A DYNAMIC DUAL MODEL UNDER STATE-CONTINGENT PRODUCTION UNCERTAINTY

Teresa Serra[1], Spiro Stefanou[2,3] and Alfons Oude Lansink[3]

[1]Centre de Recerca en Economia i Desenvolupament Agroalimentari (CREDA-UPC-IRTA) Spain, teresa.serra-devesa@upc.edu; [2]Department of Agricultural Economics and Rural Sociology, Penn State University, USA; [3]Business Economics Group, Wageningen University, Netherlands.

**Paper prepared for presentation at the 114th EAAE Seminar
'Structural Change in Agriculture', Berlin, Germany, April 15 - 16, 2010**

Abstract

In this paper we assess how production costs and capital accumulation patterns in agriculture have evolved over time, by paying special attention to the influence of risk. A dynamic state-contingent cost minimization approach is applied to assess production decisions in US agriculture over the last century. Results suggest the relevance of allowing for the stochastic nature of the production function which permits to capture both the differences in the costs of producing under different states of nature, the differences in the evolution of these costs over time, as well as the differential impacts of different states of nature on investment decisions.

Keywords: risk, state-contingent, dynamic model, investment decisions.

JEL classification: D21

1. Introduction

The influence of risk on agricultural production decisions has been addressed widely in the literature in both proposing theoretical modeling and empirical assessments and investigations. As a result of unpredictable weather conditions, pest infestations, unstable markets, etc., risk effects have been of special interest in agriculture (Chavas and Holt, 1996; Moschini and Hennessy, 2001). A range of different techniques have been developed to model risk and risk preferences. A priori probability assessments have usually served as risk assessment tools, which are known to lead to potentially serious biases (Camerer, 1995). Risk preferences have been generally measured within the expected utility model (Saha et al., 1994) which has also been questioned as a useful tool to adequately represent economic agents' risk attitudes (see Rabin, 2000; Just and Peterson, 2003).

Technical change in agriculture can contribute to mitigate risk by means of improved management techniques, introduction of genetic varieties that are more resistant to weather fluctuations, or improvement in feeding practices. In addition to the output enhancing prospects of many technological innovations, a tangible consequence of technological progress in agriculture are changes in the cost of facing production risk.

Pope and Chavas (1994) demonstrate that cost minimization cannot be adequately characterized by expected output alone under risk aversion, because the role of risk management in input use can be relevant. Chambers and Quiggin (1998, 2000) propose an alternative characterization of choice under uncertainty by representing the stochastic technology using a state-contingent input correspondence and they show that under a state-contingent approach a standard cost minimization problem applies irrespective of risk preferences.

The state-contingent approach is based on the assumption that production under uncertainty can be represented by differentiating outputs according to the state of nature in which they are realized and has its foundations in Debreu (1959) and Arrow (1965). The state-contingent approach offers two main advantages over more traditional methods. First, it does not require a probability assessment of output risk and second, it is applicable independently of the risk preferences of the decision maker.

In spite of its appeal as a tool to model production risk, the state-contingent approach has seen very few empirical applications. O'Donnell and Griffiths (2006) and Chavas (2008) constitute two notable exceptions. O'Donnell and Griffiths (2006) propose an approach based on a finite mixtures framework to estimate a state-contingent production frontier. Developing a methodology to specify and estimate cost-minimizing input choices, Chavas (2008) results provide evidence that the cost of facing production risk has declined in US agriculture over the last few decades as a result of technological progress.

The innovative work by Chavas (2008) does not explicitly model investment demand and associated dynamics. By working with a static cost minimization framework, capital is assumed to be a fixed input. However, the role of uncertainty on production decision making and investment patterns remains an open question. The use of a state-contingent framework is particularly useful to introduce production risk in dynamic models, since their complexity makes it difficult to model risk and risk attitudes by means of an expected utility model. In this paper we advance a dynamic state-contingent cost minimization approach, to assess production decisions in US agriculture over the last century and determine how the costs of producing under different states of nature have changed over time.

Previous research has analyzed capital accumulation in US agriculture by paying a special attention to the relevance of the role of input prices in signaling technological progress (Olmstead and Rhode, 1993; Thirtle et al., 2002). While the influence of price risk and uncertainty on capital investment in agriculture has been assessed by previous research (Luh and Stefanou, 1996; Pietola and Myers, 2000; Sckokai, 2005; Serra et al., 2009), the role of risk on US agriculture capital accumulation patterns has not been studied using the state-contingent methodology. This paper contributes to previous literature by providing insights on this issue.

The next section presents a dynamic dual model of dynamic decision making under intertemporal cost minimization in a state-contingent setting and measures the state-contingent, ex-ante output by simulating an output distribution using the ex-post observations. In the empirical specification section the model is specified following Epstein (1981). The empirical application section presents model estimation results which are based on an augmented version of the data found in Thirtle et al. (2002). The paper concludes with the concluding remarks section.

2. The Model

Focusing on the production function specification, we follow Chambers and Quiggin (1996, 1997, 2000) by representing the stochastic technology using a state-contingent approach. Assume a single-output firm. Uncertainty is represented by a set of states of nature $\Omega=\{1,2,...,S\}$. Let $\mathbf{x}=(x_1,...,x_n)' \in \mathrm{R}_+^n$ be a vector of variable inputs and $\mathbf{k}=(k_1,...,k_m)' \in \mathrm{R}_+^m$ a vector of quasi-fixed inputs. These inputs are assumed to be allocated before uncertainty is resolved. While variable input quantities are assumed to be adjusted at no cost, nonzero adjustment costs are supposed for quasi-fixed factors. These capital adjustment costs are expressed as a reduction in output that results from diverting resources away from production when gross investments $\mathbf{I}=(I_1,...,I_m)' \in \mathrm{R}_+^m$ take place (Brechling, 1975). Inputs $\mathbf{x}$ and $\mathbf{k}$ are devoted to produce the state-contingent output $\mathbf{y}=(y_1,...,y_s)' \in \mathrm{R}_+^s$, where y_s is the quantity of output realized under the sth state of nature when the producer has chosen the ex-ante input-output combination $(\mathbf{x},\mathbf{k},\mathbf{I},\mathbf{y})$.

