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Dynamic Contingent Valuation and Choice Modelling for Ecosystem Services

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Abstract:

Non market valuation and bio economic modelling are combined in a dynamic model of ecosystem services. A mathematical proof demonstrates that the imputed price of natural capital contains all non market values and that scarcity rent is the total value of ecosystem services. A dynamic demand system, including characteristics is derived. New methods are developed for dynamic welfare analysis and both revealed and stated preference methods are proposed for estimating the price of natural capital. Estimation is simple if we avoid surveying consumers who degrade the ecosystem and instead consult owners who accrue the scarcity rent and conserve for the future.

Keywords: Non market valuation, ecosystem services, Lancaster demand, welfare analysis, analytical solutions

JEL Classification Codes: Q57, Q51, Q56

Introduction

Non market valuation identifies many values—use values, including current use and option values for future use, bequest values and existence values. In the names, we recognise stocks and flows. Current use is a flow. Future use is the flow from stocks conserved for our future [1]. Bequests and existence are values we have for the stocks themselves [2]. Indeed, our environment is a system of stocks and flows. Ecosystem services are flows. Greenhouse gases, biodiversity, wildlife, national parks, old growth forests—all are stocks. Yet our methods for non market valuation, the travel cost method, hedonic pricing, contingent valuation and choice modeling, do not model stocks and flows [3], [4], [5], [6].

We often model stocks and flows as inputs into a dynamic production process. Stocks, such as minerals in the ground, fish swimming in the ocean and wildlife roaming in the wilderness, or trees before they are cut and land before it is degraded, are transformed into commodities for consumers. Stocks are often overexploited because they have no market prices. Instead, we solve bio economic models and impute the prices that should be paid for using stocks now instead of conserving for the future [7]. Recently, bio economic models have been used to value ecosystem services [8], [9], [10], [11], [12], [13]. However, few ecosystem services are used in the production of commodities. Most are public goods. In general, we can't value ecosystem services without first measuring the values that people have for the environment.

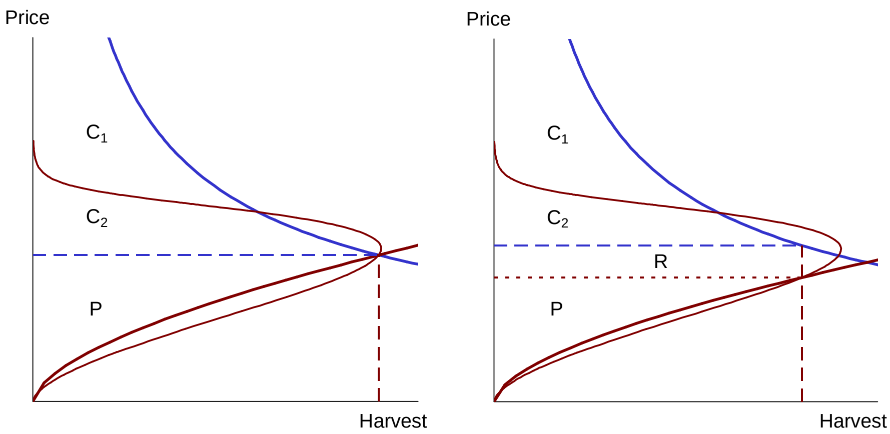
We have a dilemma. To find non market values, we must solve a bio economic model with stocks and flows. To solve the model we must know the utility that people gain from the environment. Utility can't be observed so we must infer utility from demand for ecosystem services or from willingness to pay. To estimate demand or willingness to pay we must use the appropriate estimating equations. To derive the estimating equations, we must first know the solution to the model. In other words, we must find the analytical solution.

Non market valuation relies on static consumer theory because the analytical solution is well understood. Demand and willingness to pay equations are derived using duality without directly specifying or solving the model. Analytical solutions for bio economic models are less well understood. Before using duality, we must learn the basic structure of the solution and the variables therein.

This paper incorporates non market valuation into a bio economic model of ecosystem services. Because the concepts of valuation differ—non market valuation elicits people's willingness to pay and bio economic modelling imputes the prices of stocks—the following section explores the two concepts. Next a general model of non market valuation with stocks and flows is introduced and the non market values are identified. Then the unique analytical solution is found for a special case of the general model. Dynamic methods for welfare analysis are derived and contrasted with static methods currently used in policy analysis. Finally, revealed preference and stated preference methods are proposed for estimating the non market values of ecosystem services.

Bio economic Models and Non market Valuation

Consider non market valuation applied to a dynamic production process. Figure 1 shows two steady states for a renewable resource, one with open access and another with optimal management. Often with open access, resources are overexploited and destroyed, but this example allows a comparison of static and dynamic methods of non market valuation. For ease of explanation, suppose the problem is deforestation.



Open Access Optimal Management

Figure 1. Steady State Deforestation

The demand curves slope downward and show consumers' marginal willingness to pay for fuel wood, after it is harvested and transported to market. The short-run supply curves slope upward and show woodcutters' marginal effort costs. Effort costs include the costs for saws, labour and transport. The long-run supply curves slope upward and then bend backward as costs rise but harvest diminishes. These show woodcutters' marginal effort costs at all possible steady states.

Open access is shown in the first panel. The harvest of fuel wood is near the maximum sustainable harvest for the forest. The price and harvest are determined where the short-run supply curve and the demand curve intersect. With open access, the future is ignored. Optimal management is shown in the second panel. Harvest is less and the biomass of the forest is greater. Less harvest and more biomass reduce the effort costs and the short-run supply curve shifts down. If harvest were at the intersection of the demand and short-run supply curves, biomass would decrease and the short-run supply curve would shift up. The system would move to the open access steady state. Instead, optimal management maximizes the benefits now and in the future by harvesting less and conserving the forest. There is a gap between the price of fuel wood and the marginal effort cost. This gap is the marginal user cost.

What is the non market value of ecosystem services from an open access forest? One method calculates the sum of consumer and producer surpluses [14]. In the first panel of Figure 1, consumer surplus is $C_1 + C_2$, the area below the demand curve and above the price. Producer surplus is P, the area below the price and above the short-run supply curve. Adding up the surpluses gives a large value for ecosystem services. Might the value be zero? A bio economic model would count consumer surplus as the contribution of consumers and producer surplus as the contribution of producers but the contribution of the forest is destroyed by open access.

What is the non market value if perfect institutions are implemented and the forest becomes optimally managed? One method calculates the environmental benefits and costs of the change from open access to optimal management [15]. Comparing the first and second panels in Figure 1, consumer surplus decreases and producer surplus may increase or decrease, depending on the shift in the short-run supply curve. After the change, rent accrues to the owners of the forest. In the second panel, rent is R, the area equal to the marginal user cost multiplied by the optimal harvest. Overall, benefits to society will increase, but by less than the rent. An environmental benefit-cost analysis would calculate a relatively small value for ecosystem services. Might the value be larger? A bio economic model would identify the rent as the total value of ecosystem services from an optimally managed forest.

Which of these methods should we use? Or, more precisely, which of society's benefits are contributed by the ecosystem? Producer surplus is usually considered to be the return to entrepreneurship and a contribution to society by producers. An even more difficult question is whether consumer surplus is a contribution by consumers or by the ecosystem. Some ecosystem services are essential for life—gravity, sunlight, atmospheric filtering of radiation, photosynthesis, nutrient cycling, rainfall. Most essential services have open access and are inexpensive or free. If necessary, however, consumers would spend all they have. Therefore, the consumer surplus of essential services is the wealth of the world. We are left with a conundrum. Is life a contribution by the ecosystem or by the people who live in it?

Lifetime Utility from Ecosystem Services

Renewable resources like fuelwood are harvested and the products sold in markets. These are rival in use. Other ecosystems services are not, and may never be, sold in markets. These are non rival in use. A partial list of ecosystem services from rival to non rival is:

- food from agriculture;
- harvest from a capture fishery;
- fuel wood cutting from communal woodlots;
- recreational fishing;
- tourism in national parks;
- amenities from old growth forests
- medicines from nature;
- clean air;
- biodiversity and resilience;
- global temperatures;
- sunlight.

For rival services, non market values might be imputed from commodity supply and demand curves, as in Figure 1. For non rival services, a more general model is needed.

Suppose people act as if their objective is to maximise utility, now and in the future, subject to an economic constraint for manufactured capital and an ecosystem constraint for natural capital.

$$J(M_0, N_0) = \max_{Q_1, Q_2} \int_0^T U(t, Q_1, Q_2, N)dt + V(T, M_T, N_T)$$

$$\text{subject to :}$$

$$\dot{M} = F(M, N, Q_1, Q_2)$$
$$\dot{N} = G(N, Q_2)$$

This model can be interpreted as a model of endogenous growth in an economy dependent upon the ecosystem [16]. We will interpret it as a model of individual decisions. Lifetime utility, *J*, depends upon initial endowments of manufactured capital, *M*, and natural capital, *N*. Manufactured capital is an aggregate of all productive assets other than natural capital. Natural capital includes all stocks that provide flows of ecosystem services. People maximise lifetime utility by choosing commodities, Q_1, and the flow of ecosystem services, Q_2. With natural capital, these determine current utility, *U*, at each age in people's lives, *t*. People consume until, at the end, *T*, they bequeath manufactured and natural capital to future generations and gain utility *V*. Manufactured capital and natural capital evolve over time according to differential equations. Manufactured capital increases with net production *F* and natural capital increases with net growth *G*. Natural capital can be modelled as beneficial ecosystem stocks. Pollutants such as sulphur dioxide and green house gases degrade beneficial stocks such as clean air and comfortable temperatures.

In the model, we can identify the non market values. Current use value is the utility of ecosystem services at time *t*. Future use values are utilities of ecosystem services after time *t*. The bequest value is utility at time *T*. Existence value includes the bequest value plus the utility of natural capital during people's lifetimes.

Maximizing lifetime utility subject to constraints is equivalent to maximizing a dynamic measure of utility that accounts for changes in capital.

$$H(t,M,N)=\max_{Q_1,Q_2}\left[U(t,Q_1,Q_2,N)+\lambda F(M,N,Q_1,Q_2)+\psi G(N,Q_2)\right]$$

This is the Hamiltonian. On the right-hand side, the first term is current utility at time *t*. The second term is total user costs of manufactured capital and the third term is total user costs of natural capital. Total user costs, like other total costs, are a price times a quantity. The prices are the marginal user cost of manufactured capital, λ, and the marginal user cost of natural capital, ψ. The quantities are the net production from manufactured capital, *F*, and the net growth in natural capital, *G*. If natural capital is degrading, the net growth is negative and total user costs subtract from current utility to account for costs to the future. If the natural capital is renewing, the net growth is positive and total user costs should be called total user benefits. Total user benefits add to current utility to account for benefits in the future.

Because maximizing the Hamiltonian is equivalent to maximizing lifetime utility, it must also contain the non market values. Current use value is in current utility. A small portion of existence value may also be in current utility. Otherwise future use, bequest and existence values must be in the total user costs of manufactured and natural capital

People make two decisions in each time period, with two optimality conditions for commodities and the flow of ecosystem services.

$$\frac{\partial U}{\partial Q_1}+\lambda\frac{\partial F}{\partial Q_1}=0$$
$$\frac{\partial U}{\partial Q_2}+\lambda\frac{\partial F}{\partial Q_2}+\psi\frac{\partial G}{\partial Q_2}=0$$

The first optimality condition for commodities generalizes the conditions from a static model of consumer demand. The marginal utility of consumption is compared with the value of the marginal product. The marginal product is valued at the marginal user cost of manufactured capital. The second condition for the flow of ecosystem services has no counterpart in a static model and generalizes the optimality condition for harvest of a natural resource which was illustrated in Figure 1. Marginal utility of ecosystem services is compared with the value of the marginal product plus the value of marginal growth. The marginal product and marginal growth are valued at the marginal user costs. Because this condition determines the optimal allocation of ecosystem services over time, the marginal value of future uses, bequests and existence must be contained in the marginal user costs.

