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DEPRECIATION ERODES THE COASE CONJECTURE

by

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Depreciation Erodes the Coase Conjecture[1]

Larry Karp[2]

Abstract

If a durable good monopolist produces at constant marginal costs and the good depreciates, there exists a family of Strong Markov Perfect Equilibrium (SMPE) with an infinitesimal period of commitment. One member of this family entails instantaneous production of the level of stock produced in a competitive equilibrium; this is consistent with the Coase Conjecture. Other SMPE in the family entail steady state production at a stock level lower than in the competitive equilibrium. In these equilibria, there may be a jump to the steady state, or the steady state may be approached asymptotically. Monopoly profits are positive in these equilibria, and the Coase Conjecture fails. We contrast this result to other papers which use *non-Markov* strategies to construct multiple equilibria.

JEL classification numbers: D42, L12, Q39

Key words: Coase Conjecture, Depreciation, Multiple Markov Equilibria

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Depreciation Erodes the Coase Conjecture

The Coase Conjecture [Coase (1972)] states that as the period over which a durable goods monopolist is able to make binding commitments diminishes, the monopolist's ability to exert market power decreases. Moreover, in the limit, as the period of commitment becomes infinitesimal, market power vanishes and the monopolist reproduces the competitive outcome. There are two senses in which the Conjecture might be incorrect. First, the competitive equilibrium might be only one of many possible equilibrium outcomes for the monopolist who has an infinitesimal period of commitment. There may be no compelling reason to suppose that the competitive equilibrium represents the most plausible of these outcomes. Second, the competitive equilibrium might not be a equilibrium outcome for the monopolist. We describe a situation where the Conjecture fails in (only) the first sense.[1]

The failure depends on the durable good being imperfectly durable, which we model using a constant positive decay rate. In this case, a steady state stock level implies a positive rate of production in the long run, and this is consistent with long run monopoly profits. There exists a continuum of steady states which satisfy our criteria for equilibrium; this in turn implies the existence of a continuum of equilibrium trajectories. If, on the other hand, the good is infinitely durable, any steady state must involve a zero rate of production and zero flow of profits in the long run. Thus, zero depreciation implies that there is a unique

[1] There are a number of situations where the Conjecture fails for the second reason. These include: production costs that are convex in the *rate of production* [Kahn (1987), Malueg and Solow (1990)]; a constraint on the rate of production [Bulow (1982)];or production costs that depend on *cumulative production*, together with a durable good that depreciates [Karp (1993)]. In none of these cases is the competitive equilibrium also an equilibrium for a monopolist with an infinitesimal period of commitment. Bagnoli *et al.* (1989) describe another situation where the Conjecture fails, and Gul *et al.* (1986) provide conditions under which the Conjecture holds.

steady state, and with our definition of equilibrium, a unique equilibrium path.

Bond and Samuelson (1984) show by example that if the durable good depreciates and the monopolist's period of commitment is infinitesimal, then the competitive equilibrium constitutes a Strong Markov Perfect Equilibrium (SMPE). The modifier "Strong Markov" means that all agents condition their current actions and/or their beliefs about the future on only the current state variable, which in this case is the stock of the durable good. "Perfect" means that the continuation of the original strategies and/or beliefs constitute an equilibrium even if the state has departed from its equilibrium trajectory (following, for example, a deviation by some agent in the past or a random shock). However, their analysis may (incorrectly) suggest that the competitive equilibrium is the only SMPE for their example. Our simpler and more general approach allows us to demonstrate the existence of, and to characterize, a continuum of SMPE.

There are at least three reasons why economists have been interested in the Coase Conjecture. First, the Conjecture has important welfare implications in markets where monopolists produce durable goods. Second, the durable goods monopoly model provides a useful analogy for situations where a strategic agent is constrained by the beliefs of non-strategic agents who have rational expectations. (For example, the Conjecture improves our intuition about why, in some circumstances, a government that cannot make commitments about the future has limited ability to influence private agents.) Third, there is a close parallel between certain bargaining problems and the durable goods monopoly. Fudenberg and Tirole's (1993) text on game theory studies the Coase Conjecture exclusively in the context of a bargaining problem. Our extension of the durable goods monopoly should

interest economists who care about the Conjecture for the first two reasons. We have not, however, discovered the bargaining analog of the durable goods model with depreciation.[2]

The next two sections present the model and characterize the set of SMPE. The following section discusses the result in relation to existing literature. The conclusion provides a summary.

The Model and Basic Result

The usual procedure in modelling the durable goods monopolist begins with a discrete stage problem, in which the monopolist's period of commitment is $\varepsilon > 0$, and then studies the limiting form as $\varepsilon \to 0$. In the interests of simplicity, and because we care only about the problem with an infinitesimal period of commitment, we begin with a continuous time model. The reader can easily verify that our basic equilibrium conditions, equations (2) and (4) below, are the limiting form of the equilibrium conditions to the discrete stage model.

We proceed in three steps to obtain the non-uniqueness result. First, we use dynamic programming to construct a family of candidate SMPE. We then note that for each member of this family, the candidate is patently "unreasonable" over an interval of state space, so we modify the candidate to overcome this objection. This modification does not eliminate any steady state level of output; in particular, it includes the steady state of the monopolist who can precommit. Finally, we verify that the monopolist has no incentive to deviate from the

[2] We think that it is likely that there is an analogy waiting to be discovered. Olsen (1992) shows that when the standard Coasian model is extended to include learning by doing, there is a parallel interpretation as a bargaining problem. This kind of result, in addition to the literature surveyed in Fudenberg and Tirole, suggests that there may also be a bargaining interpretation to the durable goods model with depreciation.

(modified) candidate, so that it does represent a SMPE. This is stated as Proposition 1.

The stock of the durable good at t is Q_t, the rate of production is q_t, and the stock depreciates at constant rate $\delta \geq 0$. We suppress time subscripts where convenient. The equation of motion for Q is

$$\dot{Q} = q - \delta Q . \tag{1}$$

The inverse demand for services (the implicit rental rate) is F(Q),[3] which is exogenous. The interest rate common to all agents is r. The equilibrium price for a unit of the durable good at t must satisfy

$$P_t = \int_t^{\infty} e^{(r+\delta)(t-\tau)} F(Q_\tau) d\tau \quad \Rightarrow \quad \dot{P} = (r+\delta)\ P - F(Q) . \tag{2}$$

so that in equilibrium buyers' beliefs about the value of a unit of the good are confirmed. The constant average cost of production is c.

