# Yield and Area Elasticities.
# A Cost Function Approach with Uncertainty

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**Abstract**

This paper develops a method to jointly estimate crop yield elasticities and area elasticities with respect to output prices based on a theoretically consistent model. The model uses a duality theory approach for the multi-output and multi-input firm, and introduces uncertainty in the level of target output which conditions the cost minimization problem, in the output prices and in the conditional input demand functions. The underlying production technology is conditioned on fixed inputs, both allocatable and non-allocatable. Up to our knowledge, there have been no theoretical developments of this type of models for multioutput technologies. Our approach is also novel because no previous model of this type has introduced the effects of allocatable fixed inputs. We provide an empirical application of this theoretical framework using State-level data and approximating the dual cost function by a normalized quadratic flexible functional form. We derive expressions for the elasticities of interest conditional on the function specification assumed.

## 1. Introduction

Recent developments in the literature have emphasized the environmental effects of agriculture, such as the land use change at a global scale induced by biofuels policies; the additional greenhouse gas (GHG) emissions generated by such policies; and the consequent increase in food prices due to the requirement of a higher production to satisfy the demand of feedstocks for the biofuels industry (Searchinger et al., 2008; Dumortier and Hayes, 2009).

Searchinger et al. (2008) showed that current mandates on the utilization of biofuels are capable of inducing land use changes in the US and foreign agriculture that result in a longer payback period of GHG emissions than previous estimates. Farigone et al. (2008) showed that the effects of biofuels on carbon savings are very sensitive to the type of land and feedstock used to produce renewable fuels. Righelato and Spracklen (2007) concluded that if the objective of biofuels policies is to mitigate global warming induced by carbon-dioxide, policy makers should concentrate first on improving the efficiency on the energy use because a small substitution of fossil fuels by renewable sources would require the conversion of large areas of forests. The increased demand for feedstocks by the biofuels industry and for food from developing countries has driven up food prices. The extent to which this extra demand will be satisfied in the near future with more or less land conversion depends on how yields react to these price changes (Keeney and Hertel, 2009).

In this context, the following elements are important: first, small changes in crop yields have a great impact on the payback period of GHG emissions induced by agriculture, and also on the quantity of new land that is brought into agriculture to satisfy an increasing demand of agricultural products. Second, the allocation decision of land to competing enterprises (cash crops, pasture, forestry and others land uses) is very sensitive to price shocks.

Therefore, the effects of biofuel and climate policies and, in particular, their environmental consequences are inherently related to both the supply of agricultural products and the change in commodity prices. This brings supply response models into the picture. Supply response models in agriculture require evaluation of both the extensive and intensive margin. The intensive margin or "intensification" accounts for the increase in production due to reallocation of inputs without changing the area dedicated to each crop; in the case of agriculture, intensification is equivalent to an increase in yields. The extensive margin or "extensification" measures the change in production derived from the reallocation of land among different crops; this is usually defined as land-use change.

We propose a method to jointly estimate both the intensive and the extensive margin within a model framework that is theoretically consistent and capable of evaluating real world data. The intensive margin is estimated through the yield elasticity with respect to crop price (yield-price elasticity) and the extensive margin through the area elasticity with respect to crop price (area-price elasticity).

Our theoretical model closely follows the two-step decision approach of Chambers and Just (1989) for dual profit maximization when fixed but allocatable inputs are present. However, we apply the equivalent duality set up but for the case of cost minimization, and then introduce uncertainty in the level of target output, output prices and in the conditional input demands.

The empirical approach used in this paper relies heavily on duality theory and the treatment of uncertainty. Duality theory was introduced in the mid-fifties through the seminal work by Ronald Shephard (Shephard, 1953) and its use in the economic science has been extensive since then. Moreover, with the contribution of Christensen, Jorgenson and Lau (1971), Diewert (1971), Diewert and Wales

(1971), Lau (1974), and McFadden (1978), several types of flexible functional forms were developed with enormous virtues in describing the data.