Further optimality conditions define the evolution of the marginal user costs and their terminal values.

$$\dot{\lambda} = -\lambda \frac{\partial F}{\partial M}; \quad \lambda_T = \frac{\partial V_T}{\partial M_T}$$
$$\dot{\psi} = -\frac{\partial U}{\partial N} - \lambda \frac{\partial F}{\partial N} - \psi \frac{\partial G}{\partial N}; \quad \psi_T = \frac{\partial V_T}{\partial N_T}$$

Production depends upon both manufactured and natural capital and links the marginal user costs. For the most part, however, the marginal values of future uses, bequests and existence are contained in the marginal user cost of natural capital.

If manufactured capital is in units of \$, then its marginal user cost is in units of utils/\$. Natural capital is a vector of stocks which may be measured in many different units. For example, if natural capital is in units of tons of biomass, then its marginal user cost is in units of utils/ton. The ratio of marginal user costs gives the price of natural capital in \$/ton.

$$\pi = \frac{\psi}{\lambda}; \quad \dot{\pi} = \left(\frac{\dot{\psi}}{\psi} - \frac{\dot{\lambda}}{\lambda}\right)\pi; \quad \pi_T = \frac{\partial V/\partial N_T}{\partial V/\partial M_T}$$

This price measures the relative scarcity of natural capital. It contains all non market values. Therefore, dynamic non market valuation of ecosystem services is a matter of quantifying the price of natural capital.

A Dynamic Lancaster Demand System for Ecosystem Services

The price of natural capital is the ratio of two marginal utilities. We don't know and can't observe people's utility. Instead we must estimate demand or willingness to pay and infer utility from the estimation. The estimating equations must be derived from an analytical solution. If we understood the structure of the solution, we could use duality theory for the general model. Alternatively, we can solve a special case. To simplify, eliminate natural capital from current utility and from the production function. As a consequence, existence values become the same as bequest values and natural capital becomes a perfect substitute for manufactured capital in production. In addition, production and growth will be linear.

This special case is still complex, however. Ecosystem services and consumption produce the characteristics of prosperity and good health which give people utility. Ecosystem services can vary along a continuum from rival to non rival. Manufactured capital and natural capital can be substitutes or complements in bequests. The solution will contain lifetime utility and expenditure functions and a dynamic Lancaster demand system. It will show how dynamic duality can be applied and give new results for welfare analysis. Most importantly, the solution will provide equations for estimating the price of natural capital.

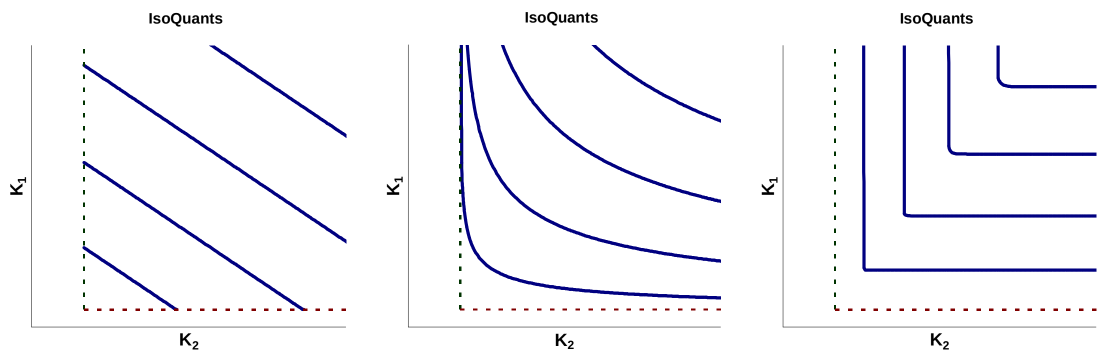
Assume the following functional forms.

$$U(t,Q_1,Q_2) = e^{-\rho t}\left[\beta_1 (K_1 - \chi_1)^{-\nu} + \beta_2 (K_2 - \chi_2)^{-\nu}\right]^{\frac{-\alpha}{\nu}}$$
$$K_1 = k_{11}Q_1 + k_{12}Q_2$$
$$K_2 = k_{21}Q_1 + k_{22}Q_2$$
$$Q_1 \geq 0; \quad Q_2 \geq 0$$

$$F(M,N,Q_1,Q_2) = rM + Y - p_1Q_1 - p_2Q_2$$
$$G(N,Q_2) = gN - cQ_2$$

$$p_1 = e^{-(r-g)(T-t)} p_{1T}$$
$$p_2 = e^{-(r-g)(T-t)} p_{2T}$$

Utility of consumption is a generalized constant elasticity of substitution function [17]. Parameter ν is the substitution parameter. As it varies from -1 to ∞, the elasticity of substitution, $\sigma = 1/(\nu+1)$, varies from ∞, for perfect substitutes, to 0, for perfect complements. Figure 2 shows threes sets of isoquants for elasticities of substitution of ∞, 1 and 0.

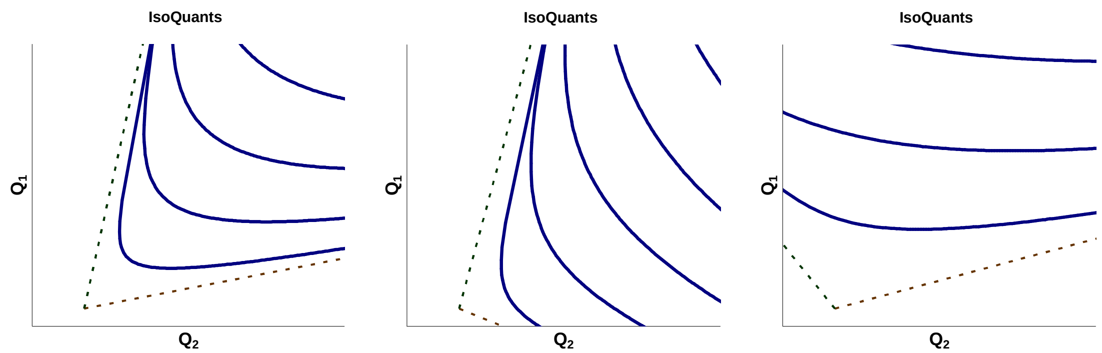


Perfect Substitutes Substitutes Perfect Complements

Figure 2. Isoquants for Prosperity and Good Health

When the elasticity of substitution is infinite, isoquants are linear. When the elasticity of substitution is one, isoquants are of the Stone-Geary type. When the elasticity of substitution is 0, isoquants are of the Leontief type. Subsistence quantities of characteristics are χ_1 and χ_2. These are shown in Figure 2 as the dotted lines which effectively shift the origin away from zero. Suppose the first characteristic is for feelings of prosperity and the second characteristic is for feelings of good health. Good health is shown as more necessary for survival than prosperity with the origin shifted further to the right. Elasticities of prosperity and good health are β_1 and β_2. These change the slopes and curvature of the isoquants. Two additional parameters are the nominal rate of time preference, ρ, and the elasticity of current utility, α. These two parameters define a monotonic transformation of utility and change the spacing among the isoquants. In a static demand system, these two parameters disappear, but in the dynamic system they will prove necessary.

Characteristics are produced by consumption and ecosystem services [18], which also have isoquants. These can be exactly the same as the isoquants for characteristics, or they may differ in complicated ways.



Both Essential   Commodities Not Essential   Services Not Essential

Figure 3. Isoquants for Commodities and Ecosystem Services

Commodities and ecosystem services may both be essential for the production of characteristics or only one may be essential, depending upon the production relationships. For example, if commodities tend to reduce good health and ecosystem services tend to reduce prosperity, both are essential. However, if ecosystem services produce both prosperity and good health, commodities may not be essential. Or if commodities produce both prosperity and good health, ecosystem services may not be essential. Corner solutions are possible and commodities and ecosystem services must be constrained from becoming negative.

Net production of manufactured capital is very simple with investment income, earned income and expenditures. Investment of manufactured capital accrues interest at the rate *r*. Earned income is *Y*. Expenditures on commodities and ecosystem services are at prices p_1 and p_2, which grow at the rate of inflation, $r-g$. The price for ecosystem services does not include the non market value of natural capital, but only the effort costs of extracting the services. Net growth is also very simple. Natural capital grows at rate *g*. Parameter *c* is the congestion parameter which varies from 0 to 1. When *c* is 0, ecosystem services are completely non rival and pure public goods. When *c* is 1, ecosystem services are completely rival and perfectly exclusive goods. In between, there is congestion and ecosystem services become club goods [19].

With these assumptions, people's lifetime utility has a unique solution.

$$J(t,M,N) = A_\delta(t)U(t,M,N) + V(T,M_T,N_T)$$
$$\text{where:}$$

$$U(t,M,N) = e^{-\rho t}A_\delta(t)^{-\alpha}B^{-\alpha}E(t,M,N)^{\alpha}$$

$$B = \left[\beta_1\left(\frac{\kappa_1}{\beta_1}\right)^{\frac{\nu}{\nu+1}} + \beta_2\left(\frac{\kappa_2}{\beta_2}\right)^{\frac{\nu}{\nu+1}}\right]^{\frac{\nu+1}{\nu}}$$

$$E(t,M,N) = C(t,M) + \pi D(t,N)$$
$$C(t,M) = M - e^{-r(T-t)}M_T + A_r(t)Y - A_g(t)[(p_1-\zeta_1)\gamma_1 + (p_2-\zeta_2)\gamma_2]$$
$$D(t,N) = N - e^{-g(T-t)}N_T - A_g(t)c\gamma_2$$

$$A_\delta(t) = \tfrac{1}{\delta}\left(1-e^{-\delta(T-t)}\right); \quad \delta = \tfrac{\rho-\alpha g}{1-\alpha}$$
$$A_r(t) = \tfrac{1}{r}\left(1-e^{-r(T-t)}\right)$$
$$A_g(t) = \tfrac{1}{g}\left(1-e^{-g(T-t)}\right)$$

$$\kappa_1 = \frac{(p_1-\zeta_1)k_{22} - (p_2+\pi c-\zeta_2)k_{21}}{k_{11}k_{22}-k_{12}k_{21}}$$
$$\kappa_2 = \frac{(p_2+\pi c-\zeta_2)k_{11} - (p_1-\zeta_1)k_{12}}{k_{11}k_{22}-k_{12}k_{21}}$$

$$\pi = e^{-(r-g)(T-t)}\pi_T; \quad \pi_T = \frac{\partial V/\partial N_T}{\partial V/\partial M_T}$$

$$\alpha \neq 1; \quad \delta \neq 0; \quad \nu \neq 0; \quad T < \infty$$

Proof is in the Appendix. A glossary of symbols is in Table 1. As before, people's lifetime utility, J, equals a lifetime's utility from consumption plus the utility of