We assume that F(Q) is strictly decreasing and continuous over the interval $[0, Q_0]$, where Q_0 solves $F(Q_0) = (r+\delta)c$; Q_0 is the steady state that equates price and marginal cost, i.e. the competitive steady state. This assumption corresponds to the "no-gap" case in the standard (no depreciation) durable goods monopoly problem. The term "no-gap" refers to the fact that there is no gap between the cost of production and the reservation price of the buyer with the lowest valuation which is no less than the cost of production. The assumption simplifies the exposition, and we also consider it empirically reasonable. We describe the

[3] We have in mind the case where the durable good is a producer good, so that F(Q) is the value of the marginal product of a machine. Of course the model is also appropriate for the case of consumer durable goods.

alternative "gap" case, and we discuss how this alters our results, in Appendix B. Fudenberg and Tirole, chapter 10, discuss the "no-gap" and "gap" distinction for the standard durable goods model.

To obtain a SMPE we need to find a function P(Q) such that when the monopolist solves the control problem

$$\max_{q \geq 0} \int_t^{\infty} e^{r(t-\tau)} \, [P(Q_\tau)-c] \; q_\tau d\tau \tag{3}$$

subject to (1) with Q_t given, equation (2) is satisfied.[4] In this section we restrict attention to continuous functions P(Q). The reason for this restriction is discussed in a following section. We will occasionally refer to the function P(Q) as an equilibrium, by which we mean that there are equilibrium beliefs by buyers and equilibrium behavior by the monopolist that support P(Q).

We show that there exists a family of such functions, one member of which is the trivial function $P^0(Q) \equiv c$, which corresponds to the competitive equilibrium. (The reason for

[4] If this claim is not obvious, the reader should write down the discrete stage version of (1) and (2), denoting ε as the period of commitment, and $q_t\varepsilon$ as the amount produced and sold at the beginning of period t. Denote the equilibrium price function, induced by the buyers' expectations of the monopolist's *future* sales strategy as $P(Q_{t+\varepsilon};\varepsilon)$. If the stock at the beginning of period t is Q_t and sales in that period are $q_t\varepsilon$, buyers are willing to pay $F(Q_t+q_t\varepsilon)\varepsilon + e^{-(r+\delta)\varepsilon}P[e^{-\delta\varepsilon}(Q_t + q_t\varepsilon);\varepsilon]$. This expression reveals that the monopolist's ability to affect the current price by choosing q_t, given that the function $P(\cdot)$ depends on *future* sales, is of the same order of magnitude as ε. Therefore as $\varepsilon \to 0$ the monopolist takes the current price as given, and solves the control problem (3). This does not necessarily imply that the monopolist reproduces the competitive equilibrium. The competitive seller takes the *price trajectory* as given. The monopolist with a 0 period of commitment takes the *price function* as given; when this is a non-trivial function of Q, the monopolist is able to affect the price trajectory indirectly by controlling Q.

using the superscript 0 to denote the competitive outcome will become clear in a moment.) In this case the monopolist can do no better than to produce in the first instant the quantity Q_0 (defined above) which solves $F(Q) = (r+\delta)c$; thereafter the monopolist produces at the rate δQ_0, and maintains the competitive steady state. Other SMPE may involve slower adjustment to the steady state, lower steady state levels of production, and positive monopoly profits.

Step 1. We construct a candidate SMPE using dynamic programming. Define J(Q) as the value of the monopolist's program when the existing stock is Q. The dynamic programming equation for the monopolist's problem is

$$r\,J(Q) \;=\; \max_{q\geq 0}\; \left[P(Q) - c + J_Q(Q)\right] q \;-\; J_Q(Q)\,\delta Q\;. \tag{4}$$

In order for the monopolist to be willing to produce at a positive, finite rate, the term that multiplies q in (4) must vanish:

$$P(Q) \;=\; c - J_Q(Q)\;. \tag{5}$$

This implies

$$rJ(Q) \;=\; -\delta Q J_Q(Q) \;\Rightarrow\; J(Q)=kQ^{-r/\delta}\;. \tag{6}$$

The parameter k is an arbitrary constant of integration. The differential equation in (6) shows the importance of δ. For $\delta = 0$, this equation implies that $J(Q) = 0$, which implies that $P(Q) = P^0$, whenever the monopolist is producing at a positive finite rate.

Differentiating J(Q) given in (6) and substituting the result into (5) implies that the equilibrium price function, indexed by k, is

$$P^{k}(Q) = c + \frac{rk}{\delta} Q^{-(r+\delta)/\delta} . \tag{7}$$

For k=0, the monopolist reproduces the competitive equilibrium, even when $\delta > 0$. This is consistent with Bond and Samuelson's (1984) result that there is a SMPE that reproduces the competitive outcome when the durable good depreciates and the period of commitment is infinitesimal.

We now calculate a control rule, $q^k(Q)$, that supports the price $P^k(Q)$ and satisfies (2) on the interval $[0,Q_0]$. (For $Q > Q_0$ competitive sellers would set $q = 0$, as would the monopolist.) We substitute P^k for P in the differential equation in (2), and equate the result to the time derivative of P^k. Rearranging this yields the control rule

$$q^{k}(Q) = \frac{\delta^2}{rk}\left[\frac{F'(Q)}{r+\delta} - c\right] Q^{(r+2\delta)/\delta} \geq 0 . \tag{8}$$

This verifies that the non-negativity constraint on production is satisfied over $[0,Q_0]$.

<u>Step 2</u> By construction, the price function P^k and associated control rule q^k are consistent, in that they satisfy the monopolist's first order condition and in equilibrium the consumers' expectations are born out. By definition the equilibrium is Strong Markov: the control rule, given by (8), depends only on the stock of the good, and since the price function also depends only on the stock, so must the buyers' expectations which induce that function. Moreover, the equilibrium is Perfect: any past deviation or random shock which causes the

state to leave its equilibrium trajectory does not alter the monopolist's control problem, so (8) remains an equilibrium sales rule and (7) an equilibrium price function.

