considers the time-varying covariance between spot and futures returns[3],

[3] Consistent with traditional optimal hedge ratios, $b_{i,t-3} = \frac{Cov(S_i,F_i)}{Var\ (F_i)}$ per Manfredo et al. (2000)

without taking into account the existing relationship between the different soybean products. These settings are compared to the case of naive hedging, which is equivalent to the hedge ratio being equal to 1 (i.e. agents take equal but opposite positions in the futures contracts to the corresponding cash position). Results are presented in table 4 below.

Table 4.

Average Hedge Ratio *RSDC Model* - **In Sample**

	Soybean	*Soybean Meal*	*Soybean Oil*
Regime 1	0.9456	1.0432	0.9296
Regime 2	0.5944	0.9088	0.8618

Average Hedge Ratio *RSDC Model* - **Out of Sample**

	Soybean	*Soybean Meal*	*Soybean Oil*
Regime 1	1.0436	1.0589	0.9527
Regime 2	0.7802	0.8946	0.8513

Average Hedge Ratio *Simple Hedge* - **In Sample**

	Soybean	*Soybean Meal*	*Soybean Oil*
Regime 1	0.8977	1.0318	0.9504
Regime 2	0.8485	0.9745	0.9128

Average Hedge Ratio *Simple Hedge* - **Out of Sample**

	Soybean	*Soybean Meal*	*Soybean Oil*
Regime 1	1.0288	1.0619	0.9726
Regime 2	0.9724	1.0037	0.9193

As may be noted, the difference in average hedge ratios between the two regimes is larger when the model takes into account the different dynamic relationships between soybean, soybean meal and soybean oil than for the case of a simple hedge consisting of a single product.

The following tables 5 and 6 contain the hedging effectiveness[4] provided by the two methods estimated (i.e. two regimes from the RSDC model and from the univariate cash futures cov/var quotient), along with the naïve hedging method (i.e. hedge ratio equal to 1) being compared to the case of the soybean complex not being hedged. Table 5 contains the average, variance and the hedging effectiveness for hedge ratios from the in sample data, and Table 6 contains the same statistics for the out of sample data. In both cases, there is an improved hedging effectiveness by using the regime switching model of dynamic correlations.

[4] Percentage reduction in the variance of the hedged margin with respect to the unhedged margin, equal to $1 - \frac{Var(hedged)}{Var(unhedged)}$, per Manfredo et al. (2000).

Table 5. Hedging Effectiveness - In Sample

Model		Mean	Variance	Percent Reduction
Unhedged		1.2665	0.1541	
Naive		1.2477	0.0641	*58.4024*
Simple	Regime 1	1.2433	0.0762	50.5129
	Regime 2	1.2436	0.0731	52.5328
	Combined	1.2426	0.0675	*56.2110*
RSDC	Regime 1	1.2469	0.0712	53.7724
	Regime 2	1.2396	0.1092	29.1071
	Combined	1.2293	0.0596	*61.3272*

Table 6.

Hedging Effectiveness - Out of Sample

Model		Mean	Variance	Percent Reduction
Unhedged		1.3459	0.1859	
Naive		1.2688	0.0125	*93.27*
Simple	Regime 1	1.2700	0.0182	90.24
	Regime 2	1.2742	0.0156	91.63
	Combined	1.2729	0.0154	*91.73*
RSDC	Regime 1	1.2686	0.0172	90.74
	Regime 2	1.2876	0.0234	87.42
	Combined	1.2826	0.0112	*94.00*

Results show that there is an improvement in using the model with Time Varying Correlations within a Regime Switching context when compared to the simple hedging model and the naïve hedging method. Improvement of over 3 percentage points are obtained in comparison of this former model to the naive model for in sample data, yet only a bit more than half a percentage point for out of sample data. Perhaps more data may be required in this latter case to obtain an improved variance reduction of the hedge ratio.

Discussions & Conclusions

We determined time-varying hedge ratios in a multiproduct setting using a multivariate state dependent model of regime switching dynamic correlations. The setting consisted of a soybean process, such that soybeans are purchased as input and production of soybean meal and soybean oil are sold as output. The model enabled to depict the time-varying correlations for multiple series of cash and future prices in two different regimes (i.e. the conditional correlation is not constant in this multivariate model). This provided an improved characterization of the multiproduct dynamic hedging process as it captures the evolution of the cash/futures correlation matrix when the model switches from a regime of low correlation to one of higher correlation or vice-versa. The model switches regimes according to a Markov chain process and does not have a dimensionality problem for larger numbers of series, as does the more conventional BEKK model.

Results show that there is an improvement in using the model with Time Varying Correlations within a Regime Switching context when compared to the simple hedging model and the naïve hedging method. In addition, the introduction of use to stock ratios of soybeans in the state dependent probabilities provided a mild improvement over the dynamic process of constant transition probabilities, yet the correlations estimated between regimes arrived at very similar results between the two methods. Perhaps estimation with other relevant variables such as soybeans meal and soybeans oil use to stock ratio may show more improvement. However, results indicate that the model applied, which may include state dependent related factors, provides a reasonable estimator of time varying correlations for the computation of multiproduct optimal dynamic hedge ratios with better outcome than the naive hedge.