Cost minimization problems (CMP) are sometimes preferred to primal approaches[1] since they are general enough to treat several different economic environments: perfect competition, monopoly, and non-profit objectives (Pope and Chavas, 1994); and by using input prices instead of input quantities as explanatory variables in estimation, a potential source of simultaneity is removed (Moschini, 2001). Dual approaches can analyze multioutput technologies in a straightforward fashion, while primal approaches are usually restricted to single output production problems. Also, the flexible representation of the problem with a cost or profit function yields a more tractable system to be estimated than primal approaches, in which the profit maximizing output supplies and input demands are derived from a set of nonlinear first-order conditions associated with a production function previously specified (Just, 1993)

However, when agriculture is the industry of interest, one has to account for the existence of production risks, which affects directly the output level that is conditioning the CMP. The decision maker faces a great deal of uncertainty on the output level that will be produced at the end of the period, uncertainty that is inherent to agricultural production (such as weather and pests). One group of approaches of cost minimization have ignored this source of uncertainty by conditioning the CMP and the input demand functions on the observed level of output. Pope and Just (1996) called these the "traditional approaches" and its cost function the "ex-post cost function." However, when the uncertainty in output is recognized in the analysis, the cost function obtained as the solution of the CMP is the so called "ex-ante cost function." The treatment of uncertainty in cost function analysis was studied, among others, by Pope and Chavas (1994), Pope and Just (1996, 1998), Chambers and Quiggin (1998), Moschini (2001) and Chavas (2008). In this paper, we treat uncertainty in production in a way that is close to that of Moschini (2001), given that in his analysis, the stochastic nature of the input demand functions is explicitly considered.

We contribute to the literature with a method to jointly estimate yield and area elasticites with respect to crop prices using available agricultural data. The model presented here belongs to the family of cost function approaches under production uncertainty that we extend and treat as a supply response model in agriculture. We follow Moschini (2001) to remove the errors-in-variables problem arising in this type of model when uncertainty is introduced. Relying on duality theory, we approximate a cost function by a flexible functional form and incorporate uncertainty in the level of conditional output, output prices and conditional input demand functions. The cost function is a multi-output multi-input normalized quadratic that we modify to incorporate allocatable and non-allocatable fixed inputs. Yield and area elasticities are theoretically derived but conditional on the functional form specified. Joint estimation of a system of conditional input demand functions and output supplies generates the parameters needed to calculate these elasticities.

## 2. Literature Review on Yield and Area Elasticities

While yield and area response to output prices was a widely analyzed topic in the 70´s and 80´s which provided several empirical estimations, this issue has been essentially ignored in recent years. A thorough review of the literature also shows that the efforts to estimate yield and area elasticities with

[1] The primal approach in production theory is the optimization problem where the objective function is the profit of the firm given input and output prices and a previously specified production function.

respect to crop prices have usually consisted of a separate estimation, with a few exceptions (Pomareda and Samayoa, 1979; Guyomard, Baudry and Carpentier, 1996; Arnade and Kelch, 2007).

Regarding yield elasticites with respect to output prices, pioneering work by Houck and Gallagher (1976) found clear evidence of a positive elasticity for US corn in the period 1951-1971. Choi and Helmberger (1993) also found a positive relationship for US corn, soybean and wheat in the period 1964-1988. However, Menz and Pardey (1983) pursued an analysis similar to that of Choi and Helmberger but for a longer period and found that the yield-price elasticity was not significant for following 10-years period. Kaufmann and Snell (1997) found a positive elasticity for US corn, but it was close to zero. Lyons and Thompson (1981) used cross-country data and found a positive and significant response of yields to corn price. However, there are some studies that did not encounter a positive relationship between yields and price for US corn, including Reed and Riggins (1982) and Ash and Lin (1987). Keeney and Hertel (2008) suggest that a possible explanation for the lack of response found in these studies could be the plateau-like relationship between yields and fertilizer, especially for those studies which rely heavily on a primal specification of the technology, such as Ash and Lin (1987). Also, the estimation of supply response in single equation models, as in Reed and Riggins (1982), fails to acknowledge the effect of land substitution by other crops.

The lack of updated estimates of the yield response to output prices puts more pressure in obtaining these estimates for the following reasons: new estimates are highly relevant for practitioners; for example Keeney and Hertel (2008) extensively review the literature on yield-price elasticities of several crops showing the lack of recent estimations. Then Keeney and Hertel (2009) approximated the long run corn yield-price elasticity by the average of a series of studies from the 70´s and 90´s and showed how land use impacts are highly sensitive to the yield elasticity assumption. Estimations for other crops, such as soybeans and wheat, are even more difficult to find.

Area elasticities with respect to output prices have also been a widely analyzed topic in agricultural economics literature. Early work by Houck and Ryan (1972) and Lee and Helmberger (1985) focused on the effects of both market prices and farm programs payments on acreage variation; they analyzed specific crops (corn and soybeans) in US regions. Gardner (1976) estimated supply and acreage elasticities for US soybeans (1950-1974) and cotton (1911-1933) with respect to their respective future prices. Chavas and Hold (1990) set up an acreage response model under expected utility maximization using 1954-1985 US corn and soybeans data. Davison and Crowder (1991) estimated soybean acreage elasticities with respect to expected net-returns (constructed from future prices, farm programs payments and variable production costs) for the US Northeast region. More recent studies such as Lee and Kennedy (2008) and Bridges and Tenkorang (2009), estimated the response of acreage to market and government incentives using the Rotterdam model and a translog flexible functional form, respectively.

