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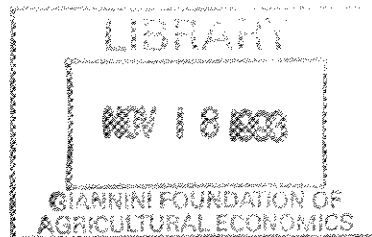
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ON THE CREATION AND DESTRUCTION OF PUBLIC GOODS:  
THE MATTER OF SEQUENCING

by

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**THE MATTER OF SEQUENCING**

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## Abstract

We consider sequences where a subset of public goods are systematically being created or destroyed. For the case of strict Hicksian substitutes between all pairs of this subset of public goods, we show that willingness to pay for an increase in a particular public good is strictly decreasing the farther out in a sequence it is valued. For the destruction of public goods the reverse is true for willingness to accept compensation. Thus, sequencing has opposite implications for the assessment of the benefits of providing public goods and for the assessment of the damages from destroying them.

## Introduction

The issue of valuing public goods, particularly environmental amenities, has moved to the forefront of many recent policy discussions. One of the most disturbing aspects of the efforts of environmental economists to place dollar values on public goods is the casual observation that if one summed the apparent valuations of the public for individual environmental amenities, it would quickly exceed disposable household income.<sup>1</sup> Hoehn and Randall (1989) have theoretically shown in the context of the benefit assessment of new projects, that the problem lies in the adding together of individually derived willingness to pay estimates. If those projects are substitutes and compete with each other for the agent's income, the sum of independently derived willingness to pay estimates will exceed, often by a large amount (Hoehn, 1991), the agent's willingness to pay for the projects taken together as a group. Hoehn and Randall's work represents the formalization of an old concern: given a number of potential policies which by themselves appear to be beneficial to undertake, how many should actually be undertaken and in what order should those chosen be undertaken.

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<sup>1</sup>This observation is typically made by critics (e.g., Cummings, 1989; d'Arge, 1989; Harrison and Hausman, 1989; Kahneman and Knetsch, 1992; and Phillips and Zeckhauser, 1989) of using contingent valuation (Mitchell and Carson, 1989) to place a dollar value on physical injuries to publicly owned natural resources.

This paper extends that work by looking at both willingness to pay (WTP) and willingness to accept (WTA) compensation sequences for public goods and at the relationship between such sequences.<sup>2</sup> It uses a stronger, yet generally plausible assumption, that the public goods of interest are strict Hicksian substitutes, which greatly simplifies our analysis and makes it more intuitive.

### Formulation of the Problem

Suppose we have the following:

$X \in R^n$  = Levels of  $n$  Private Goods

$Q \in R^k$  = Levels of  $k$  Publicly Provided Goods of Interest

$Z \in R^m$  = Levels of  $m$  Other Publicly Provided Goods<sup>3</sup>

$p \in R^n$  = Prices of  $n$  Private Goods

$y$  = Consumer's income

Consumer satisfaction is derived from both marketed and non-marketed goods. Preferences over these goods can be represented by a utility function:

$U(X, Q, Z): R^{n+m+k} \rightarrow R^1$  which possesses the properties

i) twice continuously differentiable,

ii) strictly quasi-concave, and

iii) strictly increasing in  $X, Q$  and  $Z$ .

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<sup>2</sup>As such, this paper can be seen to follow a line of papers (Willig, 1976; Randall and Stoll, 1980; Hanemann, 1991) which have examined issues related to the nature of the divergence between willingness to pay and willingness to accept compensation.

<sup>3</sup> It is possible for  $m$  to equal zero implying all public goods are of interest.

We will focus our attention on the relationships between the different public goods of interest to the benefit or damage assessment. The other  $m$  public goods will remain fixed at  $Z = \hat{Z}$  throughout the entire analysis and therefore can be considered parameterized in the utility function. We can thus simply consider the utility function:

$$U(X, Q): R^{n+k} \rightarrow R^1,$$

where  $U(X, Q) = U(X, Q; \hat{Z})$ . We assume that consumers freely choose the quantities of marketed goods subject to a budget constraint, but that public goods are different because they are provided in fixed quantities.