References:

Anderson, R. and J.P. Danthine (1980), "Hedging and Joint Production: Theory and Illustration", *Journal of Finance*, 35: 487 – 498.

Baillie, R.T. and R.J. Myers (1991), "Bivariate GARCH estimation of the Optimal Commodity Futures Hedge", *Journal of Applied Econometrics,* 6, 109-124.

Collins, R.A. (2000), "The risk management effectiveness of multivariate hedging models in the U.S. Soy Complex", *The Journal of Futures Markets,* 20, 189-204

Engle, R., (2002) "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models", *Journal of Business & Economic Statistics*, 20(3): 339-350.

Fackler, P.L. and K.P. McNew (1993), "Multiproduct hedging: Theory, Estimation, and an Application", *Review of Agricultural Economics,* 15, 521-535.

Garcia, P., J. Roh, R.M. Leuthold (1995), "Simultaneously Determined, Time Varying Hedge Ratios in the Soy Complex", *Applied Economics,* 27, 1127-1134

Haigh, M.S., M.T. Holt (2000), "Hedging multiple price uncertainty in international grain trade", *American Journal of Agricultural Economics,* 82, 881-896.

Haigh, M.S., M.T. Holt (2002), "Combining time-varying and dynamic multi-period optimal hedging models", *European Review of Agricultural Economics,* 29, 471-500.

Lee, H.T., J.K. Yoder, R.C. Mittelhammer, J.J. McCluskey (2006), "A Random coefficient autoregressive Markov regime switching model for dynamic futures hedging", *The Journal of Futures Markets,* 26, 103-129.

Lee, H.T., J.K. Yoder, (2007a), "A bivariate Markov regime switching GARCH approach to estimate varying minimum variance hedge ratios", *Applied Economics,* 39, 1253-1265.

Lee, H.T., J.K. Yoder, (2007b), "Optimal Hedging with a Regime-Switching Time-Varying Correlation GARCH model", *The Journal of Futures Markets,* 27, 495-516

Manfredo, M.R., P. Garcia, R.M. Leuthold (2000), "Time_Varying Multiproduct Hedge Ratio estimation in the Soybean Complex: A Simplified Approach". *Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management*. Chicago, Illinois, April 17-18, 2000.

Myers, R.J., and S.R. Thompson (1989), "Generalized Optimal Hedge Ratio Estimation", *American Journal of Agricultural Economics,* 71, 858-868.

Noussinov, M.A., R.M. Leuthold (1999), "Optimal hedging strategies for the U.S. cattle feeder", *Journal of Agribusiness,* 17, 1-19

Pelletier, D.,(2006),"Regime Switching for Dynamic Correlations", *Journal of Econometrics*, 131:445-73

Peterson, P., R. Leuthold (1987), "A Portfolio approach to Optimal Hedging for a Commercial Feedlot", *The Journal of Futures Markets,* 7, 443-457.

Taylor, S., (1986). "Modelling Financial Time Series". Wiley, New York.

Tejeda, H.A., B.K. Goodwin, D. Pelletier (2009), "A State Dependent Regime Switching Model of Dynamic Correlations", *Selected paper presented at the AAEA Annual meeting*, Milwaukee, WI., July 26-28, 2009.

Tse, Y.K., K.C. Tsui (2002), "A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time Varying Correlations", *Journal of Business and Statistics*, 20: 351-362

Tzang, D., R. Leuthold (1990), "Hedge ratios under inherent risk reduction in a commodity complex", *The Journal of Futures Markets,* 10, 497-504

Appendix 1

$$b_{b,t-3} = \frac{\begin{gathered} \mathrm{Cov}(F_bS_m)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mF_o) - \mathrm{Cov}(F_bS_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mF_o) + \mathrm{Cov}(F_bS_o)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mF_o) + \\ \mathrm{Cov}(F_bS_b)\mathrm{Var}(F_o)\mathrm{Var}(F_m) - \mathrm{Cov}(F_bS_o)\mathrm{Var}(F_o)\mathrm{Var}(F_m) - \mathrm{Cov}(S_mF_b)\mathrm{Var}(F_o)\mathrm{Var}(F_m) + \\ \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oS_o)\mathrm{Var}(F_m) + \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oS_m)\mathrm{Var}(F_m) + \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_b) - \\ \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_o) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_m) - \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_oS_o) + \\ \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_oS_b) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mS_m)\mathrm{Var}(F_o) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mS_o)\mathrm{Var}(F_o) - \\ \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mS_b)\mathrm{Var}(F_o) - \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_oS_m) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oS_b)\mathrm{Var}(F_m) \end{gathered}}{|D|}$$