## 3. Theoretical Model

Assume there exists a representative farmer whose problem is to maximize his or her expected profits, where uncertainty is given by the stochastic nature of agricultural production and the fact that output prices are unknown at the moment of the planting decision. The farmer's problem can be described as follows

$$max_{[\boldsymbol{x}]}\{E[\widetilde{\boldsymbol{p}}.f(\boldsymbol{x},\boldsymbol{L},\boldsymbol{Z};\boldsymbol{\theta_0},\tilde{\eta}) - \boldsymbol{w}.\boldsymbol{x}]\} \tag{1}$$

where $\boldsymbol{x}$ is a vector of $n$ variable input quantities to be determined, $\boldsymbol{w}$ is a vector of $n$ variable input prices, $\widetilde{\boldsymbol{p}}$ is a vector of $m$ unobserved prices, and $f(.)$ is the farmer's stochastic technology function which is constrained by a vector of allocatable and non-allocatable fixed inputs $\boldsymbol{L}$ and $\boldsymbol{Z}$**,** respectively. From now on, $\boldsymbol{L} = \{L_1, \dots, L_m\}$ denotes the vector of fixed inputs allocated to the $m$ crops and not the total quantity of the input. The parameter $\boldsymbol{\theta_0}$ is a vector of production function parameters to be estimated and $\tilde{\eta}$ is the random error of the production technology. E[.] is the expectation operator taken over the random error of the production technology and the random error of the unobserved prices, which we assume to be independent from each other.

A profit maximization problem can be equivalently specified as one in which the agent chooses the optimal level of target output which maximizes the difference between revenues and minimized costs. When the underlying technology is constrained by fixed inputs, the alternative specification of the program can be written as follows

$$max_{[\overline{\boldsymbol{y}}]}\{E[\widetilde{\boldsymbol{p}}].\overline{\boldsymbol{y}} - C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})\}$$

where $\overline{\boldsymbol{y}}$ is the vector of target expected output; $\boldsymbol{L}$, $\boldsymbol{Z}$ and $\boldsymbol{w}$ are as defined above; E[.] is the expectation operator taken over the random error of the unobserved prices; $\boldsymbol{\theta}$ a vector of parameters to be estimated which includes the parameters of the distribution of the random variable $\tilde{\eta}$; and $C(.)$ is the cost function consistent with the following program $C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) \equiv min_{[\boldsymbol{x}]}\{\boldsymbol{w}.\boldsymbol{x} \;\; s.t. \;\; f(\boldsymbol{x}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) \geq \overline{\boldsymbol{y}}\}$.

The presence of allocatable fixed inputs gives rise to a setup of the optimization program consisting of two stages (Chambers and Just, 1989). In the first stage, the farmer solves a cost minimization problem conditional on a specific level of expected (and unobserved) output, a specific allocation of allocatable fixed inputs $\boldsymbol{L} = \{L_1, \dots, L_m\}$ and a given vector of non-allocatable fixed inputs:

$$C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) \equiv min_{[\boldsymbol{x}]}\{\boldsymbol{w}.\boldsymbol{x} \;\; s.t. \;\; f(\boldsymbol{x}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) \geq \overline{\boldsymbol{y}}\} \qquad (2)$$

where the variables and parameters are as described above, in particular $\boldsymbol{L} = \{L_1, \dots, L_m\}$. The solution to this problem is a set of conditional input demand functions $\boldsymbol{x}^* \equiv h(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})$ and a cost function $C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) \equiv \boldsymbol{w}.h(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})$. While CMP directly provide solutions for the conditional input demands, conditional output supply can be obtained by assuming profit maximization behavior which we already assumed in problem (1). In the context of our model, where uncertainty is present in output prices and in output level, the profit maximization assumption implies $E[\widetilde{\boldsymbol{p}}] - MC(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) = 0$, where $MC(.)$ is the vector of marginal costs. By jointly solving this system of equations for the vector of expected output levels $\overline{\boldsymbol{y}}$, we obtain the vector of output supply function: $\overline{\boldsymbol{y}} = s(\overline{\boldsymbol{p}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})$, where we denote $\overline{\boldsymbol{p}} = E[\widetilde{\boldsymbol{p}}]$[2].

