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by

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ABSTRACT

This paper considers the effect of market structure on the durability of goods. A previous model is analyzed to provide conditions under which a monopolist provides less durable goods and a lower present value of services than the social optimum.

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A COMMENT ON MARKET STRUCTURE AND THE DURABILITY OF GOODS

A number of papers (see Abel for references) have discussed the relation between market structure and the durability of goods. These analyses concentrate on comparison of steady-state optima for the various market structures. We extend the analysis by considering the second variation around a steady state. We show that the Hessian of the dynamic problem evaluated at the steady state can be decomposed into two asymmetric matrices. The quadratic forms of these matrices give, respectively, the second-order terms of the Taylor expansion of the benefit of an instantaneous arrival at a neighboring (suboptimal) steady state (the "static benefit") and the benefit of the journey (the "adjustment benefit"). The sum of these benefits must be negative, but either may be positive.

There are two reasons for considering this decomposition into static and adjustment benefits. First, it provides a way of evaluating a government policy that forces an industry to move away from its optimal steady state. By the necessary conditions for optimality, the first term in the Taylor expansion of the value of the change is 0; the first-order terms in the Taylor expansions of the static and adjustment benefits must just offset each other and can, therefore, be ignored. Hereafter, adjustment benefit and static benefit refer to the second-order terms in the Taylor expansions of, respectively, adjustment and static benefits. By the sufficient conditions for local optimality, the second term of the Taylor expansion of the value of the change is negative. The decomposition leads to an easily checked sufficient condition which insures that the adjustment benefit term is positive, i.e., static analysis overstates the firm's loss resulting from the change.

The second use of the decomposition is to shed some light on a problem encountered by Abel. His Proposition 3 does not determine whether a monopolist provides a lower present value of services than is provided by a social planner. The ambiguity is caused by the indeterminacy of the sign of the determinant of the matrix A, which is obtained by totally differentiating the first-order conditions evaluated at the steady state. This indeterminacy is surprising because intuition suggests that a monopolist would produce less than the social optimum.

We show that A is the part of the decomposition that gives the static benefit of a change in controls. Sufficient conditions for $|A| > 0$ are (1) the cost function is convex in output and durability, (2) the marginal cost of output falls as durability increases, and (3) a transformation of the decay function (defined below) is concave in durability. The above conditions on the cost and decay functions are plausible; but, as the derivation shows, they are much too restrictive. Hence, the result should hold even if the conditions are not satisfied. The conditions also imply that the monopolist will produce less durable goods at a lower rate than is socially optimal. This in turn implies that, if the social planner considers forcing the monopolist toward the social optimum, the monopolist's adjustment benefit will be positive.

We adopt Abel's model and notation. Let X_s be the output of a durable good at time s and D_s be an index of its durability. Define $\phi(t, D)$ as the fraction of a good of durability D that survives to age t. Let Q_t be the stock of the good available at time t, so

$$Q_t = \int_{-\infty}^{t} X_s \phi(t - s, D_s)\, ds.$$

Let $R(Q)$ be the revenue (social welfare) function for the monopolist (social planner). If $C(D_t, X_t)$ is the cost of producing the flow X with durability D and r is the interest rate, the problem at t_0 is to maximize over X_t and D_t

$$\int_{t_0}^{\infty} e^{-r(t-t_0)}[R(Q_t) - C(D_t, X_t)]\, dt.$$

The following definitions are also used

$$\rho(D) = \int_0^{\infty} \phi(s, D)\, e^{-rs}\, ds$$

$$\gamma(D) = \int_0^{\infty} \phi(s, D)\, ds.$$

The Appendix (available upon request) shows that the Hessian H of the dynamic problem evaluated (for constant perturbations) at a steady state can be written as

$$H = \frac{A}{r} - R''J \tag{1}$$

where

$$A_{11} = R''\rho\gamma - C_{XX}$$

$$A_{12} = R''X\rho\gamma' + R'\rho' - C_{XD}$$

$$A_{21} = R''X\rho'\gamma + R'\rho' - C_{DX}$$

$$A_{22} = R'X\rho'' + R''X^2\rho'\gamma' - C_{DD}$$

$$J_{11} = \int_0^{\infty} e^{-rt} \int_0^{\infty} e^{-rs}\, \phi(s, D) \int_{t+s}^{\infty} \phi(z, D)\, dz\, ds\, dt$$

$$J_{12} = X \int_0^\infty e^{-rt} \int_0^\infty e^{-rs} \phi(s, D) \int_{t+s}^\infty \phi_D(z, D)\, dz\, ds\, dt$$

$$J_{21} = X \int_0^\infty e^{-rt} \int_0^\infty e^{-rs} \phi_D(s, D) \int_{t+s}^\infty \phi(z, D)\, dz\, ds\, dt$$

$$J_{22} = X^2 \int_0^\infty e^{-rt} \int_0^\infty e^{-rs} \phi_D(s, D) \int_{t+s}^\infty \phi_D(z, D)\, dz\, ds\, dt.$$

Note that the elements of J are all positive.

