Do Theoretical Restrictions Matter for the Translog Stochastic Production Function? Evidence from the Kansas Farm Sector

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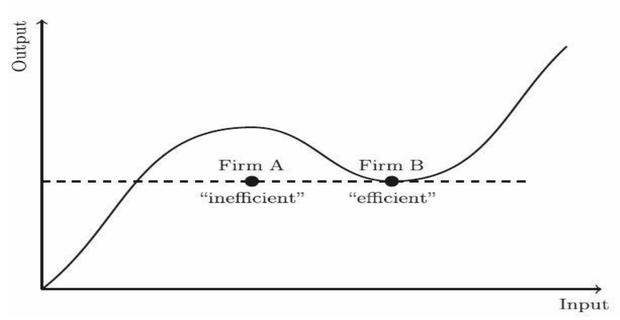
Introduction:

Microeconomic theory suggests that production functions should be monotonically increasing in all inputs. A reasonable interpretation of the relative technical efficiencies of firms is impossible if the efficiencies are estimated from a stochastic production frontier (SPF) that violates the monotonicity assumption.

However, many empirical applications of the SPF to the farm sector often present results in which the monotonicity condition is not fulfilled.

Objective:

- To assess the importance of imposing monotonicity in the estimation of a translog stochastic production frontier.
- We accomplish this by comparing the mean technical efficiency scores, partial input elasticities, and in-sample and out-of-sample predictions of the unrestricted and restricted translog stochastic production frontiers for the Kansas farm sector.



Firm A is technically inefficient, it is below the frontier. Firm B is efficient, it is on the frontier. Firm B uses more input to produce the same output as firm A. This illustrates that the relative efficiency estimates based on a non-monotone production function cannot be reasonably interpreted

Figure 1. Non-monotone production frontier

Methods: Three-Step Estimation

1. Estimate Unrestricted Frontier

$$\ln y = \ln f(\mathbf{x}, \beta) - u + v, u = \mathbf{z}'\delta + \varepsilon$$

$\Rightarrow$ unrestricted $\hat{\beta}$ parameters

$\Rightarrow$ their covariance matrix $\sum_{\beta}$

2. Minimum Distance Estimation

$$\hat{\beta}^0 = \operatorname{argmin}\left[(\hat{\beta}^0 - \hat{\beta})' \sum_{\beta}^{-1} (\hat{\beta}^0 - \hat{\beta}) \right]$$

$$s.t.\ MP_i(\mathbf{x}, \hat{\beta}^0) \geq 0 \forall i, \mathbf{x}$$

3. Final Frontier Estimation

Calculate frontier output of each firm and estimate frontier

$$\Rightarrow y^{max} = f(\mathbf{x}, \hat{\beta}^0)$$

$$\Rightarrow \ln y = \alpha_0 + \alpha_1 \ln y^{max} - u + v, u = \mathbf{z}'\delta^0 + \varepsilon$$

Data

- 1 output (gross farm income) and 3 inputs (capital, labour and purchased inputs) for 564 farms for the period 1993 to 2007. All variables are measured in real dollar values with year 2007 as the base year
- 2 variables are used to explain inefficiency (farm size and specialization)

The Empirical Translog SPF Model

$$\ln y = \ln f(\mathbf{x}, \boldsymbol{\beta}) = \beta_0 + \sum_{i=1}^{n} \beta_i \ln x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} \ln x_i \ln x_j, \text{with } \beta_{ij} = \beta_{ji}$$

Empirical Results:

Table 1. Unrestricted Stochastic Frontier Estimation

	Estimate	Std. Error	t value	Pr(>|t|)
a_0	2.509	1.095	2.291	0.022
a_K	0.400	0.173	2.312	0.021
a_L	-0.388	0.187	-2.077	0.038
a_P	0.811	0.135	6.016	0.000
a_T	-0.037	0.016	-2.256	0.024
b_K_K	0.045	0.022	2.098	0.036
b_K_L	0.038	0.017	2.241	0.025
b_K_P	-0.059	0.015	-3.882	0.000
b_K_T	-0.003	0.002	-1.677	0.094
b_L_L	-0.064	0.024	-2.617	0.009
b_L_P	0.005	0.016	0.316	0.752
b_L_T	0.000	0.002	0.120	0.904
b_P_P	0.016	0.015	1.063	0.288
b_P_T	0.006	0.002	3.578	0.000
b_T_T	0.002	0.001	3.693	0.000
Z_Dvs	1.181	0.019	63.303	0.000
Z_Ds	0.710	0.015	46.132	0.000
Z_Dm	0.390	0.013	31.190	0.000
Z_Sliv	0.018	0.008	2.410	0.016
Z_Smix	0.012	0.006	2.104	0.035
sigmaSq	0.041	0.001	56.617	0.000
gamma	0.402	0.070	5.776	0.000

The production function is monotonically increasing at 5403 out of 7579 observations (71.3%). It is quasiconcave at 4779 out of 7579 observations (63.1%)

Note: K is capital, L is labor, P is purchased inputs and T is time. Dummies for farm size categories and specializations are: Dvs for very small farms, Ds for small farms and Dm for medium sized farms; Sliv is for livestock farms and Smix is for farms with both livestock and crops.