However, other considerations imply that there is an upper bound on k; and even if k is below that bound, there is a region of state space (an interval of Q), which depends on the value of k, for which the candidate proposed above is unreasonable. To explain these points we concentrate on a particular class of rental functions, F(Q), defined by Assumption 1, below. This restriction is made only to simplify exposition, since it makes it unnecessary to consider many special cases. As we point out below, the restriction is not necessary to obtain our chief results.

<u>Definition 1:</u> The parameter k_* is a value of k such that P^k is tangent to $F/(r+\delta)$, and Q_* is the value of Q at the tangency.•

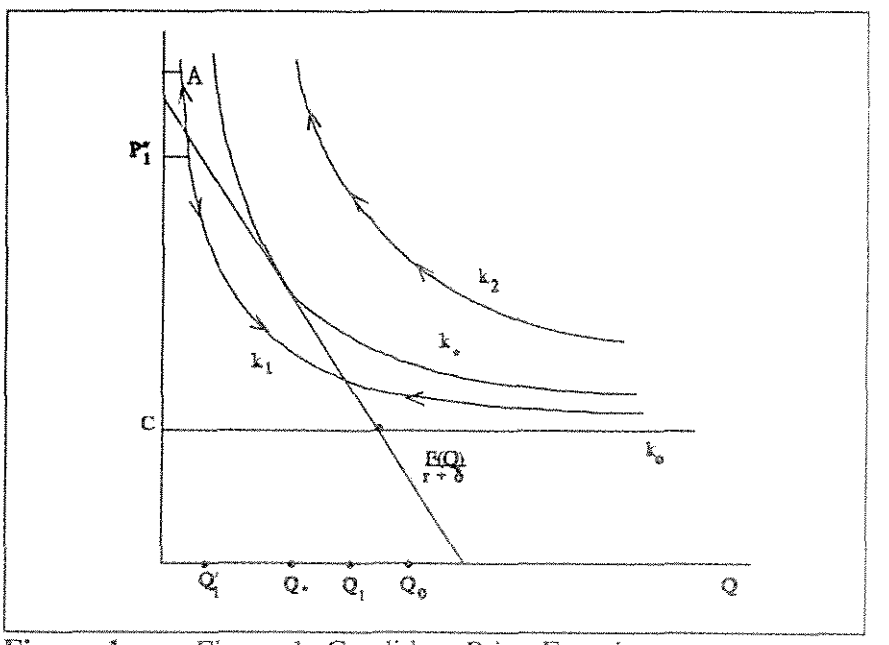

Figure 1 Figure 1: Candidate Price Functions
$0 = k_0 < k_1 < k_* < k_2$

<u>Assumption 1:</u> There is a unique value for k_* and Q_*.[5] For $k < k_*$, P^k and $F/(r+\delta)$ intersect at exactly two points. For $k > k_*$, P^k lies above $F/(r+\delta)$.•

Figure 1 graphs $F(Q)/(r+\delta)$ for linear F, and shows the graphs of P^k for four values of k: $0 = k_0 < k_1 < k_* < k_2$. Any reasonable equilibrium requires $k \leq k_*$. Larger values, such as k_2 in Figure 1, imply that for all values of Q price is greater than $F/(r+\delta)$, which by (2) implies that P is rising. The arrows on the curve labelled k_2 illustrate this. Then, by (7), P becomes unbounded. In this equilibrium agents always buy the durable good because of the expectation of future capital gains. This "Ponzi equilibrium" is consistent with the model, but it seems unreasonable, and we rule it out by requiring that $k \leq k_*$.

Even for $k \leq k_*$, the candidate equilibrium prescribes that price becomes unbounded if the initial value of Q is sufficiently small. The curve labelled k_1 in Figure 1 illustrates this. Figure 1 shows Q_1' as the smaller, and Q_1 as the larger, of the two values of Q at the intersection of the graphs of $F/(r+\delta)$ and P^k when $k = k_1$. For $Q < Q_1'$ or $Q > Q_1$, $P > F/(r+\delta)$ and, as the arrows indicate, price rises. The point Q_1' is an unstable steady state, and Q_1 is a stable steady state. When $k = k_1$, for any value of $Q < Q_1'$, price rises and the stock falls. This leads to another Ponzi equilibrium. To avoid this outcome we construct a modified candidate price function as follows: For any $0 < k < k_*$, define the points (Q_k', P_k') and (Q_k, P_k) as, respectively the smallest (minimum Q) and the largest intersection of the curves P^k and $F/(r+\delta)$. Take (Q_k'', P_k'') as a point on the curve P^k, with Q_k'' in the interval $[Q_k', Q_k]$.

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[5] The tangency point Q_* solves $h(Q) = g(Q)$, where $h(Q) \equiv F(Q) - (r+\delta)c$ and $g(Q) \equiv -\delta F'(Q)Q/(r+\delta)$. For each Q_* there is associated a unique k_*. Therefore Q_* and k_* are unique iff there is a unique solution to $h(Q) = g(Q)$.

Define the modified candidate for the price function as

$$\hat{P}^{k}(Q) \equiv \begin{Bmatrix} P^{k}(Q) & for & Q \geq Q_k'' \\ P_k'' & for & Q < Q_k'' \end{Bmatrix}. \tag{9}$$

The modification flattens the original candidate at a point in the interval $[Q_k', Q_k]$ .An example of this function is shown in Figure 1. The graph of the modified price function $\hat{P}^k$ for $k = k_1$ is the horizontal line at P_1'', and the part of the curve labelled k_1 below this line. The sales rule that supports this candidate is

$$\hat{q}^{k}(Q) \equiv \begin{Bmatrix} q^{k} \; for \; Q \geq Q_k'' \\ \infty \; for \; Q < Q_k'' \end{Bmatrix}. \tag{10}$$

The expression $\hat{q}^k = \infty$ means that sales rate is infinite for an instant, causing the stock to jump to Q_k''.