In the second stage, given the solution to the first stage, the farmer chooses the specific allocation of allocatable fixed inputs which minimizes the cost function obtained in the first step subject to the resource constraint, that is

$$min_{[L_1,\dots,L_m]} \; \{ C(\overline{\boldsymbol{y}}, \boldsymbol{w}, L_1, \dots, L_m, \boldsymbol{Z}; \boldsymbol{\theta}) \;\; s.t. \;\; L_1 + \cdots + L_m = \bar{L} \}$$

[2] This is equivalent to assume that after the farmer finds the optimal conditional demands, he or she solves a profit maximization problem of the form $max_{[\overline{\boldsymbol{y}}]}\{E[\widetilde{\boldsymbol{p}}].\overline{\boldsymbol{y}} - C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})\}$ whose first-order conditions are $E[\widetilde{\boldsymbol{p}}] - MC(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) = 0$; and from this system, conditional output supplies are obtained (Moschini 2001).

where $\bar{L}$ is a vector of total quantities available of the fixed but allocatable inputs.

The two stage approach of the decision process is based on realistic farmer behavior since the choice of allocatable fixed inputs (such as land) to each crop is done in one part of the year and is influenced by factors like expected prices; however, other factors that affect the crop production occur later in the production season such as fertilizer, herbicides, seeds and weather (Arnade and Kelch, 2007).

While so far we have mentioned two sources of uncertainty, the output levels and output prices, there is also uncertainty in the level of input demands, at least from the point of view of the econometrician. To incorporate this aspect in our model we follow the additive general error model (AGEM) introduced by McElroy (1987). The AGEM considers that the conditional input demands while observed with certainty by the producer, they are observed with an error by the econometrician. That is

$$\boldsymbol{x} = h(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon}$$

where the joint distribution of the error structure $\boldsymbol{\varepsilon}$ is independent from output levels, output prices and input prices. The cost function that is theoretically consistent with this error specification becomes

$$C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) = \boldsymbol{w}.\boldsymbol{x} = \boldsymbol{w}.h(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{w}.\boldsymbol{\varepsilon}$$

and the underlying technology specification is $f(\boldsymbol{x} - \boldsymbol{\varepsilon}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta_0}, \tilde{\eta})$. This gives us then, an internally consistent way of introducing an error structure to the input demand system.

Given the theoretical model described above, we now proceed to describe how we conduct the estimation procedure and how we derive our desired elasticities.

Traditional approaches using duality theory have estimated the input demand system treating the unobserved output $\overline{\boldsymbol{y}}$ as observed. However, Moschini (2001) argues that this procedure induces an errors-in-variables problem that yields inconsistent parameter estimates. He proposes an alternative estimation procedure that, by relying on the profit maximization assumption, the errors-in-variables problem is completely removed. This procedure consists of using the output supply equations given by $\overline{\boldsymbol{y}} = s(\overline{\boldsymbol{p}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})$ to substitute for the unobserved output level in each input demand equations, such that the system of equations to be estimated becomes

$$\begin{aligned} \boldsymbol{x} &= h(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon} \\ &= h(s(\overline{\boldsymbol{p}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}), \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon} \end{aligned}$$

This system depends only on observable values and on expected prices. But if we assume that farmers form their price expectations with observed future prices at the planting moment, then we can replace $\overline{\boldsymbol{p}} = E[\tilde{\boldsymbol{p}}]$ by $\boldsymbol{f}_{oT}$**,** where $\boldsymbol{f}_{oT}$ is a vector of observed future prices at time *t=0* for delivery at time *t=T*. Therefore, the system that we are estimating becomes

$$\begin{aligned} \boldsymbol{x} &= h(s(\boldsymbol{f}_{oT}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}), \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{\varepsilon} \\ \boldsymbol{y} &= s(\boldsymbol{f}_{oT}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta}) + \boldsymbol{u} \end{aligned} \tag{3}$$

Note that we add the system of supply equations, which shares several parameters in common with the input demand system, so that parameter restrictions will be imposed for estimation. Also note that $\boldsymbol{y}$ is the vector of observed output levels.