Table 1: Glossary of Symbols

Description	Symbol	Description	Symbol
Utility		*Rates*	
Lifetime	J	Congestion	c
Dynamic	H	Nominal time preference	ρ
Current	U	Real time preference	δ
Bequests	V	Interest	r
Stocks		Growth	g
Wealth	W	Inflation	$r-g$
Manufactured capital	M	*Time*	
Natural capital	N	Birth	0
Current flows		Current age	t
Commodities	Q_1	Death	T
Ecosystem services	Q_2	*Elasticities*	
Earned income	Y	Current utility	α
Lifetime flows		Prosperity	β_1
Commodities	C	Good health	β_2
Ecosystem services	D	Bequests	ω
Expenditures	E	Manufactured capital	ϕ_1
Annuities		Natural capital	ϕ_2
Time preference	A_δ	*Substitution parameters*	
Interest	A_r	Current utility	ν
Growth	A_g	Bequests	μ
Substitution factor	B	*Subsistence parameters*	
Prices		Commodities	γ_1
Commodities	p_1	Ecosystem services	γ_2
Effort for ecosystem services	p_2	Prosperity	χ_1
Prosperity	κ_1	Good health	χ_2
Good health	κ_2	Manufactured capital	η_1
Natural capital	π	Natural capital	η_2
Reduced costs		*Characteristics*	
Commodities	ζ_1	Prosperity from commodities	k_{11}
Effort for ecosystem services	ζ_2	Prosperity from services	k_{12}
Marginal user costs		Health from commodities	k_{21}
Manufactured capital	λ	Health from services	k_{22}
Natural capital	ψ	Bequest weighting	θ

bequests, V. Therefore, the first term on the right-hand side is a lifetime's utility of consumption. Current utility, U, is integrated over time by multiplying by the annuity factor A_δ. Within the annuity factor, δ is the real rate of time preference. The nominal rate of time preference must be positive, but the real rate may be positive or negative. Within current utility, B is the substitution factor. B is dual to the isoquants shown in Figures 2 and 3. When the isoquants are linear, B is Leontief and when the isoquants are Leontief, B is linear. Within the substitution factor, κ_1 and κ_2 are the prices of prosperity and good health. These are calculated from the net prices of commodities and ecosystem services using the coefficients for the production of characteristics. Net prices subtract any reduced costs. The net price of commodities is $p_1 - \zeta_1$. The reduced cost, ζ_1, shows how much the price must be reduced before commodities will enter the optimal solution. It will be zero if commodities are already in the solution, but may be positive if commodities are constrained from becoming negative. The net price for ecosystem services is $p_2 + \pi c - \zeta_2$, which includes an individual's price of natural capital. An individual's price equals society's price, π, multiplied by the proportion of ecosystem services destroyed by consumption, c. Also within current utility, E is lifetime expenditures. Within expenditures, C is lifetime consumption of commodities and πD is lifetime rent, where D is lifetime consumption of ecosystem services. Expenditures and consumption are above subsistence.

During people's lifetimes, demand for prosperity and good health is a dynamic version of a generalized constant elasticity of substitution system.

$$K_1 - \chi_1 = b_1 \frac{E}{A_\delta}; \quad b_1 = \left(\frac{\kappa_1}{\beta_1}\right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}}$$

$$K_2 - \chi_2 = b_2 \frac{E}{A_\delta}; \quad b_2 = \left(\frac{\kappa_2}{\beta_2}\right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}}$$

On the left hand sides, prosperity and good health are above subsistence. On the right-hand sides, lifetime expenditures above subsistence are divided by the annuity factor and converted into current expenditures. Current expenditures are apportioned between prosperity and good health by shares b_1 and b_2. These shares can be transformed by the coefficients for the production of characteristics to become shares in a demand system for commodities and ecosystem services.

$$Q_1 - \gamma_1 = q_1 \frac{E}{A_\delta}; \quad \gamma_1 = \frac{\chi_1 k_{22} - \chi_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}}; \quad q_1 = \frac{b_1 k_{22} - b_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}}$$
$$Q_2 - \gamma_2 = q_2 \frac{E}{A_\delta}; \quad \gamma_2 = \frac{\chi_2 k_{11} - \chi_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}}; \quad q_2 = \frac{b_2 k_{11} - b_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}}$$

Expenditures grow over time at the nominal rate $r - \delta$. If the interest rate exceeds the real rate of time preference, people save for the future and spend later. The inflation rate is $r - g$ and demand grows at the real rate $g - \delta$. In a steady state, all of the rates are equal, $\rho = r = g = \delta$. Demand becomes constant and indistinguishable from a static demand system.

The solution defines a true measure of wealth that includes both manufactured and natural capital.

$$W = M + \pi N$$

The change in wealth is a true measure of savings. It equals production and earned income above subsistence minus expenditures above subsistence.

$$\dot{W} = rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta}$$

It can be rearranged into the change in manufactured capital plus the change in natural capital.

$$\dot{W} = \left[rM + Y - \left(\frac{C}{A_\delta} + (p_1 - \zeta_1)\gamma_1 + (p_2 - \zeta_2)\gamma_2 \right) \right] + \pi \left[rN - \left(\frac{D}{A_\delta} + c\gamma_2 \right) \right]$$

This version of the change in wealth could be used in green accounting to adjust a country's national accounts for ecosystem services [20]. A typical set of national accounts measures the costs and benefits from manufactured capital. Gross domestic product is adjusted by foreign income to become gross net income. This corresponds to production from manufactured capital plus earned income, $rM + Y$. Next, consumption is subtracted to get gross national savings and then depreciation on manufactured capital is subtracted to get net national savings. In the change in wealth, consumption and depreciation correspond to $C / A_\delta + (p_1 - \zeta_1)\gamma_1 + (p_2 - \zeta_2)\gamma_2$. Green accounting subtracts the rent paid on the extraction of exhaustible resources to get adjusted net savings. In the change in wealth, rent is $\pi(D / A_\delta + c\gamma_2)$. To account for ecosystem services, the value of production from natural capital, πrN, should also be added. Renewable natural capital may have a high price but cost nothing if used sustainably.

How does the price of natural capital affect people? Consider the maximized Hamiltonian as a dynamic measure of utility.

$$H(t, M, N) = e^{-\rho t} A_\delta^{-\alpha} B^{-\alpha} E^{\alpha} + \lambda\left[rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta}\right]$$

$$\lambda = \alpha e^{-\rho t} A_\delta^{1-\alpha} B^{-\alpha} E^{\alpha - 1}$$

The first term on the right hand side is the current utility of consumption. The second term is the total user costs of both manufactured and natural capital. Dividing by the marginal user cost of manufactured capital converts the Hamiltonian into a money measure, in units of dollars per time period. Rearranging allows a new interpretation.

$$\frac{H}{\lambda} = \left(\tfrac{1-\alpha}{\alpha}\right)\frac{C + \pi D}{A_\delta} - ((p_1 - \zeta_1)\gamma_1 + (p_2 + \pi c - \zeta_2)\gamma_2) + r[M + \pi N] + Y$$

The first and second terms on the right hand side can be thought of as a surplus. The first term is an expenditure measure of the current utility of consumption minus actual expenditures, all above subsistence. The second term subtracts subsistence expenditures. The remaining terms are total income. In this interpretation, natural capital increases people's welfare by increasing both income and expenditures. Expenditures increase because people act as owners who accrue rent and count natural capital as part of their wealth.

Welfare Analysis for Ecosystem Services

Will a policy create a better future? Investing in environmental quality will change the current stock of natural capital. Granting a subsidy, imposing a tax or creating a market in transferable quotas will change the price of effort for extracting ecosystem services. These changes will alter the evolution of manufactured capital, natural capital and the price of natural capital. Therefore, welfare analysis must compare people's lifetime utilities before and after the changes.

$$J(t, M - WTP, N, p) = J\left(t, M, N + \Delta_N, p + \Delta_{p-\zeta}\right)$$
$$J(t, M, N, p) = J\left(t, M + WTA, N + \Delta_N, p + \Delta_{p-\zeta}\right)$$

In these welfare equations, policies may change natural capital, Δ_N, the price of effort above reduced costs, $\Delta_{p\text{-}\zeta}$, or both. Willingness to pay, *WTP*, is an equivalent variation—the amount people are willing to pay to avoid the changes. Willingness to accept, *WTA*, is a compensating variation—the amount people would accept to allow the changes [21], [22]. The evolution of manufactured capital, natural capital and the price of natural capital will be different for lifetime utilities on the left hand and right hand sides of the equations. In general, these equations are nonlinear and difficult to solve for *WTP* and *WTA*.

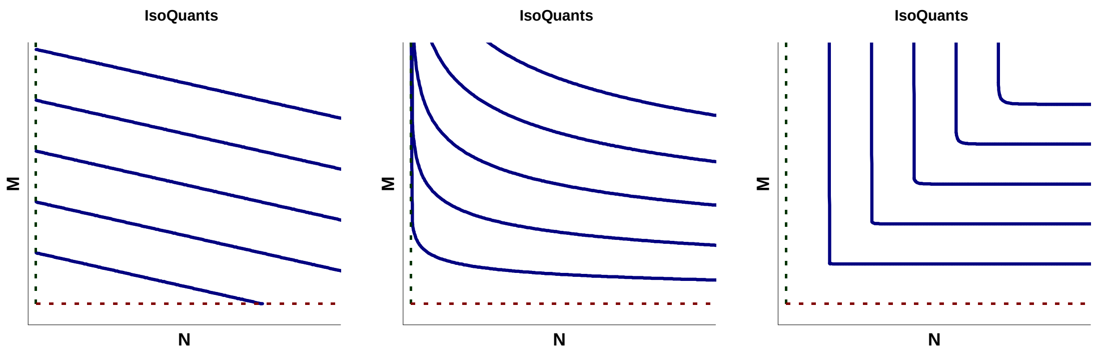
An important special case, however, is trivially easy to solve. Manufactured capital and natural capital are assumed to be perfect substitutes in production. Therefore, if a policy changes only the stock of natural capital, *WTP* and *WTA* are changes in manufactured capital which perfectly offset the change in natural capital.

$$WTP = WTA = -\pi\Delta_N$$

However, if a policy also changes the price of effort, people's utility of bequests must be known before *WTP* and *WTA* can be calculated. Assume the utility of bequests is a constant elasticity of substitution function.

$$V(T, M_T, N_T) = e^{-\rho T}\theta\left[\phi_1(M_T - \eta_1)^{-\mu} + \phi_2(N_T - \eta_2)^{-\mu}\right]^{\frac{-\omega}{\mu}}$$

Parameter μ is the substitution parameter, ω is the elasticity of the utility of bequests, ϕ_1 and ϕ_2 are elasticities of manufactured capital and natural capital, η_1 and η_2 are subsistence stocks and θ is a weight for the utility of bequests within lifetime utility.



Perfect Substitutes Substitutes Perfect Complements

Figure 3. Isoquants for Manufactured Capital and Natural Capital

Manufactured capital and natural capital can vary from perfect substitutes to perfect complements. In addition, decreasing marginal utility may increase the spacing of the isoquants. Using this utility of bequests, lifetime utility can be converted into a money measure, in units of dollars per lifetime.

$$\frac{J(t,M,N)\omega}{\lambda} = \left(\tfrac{\omega-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}} A_\delta B^{\frac{\alpha}{\alpha-1}} - A_g\left[(p_1-\zeta_1)\gamma_1 + (p_2+\pi c-\zeta_2)\gamma_2\right] + W + A_r Y - e^{-r(T-t)}\eta_1 - e^{-g(T-t)}\pi\eta_2$$

Derivation is in the Appendix. On the right hand side, the first two terms are a surplus—an expenditure measure of a lifetime's utility of consumption minus actual expenditures. The remaining terms are current wealth plus the present value of future earned income. Lifetime utility is zero at subsistence and positive above.