For given $k < k_*$, there are a continuum of $\hat{P}^k$, which depend on the choice of (Q_k'', P_k''). That is, $\hat{P}^k$ depends on the height at which we flatten the original function P^k. To avoid excessive notation, we do not make this dependence explicit. However, it is important to note that we cannot choose (Q_k'', P_k'') strictly above (to the left) of (Q_k', P_k'). The reason can be seen by examining the horizontal line that intersects the curve k_1 at point A in Figure 1. Point A would be a (stable) steady state [under sales rule (10)][6], so at A buyers do not anticipate

[6] We can construct other sales rules that support (9) and which result in stable steady states to the left of point A, but this modification would not change our conclusion.

capital gains. This can not be an equilibrium, since price exceeds the present value of the stream of implicit rents at A. If we were to flatten the price function at a level strictly below (Q_k, P_k) we would also obtain a stable steady state where there are no capital gains and sales price is above the present value of the stream of implicit rents. This explains why we restrict Q_k'' to lie in the interval $[Q_k', Q_k]$.

The modification expressed by (9) and (10) eliminates the Ponzi features of the original candidate equilibrium, and it retains consistency.

Step 3 Finally, we verify that the modified candidate given by (9) and (10) is an equilibrium. The discussion thus far has used only the first order condition of the monopolist's problem, equation (5), the buyers' rational expectations constraint, given by (2), and the elimination of Ponzi equilibria. Because of the linearity of the monopolist's control problem, the first order condition is not sufficient for a maximum. Our final step is to show that the monopolist would have no incentive to deviate from the proposed equilibrium. Once this is done, we have the basic result of this section, which we state as

Proposition 1: Suppose that F is continuous, k is chosen so that P^k intersects $F/(r+\delta)$, and Q_k'' is chosen to lie in an interval over which P^k is not above $F/(r+\delta)$. In this case, the price function (9) and the sales rule (10) constitute a SMPE for the durable good monopolist with an infinitesimal period of commitment.◦

All proofs are contained in Appendix A. Note that the conditions of Proposition 1 are weaker than Assumption 1.

Analysis of the Model

We discuss the implications of the model in this section. An immediate consequence of Proposition 1 is that any steady state in the interval $(0, Q_0]$, including that of the precommitted monopolist, can be supported by a SMPE (Proposition 2). If we require the equilibrium to be stable (in a sense defined below), monopoly profits in a SMPE are lower than under the precommitted monopolist (Proposition 3).

Since all values of $Q \in (0,Q_0]$ are points of intersection between P^k and $F/(r+\delta)$ for some k we have

Proposition 2: If F is continuous, any level of output in the interval $(0, Q_0]$ and corresponding price on the curve $F/(r+\delta)$ can be supported as a steady state in a SMPE.•

The interval $(0, Q_0]$ includes the steady state under the precommitted monopolist, which we denote as Q_{pm}. Therefore the implication of Proposition 2 is that the inability to make commitments and restriction to SMPE do not necessarily reduce the monopolist's payoff. By further restricting the set of "reasonable" equilibria, we can overturn Proposition 2. As we noted above, points like Q_1' in Figure 1 are unstable steady states. By excluding such points we eliminate Q_{pm} as an equilibrium steady state.

<u>Definition 2</u>: We designate an equilibrium as *stable* if altering the initial condition does not change the steady state.•

Proposition 3: A (continuous) stable SMPE cannot support the precommitted monopolist's steady state Q_{pm}.•

We include the qualifier "continuous" to remind the reader that (9) and (10) are constructed under the assumption that buyers' beliefs induce a continuous function P(Q). In the next section we explain why this qualification is unimportant. If the assumption of stability is considered a reasonable requirement for equilibrium, Proposition 3 implies that the inability to make commitments results in a loss of market power.[7] The assumption of stability reduces the set of equilibria, but still leaves a continuum of possibilities.

If F(Q) satisfies Assumption 1 it is easy to show that the set of stable steady states consists of $[Q_*, Q_0]$. The set of stable (continuous) SMPE resembles the equilibrium predicted by the Coase Conjecture, but has important differences. Under Assumption 1 the steady state is always higher than under precommitted monopoly, but, contrary to the Coase Conjecture, it may be lower than under perfect competition. If the initial stock of Q is small, there is a jump in the stock. In some of these equilibria the initial jump is to the steady state (if $Q_k'' = Q_k$). In other equilibria (where $Q_k'' < Q_k$) the jump is to a level lower than the steady state, and the stock increases asymptotically to the steady state. This last possibility means that consumer welfare is not necessarily higher in a SMPE than under the precommitted monopolist. The level of the stock immediately after the jump may be *higher* under the precommitted monopolist. Although steady state consumer welfare is always higher in a stable SMPE, it may take a long time to get close to the steady state. During much of the transition, consumer welfare could then be lower in a SMPE. In that case, the present value at time 0 of consumer welfare would be higher under precommitted monopoly. This is more

[7] We think that stability is a reasonable requirement, for essentially the same reasons that we regard equilibria supported by punishment strategies as unreasonable *in this context*. We return to this issue in the next section. See also note 11.

likely to occur if consumers' discount rate is higher than the monopolist's.

It is also apparent that if stability is viewed as a reasonable property for equilibria, the monopolist has an incentive to build-in obsolescence. This incentive is absent for the competitive firm or the monopolist who can precommit. This point has been recognized previously [e.g. Bulow (1986) and Bond and Samuelson (1984)] for particular SMPE, but it also holds for a much more general class.

For example, if $F = 1 - Q$, the steady state under competition is $Q_0 = 1 - (r+\delta)c$, and the steady state under the monopolist who can make binding commitments is $Q_{pm} = Q_0/2$. The smallest stable SMPE steady state is $Q_* = (r + \delta)Q_0/(r + 2\delta)$. As $r \to 0$ the inability to make commitments results in negligible loss to the monopolist. If $c = 0$, so that Q_0 is independent of δ, then as $\delta \to \infty$, the inability to make commitments again results in negligible loss to the monopolist. However as $\delta \to 0$, monopoly profits are 0.[8] Since profits are 0 for $\delta = 0$ and can be positive when $\delta > 0$, potential profits are certainly increasing for small δ: the monopolist has an incentive to build-in obsolescence. When $c = 0$ it is easy to show that maximal profits (i.e., profits under the monopolist's preferred SMPE) are monotonically increasing in δ, so the monopolist would like to set $\delta = \infty$. For positive c, it must be the case that $(1-rc)/c > \delta$ in order for it to be profitable to produce the good at all; thus when production costs are positive the monopolist's optimal level of δ is positive and finite. The optimal level of δ is 0 for the monopolist who can pre-commit and for the social planner, when $c > 0$.