But our main objective is not to just estimate the above system of equations but to jointly estimate yield and area elasticities with respect to output prices from the above model. To this end, we recognize that all the required elasticities can be obtained by manipulating the system of output supply equations $\overline{\boldsymbol{y}} = s(\boldsymbol{f}_{oT}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})$. First, the yield-price elasticity is obtained by directly taking its derivative with respect to $\boldsymbol{f}$ holding constant the area allocations, that is $\xi_{ij} = \partial s_i(\boldsymbol{f}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})/\partial f_j \,|_{\boldsymbol{L}=\overline{\boldsymbol{L}}}$. This is an yield elasticity because any increase in production of crop *i* after a change in price *j*, if there does not exist a reallocation of land, is due to an increase in the yield of crop *i*. Second, the area-price elasticity is obtained by simultaneously solving the system of output supply equations for the area vector $\boldsymbol{L}$ as a function of output, input and output prices, and non-allocatable fixed inputs; then differentiating each $\boldsymbol{L}$ with respect to each $\boldsymbol{f}$ yields the area-price elasticity, $\zeta_{ij} = \partial L_i(\overline{\boldsymbol{y}}, \boldsymbol{f}, \boldsymbol{w}, \boldsymbol{Z}; \boldsymbol{\theta})/\partial f_j$. The procedure described above indicates why it is important in our model to maintain the land allocation to different crops as a conditioning variable in the cost function.

4. **Empirical Model and Data**

To operationalize the theoretical model described above, we start with an approximation of the cost function solution to problem (2) by a normalized quadratic flexible functional form (Diewert and Wales, 1971; Lau, 1974) that is multi-output and multi-input. We extend this model to incorporate fixed inputs, both allocatable and non-allocatable. The cost function has $m$ outputs, $n$ variable inputs, one quasi-fixed input (land) allocated to the $m$ crops, $r$ non-allocatable fixed inputs and a time trend. The error structure ($\boldsymbol{\varepsilon}$) follows the McElroy AGEM. The cost function is then approximated as follows

$$C(\overline{\boldsymbol{y}}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}, t; \boldsymbol{\theta}) = \sum_i a_i w_i + \sum_k b_k \bar{y}_k + \frac{1}{2}\left[\frac{1}{w_0}\sum_i \sum_j a_{ij} w_i w_j + \sum_k \sum_m b_{km} \bar{y}_k \bar{y}_m + \sum_l \sum_s d_{ls} L_l L_s\right] + \\ + \sum_i \sum_k c_{ik} w_i \bar{y}_k + \frac{1}{2}\sum_k \sum_l e_{kl} \bar{y}_k^2 L_l + \sum_i \sum_l f_{il} w_i L_l + \sum_i \sum_r g_{ir} w_i Z_r + \sum_i w_i \varepsilon_i$$

where (*i*,*j*) = {1,…,n} indexes input prices $\boldsymbol{w}$; (*k*,*m*) = {1,…,m} indexes unobserved output $\overline{\boldsymbol{y}}$; (*l*,*s*) = {1,…,m} indexes land $L_l$ allocated; and *r* indexes the variables $Z_r$ which consist of the non-allocatable fixed inputs and a time trend. The vector parameter $\boldsymbol{\theta} = \{a_i, b_k, a_{ij}, b_{km}, c_{ik}, d_{ls}, e_{kl}, f_{il}, g_{ir}\}$ represents the set of parameters to be estimated. The normalizing price $w_0$ is the price of the numeraire good which guaranties an homogeneous of degree one cost function in $\boldsymbol{w}$, and also the homogeneity of degree zero of the compensated input demand functions in $\boldsymbol{w}$. Concavity of the cost function in $\boldsymbol{w}$ and downward slopping input demand functions can be imposed by parameter restrictions through the estimation of the Cholesky factorization coefficients of the paremeters $a_{ij}$ so as to guarantee negative semi-definiteness of the Hessian matrix in $\boldsymbol{w}$. However, we do not impose this restriction and $a_{ij}$ parameters are freely estimated. Then the mentioned properties are tested statistically. Convexity of the cost function with respect to output levels cannot be imposed due to the presence of the quadratic output interacting with land; however, we can explore whether this property holds once parameter estimates are obtained. Symmetry is imposed by parameter restriction such that $a_{ij} = a_{ji}$ and $b_{km} = b_{mk}$ for *i*≠*j* and *k*≠*m*. The first restriction is consistent with Young's theorem and assures a symmetric Hessian matrix of the cost function with respect to $\boldsymbol{w}$. The second restriction assures a symmetric Hessian matrix of the cost function with respect to $\boldsymbol{y}$.

Provided this functional form satisfies the properties of a cost function in Appendix I, there exists an underlying technology which has a cost function given by this expression. We extended the normalized quadratic specification and incorporate both allocatable and non-allocatable inputs into the cost function specification in a straightforward fashion by including interaction terms. We include interaction terms between the allocatable inputs and input prices, between the allocatable inputs and outputs levels, allocatable inputs between themselves and between non-allocatable inputs and input prices. We do not include non-allocatable inputs interacting with output levels because we assume that the output mix produced does not have an influence on the stock of non-allocatable fixed inputs (agricultural capital and family labor according to our empirical estimation).