Economists generally employ utility constant analysis when considering changes in the provision of public goods. The fundamental assumption in utility constant analysis is that the consumer solves the problem

$$\min_X p'X \quad \text{s.t.} \quad U(X, Q) \geq U^0 \quad \text{where } Q, U^0 \text{ are given.}$$

The minimal expenditure necessary to achieve the utility level  $U^0$  given market prices  $p$  and public goods provision  $Q$  is given by the restricted expenditure function  $e^*(p, Q; U^0) = p'X^h$  where  $X^h$  solves the minimization problem.

Now suppose we are interested in the willingness to pay for an increase in one component of the public goods vector  $Q$ , say  $q_l$  from level  $q_l^0$  to  $q_l^1$ . We can write willingness to pay as the difference between the minimal expenditure after the change and minimal expenditure before the change given utility level  $U^0$ ,

$$WTP_1 = e^*(p, q_1^0, Q_{-1}; U^0) - e^*(p, q_1^1, Q_{-1}; U^0) = - \int_{q_1^0}^{q_1^1} \frac{\partial e^*(p, Q; U^0)}{\partial q_1} dq_1$$

Mäler (1974) showed that under the initial assumptions on  $U(X, Q)$  stated above and positive consumption of all marketed goods, there exists an implicit price vector  $p^* \in R^k$  such that

$$\frac{\partial e^*(p, Q; U^0)}{\partial q_i} = -p_i^*(p, Q; U^0) \quad i = 1, 2, \dots, k.$$

These implicit prices can be interpreted as follows: if the consumer were solving the unrestricted problem,

$$\min_{X, Q} p'X + p^*Q \quad \text{s.t.} \quad U(X, Q) \geq U^0$$

then the implicit prices are those that would yield the solution vector  $(X, Q)$  that is the same as the  $X$  and  $Q$  from the restricted problem. We can write willingness to pay in integral form while relating to the implicit price  $p_1^*$ :

$$WTP_1 = - \int_{q_1^0}^{q_1^1} \frac{\partial e^*(p, Q; U^0)}{\partial q_1} dq_1 = \int_{q_1^0}^{q_1^1} p_1^*(p, Q; U^0) dq_1$$

The sensitivity of  $WTP_1$  to the levels of other public goods of interest can be represented by the  $k-1$  vector of partial derivatives with components:

$$\frac{\partial WTP_1(Q_{-1}; U^0)}{\partial q_j} = \frac{\partial}{\partial q_j} \left[ \int_{q_1^0}^{q_1^1} p_1^*(p, Q; U^0) dq_1 \right] \quad j = 2, 3, \dots, k.$$

Under the assumption that  $e^*(p, Q; U)$  is twice continuously differentiable and the number of other public goods of interest is finite, we have necessary conditions which allow for the exchange of integration and differentiation:

$$\frac{\partial WTP_1(Q_{-1}; U^0)}{\partial q_j} = \int_{q_1^0}^{q_1^1} \frac{\partial p_1^*(p, Q; U^0)}{\partial q_j} dq_1 \quad j = 2, 3, \dots, k.$$

With the exchange of integration and differentiation, the sign of the partial derivative of  $WTP_1$  with respect to  $q_j$  can be determined by the sign of the partial derivative of  $p_1^*$  with respect to  $q_j$ .<sup>4</sup>

### Strict Hicksian Substitutes Condition

Imagine the unrestricted situation in which consumers could choose the levels of marketed goods and the public goods of interest. Furthermore, suppose that in this case, all public goods of interest are strict Hicksian substitutes between each other. Recall the following definition of strict Hicksian substitutes:

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<sup>4</sup> We can do so provided the sign of this partial does not change in the interval  $(q_1^0, q_1^1)$ .



$$q_i, q_j \text{ are strict Hicksian substitutes if } \frac{\partial q_i(p, p^*; U)}{\partial p_j^*} > 0$$

where  $q_j(p, p^*; U)$  is the Hicksian compensated demand that results when minimizing expenditures over all goods facing prices  $(p, p^*)$  subject to a fixed level of utility  $U$ .<sup>5</sup> For our purposes, we require this condition for the public goods of interest only and place no substitute/complement restrictions on the other public goods or marketed goods. Using the strict Hicksian substitutability condition, we can proceed in relating the restricted and unrestricted sensitivity analyses following some recent results of Madden (1991).