For isoelastic demand, this matrix A is identical to Abel's. Recall that A can also be obtained by totally differentiating the first-order conditions evaluated at the steady state with respect to the controls X (the rate of production) and D (the durability). Let $y = (h, k)'$, where h and k are constant perturbations around X and D, respectively. The term $y'Ay/r$ gives the present value of the benefit to the firm of a movement from $(X, D)'$ to $(X, D)' + y$ given that Q, the stock of services, adjusts instantaneously. This is the static benefit.[1]

The term $-R''y'Jy$ can then be interpreted as the adjustment benefit, which equals the total benefit of changing controls, $y'Hy$, minus the static benefit. The following lemma is useful.

LEMMA. For a real two-by-two <u>asymmetric</u> matrix K, $|K| - |(K + K')/2| = (K_{12} - K_{21})^2/4 > 0$, where K_{ij} is the (i, j) element of K.

From the lemma, $K_{11} > 0$ and $|K| = 0$ imply that K is indefinite. Also, the inequalities $K_{11} < 0$ and $|K| > 0$ do not imply that K is negative definite. In the Appendix we show that $|J| = 0$. Since $J_{11} > 0$, this and the lemma imply that $y'Jy$ may be positive or negative. The vectors y for which $y'Jy$ is negative lie in cones in the quadrants where $hk < 0$ (Figure 1). Since $y'Hy < 0$, the vectors y such that $y'Ay > 0$ must be a proper subset of these cones. This implies that any policy which requires an industry to increase both X and D or to decrease both X and D must result in a positive adjustment benefit but a negative static benefit. A necessary condition for the opposite to hold is that the policy must require a decrease in one control and increase in the other.

To determine the relative levels of production and durability of a monopolist and a social planner, we require the sign of $|A|$. The decomposition suggests conditions which insure that $|A| > 0$ and, also, indicates that these conditions are not necessary. Therefore, we expect $|A| > 0$, even if the conditions are only "almost satisfied." The determinant of H can be written as $|H| = |A| + |\tilde{J}| + |M_1| + |M_2|$, where the first rows of M_1 and M_2 are, respecively, the first rows of A and $\tilde{J}$ and the second rows are, respectively, the second rows of $\tilde{J}$ and A; $\tilde{J} = -R''J$. Use the fact that $|H| > 0$ and $|\tilde{J}| = 0$ to write

$$0 < |A| + R''\{C_{XX}J_{22} + J_{11}\,[C_{DD} - XR'\rho''] + (J_{12} + J_{21})\,X[R'\rho' - C_{DX}]\}. \qquad (2)$$

The derivation of (2) involves a number of changes in the order of integration similar to those used in the Appendix to show that $|J| = 0$.

A sufficient condition to insure that $|A| > 0$ is to require the expression in braces to be nonnegative. One way to do this is to assume that $C_{XX} \geq 0$, $C_{DD} \geq 0$, $C_{XD} \leq 0$, and $\rho'' \leq 0$. The first two inequalities are implied by convexity. The third states that the marginal cost of output falls as durability increases. Abel's equation (8b) can be used to replace $C_{XD} \leq 0$ by the weaker requirement that $(C_D/X) - C_{DX} \geq 0$ at the steady state. The latter inequality is equivalent to $d(C_D/X)/dX \leq 0$. The assumption that $\rho'' < 0$