The specification of the cost function as the normalized quadratic functional form has implication on the structure of the underlying technology. Two important implications are related to output separability and input nonjointness. When a technology $T$ is output separable, the cost function satisfies the equality $C(\boldsymbol{w},\boldsymbol{y}) = \hat{C}(\boldsymbol{w}, g(\boldsymbol{y}))$, where $\hat{C}(\boldsymbol{w}, g)$ is the cost function for the single output technology $\hat{T}$ and $g(.)$ is a non-decreasing function of $\boldsymbol{y}$ (Chambers, 1988; Hall, 1973). We are not imposing separability with the above specification because the interaction term of $\boldsymbol{w}$ and $\boldsymbol{y}$ share the same coefficient $c_{ik}$. While the structure is not imposed, it can be statistically tested. Following Hall (1973), a test for output separability can be conducted by estimating an alternative model specification where $w_i$ and $y_k$ enter in a multiplicative separable fashion, that is substitute $\sum_i \sum_k c_{ik} w_i \bar{y}_k$ for $(\sum_i c_i w_i)(\sum_k c_k \bar{y}_k)$ and then statistically compare both models.

A multioutput technology $T$ subject to allocatable fixed inputs is input-nonjoint if and only if the joint cost function can be written as the sum of independent cost functions for each kind of output: $C(\boldsymbol{w},\boldsymbol{y},L_1,\dots,L_m) = C^1(\boldsymbol{w},y_1,L_1) + C^2\big(\boldsymbol{w},y_{2,}L_2\big) + \cdots + C^m(\boldsymbol{w},y_m,L_m)$, (Chambers and Just, 1989)[3]. Our specification does not impose nonjointness in inputs because of the presence of outputs and allocatable inputs interacting with each other; rather, we seek to statistically test for it. A test for input nonjointness when only one input is allocated to $m$ crops consists of testing the following null hypotheses

$$\partial^2 C(\boldsymbol{w},\boldsymbol{y},L_1,\dots,L_m)/\partial y_k \partial y_m = 0$$
$$\partial^2 C(\boldsymbol{w},\boldsymbol{y},L_1,\dots,L_m)/\partial L_k \partial L_m = 0, k \neq m$$
$$\partial^2 C(\boldsymbol{w},\boldsymbol{y},L_1,\dots,L_m)/\partial y_k \partial L_m = 0$$

Finally, we might be interested in exploring the returns to scale implications of our specifications. Returns to scale depend on the functional form in which outputs enters the cost function, and in the multioutput case, are calculated by relying on the notion of average incremental cost (AIC). Increasing returns to scale exist in output $k$ if the ratio of average incremental cost of producing output $k$ to the marginal cost of producing $k$ is greater than one. That is $s_k = AIC_k/MC_k > 1$, where $AIC_k = IC_k/y_k$ and $IC_k$ is incremental cost, the cost of producing all outputs minus the cost of producing all outputs except output $k$.

We propose an estimation of the above empirical model for the case of Iowa agriculture from 1960 to 2004. The specification employed consists of three variable inputs (intermediate inputs, hired labor and energy), three outputs (corn, soybeans and other crops), one allocatable fixed input (land), two non-allocatable fixed inputs (agricultural capital and family labor) and a time trend. We define energy as the

[3] The authors consider this definition because it distinguishes between apparent and true input nonjointness. This problem was first pointed out by Shumway, Pope and Nash (1984) who argued that traditional tests were not capable of making this distinction. Chambers and Just developed a test to overcome this issue.

numeraire good. The data on quantities of intermediate inputs, hired labor, agricultural capital, family labor and the prices of intermediate inputs and energy were obtained from Eldon Ball at ERS-USDA. Intermediate inputs is an aggregated variable including fertilizers, pesticides and other intermediate inputs. This provides a consistent dataset of input usage and input prices that has been widely used in the literature and has great potential because is available at the State level. Hired labor prices were obtained from USDA Farm Labor reports (since 1985 Iowa was reported jointly with Missouri). Data on output quantities of corn, soybean and other crops (wheat, oat, hay, silage corn, rye and barley) and land allocated to these crops were obtained from USDA. Quantities of other crops were calculated by taking a weighted average of production, where weights were given by the revenue generated for each crop; similar approach was taken by Arnade and Kelch (2007). Prices for corn and soybeans were obtained from the Chicago Mercantile Exchange future markets reports. The price of corn equals the average of the March 15$^{th}$ and March 30$^{th}$ of the December delivery price of each year. The price of soybeans equals the average of the March15$^{th}$ and March 30$^{th}$ of the November delivery price (Choi and Helmberger, 1993). Future prices for other crops are not available so the current year price was used; and index of prices for other crops were obtained by taking the ratio between the total revenue generated by these crops to the weighted average of production.