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[8] The elimination of Ponzi equilibria means that under Assumption 1 the set of stable steady states of SMPE is continuous in δ, even at $\delta = 0$. We can make every point in this set arbitrarily close to the competitive equilibrium by choosing δ sufficiently close to 0.

Relation to Previous Work

Our major contribution has been the construction and characterization of a class of SMPE for the durable goods monopolist with an infinitesimal period of commitment. We have shown that when the good depreciates, the equilibrium is not unique and the Coase Conjecture need not hold. In this section we discuss the relation between our result and existing literature.

It is widely recognized that if agents' beliefs are discontinuous in the variable(s) upon which they are conditioned, equilibria are likely to be non-unique; this is the basis for the Folk Theorem of repeated games, and similar results hold in dynamic games. Stokey (1981) demonstrated an analogous result for the durable goods monopolist. Ausubel and Deneckere modeled the problem as a game among a monopolist and a continuum of buyers, and also showed that there were many equilibria. Using a discrete time model, they showed that there always exists a Weak Markov Perfect Equilibrium (WMPE). As the period of commitment diminishes, monopoly profits under this WMPE approach 0. A sketch of their argument is as follows: Under an alternate trajectory, in the first period the monopolist sells nearly the first-best level, Q_{pm}, and thereafter increases the stock very slowly. Total profits can be made close to the first-best level, and continuation profits, from any stock level, are always strictly positive. If the monopolist deviates from this alternate trajectory, buyers believe he will subsequently follow the WMPE. This "punishment" can be used to support profits arbitrarily close to the first-best level, if the period of commitment is sufficiently small.

Bond and Samuelson (1987) consider the case of a durable good that depreciates. They show that there are multiple equilibria and steady states, including that of the

precommitted monopolist. We provide the gist of their argument, using our notation (and continuous time). If the initial stock Q is less than or equal to Q_{pm} (for example), the monopolist immediately sells the discrete amount Q_{pm} - Q and thereafter maintains that level by selling at rate δQ_{pm}. This is the first-best trajectory (*at time 0*), and on it the monopolist earns positive profits at every point in time. If he ever deviates by selling more than δQ_{pm}, this drives the stock above Q_{pm}. Thereafter buyers believe that $P \equiv c$ and the monopolist earns 0 profits. Since $P \equiv c$ is an equilibrium, this is a "credible threat" and it deters the monopolist from deviating from the first-best trajectory.

Note that although in this case the threat is a SMPE, the equilibrium is conditioned on histories, and is not Markov. If one were to (mistakenly) interpret this equilibrium as Markov, and graph P as a function of Q, the result might be[9] a step function, with steps at $F(Q_{pm})/(r+\delta)$ and c, and the discontinuity at Q_{pm}. However, this graph does not represent a Markov equilibrium function. To understand why, suppose to the contrary that the step function just described did represent a discontinuous *Markov* equilibrium. In that case, if the stock were ever at $Q_{pm}+\varepsilon$, for $\varepsilon > 0$, the equilibrium prescribes that the monopolist immediately sells the competitive amount and earns zero profits. However, by deviating from this path and setting $q = 0$ until the stock decays to Q_{pm}, and thereafter selling at rate δQ_{pm}, the monopolist earns positive profits. The monopolist would obviously want to deviate,

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[9] We include this discussion as a response to one reader who claimed that Bond and Samuelson (1987) had shown that discontinuous *Markov* price functions could support multiple equilibria. This reader drew their equilibria as the step function described above. Since the equilibrium is conditioned on history as well as current Q, there are many (misleading) ways that one might graph P as a function of only Q. The confusion may have been due to the fact that Bond and Samuelson described the equilibrium they proposed as "nonstationary", and said nothing about it being history dependent.

making the buyers' belief that $P \equiv c$ for $Q > Q_{pm}$ incorrect. This belief can therefore not serve as a punishment that supports the first-best trajectory.[10] In order for the "threat" to serve as a credible punishment it must be the case that buyers believe that $P \equiv c$ if $Q > Q_{pm}$, *or if Q had ever exceeded Q_{pm} in the past.* This is why the equilibrium proposed by Bond and Samuelson is history dependent. The same argument applies to any discontinuous function that is used to support a steady state equilibrium at the point of discontinuity. This does not necessarily mean that all SMPE are continuous, but it does preclude the possibility of using discontinuity as a means of "sneaking punishments in the back door". This explains why, in previous sections, we have restricted attention to beliefs which induce continuous functions P(Q).

Thus, both Ausubel and Deneckere and Bond and Samuelson obtain non-uniqueness by using a Folk-theorem type argument which relies on credible punishments. The equilibria they propose are not Markov. Our approach, in contrast, shows that even with the restriction to strong Markov behavior, equilibria are not unique. Although punishment strategies have an obvious appeal in games with a few players, they are less plausible in situations with a continuum of agents. It is harder to believe that a continuum of buyers would all dramatically revise their beliefs about the future in the event that the seller deviates by even a small amount from a proposed equilibria.[11] Markov equilibria seem more reasonable in a

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[10] This point may not be obvious, because readers are accustomed to discontinuous Markov perfect strategies in noncooperative dynamic games. Dutta and Sundaram (1993) provide an example and Karp and Newbery (1993, pg 888) discuss the general issue.

[11] Our scepticism regarding the plausibility of punishment strategies is based on the fact that they involve discontinuous - and typically large - changes in beliefs and strategies following small deviations from equilibrium behavior. We make a distinction between the coordination

situation with a continuum of buyers. Previous papers, by relying on non-Markov strategies to overturn the Coase Conjecture, have made the Conjecture appear more plausible. The fact that the Coase Conjecture need not hold even with the restriction to SMPE, demonstrates how implausible the Conjecture is.