A dynamic counterpart to the Random Utility Model [6] assumes that the utility of bequests is linear, with $\mu = -1$ and $\omega = 1$. Manufactured capital and natural capital will be perfect substitutes, with linear isoquants. In addition, marginal utility will not diminish, with uniformly spaced isoquants. As a result, the marginal user cost and the price of natural capital will be independent of wealth.

$$\lambda = e^{r(T-t)-\rho T}\theta\phi_1; \qquad \pi = e^{-(r-g)(T-t)}\frac{\phi_2}{\phi_1}$$

Lifetime utility will be linear in wealth and the welfare equations can be solved algebraically for *WTP* and *WTA*.

$$WTP = WTA = -\pi\Delta_N + \left(\tfrac{1-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}} A_\delta\left[B(p-\zeta)^{\frac{\alpha}{\alpha-1}} - B(p-\zeta+\Delta_p)^{\frac{\alpha}{\alpha-1}}\right] + A_g\left(\Delta_{p_1-\zeta_1}\gamma_1 + \Delta_{p_2-\zeta_2}\gamma_2\right)$$

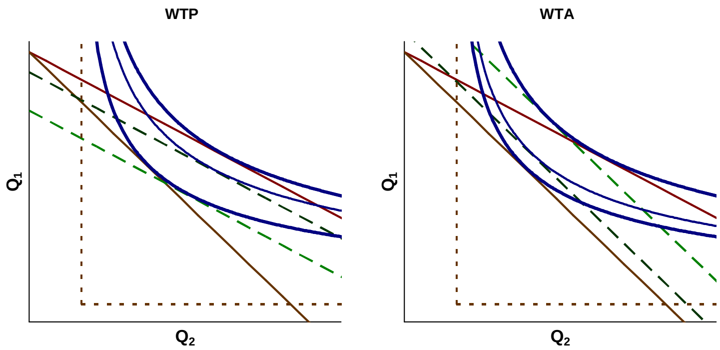
In this simple case, *WTP* equals *WTA*. On the right hand side, the first term is a change in wealth and the last two terms are the difference between surpluses. Indeed, in this case, the difference in surpluses equals the present value of all future changes in consumer surplus.

$$WTP = WTA = -\pi\Delta_N + \int_t^T e^{-r(s-t)}\left[\int_{p_1}^{p_1+\Delta_{p_1-\zeta_1}} Q_1 dp + \int_{p_2}^{p_2+\Delta_{p_2-\zeta_2}} Q_2 dp\right] ds$$

Derivation is in the Appendix. If the utility of bequests is linear and if a policy changes only the price of effort, consumer surplus is an exact measure of *WTP* and *WTA*.

In a static model, *WTP* and *WTA* depend upon ratios of the substitution factors, $B(p-\zeta)$ and $B(p-\zeta+\Delta_p)$, and if commodities are imperfect substitutes, *WTA* exceeds *WTP* [23]. In a dynamic model, *WTP* and *WTA* depend upon differences in the substitution factors. Even if commodities and ecosystem services are imperfect substitutes, *WTP* may equal *WTA*. Some examples may illustrate.

Identical characteristics and consumption. If commodities produce only prosperity and ecosystem services produce only good health, characteristics and consumption are identical. Figure 5 compares static and dynamic *WTP* and *WTA* for a price change.



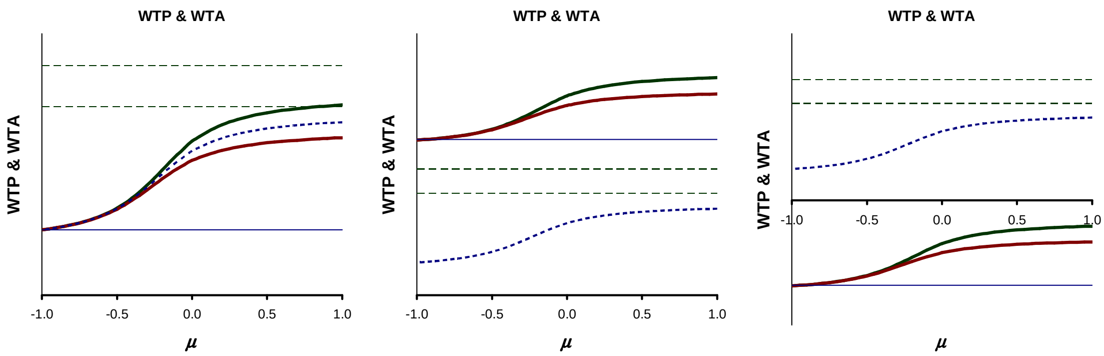
WTP WTA

Figure 5. Static and Dynamic Willingness to Pay and Willingness to Accept

The solid lines are budget constraints that would be binding in a static model. Before the price increase the budget constraint would be tangent to the highest isoquant. After the price increase the budget constraint would rotate and become tangent to the lowest isoquant. The dashed lines are parallel shifts in the budget constraints.

In the first panel, static *WTP* is the decrease in wealth required to shift the top budget constraint down to the lowest isoquant. In the second panel, static *WTA* is the increase in wealth required to shift the lowest budget constraint up to the highest isoquant. Static *WTA* exceeds static *WTP*. For ecosystem services, the budget constraints are not binding and consumers will choose isoquants between the upper and lower isoquants. In the first panel, dynamic *WTP* is the decrease in wealth for a shift from the highest isoquant to an intermediate isoquant. In the second panel, dynamic *WTA* is the increase in wealth for a shift from the lowest isoquant to another intermediate isoquant. Dynamic *WTP* and *WTA* are relatively small because consumers can rearrange an entire lifetime of consumption in response to a price increase. In Figure 5, dynamic *WTA* happens to equal dynamic *WTP*.

If the utility of bequests is nonlinear, however, *WTA* will exceed *WTP*. Figure 6 illustrates a few possibilities.



Price change only Decreased natural capital Increased natural capital

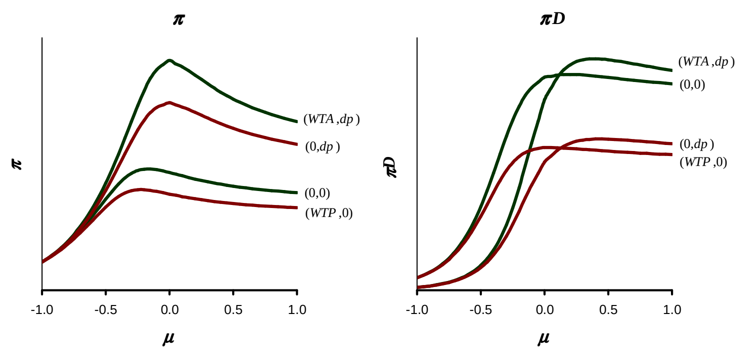
Figure 6. Dynamic Willingness to Pay and Willingness to Accept

For comparison with Figure 5, the horizontal dashed lines are static *WTA* and static *WTP*. The solid line is dynamic *WTP* and *WTA* for a linear utility of bequests. The solid curves indicate dynamic *WTP* and *WTA* as substitution parameter μ increases from -1 to 1 and the elasticity of substitution between manufactured capital and natural capital, $\sigma = 1/(\mu+1)$, decreases from ∞ to ½, with the elasticity of bequests, ω, set to 1. The dotted curve is the net present value of changes in consumer surplus.

In the first panel, a policy changes only the price of extracting ecosystem services. In the second panel, a policy also decreases natural capital and in the third panel a policy increases natural capital.

Dynamic *WTP* and *WTA* in Figure 6 are consistent with the experimental evidence. When people are offered capital items which are closer and closer substitutes, *WTA* has been observed to converge toward *WTP* [24], [25]. Very large *WTP* and *WTA* have also been observed for mining in national park and introduction of genetically modified organisms. Or consider again the example of deforestation in Figure 1. A policy to rectify open access may charge a tax on harvest or set a total allowable harvest and sell individual transferable quotas. In response, biomass will increase. This increase in natural capital will decrease *WTP* and *WTA*. As another example, effective trading in carbon credits may decrease greenhouse gases in the atmosphere. A decrease in pollution is an increase in natural capital which will decrease *WTP* and *WTA*. Indeed, *WTP* and *WTA* for slower global warming could be negative. People are both consumers of ecosystem services and owners of the ecosystem. Consumers will not wish to pay for carbon credits, but owners will wish to increase their wealth. No rational owners will pay a positive amount to forgo an increase in wealth.

The price of natural capital and scarcity rent are not affected by shifts up and down in *WTP* and *WTA*. However they are affected by a policy to change the price of ecosystem services.

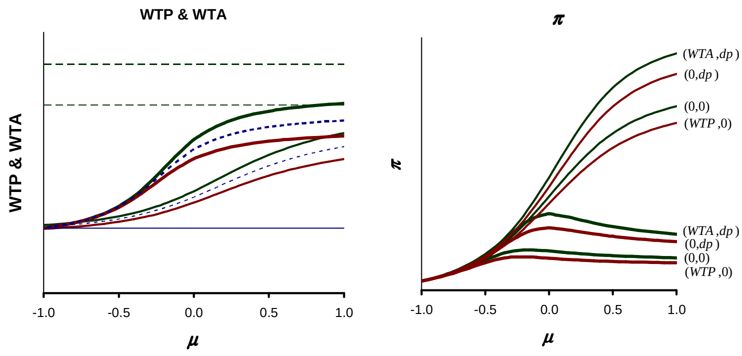


Price of natural capital   Scarcity rent above subsistence

Figure 7. The Price of Natural Capital and Scarcity Rent

The labels to the right of curves denote a lifetime utility function in the welfare equations. (*WTP*,0) denotes *WTP* with no price change. In the first welfare equation, it is compared with (0,*dp*) for no *WTP* but with a price change. (*WTA*,*dp*) denotes *WTA* with a price change. In the second welfare equation, it is compared with (0,0) for no *WTA* and no price change. The price of natural capital and scarcity rent are different for each of the four possibilities. Non market values do not exist in the vacuum of people's preferences. Instead, people's preferences are transformed through scarcity to become the price of natural capital and scarcity rent. Policies affect the relative scarcity of natural capital and alter the non market values.

The utility of bequests is even more nonlinear for a diminishing marginal utility of bequests.

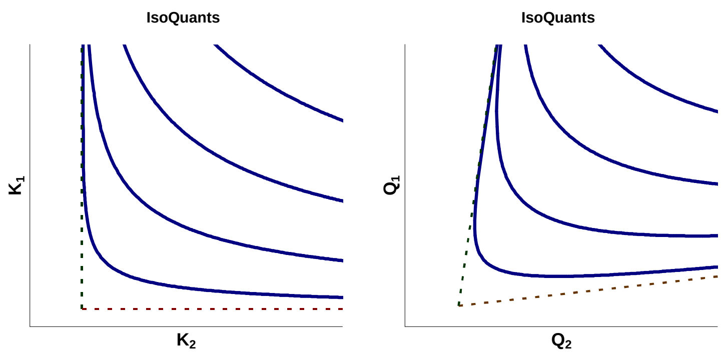


WTP and *WTA* Price of natural capital

Figure 8. Diminishing Marginal Utility of Bequests

For comparison with Figure 7, the thick curves in Figure 8 are *WTP*, *WTA* and consumer surplus for an elasticity of bequests, ω, equal to 1. The thinner curves are for ω equal to 0.5. *WTP* and *WTA* are smaller, but the price of natural capital is much larger. Given the isoquants in Figure 3, natural capital is less elastic than manufactured capital and becomes relatively scarce with diminishing marginal utility.