Our objective, here, is to make statements about the signs of the entries of the matrix of implicit price sensitivities to public goods in the restricted case. The key to doing so is being able to sign the terms of the matrix,

$$\left[ \frac{\partial p^*}{\partial Q} \right]_{kxk}$$

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<sup>5</sup> Hicksian substitutes (versus strict) satisfy the condition that the partial derivative of  $q_i$  with respect to  $p_j^*$  is non-negative rather than strictly positive as in the case of strict substitutes.

To do this, first let  $[X(p, p^*; U), Q(p, p^*; U)]$  solve the unrestricted minimization problem and assume the conditions of the implicit function theorem are satisfied. Differentiate this set of demands with respect to prices and quantities:

$$\begin{bmatrix} \frac{\partial X}{\partial p} & \frac{\partial X}{\partial Q} \\ \frac{\partial Q}{\partial p} & \frac{\partial Q}{\partial Q} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial p} & \frac{\partial x}{\partial Q} \\ \mathbf{0}_{n \times n} & \mathbf{I}_{k \times k} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial p} + \frac{\partial X}{\partial p^*} \frac{\partial p^*}{\partial p} & \frac{\partial X}{\partial p^*} \frac{\partial p^*}{\partial Q} \\ \frac{\partial Q}{\partial p} + \frac{\partial Q}{\partial p^*} \frac{\partial p^*}{\partial p} & \frac{\partial Q}{\partial p^*} \frac{\partial p^*}{\partial Q} \end{bmatrix}$$

From this set of equations we can relate the price sensitivity matrix we are interested into the public goods submatrix of the unrestricted Hicksian Substitution Matrix.

$$\begin{bmatrix} \frac{\partial p^*}{\partial Q} \end{bmatrix} = \begin{bmatrix} \frac{\partial Q}{\partial p^*} \end{bmatrix}^{-1} = \mathbf{H}_Q^{-1}.$$

We are assuming that  $\mathbf{H}_Q$  is nonsingular.  $\mathbf{H}_Q$  is a submatrix of a regular substitution matrix. Under our initial assumptions on the utility function, the entire unrestricted substitution matrix is symmetric and negative semi-definite.  $\mathbf{H}_Q$  is a special matrix which possesses the following properties:

- (i)  $\mathbf{H}_Q$  is negative definite given the assumption of nonsingularity and the property of negative semi-definiteness of the entire unrestricted substitution matrix;

(ii) all off diagonal elements of  $H_Q$  are positive; and

(iii) properties (i) and (ii) imply all elements of  $H_Q$  are non-zero and from this, it follows that  $H_Q$  is indecomposable, whereby a square matrix  $A$  is indecomposable if there does not exist a permutation matrix  $P$  such that

$$P^{-1}AP = \begin{bmatrix} A_{11} & A_{12} \\ \mathbf{0} & A_{22} \end{bmatrix}.$$

Matrices with off diagonals of like signs have been studied extensively with Takayama (1985) having summarized many of the results pertaining to them. Of particular interest in our situation is the result, given the three conditions above, that  $H^I_Q < 0$ . That is, all elements of the matrix are strictly negative.<sup>6</sup> We can now summarize the consequences of the strict Hicksian substitutability condition in the set of public goods of interest on the implicit price sensitivity matrix:

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<sup>6</sup> If we consider the weaker condition of Hicksian substitutes versus strict, we loose condition (iii) above and condition (ii) is modified to all off diagonal elements non-negative. A further result from Takayama (1985) is that given conditions (i) and all off diagonal elements non-negative, then  $H_Q^{-1} \leq 0$ .

If all  $q_p, q_j$  are strict Hicksian substitutes, then  $\left[ \frac{\partial p^*}{\partial Q} \right] < 0$ .