There is another paper which has a close technical relation to our paper, although the economic context is very different. Tsutsui and Mino (1990) explain why there may be a continuum of differentiable SMPE in a noncooperative differential game. They ascribe the non-uniqueness to an "incomplete transversality condition". There is another way of expressing this. The first order conditions of Markov equilibria (in games or control problems with a single state variable) can often be used to obtain ordinary differential equations (ODE's) that characterize the equilibrium. For example, above we obtained an ODE for the value function in (6). In many cases, however, there is no "natural boundary condition" for this ODE, and therefore no way to pin down the equilibrium. If, for example, we were told that the steady state stock was some number Q_s, we could evaluate the flow of profits at this level, π_s, and thereby obtain the boundary condition for (6), $J(Q_s) = \pi_s/r$. However, the steady state is endogenous, and in general the Markov assumption is not restrictive enough to lead to a unique value. When $\delta = 0$ there is a natural boundary condition: the competitive stock level. If the good never depreciates, the monopolist must eventually produce the competitive level; stopping production (and profits) when some

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problem, which arises whenever there are multiple equilibria (as in our model, or with punishment strategies), and the fragility (or complexity) of beliefs (which is required for punishment strategies, but not in our model). This was also the basis for the stability requirement of the previous section. If we did not require stability, we saw that a small change in behavior, which changed the level of the stock, would lead to a large change in the equilibrium outcome.

demand is unsatisfied, could not be part of a Markov equilibrium. This fact leads to a terminal condition on the state variable, and this provides the missing boundary condition to the ODE.

The importance of this observation extends beyond the model studied here. For example, if depreciation were introduced into Kahn's model, the SMPE would no longer be unique. The multiplicity of SMPE (in the durable goods monopoly model) has been overlooked because in the past people have studied the equilibrium of the infinite horizon game by taking the limit, as the horizon goes to infinity, of the finite horizon model. For the finite horizon model there is obviously a terminal condition on the monopolist's value function. This condition is a boundary condition that pins down the equilibrium. Using the inductive argument found in Bond and Samuelson (1984), we see that there is a unique equilibrium to every finite horizon game (at least in the linear case), and the unique limit[12] (as the horizon approaches infinity and the period of commitment approaches 0) of this sequence of equilibria satisfies the Coase Conjecture. We have shown that solving the model backwards from the terminal period, and then taking limits, identifies only one of a continuum of equilibria of the infinite horizon game.

The relation between the limiting equilibrium of the finite horizon model, and *a particular* equilibrium in the infinite horizon model, has recently been studied by Driskill (1994). He adds depreciation to Kahn's linear-quadratic infinite horizon model and solves for the linear-quadratic SMPE (linear price function and control rule and quadratic value

[12] Given the terminal condition implied by the final period, the difference equations which determine the parameters of the linear-quadratic equilibrium, converge to a unique limit.

function). He compares this to the equilibrium of the corresponding finite-horizon model, and demonstrates a "turnpike property": The equilibrium paths of the infinite horizon and the finite horizon models can be made arbitrarily close over an arbitrarily long finite interval, by choosing the horizon sufficiently large.

Conclusion

Introducing depreciation into the durable goods monopoly model causes the Coase Conjecture to fail. Even if production costs are constant, and agents condition their current actions and their beliefs about all agents' future actions on only the state variable, there is no reason to suppose that the monopolist will reproduce the competitive equilibrium when his period of commitment is 0. Previous results which appear to suggest that depreciation does not weaken the Coase Conjecture, identified a particular Markov equilibrium. This is equivalent to assuming a particular boundary condition. In general, however, there is no natural boundary condition to the monopolist's problem, and thus there exists a continuum of SMPE.

Previous results which show that the outcome predicted by the Coase Conjecture is only one of many possible outcomes, all relied on non-Markov equilibria. The intent of these models was to show that the Conjecture was unlikely to describe reality. However, exactly the opposite conclusion could be drawn if one regards non-Markov behavior as implausible in markets with a continuum of agents. That is, it may have appeared that non-Markov behavior was *necessary* to overturn the Conjecture. We have shown that even in a Markov equilibrium, the Conjecture need not hold.

Appendix A: Proof of Propositions

Proof of Proposition 1 By construction it is clear that for $Q \geq Q_k''$ (9) and (10) satisfy the first order condition of the monopolist's problem and that consumers' expectations are verified in equilibrium. For $Q < Q_k''$ the monopolist can do no better than to cause the stock to jump to Q_k'', since he takes demand as perfectly elastic over that region. (By the Markov assumption, delaying reaching the level Q_k'' does not alter consumers' expectation of future behavior, and thus does not alter their willingness to pay.) Again, consumers expectations are realized. An instant before the jump, no consumer would be willing to pay more than P_k''.

Since the monopolist has nothing to gain by refusing to make the prescribed jump when the initial condition for Q lies in the interval $[0, Q_k'')$, we need to consider only the case where the initial condition lies at or above Q_k''. Given the proposed price function over that region, (7), the monopolist's optimization problem can be written as the calculus of variations problem

$$\max_{\dot{Q}+\delta Q \geq 0} \int_0^\infty \left[G(Q,t) + H(Q,t)\,\dot{Q} \right] dt \tag{A1}$$

$$G \equiv e^{-rt} rkQ^{-r/\delta} \qquad H \equiv e^{-rt} \frac{rk}{\delta} Q^{-(r+\delta)/\delta}$$

We note that the following relation holds for all t and Q:

$$\frac{\partial G}{\partial Q} \equiv \frac{\partial H}{\partial t}. \tag{A2}$$

We use Figure A1 to establish that the monopolist has no incentive to defect from the candidate equilibrium. At time t = 0 the stock is at point A. The proposed equilibrium has

(for example) the stock increasing asymptotically to the steady state level Q_∞, along the solid curve labelled ABC; we denote this trajectory as Q(t). An arbitrary defection is shown as the dotted curve through ADE; we denote the defection as $Q^d(t)$.

