DO INVENTORY AND TIME-TO-DELIVERY EFFECTS VARY ACROSS FUTURES CONTRACTS? INSIGHTS FROM A SMOOTHED BAYESIAN ESTIMATOR

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Abstract

We apply a new Bayesian approach to multiple-contract futures data to allow the inventory and time-to-delivery effects on volatility to vary across contracts. We find a varying negative relationship between lumber inventories and lumber futures price volatility. The inventory effect is smaller for the most recent contracts possibly due to increasing inventories over time. While this approach reveals the downward bias on the inventory effect introduced by restricting this parameter across contracts, it does not change the time-to-delivery effect.

Key words: volatility, theory of storage, futures markets, Bayesian econometrics, lumber

1. Introduction

Explaining price movements in futures markets has attracted many economists' attention. Several studies measure the impact of economic announcements on futures price movements. See, for example, Leistikow (1989), Colling and Irwin (1990), Ederington and Lee (1993), Mann and Dowen (1996), Li and Engle (1998), Isengildina, Irwin, and Good (2006), and Karali and Thurman (2008a). Several other studies aim to explain price movements by commodity-specific fundamentals, such as storage, seasonality, or the price movement in a related commodity. These studies include Ng and Pirrong (1994), Pindyck (1994), and Smith (2005). Moreover, time-to-maturity effect is seen as another factor that determines price volatility in commodity futures markets (Samuelson, 1965). (See also, section 2.3.4 in Garcia and Leuthold (2004) for a review of other studies on volatility of futures prices.)

Our study investigates the effects of physical lumber inventories and time remaining to contract expiration on lumber futures price volatility as in the related work of Karali (2007) and Karali and Thurman (2008b). As the theory of storage suggests, price movements should be larger when inventories are small and vice versa. Like in many commodity futures markets, lumber futures contracts with different delivery dates trade simultaneously. If time series for each contract were analyzed separately, inventory effects could not be measured precisely because each contract is active for only a year or so and inventories do not change much in that time. Thus, an individual contract is traded during a roughly constant stock regime. To capture the effect of changing inventories on price behavior we need to somehow combine contracts and observe their changes over a period of several years. We accomplish this by applying the Generalized Least Squares (GLS) method developed in Karali (2007). We analyze daily settlement prices of 77 lumber futures contracts from the Chicago Mercantile Exchange (CME), from 1992 to 2005, and define volatility as the absolute value of log price changes over a day.

We extend earlier work by allowing inventory and time-to-delivery effects on volatility to vary across different contracts through a new Bayesian approach similar to the semi-parametric smooth coefficient models of Koop and Tobias (2006). This new approach allows estimation of contract-specific estimates which are "smoothed" across contracts through the use of a prior distribution that centers each contract's parameter estimates over the weighted average of the estimates for all overlapping contracts. The approach is implemented through an iterative process of single-contract conditional estimation that yields Bayesian posterior estimates from the joint distribution of parameters for all 77 contracts and still accounts for the cross-contract contemporaneous correlation of same day observations.

In conformity to the results in Karali (2007) and Karali and Thurman (2008b), we find an inverse relationship between inventory levels and lumber futures volatility. As inventory levels become smaller, lumber futures contracts become more volatile. For all contracts, the estimated inventory coefficient is negative, something that is not found for classical, unrestricted contract-specific estimates. The average across the 77 inventory coefficients is -0.772. On the other hand, when we restrict the inventory coefficient to be the same across all contracts through a very small prior variance on the smoothing prior, the inventory coefficient becomes -0.114. This result shows using fixed multiple-contract parameter estimates introduces a downward bias towards zero on the inventory effect. As for the time-to-delivery effect, we again find an inverse relation between time remaining to expiration and volatility. As the delivery date approaches, lumber futures contracts become more volatile. Like the inventory coefficients, all of the time-to-delivery coefficients are negative. However, unlike the inventory effect, the time-to-delivery effect parameters do not change much across the contracts. Further, both the average across the 77 time-to-delivery coefficients and the restricted estimate are -0.005. Thus, for this coefficient, restricting the parameter across contracts does not introduce any bias.

The empirical results of implementing this new estimation methodology to multiple, overlapping contract financial data show that introducing such flexibility to parameter estimation has important potential gains in hypothesis testing. Results show that allowing the inventory effect to differ across contracts produces very different empirical results, with the contract-specific estimates being significantly negative and the restricted multiple-contract estimator being close to zero. For the time-to-delivery effect, the increased flexibility produced little change in the parameters. This is reasonable because while the range of time-to-delivery variable is the same for all contracts, the inventory variable changes dramatically across contracts due to different time periods they cover. Thus the more flexible modeling approach allows us to analyze varying inventory effects, possibly in a nonlinear fashion.

2. Theoretical Propositions

Karali and Thurman (2008b) present a simple three-period storage model, which originated in Williams and Wright (1991). They derive the analytical solutions for optimal storage rules and perform a simulation study to show the decreasing and nonlinear relationship between the expected absolute price changes and the inventory levels. More specifically, for various levels of initial carry-in, they derive price paths for many realizations of a random shock, and compute the average price path in each period to represent the conditional mean of price in that period, $E_t(P_{t+1}|S_t)$. To motivate our theoretical propositions, we replicate their figure here. As seen in figure 1, when inventory levels become larger, the expected magnitude of price movement becomes smaller. They also show that the price response of a futures contract to a shock declines with time to delivery. As in their work, we test these hypotheses using the following linear model of volatility:

$$|\ln F_{i,t} - \ln F_{i,t-1}| = a_i + b_i S_t + c_i TTD_{i,t} + \varepsilon_{i,t}, \tag{1}$$

where $\ln F_{i,t}$ is the natural logarithm of the price of futures contract i on day t, S_t is the physical inventory level on day t, and $TTD_{i,t}$ is time to delivery, the number of trading days remaining to contract i's expiration on day t. We hypothesize that:

$$\partial(|\ln F_{i,t} - \ln F_{i,t-1}|)/\partial S_t = b_i < 0, \tag{P1}$$

$$\partial(|\ln F_{i,t} - \ln F_{i,t-1}|)/\partial TTD_{i,t} = c_i < 0, \tag{P2}$$

$$a_i \neq a_j, \quad b_i \neq b_j, \quad \text{and} \quad c_i \neq c_j. \tag{P3}$$

3. Data and Econometric Issues with Overlapping Contracts

The U.S. Census Bureau releases Monthly Wholesale Trade Reports in which the Lumber & Other Construction Materials inventory series (NAICS 4233) are included. These series are published in current dollars, and therefore we divide them by the Lumber Producer Price Index published by the Bureau of Labor Statistics to create inventory series in constant dollars. We use a cubic spline method to interpolate the resulting monthly series to obtain estimated daily inventories, which are shown in figure 2.

For the price data, we use daily settlement prices of lumber futures contracts, which are traded at the CME. The delivery months are January, March, May, July, September, and November. We study only the contracts that have full trading histories during our inventory data period, and this results in a sample period of July 14, 1992-November 15, 2005 with a total of 77 contracts. We trim the data set to include 170 observations for each contract—the number of trading days of the shortest-lived contract. On the CME, at any point in time, a total of seven contracts are listed, each with a different delivery date up to 14 months into the future. Due to our trimming procedure, the number of contracts on any given day in our sample varies from one to five. Note that all observations for a single contract are numbered from one to 170 in trading days, not actual days; no distinction is made to adjust for weekends or holidays.

Because information flows to the market affect, to some degree, all lumber contracts, price observations from the same calendar date will be correlated with each other. It is useful, however, to organize the data by matching observations in terms of TTD. To see this more clearly, consider the following structure of the data:

$$
\begin{array}{ccccccccccccccccc}
y_{1,1} & y_{1,2} & \cdots & y_{1,44} & \cdots & y_{1,86} & \cdots & y_{1,129} & \cdots & y_{1,170} & & & & & & & \\
 & & & y_{2,1} & \cdots & y_{2,43} & \cdots & y_{2,86} & \cdots & y_{2,127} & y_{2,128} & y_{2,129} & \cdots & y_{2,170} & & & \\
 & & & & & y_{3,1} & \cdots & y_{3,44} & \cdots & y_{3,85} & y_{3,86} & y_{3,87} & \cdots & y_{3,128} & y_{3,129} & \cdots & \\
 & & & & & & & y_{4,1} & \cdots & y_{4,42} & y_{4,43} & y_{4,44} & \cdots & y_{4,85} & y_{4,86} & \cdots & \\
 & & & & & & & & & & & y_{5,1} & \cdots & y_{5,42} & y_{5,43} & \cdots & \\
 & & & & & & & & & & & & & & \vdots & & \\
 & & & & & & & & & & & & & & \cdots & \cdots & \cdots \\
 & & & & & & & & & & & & & & & \cdots & \cdots \\
 & & & & & & & & & & & & & & & \cdots & y_{77,170},
\end{array}
$$