Table 2. Minimum Distance Estimation

	coef	diff	diff/std. err	adj.coef
a_0	-0.422	-0.385	2.934	2.930
a_K	0.038	0.218	0.363	0.363
a_L	-0.151	-0.810	-0.236	-0.237
a_P	0.051	0.376	0.760	0.760
a_T	-0.031	-1.907	-0.006	-0.006
b_K_K	0.008	0.372	0.037	0.037
b_K_L	0.004	0.257	0.034	0.034
b_K_P	-0.010	-0.665	-0.049	-0.049
b_K_T	-0.002	-1.000	-0.001	-0.001
b_L_L	-0.015	-0.613	-0.049	-0.049
b_L_P	0.009	0.600	-0.004	-0.004
b_L_T	-0.001	-0.556	0.001	0.001
b_P_P	0.004	0.234	0.013	0.013
b_P_T	0.004	2.400	0.002	0.002
b_T_T	0.001	2.800	0.000	0.000

This function is monotonically increasing at 7579 out of 7579 observations (100%). The function is quasiconcave at 7576 out of 7579 observations (100%).

Table 3. Final Stochastic Frontier Estimation

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.004	0.157	-0.028	0.978
LcFitted	1.000	0.012	85.910	0.000
Z_Dvs	1.187	0.018	65.362	0.000
Z_Ds	0.715	0.014	51.466	0.000
Z_Dm	0.393	0.012	33.290	0.000
Z_Sliv	0.018	0.007	2.576	0.010
Z_Smix	0.012	0.005	2.124	0.034
sigmaSq	0.041	0.001	59.536	0.000
gamma	0.379	0.060	6.272	0.000

The coefficient of intercept is virtually zero and coefficient of frontier output is virtually one

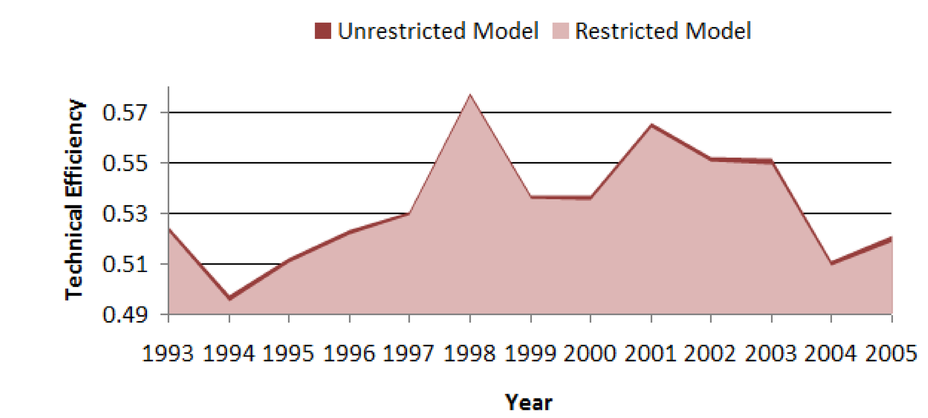


Figure 2. Average Technical Efficiency Scores

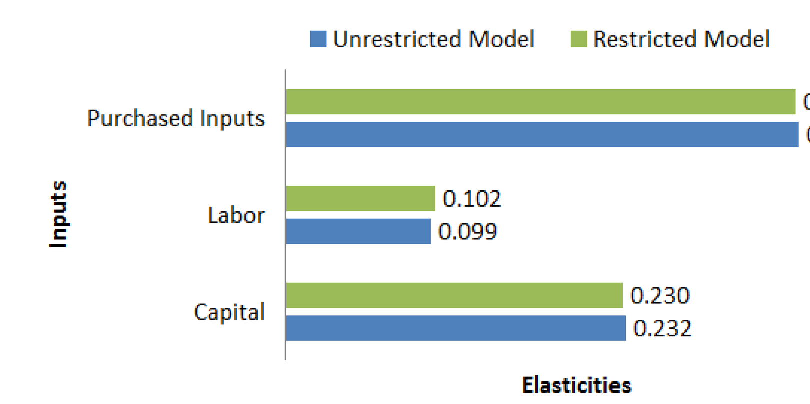
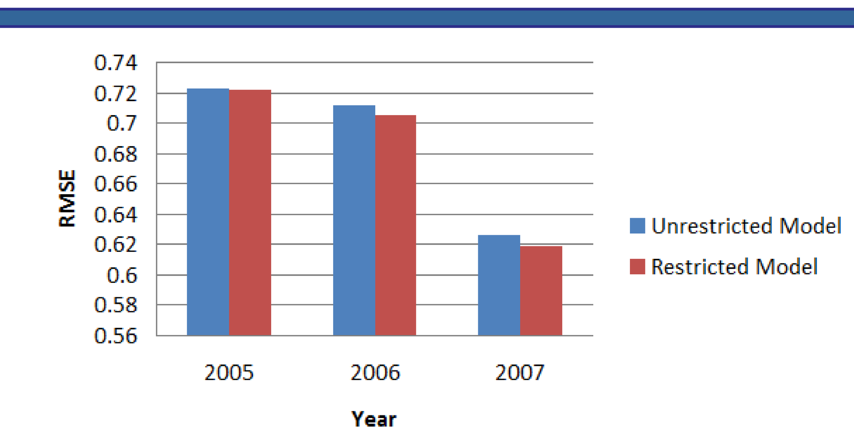


Figure 3. Input Partial Elasticities



In-sample prediction :2005 and 2006. Out of sample prediction: 2007

Figure 4. Root Mean Square Errors

Summary & Conclusion:

On average, technical efficiency scores for the unrestricted model (0.534) are slightly higher than those from the restricted model (0.528).

Imposing monotonicity decreased the partial elasticities of purchased inputs from 0.350 to 0.348 and capital from 0.232 to 0.230. The partial elasticity of labor increased from 0.099 to 0.102.

The restricted model outperforms the unrestricted model both in-sample and out of sample predictions.

Theoretical restrictions do matter in the estimation of empirical stochastic production functions. Failure to take care of those regulatory conditions may result in improper policy recommendations.

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