Under the state contingent approach, the stochastic production function specification is $f:\mathrm{R}_+^{n+m}\times\mathrm{R}_+^s$, where $(\mathbf{x},\mathbf{k},\mathbf{I},\mathbf{y})\in f$. Dual to the state-contingent input correspondence is the cost function which is defined for the case where all inputs are purchased at given prices and the firm holds static expectations on real prices, real capital rental rates, as well as on discount and depreciation rates. The intertemporally cost minimizing firms choose an investment path defined by the following optimization problem:

$$V(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})=\min_{x,I} \int_t^\infty e^{-rt}\left[\mathbf{w}'\mathbf{x}(t)+\mathbf{c}'\mathbf{k}(t)\right]dt$$

$$s.t.$$

$$\dot{\mathbf{k}} = \mathbf{I} - \delta\mathbf{k}$$

$$\mathbf{y}(t) = F\left[\mathbf{x}(t),\mathbf{k}(t),\mathbf{I}(t)\right] \ t\in\left[t,\infty\right)$$

(1)

where $V(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})$ is a value function that represents the long-run cost function starting at time t, $\mathbf{w}$ is a variable input price vector, $\mathbf{c}$ is a vector of capital rental rates, δ is a diagonal matrix

containing depreciation rates, $\dot{\mathbf{k}}$ is a vector of time derivatives of capital paths, r is the interest rate, and F is the transformation function that meets the usual regularity conditions (see Epstein and Denny, 1983; Stefanou, 1989).

The value function $V(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})$ is assumed to be real-valued, non-negative, twice continuously differentiable, non-increasing in $(\mathbf{w},\mathbf{c})$, decreasing in $\mathbf{k}$, and concave in $(\mathbf{w},\mathbf{c})$. The Hamilton-Jacobi-Bellman equation corresponding to the optimization program is:

$$rV(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})= \min_{x,I}\left[\mathbf{w}'\mathbf{x}+\mathbf{c}'\mathbf{k}+(\mathbf{I}-\delta\mathbf{k})'V_k(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})\right]+\varphi\left[\mathbf{y}-F(\mathbf{x},\mathbf{k},\mathbf{I})\right], \tag{2}$$

where φ is the Lagranger multiplier associated to the production target and is shown in Stefanou (1989) to be defined as the short-run instantaneous marginal cost, $rV(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})$ is the instantaneous imputed cost of producing $\mathbf{y}$, and subscripts denote derivatives.

The first derivatives of this expression with respect to input prices yield input demand equations:

$$\begin{aligned}\mathbf{x} &= rV_w - \dot{\mathbf{k}}V_{kw} \\ \dot{\mathbf{k}} &= V_{kc}^{-1}(rV_c - \mathbf{k}).\end{aligned} \tag{3}$$

Following Chambers and Quiggin (1998, 2000), if actual input choices do not minimize cost, under the assumption of income non-satiation, choosing $\mathbf{x}$ and $\mathbf{I}$ according to (2) will improve the welfare of the decision maker irrespective of risk preferences. Hence, under the state-contingent approach, the standard cost minimization model is applicable independently of risk attitudes.

The empirical modeling challenge is to measure the state-contingent output when only ex-post data are available as is the case in aggregated national account data series. The Chavas (2008) approach allows for technological progress by assuming that each observation on the firm can be associated to a different technology $t = 1,...,T$, where index t represents both time and

technology. At time t we observe the vectors of used inputs $\mathbf{x}_t = (x_{1t}, ..., x_{nt})'$ and $\mathbf{k}_t = (k_{1t}, ..., k_{mt})'$, the vector of gross investments $\mathbf{I}_t = (I_{1t}, ..., I_{mt})'$, input prices $\mathbf{w}_t = (w_{1t}, ..., w_{nt})'$ and $\mathbf{c}_t = (c_{1t}, ..., c_{mt})'$, and output y_t. Variable y_t is an ex-post output which is just one realization of the many ex-ante output possibilities.

Chavas (2008) recovers the ex-ante technology by defining a new random variable, e, which is a deterministic transformation of y_t capturing the relative changes of output across states of nature. The s*th* realization of e is defined as $e_s \equiv (y_s / \mu_t)^{1/\sigma_t}$, where μ_t and σ_t are positive numbers. While μ_t is assumed to capture the nature of the production technology, σ_t can be interpreted as a parameter that allows the spread of the output distribution to vary across different observations. Under this specification the relative effects of production uncertainty on each output vary across observations only through parameter σ_t.

Production uncertainty is measured by assuming an auxiliary variable z_t that under state s satisfies the following condition:

$$\ln(z_t) = \ln(o_t) + \sigma_t \ln(e_t) \tag{4}$$

This expression can be considered as an econometric model where $\ln(z_t)$ is the dependent variable, $\ln(o_t)$ is the specification of a regression line, and $\sigma_t \ln(e_t)$ is the error term, with σ_t reflecting possible heteroscedasticity. According to (4), the different states of nature have the same relative effects on output y_t than on variable z_t.

If $\ln(e_t)$ has mean zero and unit variance, $\ln(o_t)$ measures the expected value of $\ln(z_t)$ and σ_t its standard deviation. Upon selecting a parametric specification for both the mean and standard deviation, the maximum likelihood (ML) estimation procedure can be used to obtain consistent parameter estimates.

