# Forget the Hoe: Managing Invasive Plant Species with Dynamic Programming

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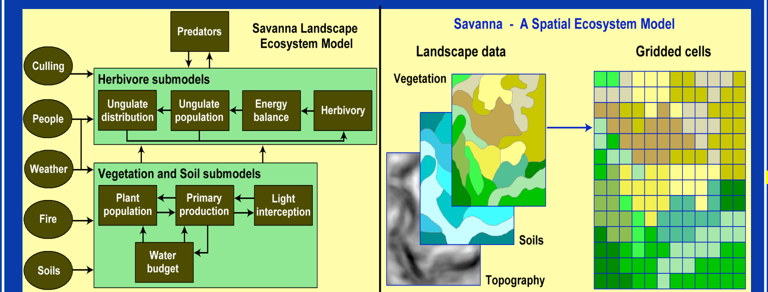
# Forget the Hoe:
# Managing an Invasive Species with Dynamic Programming

**Abstract**: Bromus Tectorum (hereafter Cheatgrass) is one of the most widespread invasive species on U.S. rangelands. Cheatgrass decreases agricultural yields, decreases animal performance, and increase the frequency and intensity of fires. We present a bio-economic model of stocking cattle on these rangelands to show that land managers can manage invasions using direct control on the invasive species, altering stocking regimes, or using combinations of direct control and changes in stocking rates.

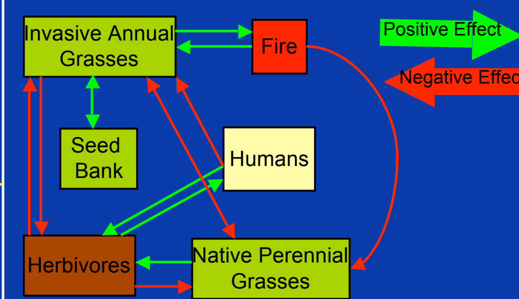
## Ecological Model




Savanna is a spatially explicit, process-based ecosystem model. It simulates competition between cheatgrass and other important plant functional groups, and interactions with large herbivores and the environment.

SAVANNA simulates field experiments that provide the data to parameterize the economic model.

## Economic Model




Conceptual Bio-Economic model. The diagram displays the economic and biological interactions expressed in the mathematical model.

$$\underset{SR,X}{Max}\,\pi(SR,X,W,G) = Max\sum_{t=0}^{T}[\delta\{P(B)\cdot B(SR_t,W_t,G_t) - C_{SR}(SR_t) - C_W(X_t;\alpha)\}]$$

s.t.

$$W_t = f(G_{t-1},S_{t-1},SM_{t-1},N_{t-1}\,|\,X_{t-1})$$

$$G_t = g(G_{t-1},W_t,SR_{t-1},SM_{t-1},N_{t-1})$$

$$S_t = \lambda S_{t-1} + \varphi(W_{t-1})$$

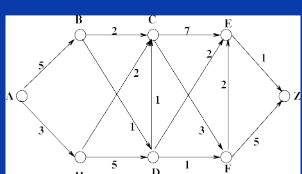
$$N_t = Q(N_{t-1},SR_{t-1};\alpha_N)$$

$$W_0 = \overline{W};G_0 = \overline{G};S_0 = \overline{S};N_0 = \overline{N}$$

| Parameter | Description |
| --- | --- |
| $\delta$ | Discount Factor |
| $P(B)$ | Stocker Beef price for season t |
| B ( ) | Beef Production Function |
| $SR_t$ | Stocking rate for season t (cattle/hectare) |
| $W_t$ | Cheatgrass biomass (kg/hectare) for season t |
| $G_t$ | Perennial biomass (kg/hectare) for season t |
| $C_{SR}$ | Per animal stocking cost |
| $C_W$ | Cheatgrass control cost |
| $X_t$ | Control decision for season t |
| $\alpha$ | Technology parameter |
| $S_t$ | Seed bank stock for season t |
| f() | Cheatgrass transition equation |
| g() | Perennial transition equation |
| $\varphi()$ | Cheatgrass seed production function |

The theoretical model is solved using Dynamic Programming

## Dynamic Programming




Dynamic Programming (Bellman, 1953) solves complex decisions by breaking the problem up into a sequence of small, simple decisions known as decision steps. This is done by defining a set of value functions, V(), where the value depends on the state(s) of the system. The Bellman equation allows researchers to solve for the set of value functions. The diagram on the left is an example of a problem that can be solved with Dynamic Programming (e.g. shortest distance problem).

Belllman equation for economic model

$$V(W_t,G_t) = \underset{SR,X}{Max}\{\pi(SR_t,X_t,W_t,G_t) + \rho V(W_{t+1},G_{t+1})\}$$

The Bellman equation is the mathematical tool used to solved Dynamic Programming problems. The Bellman equation operationalizes the Principle of Optimality. As long as the states of the problem are well defined, discrete, and finite, the Bellman equation can be solved by exploiting the recursive nature of the equation.

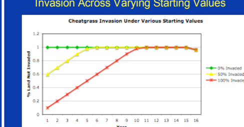
| | | Perennial Grasses State | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | State (W,G) | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1 |
| | 0 | 0,1 | 1,1 | 0,1 | 1,1 | 0,1 | 1,1 |
| Invasive | 0.2 | .2,1 | .9,1 | .2,1 | .9,1 | .2,1 | .9,1 |
| Annual | 0.4 | .33,1 | .78,1 | .33,1 | .78,1 | .33,1 | .78,1 |
| Grasses | 0.6 | .39,1 | .7,1 | .39,1 | .7,1 | .39,1 | .7,1 |
| State | 0.8 | .47,1 | .88,1 | .47,1 | .88,1 | .47,1 | .88,1 |
| | 1 | .5,1 | .9,2 | .5,1 | .9,2 | .5,1 | .9,2 |

The solution to a Dynamic Programming problem is the control that an decision maker should choose in order to maximize the given objective. Following this decision rule for each state creates an optimal path. The table at left is an example of such a decision rule. The table is intuitive and simple for land managers to use as a decision aid.

Solutions to the Dynamic Programming problems are interpreted.

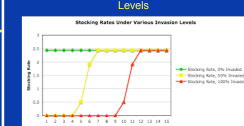
## Economic Results

Invasion Across Varying Starting Values



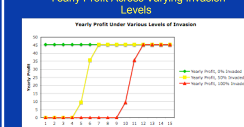

Here we see the optimal path of invasion under three different invasion starting values. In each case the present value marginal cost of reducing invasion equals the discounted stream of marginal benefits that accrue from the reduction of the marginal unit of invasion. Given the current parameterization this equality leads all paths toward un-invaded range conditions.

Stocking Rate Across Varying Invasion Levels




Dynamic Programming chooses the control (I.e. stocking rate) that satisfies the Bellman equation for each state. This “control rule” is mapped here for three different invasion starting values. When land is 100% invaded land managers should leave the range idle and allow native perrenial species to repopulate. In the other extreme case where land is not invaded a constant stocking rate equates the discounted marginal cost of growing rangeland resources with the current value marginal benefit of harvesting those resources as feed for cattle.

Yearly Profit Across Varying Invasion Levels




We can compare the flow of profits under the different starting levels of invasion. In each case we see that the starting level of invasion influences the flow of profits in a given year. In cases where land is heavily invaded managers forgo the full of extent of profits available in a given period in order to restore range conditions and earn larger profits in future profits.

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