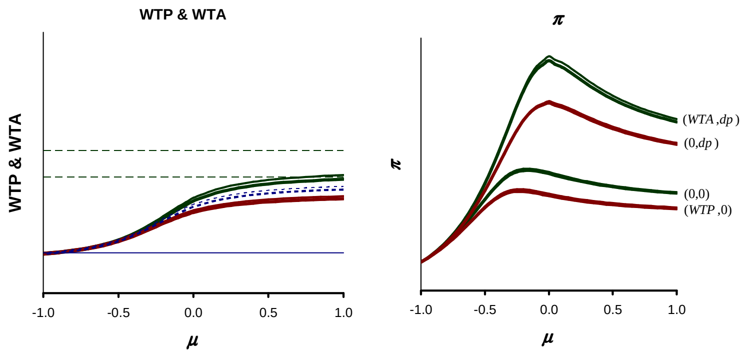
Modern Life. Suppose that mobile phones and fast cars are good for prosperity but bad for health and that a quiet life with nature is good for health but bad for prosperity. Isoquants may be as in Figure 9.



Characteristics Consumption

Figure 9. Isoquants of Modern Life

WTP, *WTA* and the price of natural capital are little affected.

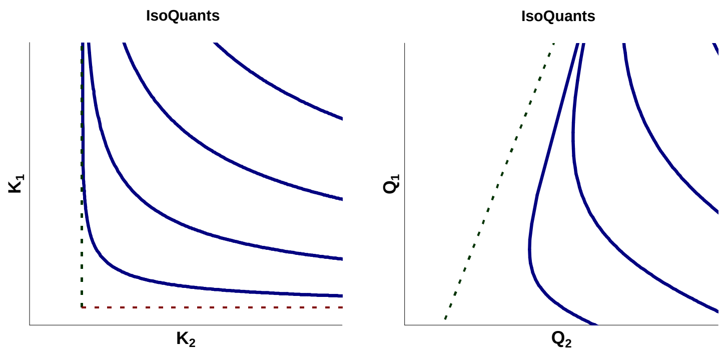


WTP and *WTA* Price of natural capital

Figure 10. Welfare and Non market Value of Modern Life

Recall that the isoquants of the utility of bequests are changing from left to right in Figure 10. In this case, an elaborate Lancaster model of characteristics is unimportant for welfare analysis or for non market valuation.

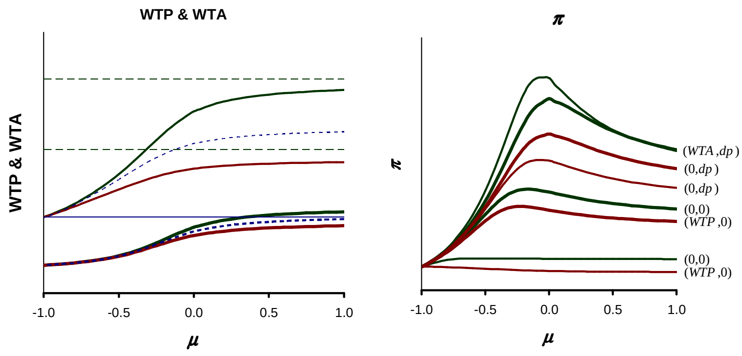
First Generation GMO. Suppose that introducing a first generation GMO is good for prosperity but may be bad for health. The isoquants of current utility rotate clockwise.



Characteristics Consumption

Figure 11. Isoquants of First Generation GMO

In this case, Q_1, as the GMO, may be zero at a corner solution. Increasing the price of natural foods, Q_2, will affect welfare and non market values.

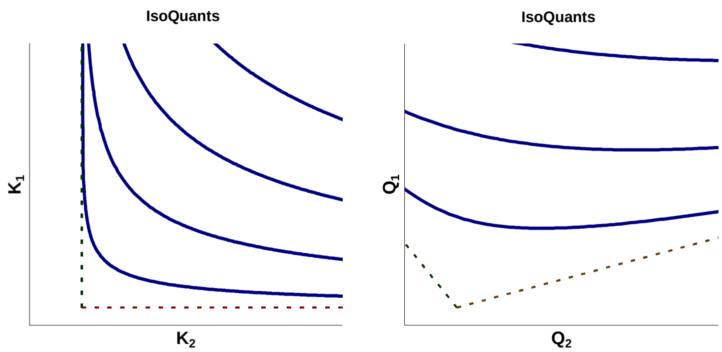


WTP and *WTA* Price of natural capital

Figure 12. Welfare and Non market Value of GMO

WTP and *WTA* are greater, and would be greater still if the degradation of natural capital is included. However greater *WTP* and *WTA* don't translate into greater prices of natural capital. For three out of four possible policies, the price of natural capital is less.

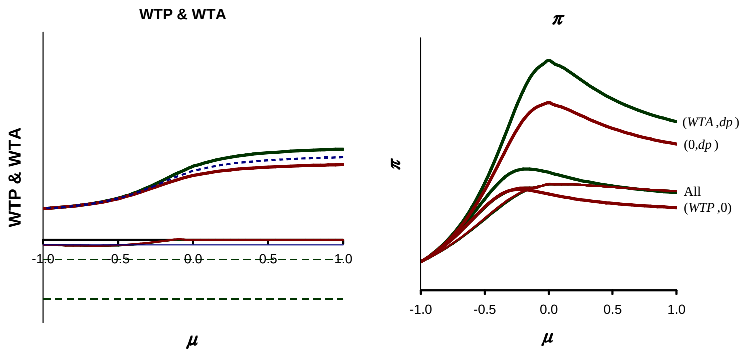
Recreational Fishing. Suppose that we are surveying recreational fishers to understand whether fish are more valuable to recreational or commercial fishers. For an occasional fisher, recreational fishing is not essential for good health and the isoquants rotate counter-clockwise.



Characteristics Consumption

Figure 13. Isoquants of Recreational Fishing

A sufficient increase in its price will cause recreational fishing, Q_2, to leave the solution and become zero. Welfare and non market values may change accordingly.



WTP and *WTA* Price of natural capital

Figure 14. Welfare and Non market Value of Recreational Fishing

Because recreational fishing leaves the solution, an increase in the price of recreational fishing has little effect. *WTP* and *WTA* are essentially zero. The price of natural capital is the same for all policies. Of course an avid recreational fisher may consider fishing to be one of life's essentials and behave differently.

Estimating the Price of Natural Capital

In contingent valuation, willingness to pay and willingness to accept are treated as non market values for an improvement in environmental quality [5], [22]. For large changes in the price of a non market good, willingness to accept can be much greater than willingness to pay [23], [24], [25]. Which is the true value? For ecosystem services, the answer is neither. Instead, willingness to pay and willingness to accept are data to be used in estimating the price of natural capital.

Demand Estimation. Many ecosystems are protected by community or government property rights but used by individuals. Examples are nature reserves with ecotourism and coral reefs with recreational fishing. For these ecosystems, estimating the dynamic demand system may reveal the price of natural capital, so long as commodities and ecosystem services are imperfect substitutes [26]. To illustrate, assume that characteristics and consumption are identical so that the prices of prosperity and good health collapse to the prices for commodities and ecosystem services. Further assume an interior solution for both commodities and ecosystem services. Table 2 shows the data that might be collected and the parameters to be estimated. As in other demand system estimation, one of the demand equations is eliminated, in this case the demand for commodities, leaving the demand for ecosystem services.

Table 2: Demand Estimates

Data	Parameters
Q_2	β_1
p_1	β_2
p_2	γ_2
E/A_δ	ν
	πc

$$Q_2 = \left(\frac{p_2 + \pi c}{\beta_2}\right)^{\frac{-1}{\nu+1}} \left[\beta_1\left(\frac{p_1}{\beta_1}\right)^{\frac{\nu}{\nu+1}} + \beta_2\left(\frac{p_2 + \pi c}{\beta_2}\right)^{\frac{\nu}{\nu+1}}\right]^{-1} \frac{E}{A_\delta} + \gamma_2$$

A travel cost survey may be required to collect the price of effort for using ecosystem services. If so, the quantity of ecosystem services, the price of commodities and annual expenditures can be collected at the same time. The estimation will reveal elasticities, the subsistence parameter for ecosystem services and the degree of substitution between commodities and ecosystem services. Unfortunately it will only reveal people's individual price of natural capital, which will be zero for pure public goods and close to zero for many ecosystem services.

Expenditure Estimation. The social price of natural capital is in the expenditure function and it may be possible to estimate expenditures rather than demand.

$$\frac{E}{A_\delta} = A_\delta^{-1}\left[M + \pi N - e^{-r(T-t)}\left[M_T + \pi_T N_T\right] + A_r Y - A_g\left((p_1 - \zeta_1)\gamma_1 + (p_2 + \pi c - \zeta_2)\gamma_2\right)\right]$$

Table 3 shows the data that might be collected and the parameters to be estimated. In addition to annual expenditures and prices, the data includes manufactured and natural capital, as well as earned income, interest and growth rates, people's age and life expectancy. Much of this data is not collected in a typical travel cost survey, but the estimation should be straightforward. Alternatively, more efficient estimates might be obtained by combining the demand and expenditure estimations in a simultaneous system. Either way, expenditure estimation reveals both the social price of natural capital and the congestion factor.

Table 3: Expenditure Estimates

Data	Parameters
E/A_δ	δ
p_1	γ_1
p_2	γ_2
M	ζ_1
N	ζ_2
Y	$M_T + \pi_T N_T$
r	π
g	c
t	
T	

Willingness to Pay and Willingness to Accept. The *WTP* and *WTA* equations contain all the parameters in the model, including the social price of natural capital. Estimation may be easy or difficult, depending upon how we construct non market valuation surveys. For example, we might ask people about a change in old growth forests. Questions might be:

Our state has 100,000 hectares of old growth forest.

- A logging company has a license to cut 1 hectare of forest. What is the maximum amount that you believe the government should pay to purchase the license?
- Another logging company wishes to cut 1 hectare of forest. What is the minimum amount that you believe the government should receive from the company?

In theory, answers to both questions will be the same. For a decrease of one hectare, the price of old growth forests equals *WTP* and *WTA*. Even if old growth forests are club goods, as long as respondents act as owners who accrue rent rather than consumers who pay a higher price, *WTP* and *WTA* are very simple. We could also ask about an increase of one hectare of forest and might expect the signs of *WTP* and *WTA* to be negative. In practice, however, these questions may be hard for people to answer. We are not asking consumers about their consumption of a public good, we are asking owners about the rent that should accrue to public property. Even more importantly, we cannot add individual responses together. Our best estimate of the social price of natural capital would be the average of all responses. Perhaps we should not survey individuals, but consult citizen juries, instead [27], [28].

More typically, surveys vary both the quantity of natural capital and the price of effort. Suppose we survey recreational fishers who are over exploiting a fishery. Questions might be:

> Stocks of sport fish off our coast are depleted. The government is considering a fee of $10 for each sport fish landed. This should reduce over fishing and increase the stocks of sport fish by 20%.
>
> - What is the maximum amount you would pay today to avoid these changes, now and in the future?
> - What is the minimum amount you would accept today to allow these changes, now and in the future?