If at time t state s occurs, one can estimate the vector of T realized values of the random variable $e:\left((z_1/o_1)^{1/\sigma_1},...,(z_T/o_T)^{1/\sigma_T}\right)$ which in turn allows us to derive the simulated state-contingent outputs:

$$\mathbf{y}_t^e = \left\{ y_{rt} : y_{rt} = y_t \frac{(z_r/o_r)^{\sigma_t/\sigma_r}}{(z_t/o_t)}; r = 1,...,T \right\} \tag{5}$$

It is important to recall that the spread of the output distribution is allowed to vary across observations through σ_t/σ_r. Under these assumptions, ex-ante outputs do not depend on the nature of the technology, μ_t (Chavas, 2008).

3. Empirical Specification

Chavas (2008) notes that consistent estimates of the mean and variance of $\ln(z_t)$ allow us to simulate the state-contingent outputs which in turn can be used to consistently estimate the dynamic cost-minimization model. A strict implementation of Chavas (2008) methodology is problematic since these ex-ante outputs, which are explanatory variables in the cost-minimization model, tend to be correlated with each other and can generate potential multicollinearity in model estimation.

Chavas (2008) proposes a parsimonious parametric specification which involves working with a reduced version of the actual state space by defining L intervals for the output variable as: $V_{1t} = \left[-\infty, b_{1t}\right]$, $V_{lt} = \left(b_{(l-1)t}, b_{lt}\right]$, $l = 2,...,L-1$ and $V_{Lt} = \left(b_{Lt}, +\infty\right]$, $t = 1,...,T$. The ex-ante outputs are classified into these intervals and the following dummy variable is computed $I_{lrt} = 1$ if

$y_{rt} \in V_{lt}$ and $I_{lrt} = 0$ otherwise.[1] Output means in each interval, $y_{lt} = \left(\sum_{r=1}^{T} I_{lrt} y_{rt} \right) \Big/ \left(\sum_{r=1}^{T} I_{lrt} \right)$,

are then taken to define the reduced ex-ante output vector: $\mathbf{y}^{L} = \left\{ y_{lt} : l = 1, ..., L \right\}$.

Since collinearity problems arise as the representation of the state-space is more accurate, we focus on a single output and we restrict the number of states of nature to two ($L=2$), where $L=1$ and $L=2$ correspond to an unfavorable, y_1, and a favorable, y_2, states of nature, respectively.

The estimation of the regression $\ln(z_t) = \ln(o_t) + \sigma_t \ln(e_t)$ employs a GARCH (1,1) specification. The dependent variable in the GARCH model is the logarithm of a partial productivity measure computed as the ratio of an index of aggregate output on a per unit of land. This productivity measure is assumed capable of capturing production uncertainty. In line with Chavas (2008), the structural part of the model is specified as a function of an aggregate machinery and land price index and a fertilizer price index, both normalized by the output price index. These indices capture the effects of market conditions on yields.[2] A research and development expenditures[3] index (RD) normalized by the output price index and on a per unit of land, and the lagged dependent variable ($\ln(z_{t-1})$) are also included in the structural part of the GARCH model and the regression is estimated using ML techniques.

The empirical specification considers one variable input x representing materials and whose price will be the numeraire in the normalized specification of the long-run cost function. Further, we distinguish between two quasi-fixed inputs (k_1 and k_2) one representing labor and the other an aggregate measure of capital. Under this specification, the value function V depends on $\mathbf{y}^{L}$, $\mathbf{c}$, and $\mathbf{k}$, where $\mathbf{c}$ is now a vector of normalized capital rental rates.

Following Epstein (1981), $V(\mathbf{y}^{L}, \mathbf{c}, \mathbf{k})$ is specified as:

---

[1] The definition of the intervals is restricted to ensure that there is at least one observation in each one.

[2] The normalized labor price index was not statistically significant and thus was discarded.

[3] Including both private and public expenditures.

$$V(\mathbf{y}^L, \mathbf{c}, \mathbf{k}) = a_0 + \begin{pmatrix} a_1' & a_2' & a_3' \end{pmatrix} \begin{pmatrix} \log \mathbf{y}^L \\ \log \mathbf{c} \\ \mathbf{k} \end{pmatrix} +$$

$$\begin{pmatrix} (\log \mathbf{y}^L)' & (\log \mathbf{c})' & (\mathbf{k})' \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{F} & \mathbf{G} \\ \mathbf{F}' & \mathbf{N} & \mathbf{0} \\ \mathbf{G}' & \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \log \mathbf{y}^L \\ \log \mathbf{c} \\ \mathbf{k} \end{pmatrix} + \mathbf{c}' \mathbf{M}^{-1} \mathbf{k}$$

(6)

where $\quad a_1 = \begin{bmatrix} a_{11} & a_{12} \end{bmatrix}'$, $\quad a_2 = \begin{bmatrix} a_{21} & a_{22} \end{bmatrix}'$, $\quad a_3 = \begin{bmatrix} a_{31} & a_{32} \end{bmatrix}'$, $\quad \mathbf{A} = \begin{bmatrix} A_{y1y1} & A_{y1y2} \\ A_{y1y2} & A_{y2y2} \end{bmatrix}$,

$$\mathbf{N} = \begin{bmatrix} N_{c1c1} & N_{c1c2} \\ N_{c1c2} & N_{c2c2} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} D_{k1k1} & D_{k1k2} \\ D_{k1k2} & D_{k2k2} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} F_{y1c1} & F_{y1c2} \\ F_{y2c1} & F_{y2c2} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} G_{y1k1} & G_{y1k2} \\ G_{y2k1} & G_{y2k2} \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} M_{k1c1} & M_{k2c1} \\ M_{k2c1} & M_{k2c2} \end{bmatrix},$$