We estimate system (3) as follows. The set of supply functions was obtained by setting each crop price $p_k$ equal to the derivative of the cost function with respect to the corresponding output level and then simultaneously solving this equality for the unobserved output level $\bar{y}_k$. The set of compensated input demands was obtained by applying Shephard's Lemma to the cost function. Given that the input demands depend on the unobserved output, inducing the aforementioned errors-in-variables problem if it is directly estimated, we eliminate this problem by substituting each unobserved output by its corresponding supply equation. So, given our normalized quadratic cost function specification, the estimated system is as follows

$$x_1 = a_1 + \sum_{j=1}^{2} a_{1j} w_j + \sum_{k=1}^{3} c_{1k} \bar{y}_k + \sum_{l=1}^{3} f_{1l} L_l + \sum_{r=1}^{2} g_{1r} Z_r + g_{1t} t + \varepsilon_1$$

$$x_2 = a_2 + \sum_{j=1}^{2} a_{2j} w_j + \sum_{k=1}^{3} c_{2k} \bar{y}_k + \sum_{l=1}^{3} f_{2l} L_l + \sum_{r=1}^{2} g_{2r} Z_r + g_{2t} t + \varepsilon_2$$

(4)

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_{11} + \sum_l e_{1l} L_l & b_{12} & b_{13} \\ b_{12} & b_{22} + \sum_l e_{2l} L_l & b_{23} \\ b_{13} & b_{23} & b_{33} + \sum_l e_{3l} L_l \end{bmatrix}^{-1} \begin{bmatrix} f_1 - b_1 - \sum_i c_{i1} w_i \\ f_2 - b_2 - \sum_i c_{i2} w_i \\ f_3 - b_3 - \sum_i c_{i3} w_i \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

In the input demands, all the $\bar{y}_k$'s are eliminated by being substituted for its corresponding expression of the output supply (given by the right hand side matrix equation in (4)). Note that we choose to jointly estimate the system of input demands and output supplies, where the dependent variables in the output supply equations are the observed levels of output. We could have chosen to estimate only the input demands instead, given that all the required parameters for the desired elasticities are available in the input demand system (once substitution of expected output is implemented). We assume that the errors $\boldsymbol{\varepsilon}$ and $\boldsymbol{u}$ are correlated and the correlation structure is given by a symmetric five-by-five matrix $\boldsymbol{\Psi}$.

We estimated this system using a nonlinear seemingly unrelated regression (nonlinear SUR) which consists of minimizing the following function

$$[\boldsymbol{X} - \varphi(\boldsymbol{f}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})]' (\boldsymbol{\Psi}^{-1} \otimes I_T) [\boldsymbol{X} - \varphi(\boldsymbol{f}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})]$$

where $\boldsymbol{X}$ is a 5T-by-1 vector of left hand side variables ($x_i$ and $y_k$) and $\varphi$ is a nonlinear function of the explanatory variables and unknown parameters $\boldsymbol{\theta}$ (which includes the parameters of the error structure). The matrix $\boldsymbol{\Psi}$ is updated at each iteration of the estimation process.

Once the estimation has been conducted, we plug the parameter estimates in our elasticity equations to obtain the desired results. The elasticity equations are provided in Appendix II.

## 5. Empirical Estimation and Results

To be completed

## 6. Concluding Remarks

The environmental effects of biofuels and climate polices, such as land-use changes, redistribution of GHG emissions worldwide and price changes, have been extensively studied in the economic literature. Also, small changes in crop yields were proven to have significant effects on the availability of food worldwide, in the reallocation of lands to different uses, and consequently in GHG emissions. The increase in supply of agricultural products occurs by a combination of productivity changes (intensive margin) and the reallocation of areas to different crops (extensive margin). Therefore, understanding the effects that price changes have on crop yields and in the land-use change is one of the most important elements to evaluate the future consequences of the mentioned policies.

In this paper we develop a method to estimate yield elasticities and area elasticities with respect to crop prices based on a theoretically consistent model. The model relies on duality theory of production and introduces uncertainty not only in the level of output which condition the cost minimization problem, but also in output prices and in the conditional input demand functions. Uncertainty in the input demand system is consistent with the additive general error model of McElroy (1987) and arises from unobservables to the researcher (not from the farmer's perspective). The cost minimization problem is also conditioned on fixed inputs, both allocatable and non-allocatable. Up to our knowledge, there have been no theoretical developments of this type of models for multioutput technologies. Our approach is also novel because no previous model of this type has introduced the effects of allocatable fixed inputs.