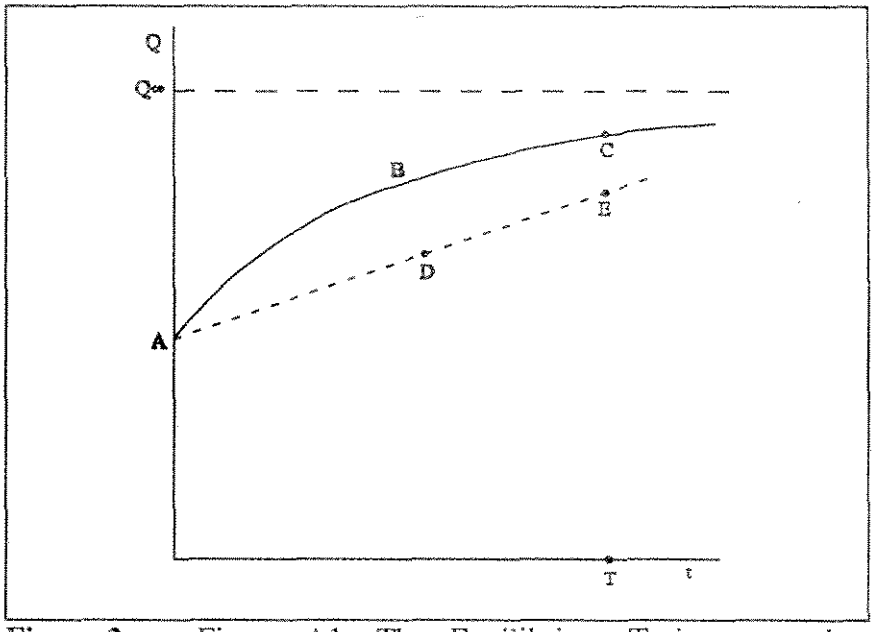

Figure 2 Figure A1: The Equilibrium Trajectory and a Defection

Define J^p as the monopolist's payoff under the proposed equilibrium and J^d as his payoff under the defection. In order to make use of a simple proof (based on Clark, pp 53 - 55), we will compare the proposed equilibrium Q(t) with defections that eventually reach Q(t). To this end, we define a modified defection, $Q^{dT}(t)$ as follows: For arbitrary T and arbitrary defection $Q^d(t)$, $Q^{dT}(t) \equiv Q^d(t)$ over [0,T); at T Q^{dT} begins a Most Rapid Approach Path (MRAP) to Q, and after reaching it, $Q^{dT}(t) \equiv Q(t)$. (For the example in the figure, $Q^d(T) < Q(T)$, so the MRAP is a vertical line, and $Q^{dT} = Q$ for all $t > T$; the argument that follows uses this fact, and needs to be modified in an obvious way for defections that lie above Q at T.) Define J^{dT} as the monopolist's payoff under the modified defection. Since the flow of monopoly profits is bounded and $r > 0$, it is obvious that $\lim_{T\to\infty} J^{dT} = J^d$. The increase in the monopolist's

payoff due to the defection is

$$J^d - J^p = \lim_{T\to\infty}(J^{dT} - J^p)$$

$$= \lim_{T\to\infty}\left[\int_0^T \left[G(Q^{dT},t)+H(Q^{dT},t)\ \dot{Q}^{dT}\right] dt - \int_0^T \left[G(Q,t)+H(Q,t)\ \dot{Q}\right] dt\right] \quad \text{(A3)}$$

$$= \lim_{T\to\infty}\left[\int_{ADEC}\left[G(Q,t)\ dt + H(Q,t)\ dQ\right] - \int_{ABC}\left[G(Q,t)\ dt + H(Q,t)\ dQ\right]\right]$$

$$= \lim_{T\to\infty}\oint_{ADECBA}[G\ dt + H\ dQ] = \lim_{T\to\infty} -\iint\left[\frac{\partial G}{\partial Q} - \frac{\partial H}{\partial t}\right] dQ\ dt = 0$$

The integrals on the third line of (A3) are line integrals; note that the points E and C are functions of T. The second equality on the fourth line follows from Green's Theorem; the double integral is over the region bounded by the curve ADECBA. The last equality uses (A2). Equation (A3) demonstrates that the monopolist has no incentive to deviate from the proposed path, so it does represent a MPE.

This proof shows that the monopolist is indifferent between deviating from and adhering to the proposed equilibrium sales trajectory. This is also the case in the equilibrium described by the Coase Conjecture.•

Proof of Proposition 2: Pick any value of $\bar{Q} \in (0, Q_0]$ and obtain k by solving $P^k(\bar{Q}) = F(\bar{Q})/(r+\delta)$. (It is obvious from (7) that such a k exists.) For this value of k, obtain $\hat{P}^k$ by flattening P^k at $\bar{Q}$. This supports $\bar{Q}$ as a steady state.•

Proof of Proposition 3: We show that the steady state for the precommitted monopolist is an

unstable steady state under $\hat{P}^k$. If the monopolist were able make binding commitments, his steady state level of output Q_{pm} solves

$$\frac{F(Q) + QF'(Q)}{r + \delta} = c . \tag{A4}$$

We define k_{pm} as the level of k such that $F(Q_{pm})/(r+\delta) = P^k(Q_{pm})$. (See Figure A2.) Using (A4) and (7) in the equality that defines k_{pm} implies

$$\begin{aligned} \frac{F'(Q)}{r + \delta} &= -\frac{rk}{\delta} Q^{-(r+2\delta)/\delta} = \frac{\delta}{\delta + r} P' \\ \Rightarrow \quad P' &< \frac{F'(Q)}{r+\delta} . \end{aligned} \tag{A5}$$

The functions in (A5) are evaluated at Q_{pm}. The second equality in the first line follows from the definition of P^k. The second line of (A5) implies that the graph of P^k that intersects $F/(r+\delta)$ at Q_{pm} is as shown in Figure A2. That is, the intersection is at an unstable point.•