with the symmetry of $\mathbf{A}$, $\mathbf{N}$, $\mathbf{D}$ and $\mathbf{M}$. The conditional demand for the numeraire variable input x_n can be expressed as:

$$x_n^* = ra_0 + ra_1' \log \mathbf{y}^L + ra_2' (\log \mathbf{c} - (\hat{\mathbf{c}}^{-1})' \mathbf{c}) + r\left(\frac{1}{2} \log \mathbf{c}' - \mathbf{c}' \hat{\mathbf{c}}^{-1}\right) \mathbf{N} \log \mathbf{c} +$$

$$r\left(\log \mathbf{c}' - \mathbf{c}' \hat{\mathbf{c}}^{-1}\right) \mathbf{F}' \log \mathbf{y}^L + a_3' (r\mathbf{k} - \dot{\mathbf{k}}) + \left(\frac{r}{2}(\log \mathbf{y}^L)' \mathbf{A} \log \mathbf{y}^L\right) + \mathbf{k}' \mathbf{D}\left(\frac{1}{2} r\mathbf{k} - \dot{\mathbf{k}}\right) +$$

(7)

$$(\log \mathbf{y}^L)' \mathbf{G}(r\mathbf{k} - \dot{\mathbf{k}})$$

and the conditional demands for the quasi-fixed assets are:

$$\dot{\mathbf{k}}^* = r\mathbf{M}\hat{\mathbf{c}}^{-1}(a_2 + \mathbf{F}' \log \mathbf{y}^L + \mathbf{N} \log \mathbf{c}) + (r\mathbf{U} - \mathbf{M})\mathbf{k}$$

(8)

Where $\hat{\mathbf{c}} = \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}$, and $\mathbf{U}$ is an identity matrix with the same size as $\mathbf{M}$. Expression (8) is a multivariate accelerator model that allows to assess the nature of the capital adjustment process and $(r\mathbf{U} \text{-} \mathbf{M})$ is the adjustment matrix showing the adjustment of capital to the steady-state capital stock. Stability is guaranteed when the adjustment matrix is negative semidefinite.

4. Empirical Application

Our model is applied to US agriculture over the period 1910-1990, and we ask how the costs of facing different production risks have been changing over time when accounting for the quasi-fixity of assets. We also provide insights on the impacts of these risks on investment decisions. An augmented version of the dataset found in Thirtle et al. (2002) is used to estimate the model. This dataset contains information on input price and quantity indices for the US agriculture (as an aggregate) and for the period 1910-1990. More specifically, this dataset contains quantity and price information on the following inputs: agricultural land, fertilizers, labor and machinery. Further details on this dataset can be obtained from the appendix to the Thirtle et al. (2002) paper. The augmentation of the Thirtle et al. (2002) series is to incorporate output, aggregate output quantity and price indices derived from the US Historical Statistics and USDA databases. The full data series and its documentation can be found in Appendices A and B in the *Journal of Productivity Analysis* 30(1): 89-98.[4]

We distinguish two quasi-fixed inputs, k_1 and k_2, and their respective prices, c_1 and c_2, representing labor, and land and machinery, respectively. To define k_2, individual quantity and price indices for land and machinery are aggregated using an expenditure-weighted geometric mean. The fertilizer index is used as a variable input series (x) whose price (w) serves as the normalization variable in the dynamic cost minimization model. The state-contingent output (y)

---

[4] A crop output and a livestock output are also available from the dataset.

is an index of aggregate agricultural production. A fixed interest rate equal to 5.5%, the average interest rate during the period analyzed, is used.[5]

The vector of state-contingent outputs is derived by defining $\ln(z_t)$ as the logarithm of the aggregate output quantity on a per unit of land. Results of the GARCH model estimation are presented in table 1 and indicate a strong and positive influence of normalized research and development expenditures on agricultural yields. An increase in the price of variable inputs (w) relative to output prices goes to the detriment of yields, while more expensive fixed inputs (c_2) relative to output prices, stimulate an increase in productivity. The results of simulating the state-contingent outputs are presented in figure 1, where it can be seen that the ratio y_1/y_2 (i.e, the ratio of unfavorable to favorable yields) shows a strong downward trend during the Great Depression. After a recovery period, the ratio returns to slightly above pre-depression levels and declines again with the farm financial crisis of the early-to-mid 1980s. This suggests that during difficult economic times, the output obtained under favorable states of nature grows quicker than the output under less advantageous conditions, which may be the result of firms adopting more conservative production practices.

The mean value of the ex-ante simulated output under state $s = 1$ (y_1) is 75.22, with 40.76 and 126.80 being the minimum and the maximum values, respectively. The mean value of output under state $s = 2$ (y_2) is 84.29 with a minimum of 46.12 and a maximum of 145.79. The realized output y, on the other hand, has a mean of 80.66 and minimum and maximium values of 43 and 142, respectively.

Equations (7) and (8) are jointly estimated by SUR and the results are presented in table 2. Two dummy variables representing the period of the Great Depression and the farm financial crisis of the 1980s were added to each equation to capture the impacts of these economic shocks on input demand. As is usual in empirical applications of dynamic dual models, the adjustment as measured by the R^2, is better for the variable input demand equation than for the quasi-fixed input demands. The Wald test for the joint significance of the model indicates that this significance cannot be rejected at the 1% confidence level.

---

[5] Interest rates data were obtained from EH net at http://www.measuringworth.org/interestrates/.