where $y_{i,j}$ indicates the jth observation on contract i. Here each row gives 170 observations on one of the 77 contracts, and each column corresponds to a calendar date within the sample period. When we line up data according to TTD, we obtain the following structure:

$$\begin{array}{ccccccccc} y_{1,1} & \cdots & y_{1,44} & \cdots & y_{1,86} & \cdots & \boxed{y_{1,129}} & \cdots & y_{1,170} \\ y_{2,1} & \cdots & y_{2,44} & \cdots & \boxed{y_{2,86}} & \cdots & y_{2,129} & \cdots & y_{2,170} \\ y_{3,1} & \cdots & \boxed{y_{3,44}} & \cdots & y_{3,86} & \cdots & y_{3,129} & \cdots & y_{3,170} \\ \boxed{y_{4,1}} & \cdots & y_{4,44} & \cdots & y_{4,86} & \cdots & y_{4,129} & \cdots & y_{4,170} \\ y_{5,1} & \cdots & y_{5,44} & \cdots & y_{5,86} & \cdots & y_{5,129} & \cdots & y_{5,170} \\ \multicolumn{9}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \\ y_{77,1} & \cdots & y_{77,44} & \cdots & y_{77,86} & \cdots & y_{77,129} & \cdots & y_{77,170} \end{array},$$

where boxed entries represent an example of observations from the same day and which, as a result, are correlated with each other. In this setting, the first column contains data on all contracts when there are 169 trading days to their expiration, and the last column shows data on all contracts when they expire. Because all boxed entries come from the same calendar day, they will be correlated with each other. This is only one example. During the sample period, at most five contracts were traded on a given day and on some days, three or four were traded. The pattern of correlated observations is irregular and is impossible to represent in a general form.

To account for contemporaneous correlation from the volatility model $|\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i}| = \alpha_i + \beta_i S_{t_j^i} + \gamma_i TTD_{i,t_j^i} + \varepsilon_{i,t_j^i}$, $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n_i$, we construct residual vectors for contracts traded on the same calendar date. Here, $m = 77$, the total number of contracts in our sample, $n_i = 170$ for all i, the number of observations on contract i, and the subscript t_j^i denotes the jth trading day of futures contract i. For example, the residual vector $\mathbf{e}_{i,\ell}$ contains residuals for contract i that come from the calendar dates on which contract ℓ was also traded. After defining residual vectors for all overlapping contracts, we stack the vectors that have the same discrepancy in delivery month. For instance, to construct the residual vector for four-month-apart contracts, we combine the vector that contains residuals for March 1994 contract that come from the trading days on which July 1994 contract was also traded with the one that contains, say, residuals for September 1996 contract that come from the days on which January 1997 contract was also traded. Thus, to estimate the correlation between contracts that are four months apart, ρ_4, first we run the regression equation

$$\begin{bmatrix} \mathbf{e}_{1,3} \\ \mathbf{e}_{2,4} \\ \mathbf{e}_{3,5} \\ \vdots \\ \mathbf{e}_{74,76} \\ \mathbf{e}_{75,77} \end{bmatrix} = \psi_4 \begin{bmatrix} \mathbf{e}_{3,1} \\ \mathbf{e}_{4,2} \\ \mathbf{e}_{5,3} \\ \vdots \\ \mathbf{e}_{76,74} \\ \mathbf{e}_{77,75} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu}_{1,3} \\ \boldsymbol{\nu}_{2,4} \\ \boldsymbol{\nu}_{3,5} \\ \vdots \\ \boldsymbol{\nu}_{74,76} \\ \boldsymbol{\nu}_{75,77} \end{bmatrix} \tag{2}$$

and then reverse the roles of the dependent and independent variables and run the regression again. We compute the square root of the product of estimated coefficients from these two regressions to obtain ρ_4. We repeat the same procedure to compute correlation coefficients between two-month apart, six-month apart, and eight-month apart contracts. Because in our data at most five contracts are traded on a given day, the most distant pair of contracts is eight months apart. The correlation and variance-covariance matrices of OLS residuals from the above specification are:

$$\boldsymbol{\rho} = \begin{bmatrix} 1 & 0.75 & 0.61 & 0.46 & 0.13 \\ 0.75 & 1 & 0.75 & 0.61 & 0.46 \\ 0.61 & 0.75 & 1 & 0.75 & 0.61 \\ 0.46 & 0.61 & 0.75 & 1 & 0.75 \\ 0.13 & 0.46 & 0.61 & 0.75 & 1 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 0.87 & 0.63 & 0.52 & 0.41 & 0.20 \\ 0.63 & 0.87 & 0.63 & 0.52 & 0.41 \\ 0.52 & 0.63 & 0.87 & 0.63 & 0.52 \\ 0.41 & 0.52 & 0.63 & 0.87 & 0.63 \\ 0.20 & 0.41 & 0.52 & 0.63 & 0.87 \end{bmatrix}. \tag{3}$$

Notice that this method assumes both covariance stationarity over time and identical covariances between contracts that have the same discrepancy in delivery month. That is, the correlation between the March and May contract residuals is assumed to be the same as that between the May and July contract residuals.

We next use the Cholesky decomposition of the contemporaneous covariance matrix, $\boldsymbol{\Sigma}$, to obtain a GLS transformation of the data set. This eliminates contemporaneous correlation among residuals.

4. Empirical Model and Bayesian Estimation

A linear volatility regression equation, with separate coefficients for each contract, is given by:

$$|\%\Delta F_{i,t_j^i}| \equiv |100 \times (\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i})| = \alpha_i + \beta_i S_{t_j^i} + \gamma_i TTD_{i,t_j^i} + \varepsilon_{i,t_j^i},$$
$$i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, n_i. \tag{4}$$

Because contracts partially overlap in time, t_j^i, the jth trading day of contract i, has a different range for each contract. Delivery date, T_i, is also different for each contract. Because there are 170 observations per contract, delivery date for contract i is t^i_{170} or $t^i_{n_i}$. Summary statistics of the variables are presented in table 1.

When we combine all contracts, the 77 regression equations in equation (4) can be written in matrix form as:

$$\boldsymbol{y} = \boldsymbol{X\theta} + \boldsymbol{\varepsilon}, \tag{5}$$

where

$$\boldsymbol{y} \equiv \begin{bmatrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \vdots \\ \boldsymbol{y_m} \end{bmatrix}, \quad \boldsymbol{X} \equiv \begin{bmatrix} \boldsymbol{\iota} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{S_1} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{TTD} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\iota} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S_2} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{TTD} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\iota} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{S_m} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{TTD} \end{bmatrix},$$

$$\boldsymbol{\theta} \equiv \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \\ \boldsymbol{\gamma} \end{bmatrix}, \quad \boldsymbol{\varepsilon} \equiv \begin{bmatrix} \boldsymbol{\varepsilon_1} \\ \boldsymbol{\varepsilon_2} \\ \vdots \\ \boldsymbol{\varepsilon_m} \end{bmatrix},$$

and

$$\boldsymbol{y_i} = \begin{bmatrix} |100 \times (\ln F_{i,t^i_1} - \ln F_{i,t^i_{1-1}})| \\ |100 \times (\ln F_{i,t^i_2} - \ln F_{i,t^i_{2-1}})| \\ \vdots \\ |100 \times (\ln F_{i,t^i_{n_i}} - \ln F_{i,t^i_{n_i-1}})| \end{bmatrix}, \boldsymbol{S_i} = \begin{bmatrix} S_{t^i_1} \\ S_{t^i_2} \\ \vdots \\ S_{t^i_{n_i}} \end{bmatrix}, \mathbf{TTD} = \begin{bmatrix} 169 \\ 168 \\ \vdots \\ 0 \end{bmatrix}, \boldsymbol{\iota} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \boldsymbol{\varepsilon_i} = \begin{bmatrix} \varepsilon_{i,t^i_1} \\ \varepsilon_{i,t^i_2} \\ \vdots \\ \varepsilon_{i,t^i_{n_i}} \end{bmatrix},$$

for $i = 1, 2, \cdots, m$, and

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix}.$$

If we define

$$\boldsymbol{\theta_i} = \begin{bmatrix} \alpha_i \\ \beta_i \\ \gamma_i \end{bmatrix}, \qquad \boldsymbol{X_i} = \begin{bmatrix} \boldsymbol{\iota} & \boldsymbol{S_i} & \boldsymbol{TTD} \end{bmatrix},$$

the equation for each contract can be shown in a more compact way as:

$$\boldsymbol{y_i} = \boldsymbol{X_i}\boldsymbol{\theta_i} + \boldsymbol{\varepsilon_i}, \qquad i = 1, 2, \cdots, m. \tag{6}$$

In related work, Karali (2007) finds that not all coefficient estimates have the expected signs when parameters are allowed to vary across contracts, as in equation (4). Then, she imposes restrictions on coefficients and forces all contracts to have the same parameter for each variable. This results in a positive and significant intercept, and negative and significant inventory and TTD coefficient estimates. However, the F-tests show that data reject these equality restrictions and that a less extreme restriction is appropriate.