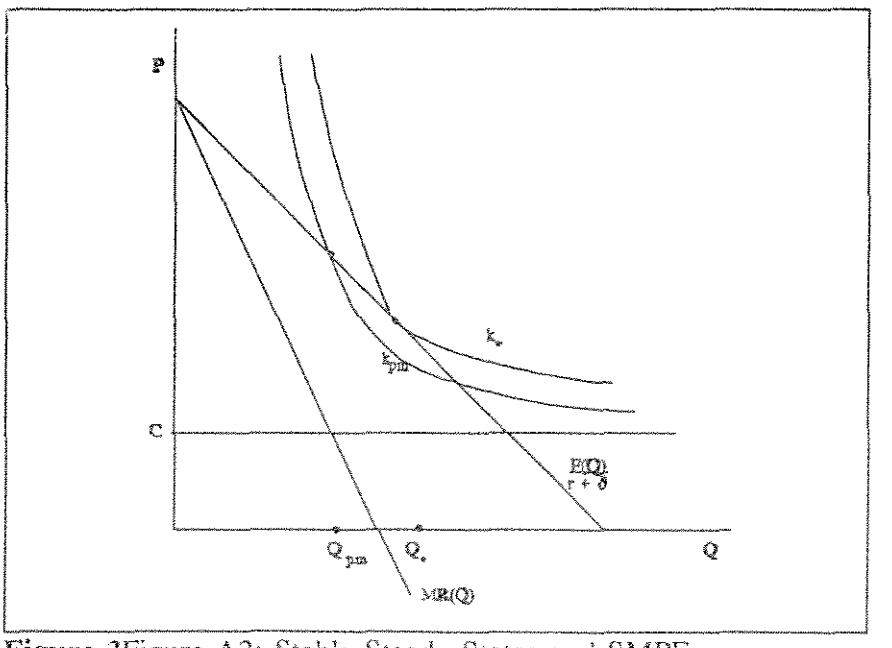

Figure 3 Figure A2: Stable Steady States and SMPE

Appendix B: The "Gap" Case

We briefly discuss how our results are changed in the "gap" case. Suppose now that F(Q) is a continuous decreasing function over $[0,\hat{Q}]$, with $\hat{Q} < Q_0$; $F(Q) < (r+\delta)c$ for $Q > \hat{Q}$. (There is a discontinuity in F(Q) at $\hat{Q}$. Alternatively, we may assume that F(Q) is not defined for $Q > \hat{Q}$.) The proof of Proposition 1 did not depend on the "no gap" assumption. This proposition continues to hold even with the gap. Proposition 2 requires the obvious modification that any output level in the interval $(0,\hat{Q}]$ can be supported as a steady state in a SMPE. However, Proposition 3, and the associated discussion, must be changed for the gap case. To understand the nature of the change, consider the simple case where F(Q) is linear for $Q \leq \hat{Q} < Q_0$, and $F(Q) < (r+\delta)c$ for $Q > \hat{Q}$. We ask the reader to refer to Figure 1, to include the value $\hat{Q}$ on the horizontal axis, and to mentally erase that portion of the curve $F/(r+\delta)$ to the right of $\hat{Q}$. We consider two possibilities: (i) $\hat{Q} > Q_*$ and (ii) $\hat{Q} < Q_*$, where Q_* is the point of tangency. In case (i) the only stable steady states must lie in the interval $[Q_*, \hat{Q}]$. Since the steady state of the precommitted monopolist lies to the left of this interval, Proposition 3 continues to hold. In case (ii), however, there is only one stable steady state stock level, in the class of equilibria we have obtained. This is the value $\hat{Q}$. Any steady state price in the interval $[c, F(\hat{Q})/(r+\delta)]$ can be supported as a SMPE. To see why, draw a vertical line at $\hat{Q}$. Using equation (2), we see that at any point on a curve $P^k(Q)$ to the right of this vertical line, price is rising. At any point on the curve $P^k(Q)$ to the left of that line, and below $F(Q)/(r+\delta)$, price must be falling. If $\hat{Q}$ is small enough, it may equal the stock level of the precommitted monopolist, in which case, Proposition 3 clearly does not hold. This is not an interesting possibility, however, since it implies that demand becomes perfectly

inelastic at such a small stock level, that the competitive and the precommitted monopoly steady states are identical.

REFERENCES

Ausubel, L., and R. Deneckere (1989): "Reputation and Bargaining in Durable Goods Monopoly," *Econometrica*, 57(3), 511-532.

Bagnoli, M., Salant, S. and Swierzbinski, J. (1989): Durable goods monopoly with discrete demand. *Journal of Political Economy, 97, 1459 - 1478.*

Bond, E., and L. Samuelson (1984): "Durable Good Monopolies with Rational Expectations and Replacement Sales," *Rand Journal of Economics*, 15, 336-345.

Bond, E. and L. Samuelson (1987): "The Coase Conjecture Need Not Hold for Durable Good Monopolies with Depreciation" *Economics Letters* 24, 93 - 97.

Bulow, J. (1982): "Durable Goods Monopolists," *Journal of Political Economy*, 90, 314-332.

Bulow, J. (1986): "The Economic Theory of Planned Obsolescence" *Quarterly Journal of Economics* 101, 729 - 749.

Clark, C. (1990): Mathematical Bioeconomics Wiley Interscience, New York.

Coase, R. (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15, 143-149.

Driskill, R. (1994): "Durable Goods Monopoly, Increasing Marginal Costs and Depreciation" Vanderbilt University working paper.

Dutta, P. and R. Sundaram (1993): "The Tragedy of the Commons?" *Economic Theory* 3, 413 - 426.

Fudenberg, D. and Tirole J. (1993): Game Theory MIT Press, Cambridge MA.

Gul, F., H. Sonnenschein, and R. Wilson (1986): "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155-190.

Kahn, C. (1987): "The Durable Goods Monopolist and Consistency with Increasing Costs,"

Econometrica, 54, 274-294.

Karp, L. (1993): "Monopoly Extraction of a Durable Nonrenewable Resource: Failure of the Coase Conjecture" *Economica*, 60, 13 - 26.

Karp, L. and D. Newbery (1993) "Intertemporal Consistency Issues in Depletable Resources" chapter 19 in *Handbook of Natural Resource and Energy Economics*, ed A. Kneese and J Sweeny, North-Holland.

Malueg, D., and J. Solow (1990): "Monopoly Production a Durable Exhaustible Resource," *Economica*, 57:29-47.

Olsen, T.E. (1992): "Durable Goods Monopoly, Learning by Doing, and the Coase Conjecture" *European Economic Review* 36, 157 - 177.

Tsutsui, S. and K. Mino (1990): "Nonlinear Strategies in Dynamic Duopolistic Competition with Sticky Prices," *Journal of Economic Theory* 52: 136 - 161.

Stokey, N. (1981). Rational Expectations and Durable Goods Pricing. *Bell Journal of Economics*, 12, 112-128.