Responses to these questions could be very interesting. Many sport fishers believe they own the fishery and will be unwilling to pay. In a typical contingent valuation survey we label them as protestors and discard their responses [5]. Given the way the questions are worded, however, owners have negative *WTP* and *WTA*. Consumers have positive *WTP* and *WTA*. Both responses are valid and our survey must allow for both. Usually, we only ask *WTP* questions because *WTA* can be embarrassingly large. But *WTP* and *WTA* are only data. We should ask for both and any divergence will help quantify the bequest value.

Assuming a linear utility of bequests may be unreasonable, but the estimating equations would be relatively simple. Otherwise, the estimating equations are in implicit form because *WTP* and *WTA* may have no algebraic solution.

$$0 = J(t, M - WTP, N, p) - J\left(t, M, N + \Delta_N, p + \Delta_p\right)$$
$$0 = J(t, M, N, p) - J\left(t, M + WTA, N + \Delta_N, p + \Delta_p\right)$$

All of the data and parameters are on the right hand sides of the equations. The estimation is an application of nonlinear regression with restrictions across equations. Table 4 shows the data to be collected and the parameters to be estimated. There are more parameters than data and it is not clear that the system is identified. To improve the efficiency of the estimates, we might combine revealed preference methods for demand or expenditure with stated preference methods for *WTP* and *WTA* [29], [30].

Table 4: WTP and WTA Estimates

Data	Parameters
WTP	α
WTA	ρ
p_1	β_1
p_2	β_2
k_{11}	γ_1
k_{12}	γ_2
k_{21}	ν
k_{22}	θ
M	ω
N	ϕ_1
Y	ϕ_2
r	η_1
g	η_2
t	μ
T	ζ_1
Δ_N	ζ_2
Δ_p	$M_T + \pi_T N_T$
	π
	c

Conclusions

Non market valuation and bio economic modelling have both been used to value ecosystem services. Yet they have different concepts of valuation. Non market valuation measures willingness to pay and, sometimes, willingness to accept. Bio economic modelling imputes marginal user costs and calculates scarcity rent. Neither is a complete model of ecosystem services. Non market valuation solves static models without stocks and flows and infers the preferences people have for the environment. Bio economic modelling solves dynamic models with stocks and flows but assumes that preferences are known. This paper combines non market valuation and bio economic modelling into a complete model and offers new conclusions.

The concept of valuation used in bio economic modelling is correct. A formal proof of existence and uniqueness for an analytical solution is also proof that non market values are contained in the price of natural capital and that scarcity rent is the total value of ecosystem services. This conclusion will be unpopular for at least two reasons. First, willingness to pay and willingness to accept are data, rather than non market values. Non market valuation studies are incomplete until the data are used to estimate the price of natural capital. Second, only scarce ecosystem services have positive prices. Abundant services have prices of zero, no matter how necessary they may be for life. Hence, the value of the world's ecosystem services is much smaller than commonly thought, certainly smaller than the world's wealth.

If possible, non market valuation studies should consult people as if they are owners of the ecosystem and avoid surveying them as if they are consumers. Owners accrue the scarcity rent and are responsible for conservation. Consumers spend money to consume and degrade the ecosystem. Policies to conserve our ecosystem will benefit owners but harm consumers. Owners respond to the social price of natural capital. Consumers respond to their individual price, which equals the social price multiplied by the proportion of ecosystem services they destroy. If consumers have a positive willingness to pay, owners will have a negative willingness to pay. Yet most non market valuation surveys allow only positive answers. People who are willing to pay a negative amount are regarded as protestors and their responses are discarded. These people are acting as owners and should be consulted, perhaps using citizen juries instead of surveys.

Welfare analysis for owners is easy. If owners are evaluating a policy to increase or decrease natural capital, they will put a price on natural capital and nominate an equal and offsetting change in manufactured capital as their willingness to pay or willingness to accept. Welfare analysis for consumers is challenging. If consumers are evaluating a policy to tax ecosystem services or create a market for tradeable permits, they will adjust their current and future consumption. Their budgets are flexible and they will nominate a willingness to pay or a willingness to accept that is smaller than predicted by static welfare analysis. Willingness to accept will usually exceed willingness to pay. The divergence, however, is not due to imperfect substitution among commodities and ecosystem services in the utility of consumption. Instead, willingness to accept exceeds willingness to pay whenever people's utility of bequests is nonlinear—whenever manufactured capital and natural capital are imperfect substitutes or people have diminishing marginal utility of bequests. Even more challenging, people are both consumers and owners. A typical policy increases the price for ecosystem services and the stock of natural capital. People's willingness to pay and willingness to accept may be positive, zero or negative. Somehow, non market valuation must sort through people's conflicting motives to find the price of natural capital.

Fortunately, estimating the price of natural capital is less challenging. Demand estimation may reveal the individual price of natural capital. Expenditure estimation may reveal the social price. Willingness to pay and willingness to accept can be used as data to estimate all parameters in the model, including the individual price and the social price. Once the social price of natural capital is estimated, it can be used to calculate scarcity rent as the total value of ecosystem services. It can be used to calculate the total user costs of degrading our natural capital and to help green our national accounts. Or it can be used directly in cost-benefit analyses to determine whether old growth forests are worth more standing than sliced into timber, whether fish are worth more for recreational or commercial fishing or in any decision about conservation versus development.

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Appendix

Analytical solution. The dynamic model of ecosystem services,

$$J(M_0, N_0) = \max_{Q_1, Q_2} \int_0^T U(t, Q_1, Q_2) dt + V(T, M_T, N_T)$$

$$\text{subject to:}$$

$$\dot{M} = F(M, N, Q_1, Q_2)$$
$$\dot{N} = G(N, Q_2)$$

with functional forms,

$$U(t, Q_1, Q_2) = e^{-\rho t}\left[\beta_1 (K_1 - \chi_1)^{-\nu} + \beta_2 (K_2 - \chi_2)^{-\nu}\right]^{\frac{-\alpha}{\nu}}$$
$$K_1 = k_{11} Q_1 + k_{12} Q_2$$
$$K_2 = k_{21} Q_1 + k_{22} Q_2$$
$$Q_1 \geq 0; \quad Q_2 \geq 0$$

$$F(M, N, Q_1, Q_2) = rM + Y - p_1 Q_1 - p_2 Q_2$$
$$G(N, Q_2) = gN - cQ_2$$

$$p_1 = e^{-(r-g)(T-t)} p_{1T}$$
$$p_2 = e^{-(r-g)(T-t)} p_{2T}$$

has an analytical solution for lifetime utility:

$$J(t, M, N) = A_\delta(t) U(t, M, N) + V(T, M_T, N_T)$$
$$\text{where:}$$

$$U(t, M, N) = e^{-\rho t} A_\delta(t)^{-\alpha} B^{-\alpha} E(t, M, N)^{\alpha}$$

$$B = \left[\beta_1 \left(\frac{\kappa_1}{\beta_1} \right)^{\frac{\nu}{\nu+1}} + \beta_2 \left(\frac{\kappa_2}{\beta_2} \right)^{\frac{\nu}{\nu+1}} \right]^{\frac{\nu+1}{\nu}}$$

$$E(t,M,N) = C(t,M) + \pi D(t,N)$$
$$C(t,M) = M - e^{-r(T-t)}M_T + A_r(t)Y - A_g(t)[(p_1 - \zeta_1)\gamma_1 + (p_2 - \zeta_2)\gamma_2]$$
$$D(t,N) = N - e^{-g(T-t)}N_T - A_g(t)c\gamma_2$$

$$A_\delta(t) = \tfrac{1}{\delta}\left(1 - e^{-\delta(T-t)}\right); \quad \delta = \tfrac{\rho - \alpha g}{1-\alpha}$$
$$A_r(t) = \tfrac{1}{r}\left(1 - e^{-r(T-t)}\right)$$
$$A_g(t) = \tfrac{1}{g}\left(1 - e^{-g(T-t)}\right)$$

$$\kappa_1 = \frac{(p_1 - \zeta_1)k_{22} - (p_2 + \pi c - \zeta_2)k_{21}}{k_{11}k_{22} - k_{12}k_{21}}$$
$$\kappa_2 = \frac{(p_2 + \pi c - \zeta_2)k_{11} - (p_1 - \zeta_1)k_{12}}{k_{11}k_{22} - k_{12}k_{21}}$$

$$\pi = e^{-(r-g)(T-t)}\pi_T; \quad \pi_T = \frac{\partial V/\partial N_T}{\partial V/\partial M_T}$$

$$\alpha \neq 1; \quad \delta \neq 0; \quad \nu \neq 0; \quad T < \infty$$

The solution is unique. It contains a dynamic demand system for characteristics,

$$K_1 - \chi_1 = b_1 \frac{E}{A_\delta}; \quad b_1 = \left(\frac{\kappa_1}{\beta_1} \right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}}$$
$$K_2 - \chi_2 = b_2 \frac{E}{A_\delta}; \quad b_2 = \left(\frac{\kappa_2}{\beta_2} \right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}}$$

and a dynamic demand system for commodities,

$$Q_1 - \gamma_1 = q_1 \frac{E}{A_\delta}; \quad \gamma_1 = \frac{\chi_1 k_{22} - \chi_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}}; \quad q_1 = \frac{b_1 k_{22} - b_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}}$$
$$Q_2 - \gamma_2 = q_2 \frac{E}{A_\delta}; \quad \gamma_2 = \frac{\chi_2 k_{11} - \chi_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}}; \quad q_2 = \frac{b_2 k_{11} - b_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}}$$

It also defines a true measure of wealth and its change over time,

$$W = M + \pi N$$
$$\dot{W} = rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta}$$

and a true measure of current utility,

$$H(t, M, N) = e^{-\rho t} A_\delta^{-\alpha} B^{-\alpha} E^{\alpha} + \lambda \left[rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta} \right]$$

$$\lambda = \alpha e^{-\rho t} A_\delta^{1-\alpha} B^{-\alpha} E^{\alpha - 1}$$

Proof. Existence and uniqueness are shown by deriving the optimality conditions and then integrating from current time t to final time T. The augmented Hamiltonian at time t is:

$$\begin{aligned} H(t, M, N) = \max_{K_1, K_2, Q_1, Q_2} \Big[& e^{-\rho t} \left[\beta_1 (K_1 - \chi_1)^{-\nu} + \beta_2 (K_2 - \chi_2)^{-\nu} \right]^{\frac{-\alpha}{\nu}} \\ & + \upsilon_1 (k_{11} Q_1 + k_{12} Q_2 - K_1) + \upsilon_2 (k_{21} Q_1 + k_{22} Q_2 - K_2) + \xi_1 Q_1 + \xi_2 Q_2 \\ & + \lambda (rM + Y - p_1 Q_1 - p_2 Q_2) + \psi (gN - cQ_2) \Big] \end{aligned}$$