In this study, we fill this gap and extend the earlier work by allowing the inventory and time-to-delivery effects to differ across contracts via a Bayesian approach. With this approach, parameter estimates are contract specific and "smoothed" via a prior distribution that centers each contract's parameter estimates over the weighted average of the estimates of all contracts. By choosing weights that give higher relative importance to nearby (and overlapping) contracts, the parameters are made to vary "smoothly" across contracts.

Our benchmark model uses a weighting scheme for prior means that forces the parameters of adjacent contracts to be close and decline as the discrepancy between contracts increases. Specifically, our model uses the following weighting matrix: $\boldsymbol{w_i} = |\ell - i|^{-1}$, for $i, \ell = 1, 2, \cdots, m$, and $w_{i\ell} = 0$ when $i = \ell$. For instance, for the 5$^{\text{th}}$ and 35$^{\text{th}}$ contracts:

$$\begin{aligned}\boldsymbol{w_5} &= \begin{bmatrix} \frac{1}{4} & \frac{1}{3} & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \cdots & \frac{1}{72} \end{bmatrix}, \\ \boldsymbol{w_{35}} &= \begin{bmatrix} \frac{1}{34} & \frac{1}{33} & \cdots & \frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & \cdots & \frac{1}{42} \end{bmatrix}. \end{aligned} \tag{7}$$

The graphical representation of weighting matrices for various contracts is given in figure 3.[1] The logic behind this weighting matrix is that adjacent contracts are more likely to move in a similar way than do the distant ones because they are traded in the same time period and are subject to same shocks. Also, note that when, for example, the 35$^{\text{th}}$ contract is traded neither the 1$^{\text{st}}$ nor the 77$^{\text{th}}$ contract is active. Therefore this weighting matrix puts declining weights on contracts as they become more separated in time.

We specify prior distributions on the regression parameters as:

$$p(\boldsymbol{\theta_i}) \sim N(\underline{\boldsymbol{\theta_i}}, \sigma_i^2 \underline{\boldsymbol{V_i}}), \qquad i = 1, 2, \cdots, m, \tag{8}$$

where N denotes the multivariate normal distribution, $\underline{\boldsymbol{\theta_i}}$ is the prior mean of the ith contract's regression parameters, and $\sigma_i^2 \underline{\boldsymbol{V_i}}$ is the prior variance-covariance matrix. We specify

[1]The weighting matrix of each contract follows this pattern. The weights are normalized when constructing prior means.

the prior distribution of σ_i^2 as an inverse gamma, or its inverse as:

$$p(\sigma_i^{-2}) \sim G(\underline{s_i}^{-2}, \underline{d_i}), \qquad i = 1, 2, \cdots, m, \tag{9}$$

where G denotes the gamma distribution, $\underline{s_i}^{-2}$ is the prior mean for the inverse error variance, and $\underline{d_i}$ is the prior degrees of freedom parameter. The prior means of the parameters, $\underline{\boldsymbol{\theta_i}}$, for contract i is computed as:

$$\underline{\boldsymbol{\theta_i}} = \frac{\sum_{\ell=1}^{m} w_{i\ell}\boldsymbol{\theta^0}}{\sum_{\ell=1}^{m} w_{i\ell}}, \tag{10}$$

where $\boldsymbol{\theta^0}$ is the matrix of starting values. We assume that the likelihood function for each contract follows a standard form after the GLS transformation, and is represented by

$$L_i(\boldsymbol{y_i^*}|\boldsymbol{\theta_i}, \sigma_i^2, \boldsymbol{X_i^*}) = (2\pi\sigma_i^2)^{-n_i/2} \exp\{-0.5\sigma_i^{-2}(\boldsymbol{y_i^*} - \boldsymbol{X_i^*\theta_i})'(\boldsymbol{y_i^*} - \boldsymbol{X_i^*\theta_i})\},$$
$$i = 1, 2, \cdots, m, \tag{11}$$

where * denotes the transformed data. It can be shown that the joint posterior is

$$p(\boldsymbol{\theta_i}, \sigma_i^2|\boldsymbol{y_i^*}, \boldsymbol{X_i^*}) \sim NG\left(\overline{\boldsymbol{\theta_i}}, \overline{\boldsymbol{V_i}}, \overline{s}_i^{\,2}, \overline{d}_i\right), \qquad i = 1, 2, \cdots, m, \tag{12}$$

where NG denotes the joint normal gamma distribution, and

$$\overline{\boldsymbol{\theta_i}} = \overline{\boldsymbol{V_i}}\left(\underline{\boldsymbol{V_i}}^{\boldsymbol{-1}}\underline{\boldsymbol{\theta_i}} + (\boldsymbol{X_i^{*\prime}X_i^*})\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}\right), \tag{13}$$

$$\overline{\boldsymbol{V_i}} = \left(\underline{\boldsymbol{V_i}}^{-1} + \boldsymbol{X_i^{*\prime}X_i^*}\right)^{-1}, \tag{14}$$

$$\overline{s}_i^{\,2} = \overline{d}_i^{\,-1}\left[\underline{d_i}\,\underline{s_i}^2 + (n_i - k_i)s_i^2 + (\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} - \underline{\boldsymbol{\theta_i}})'\left(\underline{\boldsymbol{V_i}} + (\boldsymbol{X_i^{*\prime}X_i^*})^{-1}\right)^{-1}(\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} - \underline{\boldsymbol{\theta_i}})\right], \tag{15}$$

$$\overline{d}_i = \underline{d_i} + n_i, \tag{16}$$

$$\hat{\boldsymbol{\theta}}_{\boldsymbol{i}} = (\boldsymbol{X_i^{*\prime}X_i^*})^{-1}\boldsymbol{X_i^{*\prime}y_i^*}, \tag{17}$$

$$s_i^2 = \left(\frac{1}{n_i - k_i}\right)\boldsymbol{\varepsilon_i^{*\prime}\varepsilon_i^*}. \tag{18}$$

We set $\underline{\boldsymbol{V_i}} = \boldsymbol{I_{k_i}}$, $\underline{d_i} = 5$, $\underline{s_i}^2 = 0.8\sigma_{\boldsymbol{y}}^2$, where k_i is the number of regressors for contract i and equal to three for all i. The algorithm for implementing the smoothed Bayesian estimator can be found in the appendix.

5. Empirical Results

The posterior means, standard errors, and 95% highest posterior density regions from (8)-(16), with the prior weighting matrix $\boldsymbol{w_i} = |\ell - i|^{-1}$, are presented in table 2. Further, figure 4 shows the posterior means and 95% posterior density regions for each parameter. All of the intercept estimates are positive and the density region does not include zero (figure 4(a)) unlike the results in Karali (2007). There, it is reported that 56 of 77 intercept estimates are positive and 17 of those are significant at 5% level, 52 of 77 inventory coefficients are negative and only 13 of those are statistically significant, and 72 of 77 time-to-delivery coefficients are negative and 60 of those are significant with classical methods. As table 2 shows, the average posterior mean of the intercept parameter across all contracts is 5.153 with an average posterior standard deviation of 0.918.

The posterior mean of the inventory parameter, $\overline{\beta}_i$, is negative for each contract as the theory of storage suggests. The average posterior mean of inventory parameter across the 77 contracts is -0.772 with an average posterior standard deviation of 0.206. As seen in figure 4(b), the 95% highest posterior density regions exclude zero for all contracts. Further, it is seen in table 2 that contracts traded in the beginning of the sample period have larger inventory effects. The reason is that lumber inventories increase over time (see figure 2). This result is consistent with the theory of storage. As inventories are smaller, price volatility is higher. On the other hand, with larger inventories, price volatility is lower because any shock in the market would have been absorbed by inventories. The high posterior probabilities of negative inventory effects are shown in figure 5(a), showing the near certainty of the negative signs.

For all contracts, we find a negative time-to-delivery effect on price volatility. As contracts approach delivery, futures price volatility increases—an empirical support of the Samuelson effect. The average posterior mean of time-to-delivery parameter across the 77 contracts is -0.005 with a posterior standard error of 0.001. Only 5 of the 77 95% highest posterior density upper limits are positive. Figure 5(b) shows the posterior probabilities of negative time-to-delivery effects. These probabilities are near unity for all but a few contracts.

Setting the prior variance for $\boldsymbol{\theta}$ controls how restricted the parameters are. With a larger variance, parameter estimates are allowed to change across contracts more freely. When a small prior variance is used, this restricts the parameters to be more equal across contracts. The results from a very small prior variance will be similar to the classical results with only three parameters estimated.