$$b_{m,t-3} = \frac{\begin{gathered} \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_oS_o) - \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oS_b) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_bS_b)\mathrm{Var}(F_o) - \\ \mathrm{Cov}(F_mF_b)\mathrm{Cov}(S_oF_b)\mathrm{Var}(F_o) - \mathrm{Cov}(F_bF_m)\mathrm{Cov}(S_mF_b)\mathrm{Var}(F_o) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oS_m) - \\ \mathrm{Cov}(F_oF_m)\mathrm{Cov}(F_oS_o)\mathrm{Var}(F_b) + \mathrm{Cov}(F_oF_m)\mathrm{Cov}(F_oS_b)\mathrm{Var}(F_b) + \mathrm{Cov}(F_mS_m)\mathrm{Var}(F_b)\mathrm{Var}(F_o) + \\ \mathrm{Cov}(F_mS_o)\mathrm{Var}(F_b)\mathrm{Var}(F_o) - \mathrm{Cov}(F_bS_b)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_mF_o) - \mathrm{Cov}(F_mS_b)\mathrm{Var}(F_b)\mathrm{Var}(F_o) + \\ \mathrm{Cov}(F_bS_o)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_mF_o) + \mathrm{Cov}(S_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_mF_o) - \mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_oS_m)\mathrm{Var}(F_b) + \\ \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(S_bF_m) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_mS_o) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(S_mF_m) \end{gathered}}{|D|}$$

$$b_{o,t-3} = \frac{\begin{gathered} -\mathrm{Cov}(F_oS_b)\mathrm{Var}(F_b)\mathrm{Var}(F_m) - \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_mS_b) + \mathrm{Cov}(F_oS_o)\mathrm{Var}(F_b)\mathrm{Var}(F_m) + \\ \mathrm{Cov}(F_oS_b)\mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_bF_m) + \mathrm{Cov}(F_bF_m)\mathrm{Cov}(F_bF_o)\mathrm{Cov}(S_oF_m) - \mathrm{Cov}(F_oF_m)\mathrm{Cov}(F_mF_b)\mathrm{Cov}(S_bF_b) + \\ \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(S_mF_m) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_bS_m)\mathrm{Var}(F_m) - \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_bS_o)\mathrm{Var}(F_m) + \\ \mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_bS_b)\mathrm{Var}(F_m) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_bS_m) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_bS_o) - \\ \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oS_o) + \mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_b)\mathrm{Var}(F_b) - \mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_o)\mathrm{Var}(F_b) - \\ \mathrm{Cov}(F_mF_o)\mathrm{Cov}(F_mS_m)\mathrm{Var}(F_b) + \mathrm{Cov}(S_mF_o)\mathrm{Var}(F_b)\mathrm{Var}(F_m) - \mathrm{Cov}(F_oS_m)\mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_bF_m) \end{gathered}}{|D|}$$

$|D| =$

$$\begin{gathered} \mathrm{Var}(F_o)\mathrm{Var}(F_b)\mathrm{Var}(F_m) - \mathrm{Var}(F_o)\,\mathrm{Cov}(F_mF_b)\,\mathrm{Cov}(F_mF_b) - \mathrm{Cov}(F_bF_o)\mathrm{Cov}(F_bF_o)\mathrm{Var}(F_m) - \\ \mathrm{Cov}(F_oF_m)\mathrm{Cov}(F_mF_o)\mathrm{Var}(F_b) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oF_m) + \mathrm{Cov}(F_mF_b)\mathrm{Cov}(F_oF_b)\mathrm{Cov}(F_oF_m) \end{gathered}$$

Appendix 2

The transition probability matrix Π_t:

	State 1 *Time t*	State 2
State 1	π_t^{11} P(Δ_t= 1 | Δ_{t-1}= 1, x_{t-1}; γ_1) = $\frac{\exp(x'_{t-1}\gamma_1)}{1+\exp(x'_{t-1}\gamma_1)}$	π_t^{12} = (1 - π_t^{11}) P(Δ_t= 2 | Δ_{t-1}= 1, x_{t-1}; γ_1) = 1 - $\frac{\exp(x'_{t-1}\gamma_1)}{1+\exp(x'_{t-1}\gamma_1)}$
Time t-1 **State 2**	π_t^{21} = (1 - π_t^{22}) P(Δ_t= 1| Δ_{t-1}= 2, x_{t-1}; γ_2) = 1 - $\frac{\exp(x'_{t-1}\gamma_2)}{1+\exp(x'_{t-1}\gamma_2)}$	π_t^{22} P(Δ_t= 2 | Δ_{t-1}= 2, x_{t-1}; γ_2) = $\frac{\exp(x'_{t-1}\gamma_2)}{1+\exp(x'_{t-1}\gamma_2)}$

where $x_{t-1} = (1, x_{1,t-1}, \ldots.., x_{(m-1),t-1})'$ & γ_i[5] $= (\gamma_{i1}, \gamma_{i2}, \ldots \ldots \ldots, \gamma_{i(m-1)})$;

[5] Here we use $\gamma_i$same as is if it was β_i of previous probability equations.