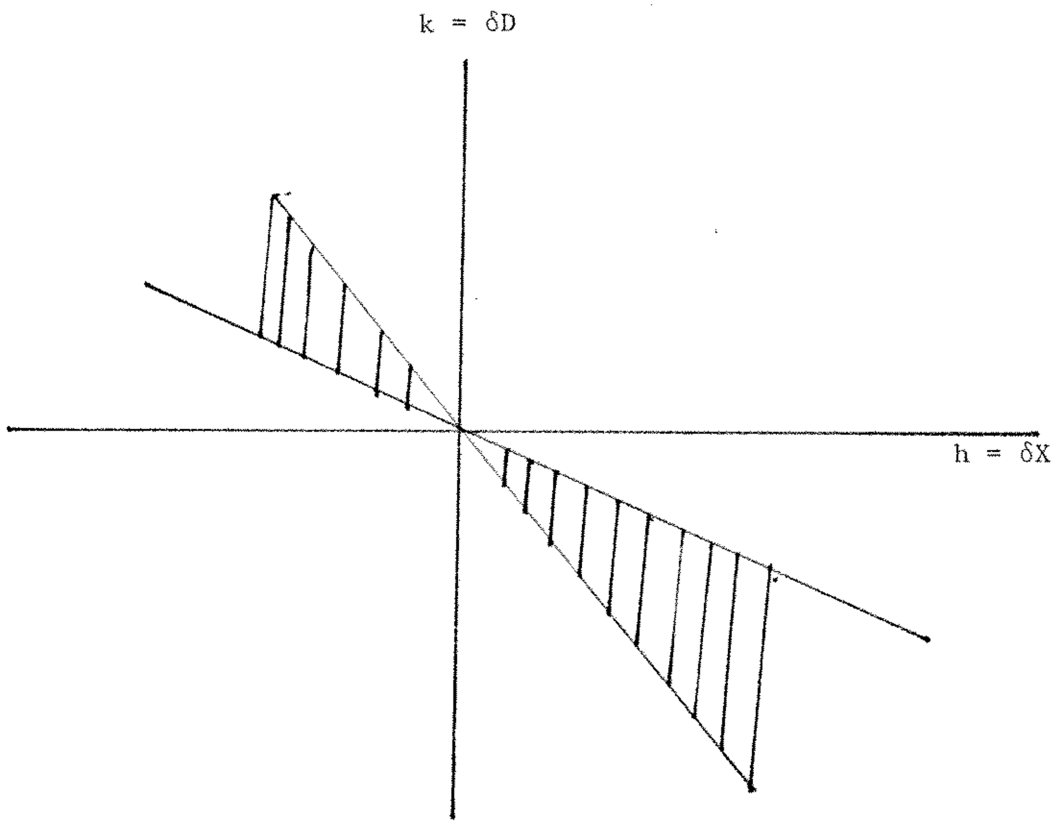

Figure 1. The shaded region gives $\{y | y'Jy \leq 0\}$.

is met by the commonly used exponential decay, $\phi(t, D) = e^{-t/D}$. Inequality (2) indicates the degree to which these assumptions are overly restrictive.

These assumptions imply that $|A| > 0$ so, from Abel's equation (A12), the social planner provides a higher present value of services than the monopolist provides. Using the stronger condition $C_{XD} \leq 0$ and Abel's equations (A3) and (A7), the durability and rate of production, D and X, are also greater for the social planner. The previous analysis implies that, if the social planner forces the monopolist to move toward the social optimum, the adjustment benefit, $-R''y'Jy$, will be positive; so static analysis overstates the cost to the monopolist.

FOOTNOTE

1. Recall that "benefit" refers to only the second-order term in the Taylor expansions. The first-order static and adjustment terms are not 0 but offset each other.

APPENDIX: Decomposition of the Hessian

We show that the Hessian can be written as the sum of two asymmetric matrices. Equations in square brackets are Abel's.

The first variations of [4] with respect to X_t and D_t are, respectively

$$\text{(A1a)}\quad \int_{t_o}^{\infty} e^{-r(t-t_o)}\{R'(Q_t)\int_{-\infty}^{t} h_s\phi(t-s,D_s)ds - C_X h_t\}dt$$

$$\text{(A1b)}\quad \int_{t_o}^{\infty} e^{-r(t-t_o)}\{R'(Q_t)\int_{-\infty}^{t} X_s k_s \phi_D(t-s,D_s)ds - C_D k_t\}dt$$

where h_s and k_s are admissible perturbations of the sample paths X_t and D_t, respectively. Changing the order of integration in (A1), and redefining the variables of integration, gives

$$\text{(A2a)}\quad \int_{t_o}^{\infty} e^{-r(t-t_o)}\left[\int_0^{\infty} e^{-rs}R'(Q_{t+s})\ \phi(s,D_t)ds - C_X\right] h_t dt$$

$$\text{(A2b)}\quad \int_{t_o}^{\infty} e^{-r(t-t_o)}\left[X_t \int_0^{\infty} e^{-rs}R'(Q_{t+s})\ \phi_D(s,D_t)ds - C_D\right] k_t dt.$$

Setting (A2a) and (A2b) equal to 0 gives [5]. If the variables are allowed to go to the steady state in (A2) we obtain [8] divided by r.