Providing new estimates on yields and area elasticities with respect to crop prices is also important because a review of the literature shows a lack of recent values for yield elasticities. It also shows, with some exceptions, that current estimates of area elasticities are available but they are not jointly obtained with the yield elasticities.

We provide an empirical application of this theoretical framework using State-level data (1960-2004) and approximating the dual cost function by a normalized quadratic flexible functional form. We derive an expression for each elasticity of interest conditional on the assumed function specification. Estimation of the empirical model is conducted by nonlinear seemingly unrelated regression.

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## 8. Appendix

### Appendix I

*Properties of the cost function:*
$C(.)$ is continuous in $\boldsymbol{w}$ because is the sum of continuous functions.
$C(.)$ is non-decreasing in $\boldsymbol{w}$ because the input demands given by $x = \partial C/\partial \boldsymbol{w}$ are non-negative.
$C(.)$ is homogeneous of degree one in $w$, that is $C(\lambda \boldsymbol{w}) = \lambda C(\boldsymbol{w})$. This is accomplished by normalizing the quadratic form in $\boldsymbol{w}$ by the price of the numeraire good $w_0$.
$C(.)$ is concave in $\boldsymbol{w}$ provided that the Hessian matrix is negative semi-definite. This property is not imposed by parameter restrictions but is tested in the empirical model.
$C(.)$ is non-decreasing in $\boldsymbol{y}$ because $MC = \partial C/\partial y$ must be greater than or equal to non-negative prices by the profit maximization condition assumed. Also the cost function tends to infinity as $\boldsymbol{y}$ tends to infinity.
$C(.)$ is convex in $\boldsymbol{y}$ provided that the Hessian matrix is positive semi-definite. This property cannot be imposed but can be tested.
$C(.)$ must be non-decreasing in $\boldsymbol{L}$. This property can be at most tested.
$C(.)$ is be convex in $\boldsymbol{L}$ provided the quadratic form $\sum_l \sum_s d_{ls} L_l L_s$ be positive semi-definite matrix.

### Appendix II

(i) Yield elasticities $\xi_{ij} = \partial s_i(\boldsymbol{f}, \boldsymbol{w}, \boldsymbol{L}, \boldsymbol{Z}; \boldsymbol{\theta})/\partial f_j \,|_{\boldsymbol{L}=\bar{\boldsymbol{L}}}$

$$\xi_{11} = A_{11}^{-1} \qquad \xi_{12} = A_{12}^{-1} \qquad \xi_{13} = A_{13}^{-1}$$
$$\xi_{21} = A_{21}^{-1} \qquad \xi_{22} = A_{22}^{-1} \qquad \xi_{23} = A_{23}^{-1}$$
$$\xi_{31} = A_{31}^{-1} \qquad \xi_{32} = A_{32}^{-1} \qquad \xi_{33} = A_{33}^{-1}$$

$$A^{-1} = \begin{bmatrix} b_{11} + \sum_l e_{1l} L_l & b_{12} & b_{13} \\ b_{12} & b_{22} + \sum_l e_{2l} L_l & b_{23} \\ b_{13} & b_{23} & b_{33} + \sum_l e_{3l} L_l \end{bmatrix}^{-1}$$

where $A_{ij}^{-1}$ is the *ij* entry of the inverse of matrix $A$,

(ii) Area elasticities $\zeta_{ij} = \partial L_i(\bar{\boldsymbol{y}}, \boldsymbol{f}, \boldsymbol{w}, \boldsymbol{Z}; \boldsymbol{\theta})/\partial f_j$

$$\zeta_{11} = B_{11}^{-1} \qquad \zeta_{12} = B_{12}^{-1} \qquad \zeta_{13} = B_{13}^{-1}$$
$$\zeta_{21} = B_{21}^{-1} \qquad \zeta_{22} = B_{22}^{-1} \qquad \zeta_{23} = B_{23}^{-1}$$
$$\zeta_{31} = B_{31}^{-1} \qquad \zeta_{32} = B_{32}^{-1} \qquad \zeta_{33} = B_{33}^{-1}$$

$$B^{-1} = \begin{bmatrix} e_{11} y_1 & e_{12} y_1 & e_{13} y_1 \\ e_{21} y_2 & e_{22} y_2 & e_{23} y_2 \\ e_{31} y_3 & e_{32} y_3 & e_{33} y_3 \end{bmatrix}^{-1}$$

where $B_{ij}^{-1}$ is the *ij* entry of the inverse of matrix $B$.