For comparison, we report the averages of the estimates across all contracts with the benchmark prior variance, $\underline{\boldsymbol{V}}^{\boldsymbol{B}} = \boldsymbol{I}$ (the last row of table 2), and with a small prior variance, $\underline{\boldsymbol{V}}^{\boldsymbol{S}} = 10^{-8}\boldsymbol{I}$, in table 3. It can be seen that the small prior variance puts downward bias towards zero on both the intercept and the inventory effect. Specifically, the intercept estimate falls from 5.153 to 2.199, and the inventory effect from 0.772 to 0.114 in magnitude. The time-to-delivery effect is not affected by this restriction. These results are reasonable because while the time-to-delivery variable is fixed for all contracts, the inventory variable changes across contracts as a result of the different time periods they span. Similarly, the intercept estimates are capturing the changes in other economic variables over time, and thus restricting the parameter does introduce a bias.

We use alternative weighting matrices to test the robustness of our results to prior mean specification. Figure 6 shows the graphical representation of prior weights for the 35$^{\text{th}}$ contract, where $w^B = |\ell - i|^{-1}$ (our benchmark model), $w^1 = |\ell - i|^{-2}$, $w^2 = \iota_m'$, and $w^3 = (m - |\ell - i|)/(m-1)$. We report the average posterior means and standard deviations of the parameters across all contracts obtained with these prior weights in table 4. Further, the posterior mean of each parameter for the 5$^{\text{th}}$ and 35$^{\text{th}}$ contracts are reported. As seen in the table, our estimates do not change much with different prior weights. Therefore, we conclude that our results are robust to prior specification with respect to the weighting of the prior mean.

6. Concluding Remarks

We implement a new Bayesian estimation methodology to investigate if the effects of lumber inventories and time remaining to delivery on lumber futures price volatility vary across contracts. We find negative inventory and time-to-delivery effects and these effects, indeed, do vary across futures contracts. While contracts traded in the beginning of our sample period exhibit larger inventory effects, contracts traded towards the end of our sample period exhibit smaller inventory effects. This may be a result of increasing lumber inventories over time. Consistent with the theory of storage and previous studies, the price variability decreases in higher inventory regimes.

This new method reveals a downward bias towards zero on the inventory effect introduced by restricted multiple-contract estimator. When the parameters are allowed to vary across contracts the average inventory effect is much larger than the restricted effect. However, there is no bias to the time-to-delivery effect and the increased flexibility produced little substantive change in the results. This is reasonable because the inventory variable changes dramatically across contracts due to different time horizons they cover while the time-to-delivery variable does not. Thus the more flexible modeling approach reveals a time-varying inventory effect.

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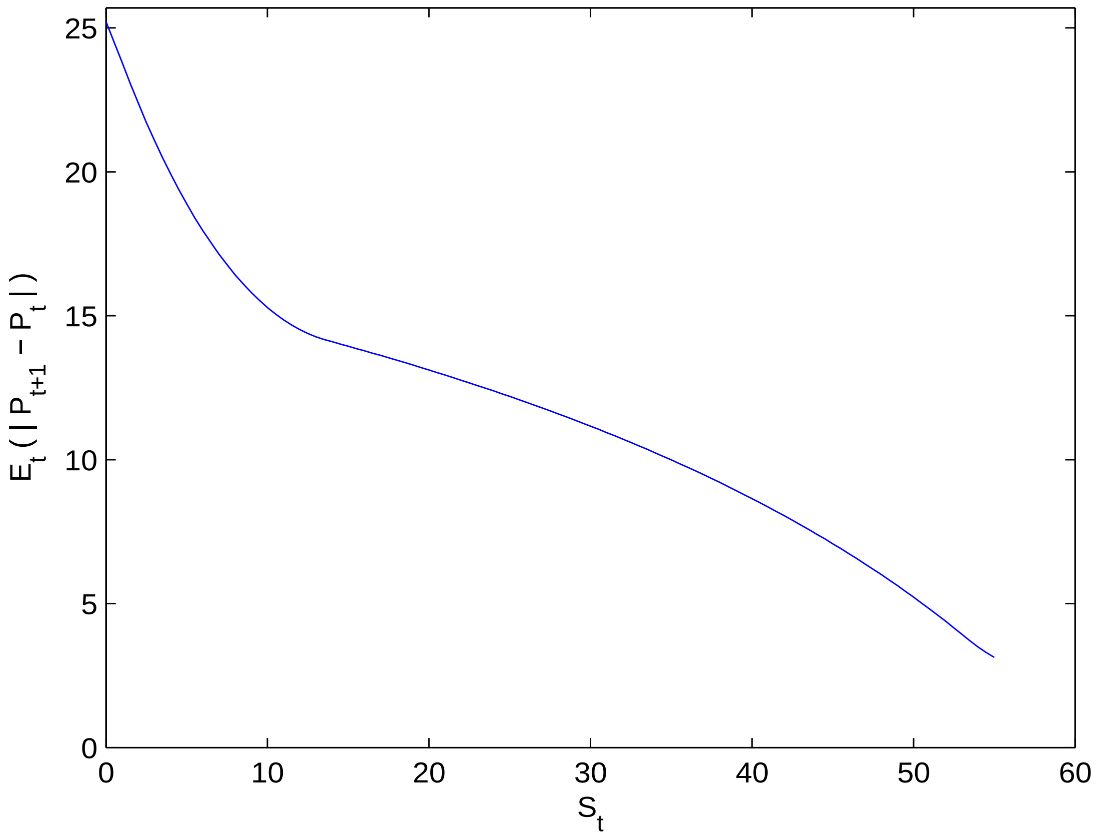


Figure 1: Expected Absolute Price Changes at Different Levels of Inventories

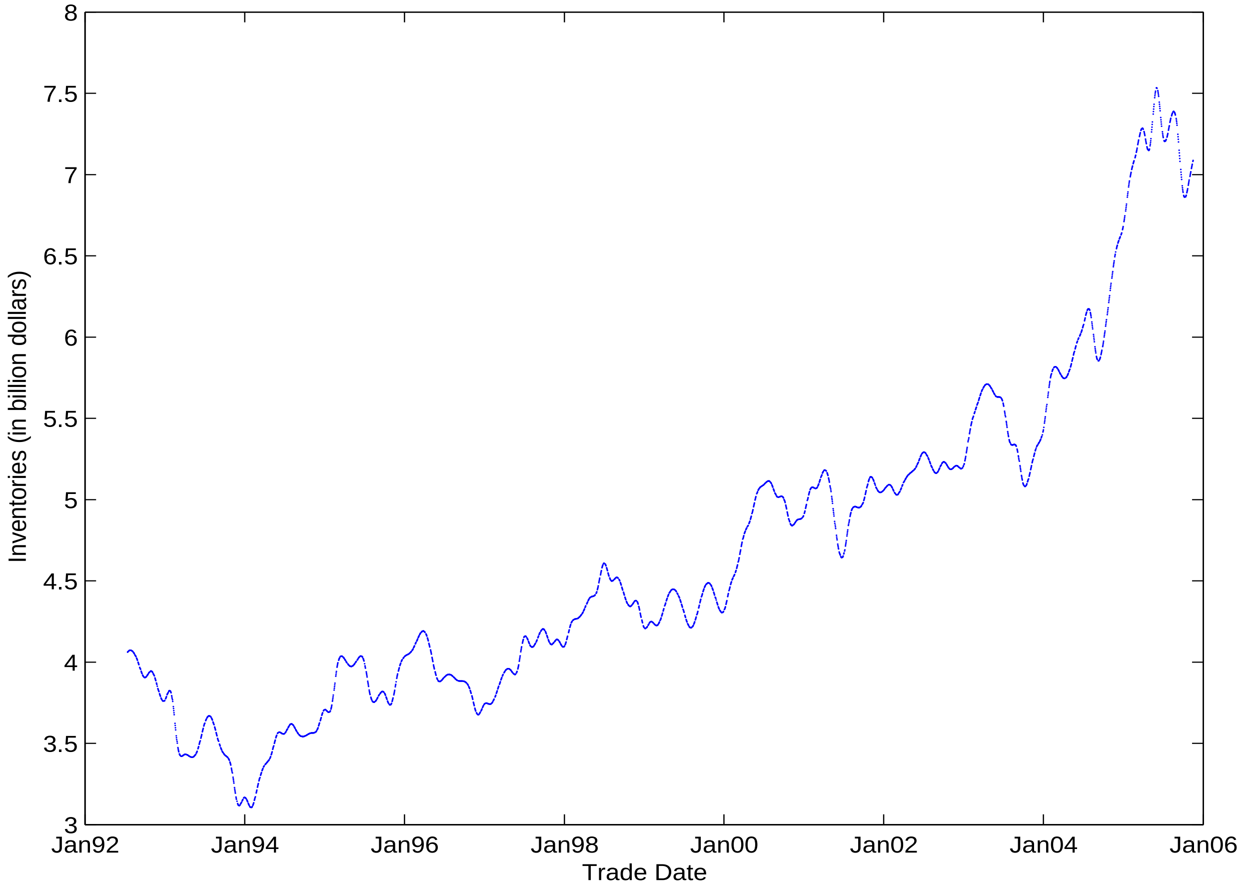
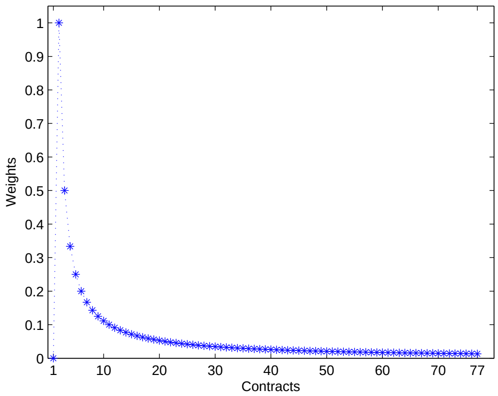
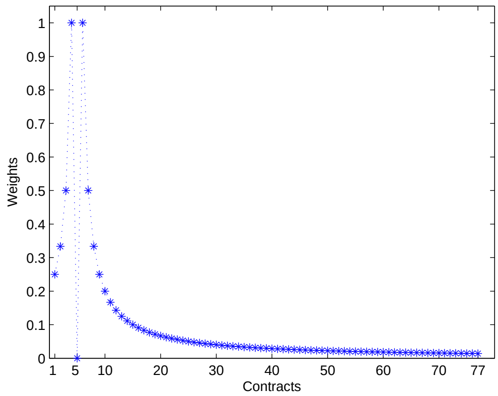