Parameter estimates lead to the following adjustment matrix that contains the capital adjustment rates:

$$(r\mathbf{U} \cdot \mathbf{M}) = \begin{pmatrix} -0.114 & \\ -0.022 & -0.022 \end{pmatrix}$$
(9)

The null hypothesis that capital fully and immediately adjusts to its long-run equilibrium (i.e. that diagonal elements of the adjustment matrix are -1, while off-diagonal entries are 0) is rejected at the 1% confidence level. It is also noteworthy that labor (k_1) requires about 9 years to adjust to long-run equilibrium, while the composite capital index (k_2), including land and machinery, requires around 46 years. The slow adjustment displayed by k_2 may be partly due to land market rigidities. The negative semidefiniteness of the adjustment matrix guarantees convergence (though very slow) to the long-run equilibrium.

From the parameter estimates we also derive the intermediate-run elasticities of the demand for capital with respect to capital prices. These elasticities are presented in table 3. The own price elasticity of k_2 is negative and statistically different from zero. The own price elasticity of k_1, negative as well, is not statistically significant. Cross price elasticities are negative too, which suggests that the two capital inputs are complementary.

Figure 2 tracks the long-run marginal costs of y_1 and y_2 over time and provides insight into improvements in technology over the last century that have led to a decline in the cost of producing an additional unit of output both under favorable and unfavorable conditions. In this regard, the dynamics of labor has been specially relevant in contributing to reduce the marginal cost of producing under unfavorable states of nature (G_{y1k1} is negative and statistically significant). It is also noteworthy that these costs registered important increases during the Great Depression. Minimum levels were registered at the beginning of the 1980s when oil prices started to decline after the 1973 and 1979 oil crises. Figure 2 also suggests a decline in the distance between the two marginal costs, implying that the extra cost of producing under unfavorable conditions relative to the favorable ones has been declining over time. This result suggests that

farmers have adopted improved risk management techniques or new technologies that were focused on reducing the marginal costs of production under unfavorable production conditions.

The ratio of the marginal cost of y_1 to the marginal cost of y_2 is presented in figure 3, indicating that producing under unfavorable states of nature is marginally more expensive than producing under more favorable ones, the ratio is greater than one. Although both the marginal costs of producing y_1 and y_2 have been declining over time, the evolution of the cost ratio has been more complex. During the Great Depression the ratio shows a definite trend upward, which is due to a faster increase in the cost of producing y_1 relative to y_2. After the Depression the ratio stabilizes and during the 1980s farm financial crisis it increases again. The evolution of this ratio is consistent with production patterns shown in figure 1. The comparison of both figures shows a negative correlation between production costs and production decisions. Increases in marginal costs (especially relevant to y_1) may be the result of firms reducing their investments during economic crises. It is important to note however, that the fluctuations experienced by the ratio of marginal costs are modest (the coefficient of variation is 0.03) and thus increases experienced by this variable are relatively small.

Our results are in contrast with Chavas (2008) who found the relative costs of producing under adverse conditions to consistently decline over time since the 1970s. However, taking a longer-run perspective and allowing for asset dynamics reveals a different story. The dynamics of land, machinery and labor over time have been more favorable to producing under good states of nature than under bad states during difficult economic times.

Using our dynamic dual model under state-contingent output uncertainty, we gain insight on the influence of production risk on investment decisions. The evolution over time of the first derivatives of net capital investments with respect to production in good and bad states of nature, $\partial \dot{k}_i / \partial y_j$, $(i, j = 1, 2)$ are presented in figures 4 and 5. While bad states of nature (y_1) discourage investments, favorable conditions (y_2) have a positive impact. The differences between the impacts of good and bad states of nature on net investments are specially pronounced at the beginning of the 20th century and the Great Depression and tend to diminish as we approach the end of the century. These results suggest that the effects of output risk on asset acquisitions in agriculture, have tended to decline over time. Improved risk managing techniques and reduced distance between the marginal costs of producing y_1 and y_2 (figure 2), are possible explanations

for such observed behavior. In spite of reduced differential impacts of production risk on investments by the end of the period analyzed, good (bad) states of nature continue to encourage (discourage) farm investments in labor, machinery and land.

Although the effects of good and bad states of nature on investment are rather symmetric, the negative influence that bad states have on labor net investments is not compensated by the magnitude of the positive effect of good states. On the other hand, the good state effects for land and machinery investments are more powerful than the disinvestment impacts of bad states. Hence, production risk is found to be specially harmful to net investments in labor.

Our results show the relevance of allowing for the stochastic nature of the production function which permits to capture both the differences in the costs of producing under different states of nature, the differences in the evolution of these costs over time, and the impacts of production risk on investment decisions. The state-contingent framework offers, in this regard, several advantages. First, it does not require a priori production risk assessments. Second, this approach does not require the measurement of economic agents' risk attitudes, since the cost minimization framework under a state-contingent approach is applicable independently of the risk preferences of the decision maker. This is especially useful when a dynamic cost minimization model is estimated, since the already substantial complexity of dynamic models is further increased if risk attitudes are to be explicitly modeled.

5. Concluding Remarks

In this paper we assess how production costs in agriculture have evolved over time. We distinguish between the costs of producing under favorable and unfavorable states of nature. We also study the impacts of production risk on farm investment decisions. To do so, we represent the stochastic nature of production using the state-contingent approach initially proposed by Chambers and Quiggin (1998, 2000) and empirically implemented by Chavas (2008). This methodology explicitly recognizes that producers commit inputs prior to uncertainty is resolved. Further, and as has been explained by previous literature, the approach offers several advantages

when modeling risk, since it does not require probability assessments of uncertain output and can be applied independently of economic agents' risk attitudes.

Chavas (2008) proposes a methodology to empirically implement the state-contingent approach. More specifically, he specifies and estimates cost-minimizing input choices with a state-contingent technology. We extend the work by Chavas to a consideration of investment demand and associated dynamics. This is specially relevant since technical change is likely to cause changes in production costs in agriculture.