It follows from this that in the case where all public goods of interest are strict Hicksian substitutes we can say the following:

$$\frac{\partial p_1^*(p, Q; U)}{\partial q_j} < 0 \quad \forall j \neq 1 \Rightarrow \int_{q_1^0}^{q_1^1} \frac{\partial p_1^*(p, Q; U)}{\partial q_j} dq_1 < 0.$$

We can summarize the effects of strict Hicksian substitution then as follows:

- Strict Hicksian Substitution: Higher levels of other public goods  $\Rightarrow$  lower  $WTP_1$ .<sup>7</sup>

#### *Effect on Willingness to Accept*

The strict Hicksian substitutability condition acts similarly on the willingness to accept compensation for  $\Delta q_1$ . The willingness to accept compensation measure is identical to willingness to pay with the exception that the initial level of Q utility is higher with willingness to accept and lower with willingness to pay.<sup>8</sup>

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<sup>7</sup> In the case of Hicksian substitutes in the public goods of interest higher levels of other public goods of interest  $\Rightarrow$  non-increasing  $WTP_1$ .

<sup>8</sup> In damage applications, the utility level in the case of willingness to accept is higher than willingness to pay because the consumer initially possesses the higher level of public good. Hanemann (1991) has shown that the difference between WTP and WTA can be quite large in some circumstances.

$$WTA_1 = -\int_{q^0}^{q^1} \frac{\partial e^*(p, Q; U)}{\partial q_1} dq_1 = \int_{q^0}^{q^1} p_1^*(p, Q; U) dq_1$$

Consequently, the same properties hold for willingness to accept as willingness to pay when considering changes in the levels of other public goods of interest. We can summarize the effects of strict Hicksian substitutability as follows:

- Strict Hicksian Substitution: Higher levels of other public goods  $\Rightarrow$  lower  $WTA_j$ .<sup>9</sup>

It further follows that under the assumption that the strict Hicksian substitutability condition is satisfied, that willingness to pay and willingness to accept must change in different valuation contexts. This is a sufficient condition, but not necessary. Other combinations of Hicksian substitutes and complements between the public goods of interest could cause changes in willingness to pay and willingness to accept values in different valuation contexts.

The only condition under which willingness to pay or willingness to accept will not be effected by changes in valuation context is  $H_Q$  a diagonal matrix. This condition is impossible in the case of strict Hicksian substitutes.

### Extension to Sequences of Public Goods

We can now extend the analysis from above and make more explicit statements about the willingness to pay for the change in  $q_j$  when placed in a valuation sequence. A valuation sequence is a sequence of the willingness to pay (or accept compensation) for each change in a given sequence of changes in public goods. For example, suppose we are considering the following

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<sup>9</sup> In the case of Hicksian substitutability between the public goods of interest, the effect higher other public goods of interest  $\Rightarrow$  non-increasing  $WTA_j$ .

changes in the  $k$  public goods:  $\Delta q_1, \Delta q_2, \Delta q_3, \dots, \Delta q_k$ . The most simple valuation sequence would be the willingness to pay for  $\Delta q_1$  followed by the willingness to pay for  $\Delta q_2$  after obtaining  $\Delta q_1$ . The next value would be the willingness to pay for  $\Delta q_3$  after obtaining  $\Delta q_1$  and  $\Delta q_2$  and so on until all changes were valued. Given that  $k$  changes being considered, there are  $k$  factorial possible valuation sequences.

Assuming the strict Hicksian substitutability condition, we can prove three useful propositions.<sup>10</sup>

**Proposition 1:** Suppose we are interested in the willingness to pay for increases in all  $k$  goods and we look at a valuation sequence where goods are valued successively. If we permute the order of valuation, then the willingness to pay for the change in  $q_1$  will be greatest when valued first in the sequence and smallest when valued last in the sequence.

**Proof:** Recall from above:

- Strict Hicksian Substitution: Higher levels of other public goods of interest  $\Rightarrow$  lower WTP for  $\Delta q_1$ .

When valued first, the lowest levels of other public goods are available and therefore WTP will be greatest. When valued last, the highest levels of other public goods are available and therefore WTP will be smallest.

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<sup>10</sup> These propositions are also valid with Hicksian substitution between the public goods of interest.