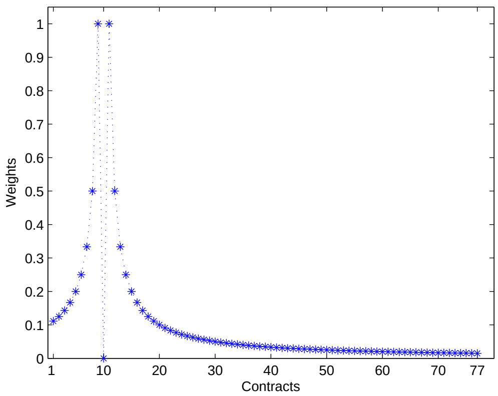
Figure 2: Lumber Inventories (billions of dollars)



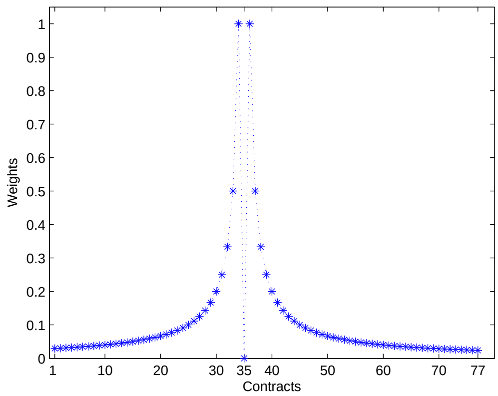
(a) w_1



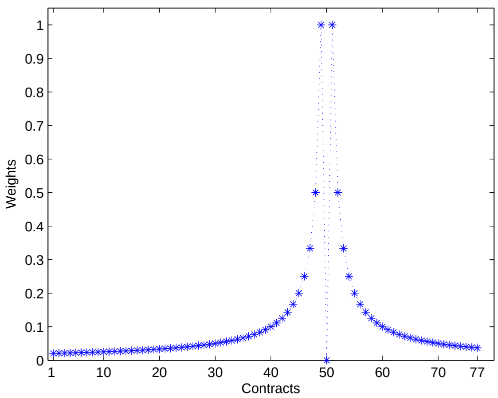
(b) w_5



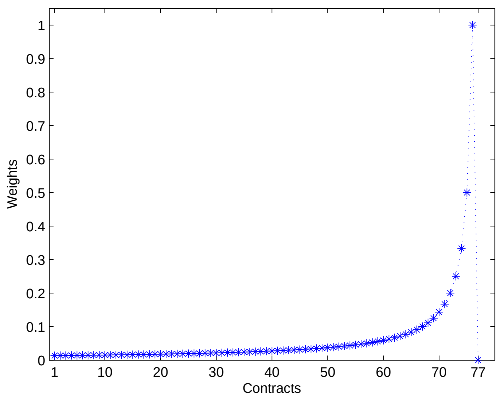
(c) w_{10}



(d) w_{35}

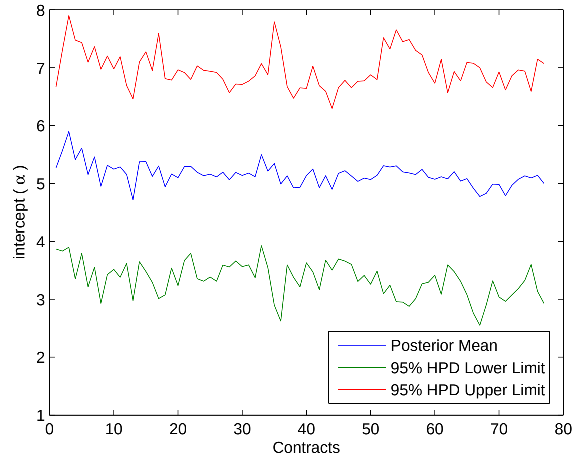


(e) w_{50}

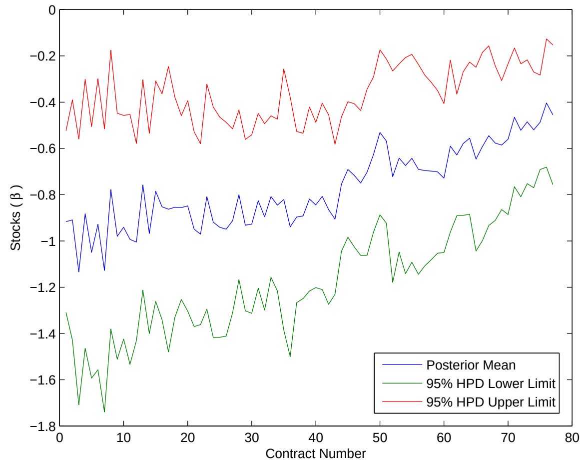


(f) w_{77}

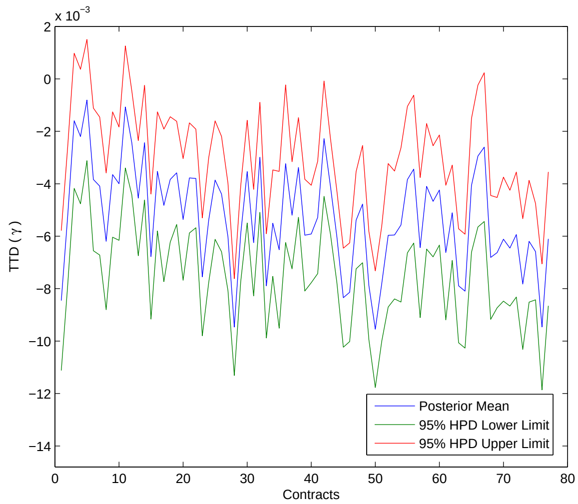
Figure 3: Prior Weights



(a) Intercept

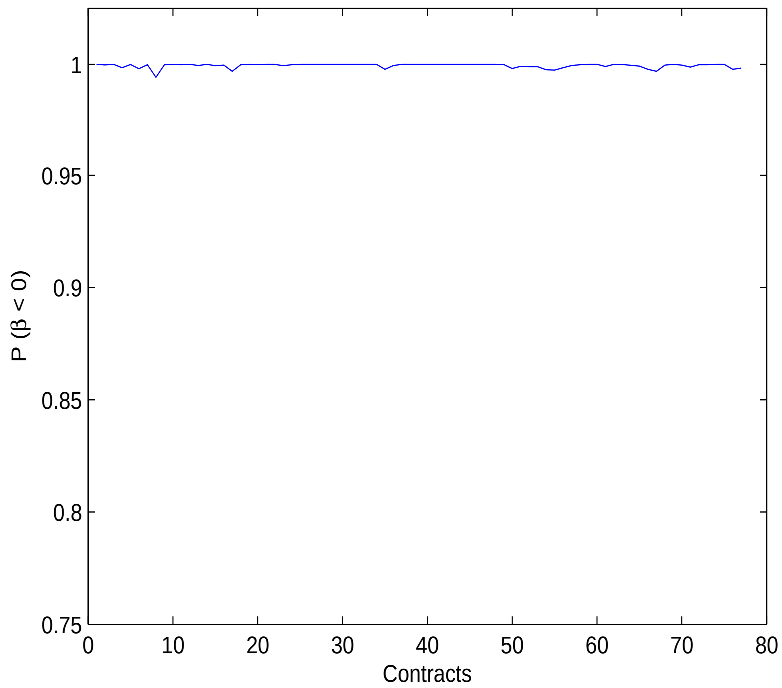


(b) Inventory Coefficient

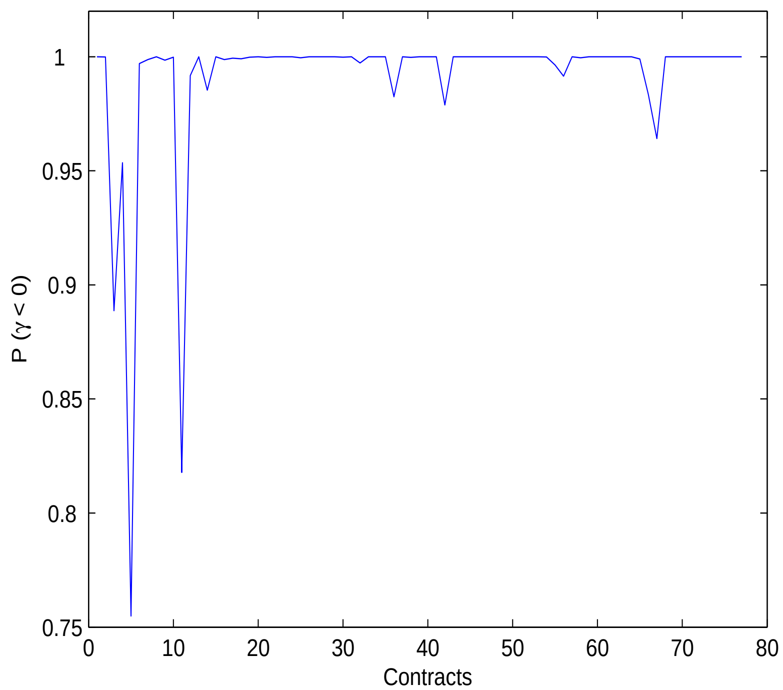


(c) Time-to-Delivery Coefficient

Figure 4: 95% Highest Posterior Density Regions

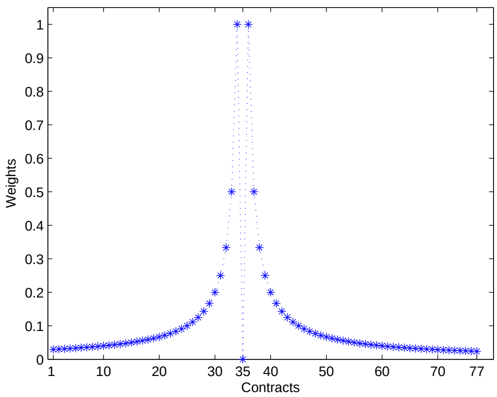


(a) Inventory Coefficient

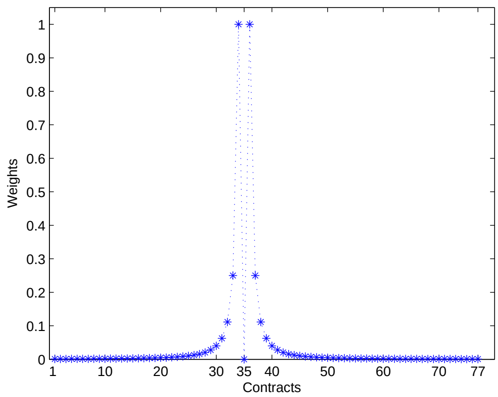


(b) Time-to-Delivery Coefficient

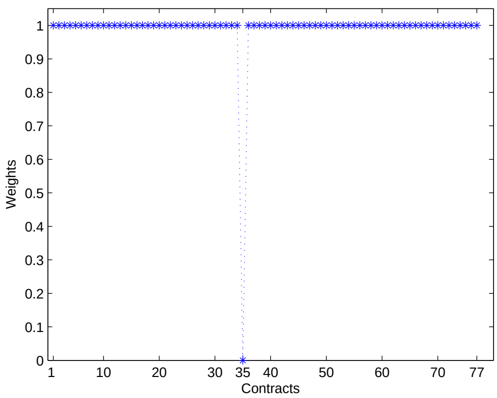
Figure 5: Posterior Probabilities



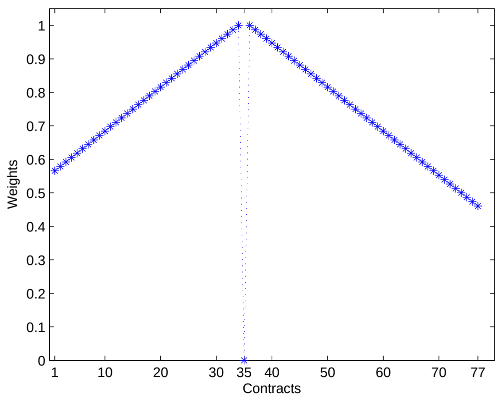
(a) w^B



(b) w^1



(c) w^2



(d) w^3

Figure 6: Various Prior Weights (35^{th} contract)

Table 1: Summary Statistics of Daily Variables

N=13,090	$\%\Delta F_{i,t_j^i}$	$\|\%\Delta F_{i,t_j^i}\|$	Inventories	TTD
Mean	-0.0105	1.2433	4.6154	84.50
Median	0	0.9739	4.3835	84.50
Min	-7.8560	0	3.1054	0
Max	14.1945	14.1945	7.5337	169
Std. Deviation	1.6097	1.0225	0.9541	49.08

Notes: $\%\Delta F_{i,t_j^i} = 100 \times (\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i})$ and $|\%\Delta F_{i,t_j^i}| = |100 \times (\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i})|$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n_i$. $m = 77$ and $n_i = 170$ for all i. The subscript t_j^i denotes the jth trading day of futures contract i. The variable $\ln F_{i,t_j^i}$ is the natural logarithm of the price on day t_j^i of the ith futures contract. Inventories are measured in billions of 1982 dollars.