A dynamic state-contingent cost minimization approach is applied to assess production decisions in US agriculture over the last century. The empirical analysis is based on an extended version of the dataset from Thirtle et al. (2002) which contains information on input and output price and quantity indices for the US agriculture as an aggregate and for the period 1910-1990.

Results derived from estimating the state-contingent outputs suggest a tendency to reduce the output produced under unfavorable conditions during difficult economic times. Parameter estimates of the dynamic dual model indicate the presence of capital adjustment costs that cause a slow convergence of capital to its long-run equilibrium.

Our results also suggest the relevance of allowing for the stochastic nature of the production function which permits to capture both the differences in the costs of producing under different states of nature, as well as the differences in the evolution of these costs over time. More specifically, we find that marginal costs (in real terms) have a declining trend that is only reversed during difficult economic situations (Great Depression and 1980s farm financial crisis) when producing under unfavorable states of nature becomes more expensive, and firms take more conservative production decisions (i.e. they tend to avoid unfavorable outcomes).

Finally, we also show the impacts of production risk on farm investment decisions. Our results suggest that while good states of nature tend to encourage investments in quasi-fixed assets, bad states of nature discourage them. Differential impacts of different states of nature on net investments, however, have tended to decline over time as risk management techniques have been improving and the extra cost of producing under bad states relative to good ones, has been declining.

References

Arrow, K. (1965). *Aspects of the Theory of Risk Bearing*. Helsinki: Yrjo Jahnsson Saatio.

Brechling, F. (1975). *Investment and Employment Decisions*. Manchester: Manchester University Press.

Camerer, C. (1995). Individual decision making. In J. H. Hagel and A. E. Roth (eds.), *The Handbook of Experimental Economics*. Princeton, NJ: Princeton University Press, 587-703.

Chambers, R. G. and Quiggin, J. (1996). Nonpoint-source pollution control as a multi-task principal-agent problem. *Journal of Public Economics* 59: 95-116.

Chambers, R. G. and Quiggin, J. (1997). Separation and hedging results with state-contingent production. *Economica* 64: 187-209.

Chambers, R. G. and Quiggin, J. (1998). Cost functions and duality for stochastic technologies. *American Journal of Agricultural Economics* 80: 288-295.

Chambers, R. G. and Quiggin, J. (2000). *Uncertainty, Production, Choice, and Agency*. Cambridge: Cambridge University Press.

Chavas, J. P. (2008). A cost approach to economic analysis under state-contingent production uncertainty. *American Journal of Agricultural Economics* 90: 435-446.

Chavas, J. P. and Holt, M. T. (1996). Economic behavior under uncertainty: A joint analysis of risk preferences and technology. *The Review of Economics and Statistics* 78: 329-335.

Debreu, G. (1959). Theory of Value. Cowles Foundation Monograph 17. New Haven, CT: Yale University Press.

Epstein, L. R. (1981). Duality theory and functional forms for dynamic factor demands. *The Review of Economic Studies* 48: 81-95.

Epstein, L. G. and Denny, M. (1983). The multivariate flexible accelerator model: its empirical restrictions and an application to U.S. manufacturing. *Econometrica* 51: 647-673.

Just, D. R. and Peterson, H. H. (2003). Diminishing marginal utility of wealth and calibration of risk in agriculture. *American Journal of Agricultural Economics* 85: 1234-1241.

Luh, Y-H. and Stefanou, S. (1996). Estimating dynamic dual models under nonstatic expectations. *American Journal of Agricultural Economics* 78: 991-1003.

Moschini, G. and Hennessy, D. (2001). Uncertainty, risk aversion, and risk management for agricultural producers. In B. L. Gardner and C. G. Rausser (eds.), *Handbook of Agricultural Economics*, volume 1A. Amsterdam: Elsevier, 88-153.

O'Donnell, C. J. and Griffiths, W. E. (2006). Estimating state-contingent production frontiers. *American Journal of Agricultural Economics* 88: 249-266.

Olmstead, A. L. and Rhode, P. (1993). Induced Innovation in American Agriculture: A Reconsideration, *Journal of Political Economy* 101: 100-118.

Pietola, K. and Myers, R. J. (2000). Investment under uncertainty and dynamic adjustment in the Finnish pork industry. *American Journal of Agricultural Economics* 82: 956-967.

Pope, R. D. and Chavas, J. P. (1994). Cost functions under production uncertainty. *American Journal of Agricultural Economics* 76: 114-127.

Rabin, M. (2000). Diminishing marginal utility of wealth cannot expain risk aversion. In D. Kahneman and A. Tversky (eds.), *Choices, Values, and Frames*. New York, NY: Cambridge University Press, 202-208.

Saha, A., Shumway, C. R. and Talpaz, H. (1994). Joint estimation of risk preference structure and technology using expo-power utility. *American Journal of Agricultural Economics* 76: 173-184.

Sckokai, P and Moro, D. (2009). Modelling the impact of the CAP Single Farm Payment on farm investment and output. *European Review of Agricultural Economics* 36: 395-423.

Serra, T., Stefanou, S., Gil, J. M. and Featherstone, A. (2009). Investment rigidity and policy measures. *European Review of Agricultural Economics* 36:103-120.

Stefanou, S. E. (1989). Returns to scale in the long run: the dynamic theory of cost. *Southern Economic Journal* 55: 570-579.

Thirtle, C. G., Schimmelpfenning, D. E. and Townsend, R. F. (2002). Induced innovation in United States agriculture, 1880-1990: time series tests and an error correction model. *American Journal of Agricultural Economics* 84: 598-614.