**Proposition 2:** Suppose we are interested in the willingness to accept compensation for a reduction in all  $k$  publicly provided goods and permute the order of valuation. Then the willingness to accept compensation for the reduction in  $q_1$  will be smallest when valued first in the sequence and greatest when valued last in the sequence.

**Proof:** Recall from above:

- Strict Hicksian Substitution: Higher levels of other public goods of interest  $\Rightarrow$  lower WTA for  $\Delta q_1$ .

This proof is similar to the proof of proposition 1. The difference between the willingness to accept sequence and the willingness to pay sequence is as we value  $\Delta q_1$  later in the sequence, the levels of other public goods are decreasing rather than increasing. The levels of other public goods are greatest when  $\Delta q_1$  is valued first and smallest when valued last. Therefore we have the opposite implication of proposition 1.

**Proposition 3:** If the public goods of interest are normal goods then willingness to pay for a good when valued first in a willingness to pay sequence is strictly less than willingness to accept compensation for the good obtained in any sequence order.

**Proof:** Assume that because consumers initially possess the higher levels of public goods, the base utility level for willingness to accept,  $U^1$  is higher than the base utility level for willingness to pay,  $U^0$ . If income elasticity is greater than zero (the normal goods condition), it can be shown that

$$\frac{\partial p_1^*}{\partial U} > 0$$

It follows then that

$$\int_{q_1^0}^{q_1^1} p_1^*(p, Q; U^0) dq_1 < \int_{q_1^0}^{q_1^1} p_1^*(p, Q; U^1) dq_1.$$

The inequality above represents the initial public goods level with willingness to pay strictly smaller than willingness to accept. By applying proposition 2, willingness to accept will become greater when placed farther out in the valuation sequence. Therefore the strict inequality will remain for any willingness to accept sequence. The size of the difference between the two measures will increase the farther out in the WTP sequence the good is valued or the farther out in a WTA sequence the good is valued.

These three propositions have important implications for damage assessment if the objective is to make the public whole in a utility sense. Because of a number of difficulties involved in asking willingness to accept questions, a willingness to pay estimate is often used to approximate the desired willingness to accept amount.<sup>11</sup> If the strict Hicksian substitutability and normal goods conditions are satisfied, willingness to pay will be strictly less than willingness to accept.

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<sup>11</sup> Mitchell and Carson (1989) discuss the issues involved in measuring willingness to accept compensation in contingent valuation surveys.



Furthermore, valuing the willingness to pay alone (i.e., first in a willingness to pay sequence) will provide the closest willingness to pay approximation to the desired willingness to accept amount regardless of where the desired change should be valued in the willingness to accept sequence.

## Conclusion

If the public goods of interest are strict Hicksian substitutes, then the value of a particular good is lower the farther out in a willingness to pay valuation sequence it is valued. The value of a good can be driven arbitrarily close to zero by placing it far enough out in a valuation sequence. The reverse is true of a willingness to accept compensation sequence. Here the farther out in a sequence a good is valued the more it is worth. If the goods of interest are also normal goods, then Hanemann's (1991) result can be used to show that willingness to pay for a particular good valued first in a sequence (or equivalently alone) will be the closest, albeit an under, estimate of willingness to accept compensation for the good valued in any sequence order if a willingness to pay measure must be used.

While the strict Hicksian substitutes assumption is sufficient for our results, it is not a necessary one. Most of our result go through with weak rather than strict inequalities with the weaker assumption that the public goods of interest are simple Hicksian substitutes. There are many substitutes/complement combinations for the public goods of interest which would also produce willingness to pay sequences where the farther out in the sequence a particular good is valued the less it is worth and willingness to accept compensation sequences where the father out in a sequence a particular good is valued the more it is worth.

Our results have substantial implications for the assessment of benefits and damages. Policymakers considering implementing several policies are likely to risk over estimating the value of providing any particular good if interactions between the goods are not taken into account. For those assessing damages using willingness to pay for the good valued as if were the first (or only) good in a sequence will underestimate the public's willingness to accept compensation for the damages but will do so less than obtaining willingness to pay for the good in any other order in a willingness to pay sequence.

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