The first-order conditions for the controls, Lagrange multipliers, states and costates are:

$$\frac{\partial H}{\partial K_1} = 0 = \alpha e^{-\rho t} \left[\beta_1 (K_1 - \chi_1)^{-\nu} + \beta_2 (K_2 - \chi_2)^{-\nu} \right]^{\frac{-\alpha}{\nu} - 1} \beta_1 (K_1 - \chi_1)^{-\nu - 1} - \upsilon_1$$
$$\frac{\partial H}{\partial K_2} = 0 = \alpha e^{-\rho t} \left[\beta_1 (K_1 - \chi_1)^{-\nu} + \beta_2 (K_2 - \chi_2)^{-\nu} \right]^{\frac{-\alpha}{\nu} - 1} \beta_2 (K_2 - \chi_2)^{-\nu - 1} - \upsilon_2$$

$$\frac{\partial H}{\partial Q_1} = 0 = \upsilon_1 k_{11} + \upsilon_2 k_{21} - \lambda p_1 + \xi_1; \quad \xi_1 Q_1 = 0; \quad \xi_1 \geq 0$$
$$\frac{\partial H}{\partial Q_2} = 0 = \upsilon_1 k_{12} + \upsilon_2 k_{22} - \lambda p_2 - \psi c + \xi_2; \quad \xi_2 Q_2 = 0; \quad \xi_2 \geq 0$$

$$\frac{\partial H}{\partial \upsilon_1} = 0 = k_{11} Q_1 + k_{12} Q_2 - K_1$$
$$\frac{\partial H}{\partial \upsilon_2} = 0 = k_{21} Q_1 + k_{22} Q_2 - K_2$$

$$-\frac{\partial H}{\partial M} = \dot{\lambda} = -\lambda r$$
$$-\frac{\partial H}{\partial N} = \dot{\psi} = -\psi g$$

$$\frac{\partial H}{\partial \lambda} = \dot{M} = rM + Y - p_1 Q_1 - p_2 Q_2$$
$$\frac{\partial H}{\partial \psi} = \dot{N} = gN - cQ_2$$

The Hamiltonian is concave and the second-order conditions are satisfied. In addition, the states satisfy initial conditions and the costates satisfy transversality conditions.

$$\lambda_T = \frac{\partial V}{\partial M_T}$$
$$\psi_T = \frac{\partial V}{\partial N_T}$$

Lifetime utility must satisfy the terminal condition.

$$J(T, M_T, N_T) = V(T, M_T, N_T)$$

To integrate the first-order conditions, first integrate the costates and obtain a particular solution using the transversality conditions.

$$\lambda = e^{r(T-t)}\lambda_T = e^{r(T-t)}\frac{\partial V}{\partial M_T}$$
$$\psi = e^{g(T-t)}\psi_T = e^{g(T-t)}\frac{\partial V}{\partial N_T}$$

The costates can be related to each other by defining the price of natural capital, π.

$$\pi = \frac{\psi}{\lambda} = e^{-(r-g)(T-t)}\pi_T = e^{-(r-g)(T-t)}\frac{\partial V / \partial N_T}{\partial V / \partial M_T}$$

The Lagrange multipliers can also be converted into the prices of characteristics, κ, and the reduced costs, ζ.

$$\kappa_1 = \frac{\upsilon_1}{\lambda}; \quad \zeta_1 = \frac{\xi_1}{\lambda}$$
$$\kappa_2 = \frac{\upsilon_2}{\lambda}; \quad \zeta_2 = \frac{\xi_2}{\lambda}$$

The first order conditions for consumption can be differentiated with respect to time to show that prices of characteristics and reduced costs grow at the rate $r-g$, the same as other prices. Solve the first order conditions for the prices of characteristics.

$$\kappa_1 = \frac{(p_1-\zeta_1)k_{22}-(p_2+\pi c-\zeta_2)k_{21}}{k_{11}k_{22}-k_{12}k_{21}}$$
$$\kappa_2 = \frac{(p_2+\pi c-\zeta_2)k_{11}-(p_1-\zeta_1)k_{12}}{k_{11}k_{22}-k_{12}k_{21}}$$

These must satisfy transversality conditions.

$$\kappa_1 = e^{-(r-g)(T-t)}\kappa_{1T}$$
$$\kappa_2 = e^{-(r-g)(T-t)}\kappa_{2T}$$

Using the prices of characteristics, the first-order conditions for characteristics can be solved as a function of a single costate.

$$K_1-\chi_1 = \left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}}\left(\frac{\kappa_1}{\beta_1}\right)^{\frac{-1}{\nu+1}}B^{\frac{\alpha+\nu}{(\alpha-1)(\nu+1)}}$$
$$K_2-\chi_2 = \left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}}\left(\frac{\kappa_2}{\beta_2}\right)^{\frac{-1}{\nu+1}}B^{\frac{\alpha+\nu}{(\alpha-1)(\nu+1)}}$$

Using transversality conditions and simplifying shows that characteristics also satisfy transversality conditions.

$$K_1-\chi_1 = e^{-(g-\delta)(T-t)}(K_{1T}-\chi_1)$$
$$K_2-\chi_2 = e^{-(g-\delta)(T-t)}(K_{2T}-\chi_2)$$

The production of characteristics can be inverted to find consumption.

$$Q_1 = \frac{K_1k_{22}-K_2k_{12}}{k_{11}k_{22}-k_{12}k_{21}}$$
$$Q_2 = \frac{K_2k_{11}-K_1k_{21}}{k_{11}k_{22}-k_{12}k_{21}}$$

Setting characteristics to subsistence gives consumption at subsistence.

$$\gamma_1 = \frac{\chi_1 k_{22} - \chi_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}}$$
$$\gamma_2 = \frac{\chi_2 k_{11} - \chi_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}}$$

Subtract subsistence from both sides of the equations for consumption.

$$Q_1 - \gamma_1 = \frac{(K_1 - \chi_1)k_{22} - (K_2 - \chi_2)k_{12}}{k_{11}k_{22} - k_{12}k_{21}}$$
$$Q_2 - \gamma_2 = \frac{(K_2 - \chi_2)k_{11} - (K_1 - \chi_1)k_{21}}{k_{11}k_{22} - k_{12}k_{21}}$$

In this form, consumption satisfies transversality conditions.

$$Q_1 - \gamma_1 = e^{-(g-\delta)(T-t)}(Q_{1T} - \gamma_1)$$
$$Q_2 - \gamma_2 = e^{-(g-\delta)(T-t)}(Q_{2T} - \gamma_2)$$

Substituting characteristics into current utility gives its dual form.

$$U(t,M,N) = e^{-\rho t}\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{\alpha}{\alpha-1}} B^{\frac{\alpha}{\alpha-1}}$$

Current utility also satisfies a transversality condition.

$$U(t,M,N) = e^{\delta(T-t)}U(T,M_T,N_T)$$

Integrate current utility over all future times, beginning at time t.

$$\begin{aligned}\int_t^T U(s,M,N)ds &= U(T,M_T,N_T)\int_t^T e^{\delta(T-s)}ds \\ &= U(T,M_T,N_T)\left(\tfrac{-1}{\delta}\right)\left(1 - e^{\delta(T-t)}\right) \\ &= A_\delta U(t,M,N)\end{aligned}$$

Therefore, lifetime utility has a simple form.

$$J(t,M,N) = e^{-\rho t}\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{\alpha}{\alpha-1}} A_\delta B^{\frac{\alpha}{\alpha-1}} + V(T,M_T,N_T)$$

Because integrating factor A_δ goes to zero at time T, lifetime utility satisfies its terminal condition. Next integrate the differential equation for natural capital, using the transversality condition for consumption of ecosystem services.

$$
\begin{aligned}
N_T &= e^{g(T-t)}\left[N - \int_t^T e^{-g(s-t)} c Q_2 ds\right] \\
&= e^{g(T-t)}\left[N - \int_t^T e^{-g(s-t)} [c(Q_2 - \gamma_2) + c\gamma_2] ds\right] \\
&= e^{g(T-t)}\left[N - A_\delta c(Q_2 - \gamma_2) - A_g c\gamma_2\right]
\end{aligned}
$$

In this result, identify lifetime consumption of ecosystem services above subsistence and solve for it.

$$
\begin{aligned}
D(t,N) &= A_\delta c(Q_2 - \gamma_2) \\
&= N - e^{-g(T-t)} N_T - A_g c\gamma_2
\end{aligned}
$$

Integrate the differential equation for manufactured capital. In this case substitute in prices less the reduced costs and note that ζQ equals zero at the optimum.

$$
\begin{aligned}
M_T &= e^{r(T-t)}\left[M - \int_t^T e^{-r(s-t)} [Y - p_1 Q_1 - p_2 Q_2] ds\right] \\
&= e^{r(T-t)}\Bigg[M - \int_t^T e^{-r(s-t)} [Y \\
&\qquad - (p_1 - \zeta_1)(Q_1 - \gamma_1) - (p_2 - \zeta_2)(Q_2 - \gamma_2) - (p_1 - \zeta_1)\gamma_1 - (p_2 - \zeta_2)\gamma_2] ds\Bigg] \\
&= e^{r(T-t)}\big[M + A_r Y \\
&\qquad - A_\delta((p_1 - \zeta_1)(Q_1 - \gamma_1) + (p_2 - \zeta_2)(Q_2 - \gamma_2)) - A_g((p_1 - \zeta_1)\gamma_1 + (p_2 - \zeta_2)\gamma_2)\big]
\end{aligned}
$$

Identify lifetime consumption of commodities above subsistence and solve for it.

$$
\begin{aligned}
C(t,M) &= A_\delta((p_1 - \zeta_1)(Q_1 - \gamma_1) + (p_2 - \zeta_2)(Q_2 - \gamma_2)) \\
&= M - e^{-r(T-t)} M_T + A_r Y - A_g[(p_1 - \zeta_1)\gamma_1 + (p_2 - \zeta_2)\gamma_2]
\end{aligned}
$$

Alternatively, use price π and combine the states to become wealth.

$$
W = M + \pi N
$$

Differentiate with respect to time to find the change in wealth. Rate g disappears and wealth grows at rate r.

$$
\dot{W} = rW + Y - p_1 Q_1 - (p_2 + \pi c) Q_2
$$

Integrate this differential equation.

$$
\begin{aligned}
W_T &= e^{r(T-t)}\left[W - \int_t^T e^{-r(s-t)}\left[Y - p_1 Q_1 - (p_2 + \pi c)Q_2\right]ds\right] \\
&= e^{r(T-t)}\left[W - \int_t^T e^{-r(s-t)}\left[Y \right.\right. \\
&\qquad \left.\left. - (p_1 - \zeta_1)(Q_1 - \gamma_1) - (p_2 + \pi c - \zeta_2)(Q_2 - \gamma_2) - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2\right]ds\right] \\
&= e^{r(T-t)}\left[W + A_r Y \right. \\
&\qquad \left. - A_\delta\left[(p_1 - \zeta_1)(Q_1 - \gamma_1) + (p_2 + \pi c - \zeta_2)(Q_2 - \gamma_2)\right] - A_g\left[(p_1 - \zeta_1)\gamma_1 + (p_2 + \pi c - \zeta_2)\gamma_2\right]\right]
\end{aligned}
$$

In this expression, lifetime expenditures above subsistence combine lifetime consumption of commodities and lifetime consumption of ecosystem services.

$$
\begin{aligned}
E(t, M, N) &= C(t, M) + \pi D(t, N) \\
&= W - e^{-r(T-t)} W_T + A_r Y - A_g\left[(p_1 - \zeta_1)\gamma_1 + (p_2 + \pi c - \zeta_2)\gamma_2\right]
\end{aligned}
$$

Substituting consumption into C and D and simplifying gives lifetime expenditures as a function of characteristics.

$$
E(t, M, N) = A_\delta\left[\kappa_1(K_1 - \chi_1) + \kappa_2(K_2 - \chi_2)\right]
$$

Substituting in characteristics gives lifetime expenditures as a function of the costate.

$$
E(t, M, N) = \left(\tfrac{1}{\alpha} e^{\rho t} \lambda\right)^{\frac{1}{\alpha-1}} A_\delta B^{\frac{\alpha}{\alpha-1}}
$$