Table 1. Parameter estimates

Parameter	Estimate	Standard error
intercept	-0.021	0.037
c_2	0.116**	0.042
w	-0.180**	0.043
$\ln(z_{t-1})$	0.708**	0.073
RD	4.029**	1.232
ARCH0	0.232E-3	0. 230E-3
ARCH1	0.369*	0.220
GARCH1	0.625**	0.190

Table 2. Parameter estimates

Parameter	Estimate	Standard error
a_0	4,349.61	13,131.00
a_{11}	-37,747.42*	21,785.60
a_{12}	37,115.71*	22,204.60
a_{21}	8,579.16**	1,592.50
a_{22}	-6,880.17**	2,004.60
a_{31}	4.98	5.37
a_{32}	-18.01*	9.84
A_{y1y1}	35,152.82	21,795.40
A_{y1y2}	-26,922.45	21,509.50
A_{y2y2}	19,008.02	22,350.20
N_{c1c1}	1,221.24**	251.20
N_{c1c2}	-394.55*	175.70
N_{c2c2}	773.21**	267.90
D_{k1k1}	-9.09E-4	3.92E-3
D_{k1k2}	0.01	7.39E-3
D_{k2k2}	-0.02	0.02
F_{y1c1}	-3,626.17**	1,334.60
F_{y1c2}	-1,194.79	2,040.80
F_{y2c1}	1,969.28	1,283.70

Where **(*) denotes statistical significance at the 5 (10) per cent confidence levels.

Table 2. Parameter estimates (continued)

Parameter	Estimate	Standard error
F_{y2c2}	3,802.44	2,050.10
G_{y1k1}	-5.20*	2.91
G_{y1k2}	-2.55	9.06
G_{y2k1}	3.45	2.32
G_{y2k2}	6.75	8.49
M_{k1c1}	0.17**	0.03
M_{k2c1}	0.02	0.01
M_{k2c2}	0.08**	0.01
D_{11}	-4.01	3.26
D_{21}	-4.47**	1.47
D_{31}	0.80	3.46
D_{12}	1.01	3.52
D_{22}	-6.67**	1.84
D_{32}	-14.05**	5.14
Wald test		16,669
(p value)		(<.001)
R squared	equation $\dot{k}_1$	0.15
	equation $\dot{k}_2$	0.31
	equation x	0.99

Where **(*) denotes statistical significance at the 5 (10) per cent confidence levels.

Table 3. Capital demand elasticities

	Mean	Standard deviation
Elasticity of k_1 with respect to c_1	-0.011	0.035
Elasticity of k_1 with respect to c_2	-0.048**	0.01
Elasticity of k_2 with respect to c_1	-0.011	0.008
Elasticity of k_2 with respect to c_2	-0.021**	0.011

Where **(*) denotes statistical significance at the 5 (10) per cent confidence levels.
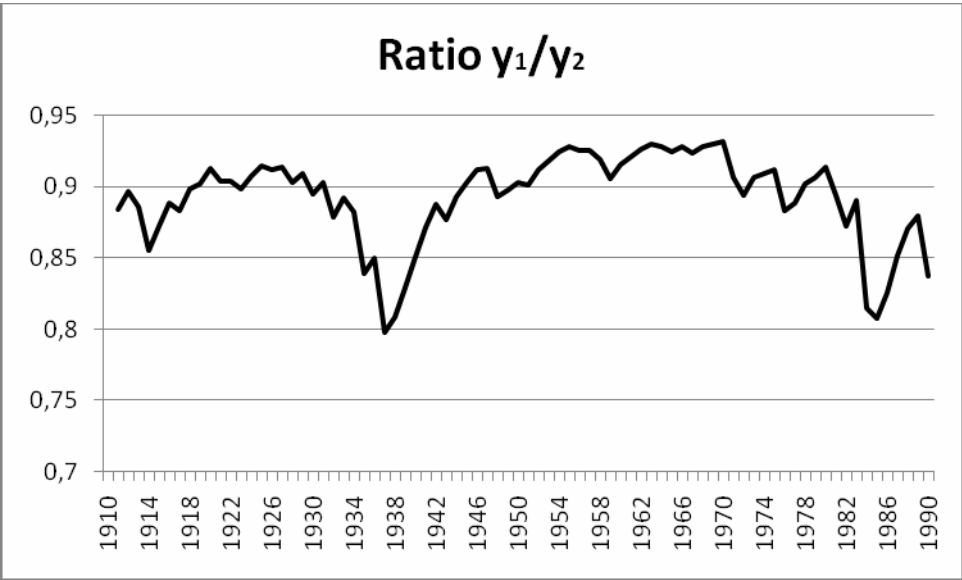
Figure 1. y_1/y_2 ratio, evolution over time

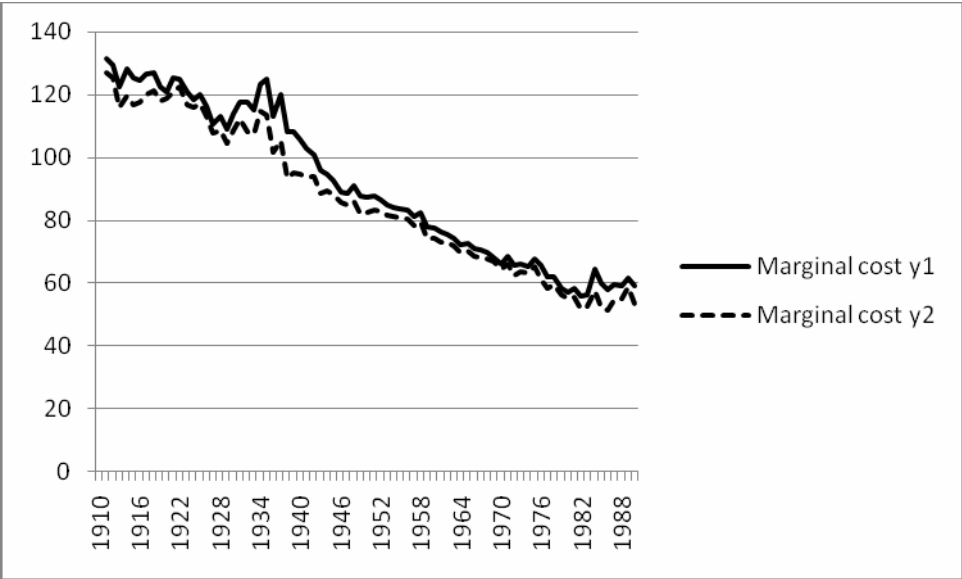**Figure 2. Evolution over time of the marginal cost of y_1 and y_2 (in constant monetary units)**