Table 2: Benchmark Model

Contract	$\overline{\alpha}_i$	$se(\overline{\alpha}_i)$	$L_{\overline{\alpha}_i}$	$U_{\overline{\alpha}_i}$	$\overline{\beta}_i$	$se(\overline{\beta}_i)$	$L_{\overline{\beta}_i}$	$U_{\overline{\beta}_i}$	$\overline{\gamma}_i$	$se(\overline{\gamma}_i)$	$L_{\overline{\gamma}_i}$	$U_{\overline{\gamma}_i}$
1	5.266	0.708	3.869	6.664	-0.916	0.199	-1.309	-0.524	-0.008	0.001	-0.011	-0.006
2	5.565	0.879	3.831	7.300	-0.909	0.263	-1.429	-0.390	-0.005	0.001	-0.008	-0.003
3	5.898	1.014	3.897	7.899	-1.134	0.291	-1.709	-0.559	-0.002	0.001	-0.004	0.001
4	5.414	1.045	3.352	7.477	-0.882	0.294	-1.464	-0.301	-0.002	0.001	-0.005	0.000
5	5.612	0.922	3.792	7.432	-1.049	0.275	-1.593	-0.505	-0.001	0.001	-0.003	0.001
6	5.153	0.983	3.213	7.094	-0.928	0.319	-1.557	-0.299	-0.004	0.001	-0.006	-0.001
7	5.458	0.965	3.553	7.363	-1.127	0.310	-1.740	-0.515	-0.004	0.001	-0.007	-0.001
8	4.950	1.025	2.928	6.973	-0.778	0.305	-1.381	-0.175	-0.006	0.001	-0.009	-0.004
9	5.313	0.957	3.425	7.202	-0.979	0.269	-1.511	-0.448	-0.004	0.001	-0.006	-0.001
10	5.247	0.877	3.516	6.978	-0.941	0.245	-1.424	-0.457	-0.004	0.001	-0.006	-0.002
11	5.285	0.965	3.381	7.190	-0.993	0.274	-1.534	-0.453	-0.001	0.001	-0.003	0.001
12	5.156	0.779	3.619	6.693	-1.005	0.216	-1.431	-0.578	-0.002	0.001	-0.004	-0.000
13	4.719	0.882	2.978	6.460	-0.758	0.230	-1.213	-0.303	-0.004	0.001	-0.007	-0.002
14	5.374	0.874	3.648	7.099	-0.967	0.219	-1.400	-0.535	-0.002	0.001	-0.005	-0.000
15	5.376	0.961	3.479	7.273	-0.785	0.241	-1.261	-0.309	-0.007	0.001	-0.009	-0.004
16	5.122	0.927	3.292	6.952	-0.852	0.247	-1.341	-0.364	-0.003	0.001	-0.006	-0.001
17	5.301	1.160	3.011	7.591	-0.863	0.312	-1.480	-0.246	-0.005	0.001	-0.008	-0.002
18	4.943	0.946	3.076	6.810	-0.854	0.241	-1.331	-0.378	-0.004	0.001	-0.006	-0.001
19	5.162	0.823	3.538	6.786	-0.856	0.201	-1.253	-0.458	-0.004	0.001	-0.005	-0.002
20	5.100	0.944	3.237	6.963	-0.848	0.231	-1.303	-0.393	-0.005	0.001	-0.008	-0.003
21	5.293	0.822	3.670	6.916	-0.949	0.213	-1.370	-0.528	-0.004	0.001	-0.006	-0.002
22	5.295	0.761	3.793	6.797	-0.971	0.198	-1.362	-0.580	-0.004	0.001	-0.006	-0.002
23	5.193	0.931	3.355	7.032	-0.808	0.246	-1.295	-0.322	-0.007	0.001	-0.001	-0.005
24	5.134	0.923	3.313	6.955	-0.919	0.252	-1.417	-0.421	-0.005	0.001	-0.008	-0.003
25	5.161	0.900	3.385	6.938	-0.941	0.241	-1.416	-0.465	-0.004	0.001	-0.006	-0.002
26	5.114	0.914	3.311	6.918	-0.949	0.234	-1.411	-0.487	-0.004	0.001	-0.007	-0.002
27	5.195	0.812	3.592	6.799	-0.913	0.201	-1.311	-0.515	-0.006	0.001	-0.008	-0.004
28	5.062	0.762	3.557	6.567	-0.800	0.186	-1.167	-0.434	-0.009	0.001	-0.011	-0.008
29	5.190	0.774	3.662	6.718	-0.932	0.188	-1.302	-0.561	-0.006	0.001	-0.008	-0.004
30	5.137	0.797	3.564	6.710	-0.927	0.195	-1.313	-0.541	-0.003	0.001	-0.005	-0.002
31	5.180	0.804	3.594	6.767	-0.826	0.191	-1.204	-0.449	-0.006	0.001	-0.008	-0.004
32	5.115	0.883	3.374	6.857	-0.896	0.204	-1.298	-0.493	-0.003	0.001	-0.005	-0.009
33	5.497	0.796	3.926	7.068	-0.808	0.177	-1.157	-0.459	-0.008	0.001	-0.010	-0.006
34	5.212	0.844	3.547	6.878	-0.845	0.188	-1.216	-0.474	-0.005	0.001	-0.007	-0.003
35	5.345	1.240	2.898	7.792	-0.821	0.286	-1.385	-0.256	-0.006	0.001	-0.009	-0.003
36	4.991	1.200	2.624	7.359	-0.939	0.284	-1.500	-0.378	-0.003	0.001	-0.006	-0.000
37	5.132	0.780	3.592	6.672	-0.897	0.187	-1.266	-0.527	-0.005	0.001	-0.007	-0.003
38	4.926	0.784	3.379	6.472	-0.892	0.181	-1.249	-0.535	-0.003	0.001	-0.005	-0.001
39	4.933	0.871	3.214	6.652	-0.819	0.201	-1.216	-0.421	-0.006	0.001	-0.008	-0.004
40	5.136	0.763	3.629	6.643	-0.844	0.181	-1.202	-0.487	-0.006	0.001	-0.008	-0.004
41	5.250	0.900	3.474	7.027	-0.807	0.204	-1.210	-0.404	-0.005	0.001	-0.007	-0.003
42	4.927	0.892	3.166	6.689	-0.864	0.207	-1.274	-0.455	-0.002	0.001	-0.004	-0.000
43	5.133	0.739	3.674	6.592	-0.906	0.164	-1.230	-0.581	-0.004	0.001	-0.006	-0.002
44	4.898	0.707	3.502	6.294	-0.754	0.147	-1.044	-0.463	-0.006	0.001	-0.008	-0.004
45	5.175	0.750	3.695	6.656	-0.691	0.148	-0.984	-0.398	-0.008	0.001	-0.010	-0.006
46	5.220	0.791	3.659	6.781	-0.716	0.157	-1.026	-0.407	-0.008	0.001	-0.010	-0.006
47	5.128	0.772	3.604	6.653	-0.749	0.159	-1.062	-0.436	-0.005	0.001	-0.007	-0.003
48	5.036	0.876	3.307	6.765	-0.703	0.182	-1.062	-0.344	-0.005	0.001	-0.007	-0.002
49	5.092	0.851	3.413	6.772	-0.627	0.170	-0.962	-0.291	-0.008	0.001	-0.010	-0.006
50	5.068	0.916	3.260	6.877	-0.530	0.181	-0.887	-0.174	-0.009	0.001	-0.012	-0.007
51	5.141	0.838	3.486	6.795	-0.568	0.180	-0.923	-0.213	-0.008	0.001	-0.010	-0.006
52	5.307	1.120	3.095	7.518	-0.722	0.231	-1.179	-0.265	-0.006	0.001	-0.009	-0.003
53	5.283	1.033	3.244	7.323	-0.641	0.206	-1.048	-0.235	-0.006	0.001	-0.008	-0.003
54	5.305	1.190	2.957	7.653	-0.674	0.237	-1.141	-0.207	-0.006	0.001	-0.008	-0.003
55	5.199	1.140	2.950	7.449	-0.643	0.228	-1.092	-0.193	-0.004	0.001	-0.007	-0.001
56	5.181	1.167	2.878	7.485	-0.690	0.230	-1.144	-0.237	-0.003	0.001	-0.006	-0.001
57	5.154	1.087	3.009	7.299	-0.695	0.209	-1.108	-0.283	-0.006	0.001	-0.009	-0.004
58	5.243	1.002	3.266	7.220	-0.698	0.194	-1.081	-0.314	-0.004	0.001	-0.006	-0.002
59	5.105	0.918	3.294	6.917	-0.701	0.178	-1.053	-0.350	-0.005	0.001	-0.007	-0.002
60	5.073	0.840	3.415	6.731	-0.728	0.163	-1.050	-0.406	-0.004	0.001	-0.006	-0.002
61	5.115	1.028	3.086	7.145	-0.590	0.188	-0.962	-0.218	-0.007	0.001	-0.009	-0.004
62	5.080	0.753	3.593	6.567	-0.628	0.133	-0.891	-0.365	-0.005	0.001	-0.007	-0.003
63	5.205	0.875	3.477	6.933	-0.579	0.157	-0.889	-0.269	-0.008	0.001	-0.010	-0.006

Continued on next page...

Table 2 – Continued

Contract	$\overline{\alpha}_i$	$se(\overline{\alpha}_i)$	$L_{\overline{\alpha}_i}$	$U_{\overline{\alpha}_i}$	$\overline{\beta}_i$	$se(\overline{\beta}_i)$	$L_{\overline{\beta}_i}$	$U_{\overline{\beta}_i}$	$\overline{\gamma}_i$	$se(\overline{\gamma}_i)$	$L_{\overline{\gamma}_i}$	$U_{\overline{\gamma}_i}$
64	5.041	0.877	3.310	6.773	-0.556	0.167	-0.885	-0.227	-0.008	0.001	-0.010	-0.006
65	5.085	1.016	3.081	7.089	-0.646	0.201	-1.044	-0.249	-0.004	0.001	-0.007	-0.001
66	4.918	1.094	2.759	7.077	-0.592	0.206	-0.999	-0.185	-0.003	0.001	-0.006	-0.000
67	4.774	1.127	2.550	6.998	-0.545	0.197	-0.933	-0.157	-0.003	0.001	-0.005	0.000
68	4.829	0.976	2.902	6.755	-0.577	0.169	-0.911	-0.243	-0.007	0.001	-0.009	-0.004
69	4.988	0.845	3.319	6.657	-0.585	0.141	-0.864	-0.307	-0.007	0.001	-0.009	-0.004
70	4.983	0.985	3.040	6.927	-0.560	0.165	-0.886	-0.234	-0.006	0.001	-0.008	-0.004
71	4.789	0.925	2.963	6.615	-0.465	0.152	-0.765	-0.165	-0.006	0.001	-0.009	-0.004
72	4.967	0.959	3.075	6.860	-0.522	0.146	-0.809	-0.234	-0.006	0.001	-0.008	-0.003
73	5.072	0.957	3.184	6.960	-0.485	0.136	-0.752	-0.217	-0.008	0.001	-0.010	-0.005
74	5.133	0.916	3.324	6.941	-0.520	0.127	-0.770	-0.269	-0.006	0.001	-0.008	-0.004
75	5.095	0.758	3.599	6.591	-0.487	0.103	-0.691	-0.283	-0.007	0.001	-0.008	-0.005
76	5.142	1.016	3.137	7.147	-0.404	0.140	-0.681	-0.126	-0.009	0.001	-0.012	-0.007
77	4.999	1.050	2.928	7.071	-0.455	0.153	-0.757	-0.153	-0.006	0.001	-0.009	-0.003
Average	5.153	0.918	3.341	6.965	-0.772	0.206	-1.179	-0.365	-0.005	0.001	-0.007	-0.003

Notes: Model: $|\%\Delta F_{i,t_j^i}| \equiv |100 \times (\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i})| = \alpha_i + \beta_i S_{t_j^i} + \gamma_i TTD_{i,t_j^i} + \varepsilon_{i,t_j^i}$, for $i = 1, \cdots, m$ and $j = 1, \cdots, n_i$. $m = 77$ and $n_i = 170$ for all i. The subscript t_j^i denotes the jth trading day of contract i. The variable $\ln F_{i,t_j^i}$ is the natural logarithm of the price on day t_j^i of futures contract i, $S_{t_j^i}$ is the lumber inventory level on day t_j^i, and TTD_{i,t_j^i} is the number of remaining days to delivery for contract i on day t_j^i. For each parameter, its posterior mean, posterior standard error, 95% highest posterior density lower and upper limits are given, respectively.