Solve for the costate.

$$
\lambda = \alpha e^{-\rho t} A_\delta^{1-\alpha} B^{-\alpha} E^{\alpha-1}
$$

Finally, use the costate to obtain the analytical solution by substituting into lifetime utility, current utility and characteristics.

$$
J(t, M, N) = e^{-\rho t} A_\delta^{1-\alpha} B^{-\alpha} E^{\alpha} + V(T, M_T, N_T)
$$

$$
U(t, M, N) = e^{-\rho t} A_\delta^{-\alpha} B^{-\alpha} E^{\alpha}
$$

$$K_1 - \chi_1 = \left(\frac{\kappa_1}{\beta_1}\right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}} \frac{E}{A_\delta}$$

$$K_2 - \chi_2 = \left(\frac{\kappa_2}{\beta_2}\right)^{\frac{-1}{\nu+1}} B^{\frac{-\nu}{\nu+1}} \frac{E}{A_\delta}$$

Substitute characteristics into consumption.

$$Q_1 - \gamma_1 = \frac{b_1 k_{22} - b_2 k_{12}}{k_{11}k_{22} - k_{12}k_{21}} \frac{E}{A_\delta}$$

$$Q_2 - \gamma_2 = \frac{b_2 k_{11} - b_1 k_{21}}{k_{11}k_{22} - k_{12}k_{21}} \frac{E}{A_\delta}$$

Substitute consumption into the change in wealth.

$$\dot{W} = rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta}$$

Current utility and the change in wealth give the maximized the Hamiltonian.

$$H(t, M, N) = e^{-\rho t} A_\delta^{-\alpha} B^{-\alpha} E^{\alpha} + \lambda\left[rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2 - \frac{E}{A_\delta}\right]$$

The solution has singularities at $\alpha = 1$, $\delta = 0$, $\nu = 0$ and $T \to \infty$. The first three singularities can be avoided by taking limits or by setting the parameters to be $\pm\varepsilon$ away from the singularity.

Steady state. In a steady state, by definition, the states and current-value costates are constant.

$$\dot{W} = 0$$

$$\frac{\partial}{\partial t}\left(e^{\rho t}\lambda\right) = 0 = e^{\rho t}\lambda(\rho - r)$$

$$\frac{\partial}{\partial t}\left(e^{\rho t}\psi\right) = 0 = e^{\rho t}\psi(\rho - g)$$

As a consequence, all rates of change are equal. Prices and expenditures above subsistence are constant.

$$\rho = r = g = \delta$$

$$\dot{p}_1 = \dot{p}_2 = \dot{\pi} = \dot{\kappa}_1 = \dot{\kappa}_2 = \dot{\zeta}_1 = \dot{\zeta}_2 = 0$$

$$\frac{E}{A_\delta} = rW + Y - (p_1 - \zeta_1)\gamma_1 - (p_2 + \pi c - \zeta_2)\gamma_2$$

Therefore demand is constant. In addition, the Hamiltonian, collapses to current utility.

$$H(t, M, N) = e^{-\rho t} A_\delta^{-\alpha} B^{-\alpha} E^\alpha$$

Welfare Analysis. Define willingness to pay and willingness to accept as equivalent and compensating variations for discrete changes in natural capital, Δ_N, and the price of effort in extracting natural capital less any reduced costs, $\Delta_{p-\zeta}$.

$$J(t, M - WTP, N, p) = J(t, M, N + \Delta_N, p + \Delta_{p-\zeta})$$
$$J(t, M, N, p) = J(t, M + WTA, N + \Delta_N, p + \Delta_{p-\zeta})$$

In general, these welfare equations are highly nonlinear and must be solved numerically, which requires a functional form for the terminal value. Assume a constant elasticity of substitution function.

$$V(T, M_T, N_T) = e^{-\rho T}\theta\left[\phi_1(M_T - \eta_1)^{-\mu} + \phi_2(N_T - \eta_2)^{-\mu}\right]^{\frac{-\omega}{\mu}}$$

There are two special cases with algebraic solutions. In the first case, assume the price of effort doesn't change. *WTP* and *WTA* are a simple exchange of manufactured capital for natural capital.

$$WTP = WTA = -\pi\Delta_N$$

To derive the second case, differentiate the terminal value to find the terminal costates.

$$\lambda_T = \omega e^{-\rho T}\theta\left[\phi_1(M_T - \eta_1)^{-\mu} + \phi_2(N_T - \eta_2)^{-\mu}\right]^{\frac{-\omega}{\mu} - 1}\phi_1(M_T - \eta_1)^{-\mu - 1}$$
$$\psi_T = \omega e^{-\rho T}\theta\left[\phi_1(M_T - \eta_1)^{-\mu} + \phi_2(N_T - \eta_2)^{-\mu}\right]^{\frac{-\omega}{\mu} - 1}\phi_2(N_T - \eta_2)^{-\mu - 1}$$

Multiply the respective costates by manufactured capital above subsistence and natural capital above subsistence and sum the results.

$$\lambda_T(M_T-\eta_1)+\psi_T(N_T-\eta_2)=\omega e^{-\rho T}\theta\left[\phi_1(M_T-\eta_1)^{-\mu}+\phi_2(N_T-\eta_2)^{-\mu}\right]^{\frac{-\omega}{\mu}}$$

Substitute in the price of natural capital to eliminate the costate on natural capital and solve for the terminal value.

$$V(T,M_T,N_T)=\frac{\lambda_T}{\omega}\left[M_T+\pi_T N_T-\eta_1-\pi_T\eta_2\right]$$

Substitute in terminal wealth and replace the terminal costate and the terminal price of natural capital with current values. Then replace discounted terminal wealth. Finally, substitute in expenditures as a function of the costate.

$$\begin{aligned}V(T,M_T,N_T)&=\frac{\lambda}{\omega}\left[e^{-r(T-t)}W_T-e^{-r(T-t)}\eta_1-e^{-g(T-t)}\pi\eta_2\right]\\&=\frac{\lambda}{\omega}\left[W+A_rY-A_g\left[(p_1-\zeta_1)\gamma_1+(p_2+\pi c-\zeta_2)\gamma_2\right]-E-e^{-r(T-t)}\eta_1-e^{-g(T-t)}\pi\eta_2\right]\\&=\frac{\lambda}{\omega}\left[W+A_rY-A_g\left[(p_1-\zeta_1)\gamma_1+(p_2+\pi c-\zeta_2)\gamma_2\right]-e^{-r(T-t)}\eta_1-e^{-g(T-t)}\pi\eta_2\right]\\&\quad-\tfrac{\alpha}{\omega}e^{-\rho t}\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{\alpha}{\alpha-1}}A_\delta B^{\frac{\alpha}{\alpha-1}}\end{aligned}$$

This form of the terminal value gives an alternative form for lifetime utility as a function of the costate.

$$\begin{aligned}J(t,M,N)&=\left(1-\tfrac{\alpha}{\omega}\right)e^{-\rho t}\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{\alpha}{\alpha-1}}A_\delta B^{\frac{\alpha}{\alpha-1}}\\&\quad+\frac{\lambda}{\omega}\left[W+A_rY-A_g\left[(p_1-\zeta_1)\gamma_1+(p_2+\pi c-\zeta_2)\gamma_2\right]-e^{-r(T-t)}\eta_1-e^{-g(T-t)}\pi\eta_2\right]\end{aligned}$$

Multiplying by ω and dividing by the costate convert lifetime utility into a money measure.

$$\begin{aligned}\frac{J(t,M,N)\omega}{\lambda}&=\left(\tfrac{\omega-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}}A_\delta B^{\frac{\alpha}{\alpha-1}}-A_g\left[(p_1-\zeta_1)\gamma_1+(p_2+\pi c-\zeta_2)\gamma_2\right]\\&\quad+W+A_rY-e^{-r(T-t)}\eta_1-e^{-g(T-t)}\pi\eta_2\end{aligned}$$

If $\mu = -1$ and $\omega = 1$, then, from their derivatives, the costates and their ratio will be independent of wealth.

$$\lambda = e^{r(T-t)-\rho T}\theta\phi_1; \qquad \psi = e^{g(T-t)-\rho T}\theta\phi_2; \qquad \pi = e^{-(r-g)(T-t)}\frac{\phi_2}{\phi_1}$$

In this case, lifetime utility is linear in wealth and the welfare equations can be solved algebraically for *WTP* and *WTA*.

$$WTP = WTA = -\pi\Delta_N$$
$$+\left(\tfrac{1-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}} A_\delta\left[B(p-\zeta)^{\frac{\alpha}{\alpha-1}} - B\left(p-\zeta+\Delta_p\right)^{\frac{\alpha}{\alpha-1}}\right] + A_g\left(\Delta_{p_1-\zeta_1}\gamma_1 + \Delta_{p_2-\zeta_2}\gamma_2\right)$$

The last two terms are the lifetime change in surplus. At a corner solution, a change in one price will cause a change in the reduced cost of the other price. The last term allows for this. The change in consumer surplus is similar to *WTP* and *WTA*.

$$\int_{p_1}^{p_1+\Delta_{p_1-\zeta_1}} Q_1 dp + \int_{p_2}^{p_2+\Delta_{p_2-\zeta_2}} Q_2 dp = \left(\tfrac{1-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}}\left[B(p-\zeta)^{\frac{\alpha}{\alpha-1}} - B\left(p-\zeta+\Delta_p\right)^{\frac{\alpha}{\alpha-1}}\right] + \Delta_{p_1-\zeta_1}\gamma_1 + \Delta_{p_2-\zeta_2}\gamma_2$$

Again allowance is made for corner solutions. A change in one price may change the reduced cost for the other price. Therefore, if the utility of bequests is linear, the present value of all future changes in consumer surplus is an exact measure of the lifetime change in surplus.

$$\int_t^T e^{-r(s-t)}\left[\int_{p_1}^{p_1+\Delta_{p_1-\zeta_1}} Q_1 dp + \int_{p_2}^{p_2+\Delta_{p_2-\zeta_2}} Q_2 dp\right] ds$$
$$= \left(\tfrac{1-\alpha}{\alpha}\right)\left(\tfrac{1}{\alpha}e^{\rho t}\lambda\right)^{\frac{1}{\alpha-1}} A_\delta\left[B(p-\zeta)^{\frac{\alpha}{\alpha-1}} - B\left(p-\zeta+\Delta_p\right)^{\frac{\alpha}{\alpha-1}}\right] + A_g\left(\Delta_{p_1-\zeta_1}\gamma_1 + \Delta_{p_2-\zeta_2}\gamma_2\right)$$

For comparison, suppose the model was in a steady state before the changes and immediately jumps to a new steady state after the changes. Using the steady state Hamiltonian, *WTP* and *WTA* would have algebraic solutions which depend upon the ratio of substitution factors.

$$WTP = \left(1 - \frac{B(p-\zeta)}{B(p-\zeta+\Delta_p)}\right)\left[W + \tfrac{1}{r}(Y - p_1\gamma_1 + p_2\gamma_2)\right]$$
$$+ \frac{B(p-\zeta)}{B(p-\zeta+\Delta_p)}\left[\tfrac{1}{r}\left(\Delta_{p_1-\zeta_1}\gamma_1 + \Delta_{p_2-\zeta_2}\gamma_2\right) - \pi\Delta_N\right]$$
$$WTA = \frac{B(p-\zeta+\Delta_p)}{B(p-\zeta)}WTP$$

These equations have the same general form as equations for static *WTP* and *WTA*.