Since we are only interested in comparing steady state paths, we restrict our attention to constant perturbations of the form

(A3a) $h_t = \begin{cases} h & \text{for } t \geq t_o \\ o & \text{for } t < t_o \end{cases}$

(A3b) $k_t = \begin{cases} k & \text{for } t \geq t_o \\ o & \text{for } t < t_o \end{cases}$

where h and k are arbitrary. We choose as the sample path the steady state values $X_t = X$, $D_t = D$, $Q_t = Q$.

Details of the calculations of the (1,1) element of H are given. The other elements of H are obtained by performing similar manipulations. Consider the second variation of the value function around X, obtained by taking the first variation of (A2a) around X. (Alternatively, use (A1a) and reverse the order of integration in the result.) This is

$$\text{(A4)} \quad \int_{t_o}^{\infty} e^{-r(t-t_o)} h_t \Big[\int_o^{\infty} e^{-rs} R''(Q_{t+s}) \; \{ \int_{t_o}^{t+s} h_y \phi(t+s-y, D_y) dy \} \cdot \phi(s, D_t) ds - C_{XX} \Big] dt.$$

Use (A3a) and the fact that X, D and Q are constant over (t_o, ∞) to rewrite this as

$$\text{(A5)} \quad h^2 \int_{t_o}^{\infty} e^{-r(t-t_o)} \Big[R''(Q) \int_o^{\infty} e^{-rs} \; \{ \int_{t_o}^{t+s} \phi(t+s-y, D) dy \} \cdot \phi(s, D) ds - C_{XX} \Big] dt.$$

Define the new variable of integration $z = t+s-y$ and rewrite (A5) as

(A6) $$h^2\int_{t_o}^{\infty} e^{-r(t-t_o)}\left[R''\int_o^{\infty}e^{-rs}\phi(s,D)\{\int_o^{t+s-t_o} \phi(z,D)dz\}ds - C_{XX}\right]dt =$$

$$h^2\int_{t_o}^{\infty} e^{-r(t-t_o)}\left[R''\int_o^{\infty}e^{-rs}\phi(s,D)\{\int_o^{\infty} \phi(z,D)dz -\right.$$

$$\left.\int_{t-t_o+s}^{\infty}\phi(z,D)dz\}ds - c_{XX}\right]dt = h^2[A_{11}/r - R''J_{11}],$$

where the definitions of A_{11} and J_{11} are given in the text.

Abel's definitions [6] and [7] are used to obtain the expression for A_{11}, the (1,1) element of A.

The (2,2) element of H is obtained by taking the variation around D of (A2b). Following the same sequence of operations used above gives $H_{22} = A_{22}/r - R''J_{22}$, where A_{22} and J_{22} are given in the text.

The off-diagonal terms of H can be obtained in two ways. One way uses (A1) and reverses a particular pair of integrations to obtain an expression for H_{12} that looks like A_{12} + other terms; a different pair of integrations is interchanged to obtain H_{21}. The more direct method uses expressions (A2). To obtain H_{12}, take the variation around D of (A2a); to obtain H_{21}, take the variation around X of (A2b). Manipulate the resulting expressions using the steps described above. The results are given in the text.

The matrix H can be written as the sum of two asymmetric matrices: $H = A/r - R''J$. The indefiniteness of J remains to be shown. By inspection, the diagonal elements of J are positive, so it suffices to show that $|J| = 0$. Then the lemma implies the result. Consider the product of the main diagonal elements. This may be written as

$$X^2\int_0^{\infty} e^{-rt}\int_0^{\infty} e^{-rs}\phi(s,D)\int_{t+s}^{\infty}\phi(z,D)\int_0^{\infty} e^{-rt'}\cdot$$

$$\int_0^{\infty} e^{-rs'}\phi_D(s',D)\int_{t'+s'}^{\infty}\phi_D(z',D)dz'ds'dt'dz\ ds\ dt =$$

$$X^2\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-rt}e^{-rs}\phi(s,D)e^{-rt'}e^{rs'}\phi_D(s',D)\cdot$$

$$[\int_{t'+s'}^{\infty}\int_{t+s}^{\infty}\phi(z,D)\phi_D(z',D)dz\ dz']ds'dt'ds\ dt.$$

The same change in the order of integration can be made on the product of the off-diagonal elements of J. The result is an expression equivalent to the above. Hence $|J| = 0$.

REFERENCES

Abel, A. B. "Market Structure and the Durability of Goods," Review of Economic Studies 50(1983):625-637.