Figure 3. Ratio of the marginal cost of y_1 with respect to y_2
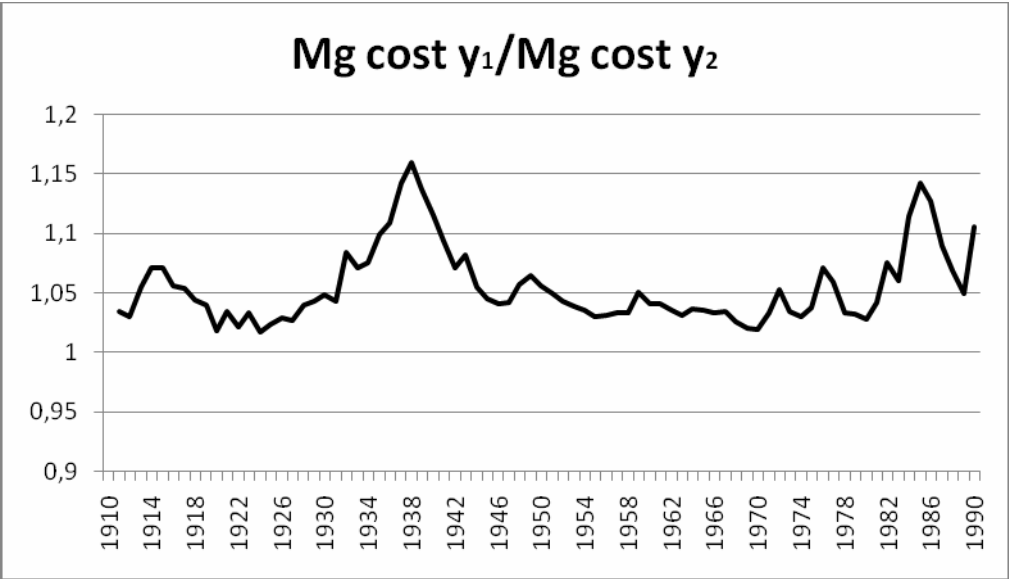

Mg cost y_1/Mg cost y_2

Figure 4. Evolution over time of $\partial \dot{k}_1 / \partial y_j$, $(i, j = 1, 2)$ **(in constant monetary units)**
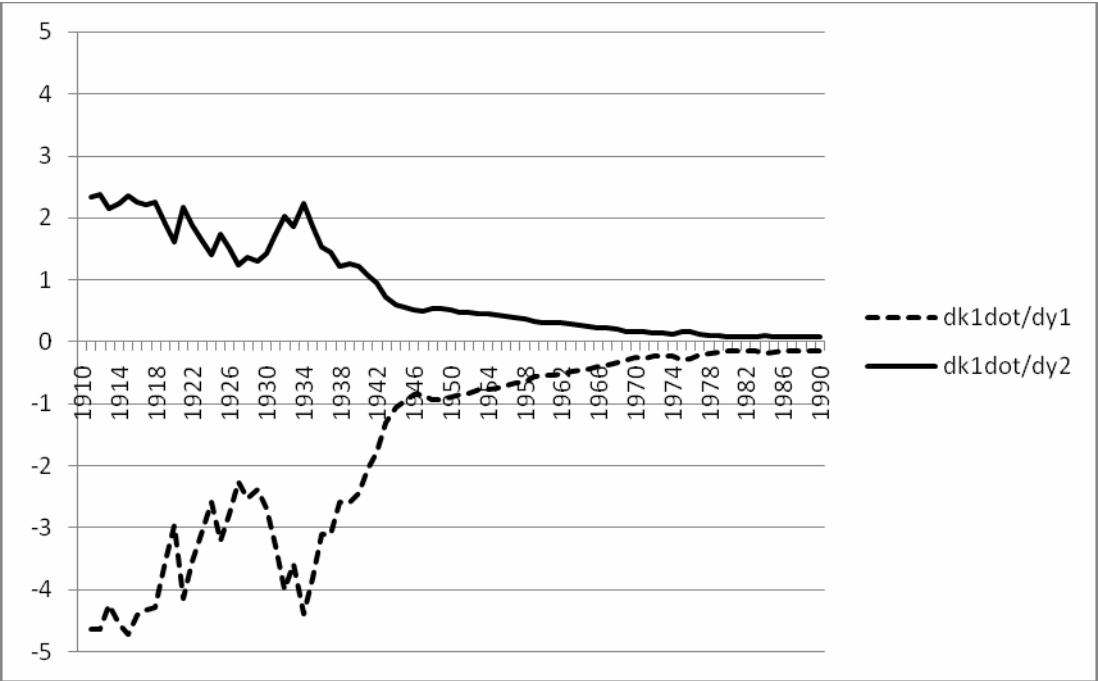
Figure 5. Evolution over time of $\partial \dot{k}_2 / \partial y_j, \ (i, j = 1, 2)$ (in constant monetary units)