Table 3: Prior Variance Sensitivity

	$\overline{\alpha}_i^A$	$se(\overline{\alpha}_i^A)$	$L_{\overline{\alpha}_i^A}$	$U_{\overline{\alpha}_i^A}$	$\overline{\beta}_i^A$	$se(\overline{\beta}_i^A)$	$L_{\overline{\beta}_i^A}$	$U_{\overline{\beta}_i^A}$	$\overline{\gamma}_i^A$	$se(\overline{\gamma}_i^A)$	$L_{\overline{\gamma}_i^A}$	$U_{\overline{\gamma}_i^A}$
$\underline{V}^B = I$	5.1532	0.9180	3.3414	6.9650	-0.7718	0.2063	-1.1789	-0.3647	-0.0052	0.0012	-0.0075	-0.0030
$\underline{V}^S = 10^{-8}I$	2.1987	0.0001	2.1985	2.1989	-0.1138	0.0001	-0.1140	-0.1136	-0.0050	0.0001	-0.0052	-0.0048

Notes: Model: $|\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i}| = \alpha_i + \beta_i S_{t_j^i} + \gamma_i TTD_{i,t_j^i} + \varepsilon_{i,t_j^i}$, for $i = 1, \cdots, m$ and $j = 1, \cdots, n_i$. $m = 77$ and $n_i = 170$ for all i. The subscript t_j^i denotes the jth trading day of contract i. The variable $\ln F_{i,t_j^i}$ is the natural logarithm of the price on day t_j^i of futures contract i, $S_{t_j^i}$ is the lumber inventory level on day t_j^i, and TTD_{i,t_j^i} is the number of remaining days to delivery for contract i on day t_j^i. For each parameter, the averages of its posterior mean, posterior standard error, 95% highest posterior density lower and upper limits are given, respectively.

Table 4: Sensitivity to Prior Mean Specification

	$\overline{\alpha}_i^A$	$se(\overline{\alpha}_i^A)$	$\overline{\alpha}_5$	$\overline{\alpha}_{35}$	$\overline{\beta}_i^A$	$se(\overline{\beta}_i^A)$	$\overline{\beta}_5$	$\overline{\beta}_{35}$	$\overline{\gamma}_i^A$	$se(\overline{\gamma}_i^A)$	$\overline{\gamma}_5$	$\overline{\gamma}_{35}$
w^B	5.153	0.918	5.612	5.345	-0.772	0.206	-1.049	-0.821	-0.005	0.001	-0.001	-0.006
w^1	5.191	0.918	6.801	5.497	-0.806	0.206	-1.403	-0.856	-0.005	0.001	-0.001	-0.006
w^2	5.040	0.918	5.345	5.228	-0.743	0.206	-0.970	-0.794	-0.005	0.001	-0.001	-0.006
w^3	5.069	0.918	5.400	5.257	-0.750	0.206	-0.986	-0.801	-0.005	0.001	-0.001	-0.006

Notes: $w^B = |\ell - i|^{-1}$, $w^1 = |\ell - i|^{-2}$, $w^2 = \iota'_m$, $w^3 = (m - |\ell - i|)/(m-1)$ Model: $|\ln F_{i,t_j^i} - \ln F_{i,t_{j-1}^i}| = \alpha_i + \beta_i S_{t_j^i} + \gamma_i TTD_{i,t_j^i} + \varepsilon_{i,t_j^i}$, for $i = 1, \cdots, m$ and $j = 1, \cdots, n_i$. $m = 77$ and $n_i = 170$ for all i. The subscript t_j^i denotes the jth trading day of contract i. The variable $\ln F_{i,t_j^i}$ is the natural logarithm of the price on day t_j^i of futures contract i, $S_{t_j^i}$ is the lumber inventory level on day t_j^i, and TTD_{i,t_j^i} is the number of remaining days to delivery for contract i on day t_j^i. For each parameter, the averages of its posterior mean and posterior standard error are given as well as the posterior means for the 5$^{\text{th}}$ and 35$^{\text{th}}$ contracts.

SUPPLEMENTARY APPENDIX

Algorithm for Implementing the Smoothed Bayesian Estimator:

Assume the restricted OLS parameter estimates from equation (5) as the starting values, $\boldsymbol{\theta^0}$, where

$$\boldsymbol{\theta^0} = \begin{bmatrix} \boldsymbol{\theta^0_1} & \boldsymbol{\theta^0_2} & \cdots & \boldsymbol{\theta^0_m} \end{bmatrix} = \begin{bmatrix} \alpha_1^0 & \alpha_2^0 & \cdots & \alpha_m^0 \\ \beta_1^0 & \beta_2^0 & \cdots & \beta_m^0 \\ \gamma_1^0 & \gamma_2^0 & \cdots & \gamma_m^0 \end{bmatrix}.$$

A. Outer loop: iterations (repeat until convergence)

B. Inner loop: contracts

1. Use $\boldsymbol{\theta^0}$ to compute variance-covariance matrix of residuals, $\boldsymbol{\Sigma}$.
2. Use Cholesky factor of $\boldsymbol{\Sigma}$ to apply GLS transformation to data.
3. Pull out one contract, i, from transformed data and estimate equation (6) via OLS to obtain $\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}$, where

$$\hat{\boldsymbol{\theta}}_i = \begin{bmatrix} \hat{\alpha}_i \\ \hat{\beta}_i \\ \hat{\gamma}_i \end{bmatrix}.$$

4. Compute prior mean of the parameters, $\underline{\boldsymbol{\theta_i}}$, for contract i as:

$$\underline{\boldsymbol{\theta_i}} = \frac{\sum_{\ell=1}^{m} w_{i\ell}\boldsymbol{\theta^0}}{\sum_{\ell=1}^{m} w_{i\ell}}.$$

5. Compute posterior mean of the parameter vector, $\overline{\boldsymbol{\theta_i}}$, for contract i by Bayesian estimator for normal gamma distribution:

$$\overline{\boldsymbol{\theta_i}} = \overline{\boldsymbol{V_i}} \left(\underline{\boldsymbol{V_i}}^{\boldsymbol{-1}} \underline{\boldsymbol{\theta_i}} + (\boldsymbol{X_i^{*\prime}} \boldsymbol{X_i^*}) \hat{\boldsymbol{\theta}}_{\boldsymbol{i}} \right),$$

where

$$\overline{\boldsymbol{V_i}} = \left(\underline{\boldsymbol{V_i}}^{-1} + \boldsymbol{X_i^{*\prime}} \boldsymbol{X_i^*} \right)^{-1}.$$

6. Compute standard errors of the posterior means for contract i:

$$se(\overline{\boldsymbol{\theta_i}}) = \sqrt{diag\left(\overline{s}_i{}^2\overline{\boldsymbol{V_i}}\right)},$$

where *diag* is an operant that selects the diagonal elements of a matrix and

$$\begin{aligned}
\overline{s}_i{}^2 &= \overline{d}_i{}^{-1}\left[\underline{d_i}\,\underline{s_i}{}^2 + (n_i - k_i)s_i^2 + (\boldsymbol{\hat{\theta}_i} - \underline{\boldsymbol{\theta_i}})'\left(\underline{\boldsymbol{V_i}} + (\boldsymbol{X_i^{*\prime}X_i^{*}})^{-1}\right)^{-1}(\boldsymbol{\hat{\theta}_i} - \underline{\boldsymbol{\theta_i}})\right] \\
\overline{d}_i &= \underline{d_i} + n_i \\
s_i^2 &= \left(\frac{1}{n_i - k_i}\right)\boldsymbol{\varepsilon_i^{*\prime}\varepsilon_i^{*}}.
\end{aligned}$$

7. Replace starting values for contract i, $\boldsymbol{\theta_i^0}$ in $\boldsymbol{\theta^0}$, with posterior mean, $\overline{\boldsymbol{\theta_i}}$.
8. Go back to step 1 and repeat the same procedure for the next contract.

B′. End the loop over the contracts and go to the next step.

A′. If

$$\frac{\|\overline{\boldsymbol{\theta}}^{\boldsymbol{(h)}} - \overline{\boldsymbol{\theta}}^{\boldsymbol{(h-1)}}\|}{1 + \|\overline{\boldsymbol{\theta}}^{\boldsymbol{(h)}}\|} < \tau,$$

where $\overline{\boldsymbol{\theta}}^{\boldsymbol{(h)}}$ is the posterior means of the parameters from the h^{th} iteration, and τ is the tolerance criteria, then stop and use $\overline{\boldsymbol{\theta}}^{\boldsymbol{(h)}}$ as the final parameter estimates. If this convergence criteria is not met, then go to the next iteration, $h + 1$. The convergence parameter τ is normally set to something between 10^{-3} and 10^{-6}.