# Incomplete Demand Systems, Corner Solutions, and Welfare Measurement

Roger H. von Haefen

This paper demonstrates how corner solutions raise difficulties for the specification, estimation, and use of incomplete demand systems for welfare measurement with disaggregate consumption data, as is common in the outdoor recreation literature. A simple analytical model of consumer behavior is used to elucidate the potential biases for welfare measurement arising from modeling the demand for $M$ goods as a function of $M + N$ prices ($N > 1$) and income when individuals do not consume all goods in strictly positive quantities. Results from a Monte Carlo experiment suggest that these biases can be substantial for large-scale policy shocks when prices are highly correlated.

***Key Words:*** demand systems, welfare analysis, corner solutions, microeconometrics

Applied researchers are often interested in developing empirical demand models for a subset of goods entering preferences. A practical issue arising in these situations is the treatment of the remaining goods whose demands are not explicitly modeled. The dominant strategies for resolving this issue in applied microeconomic analysis involve either separability or Hicksian composite commodity assumptions. Both of these approaches imply restrictions on preferences or prices that may not hold empirically. When they do not, Epstein (1982) has proposed an incomplete demand system strategy in which the analyst models the demand for the goods of interest as functions of their own prices, the remaining goods prices, and income. LaFrance and Hanemann (1989) prove that if the incomplete demand system satisfies a set of regularity conditions analogous to the classical integrability conditions for complete demand systems, it is consistent with a rational preference ordering and can in principle be used to generate Hicksian welfare measures for changes in the prices of the goods whose demands are explicitly modeled.

To illustrate the potential usefulness of the incomplete demand system framework, consider a common empirical problem in the outdoor recreation literature [see e.g., Herriges and Kling (1999) for a review]. Analysts are often interested in valuing the access to a single recreation site that is located in a larger geographic region containing several sites. The varying proximity of individuals to these sites, as well as differences in each individual's opportunity cost of time, suggest that the sites' implicit prices or travel costs vary across the target population. Data limitations often imply that the analyst only has trip data for the site of interest. This combination of preference linkages, price variations, and data limitations suggest that employing separability assumptions or the Hicksian composite commodity theorem in a demand model for the relevant site would be inappropriate. However, the insights of Epstein and LaFrance and Hanemann suggest that the incomplete demand system approach can be used to estimate a consistent demand specification and Hicksian welfare measures for the site of interest. In fact, several authors (e.g., Gum and Martin 1975, Hof and King 1982, Caulkins, Bishop, and

Roger H. von Haefen is Associate Professor in the Department of Agricultural and Resource Economics at North Carolina State University in Raleigh, North Carolina. This research was supported by the USDA Cooperative State Research, Education, and Extension Service, Hatch Project #06843. The author thanks V. Kerry Smith, Daniel Phaneuf, David Layton, participants at the W2133 Annual Meetings in Las Vegas, Nevada, a helpful referee, and the Editor for comments and suggestions. All remaining errors are those of the author.

Bouwes 1985, Rosenthal 1987, Kling 1989, Smith 1993, Ozuna and Gomez 1994, Gurmu and Trivedi 1996, Eom and Larson 2006, Phaneuf, Carbone, and Herriges 2009) have suggested or empirically implemented demand specifications that fall under the rubric of incomplete demand system approaches. Boardman et al. (2006) summarize the conventional wisdom in this literature:

> "Estimating such a (travel cost) model is conceptually straightforward. First, select a random sample of households within the market area of the site. These are the potential visitors. Second, survey these households to determine their number of visits to the site over some period of time, all of their costs from visiting the site, their costs of visiting substitute sites, their incomes, and their other characteristics that may affect their demand. Third, specify a functional form for the demand schedule and estimate it using the survey data." (p. 354)

This paper raises difficulties with the specification, estimation, and use of the incomplete demand system framework for welfare measurement with disaggregate consumption data. An empirical regularity with individual or household level data is that consumption levels for the goods of interest and their related substitutes and complements are a mixture of interior (i.e., strictly positive valued) and corner (zero) solutions. In traditional microeconometric [i.e., random utility maximization (RUM) models of consumer behavior], Pudney (1989) and Phaneuf, Kling, and Herriges (1999) point out that observable market prices for unconsumed goods do not enter the remaining goods' demand functions. Rather, economic theory suggests that virtual prices (Neary and Roberts 1980), i.e., the prices that would drive their demands to zero, influence choice. These virtual prices are bounded from above by their corresponding market prices and depend in general on the structural parameters and unobserved heterogeneity entering preferences, income, and the prices of goods consumed in strictly positive quantities. Consequently, they are unobservable and endogenous from the analyst's perspective. Although the implications of corner solutions for the estimation of complete demand systems are now well known (see e.g., Pudney 1989), and consistent estimation and welfare calculation techniques have been developed (e.g., Wales and Woodland 1983, Lee and Pitt 1986, von Haefen, Phaneuf, and Parsons 2004), their implications have not been fully appreciated in the context of incomplete demand systems. The main thesis of this paper is that when corner solutions are present and market and virtual prices diverge, consistent estimation of incomplete demand systems is far more complex than what current empirical practice suggests. In fact, no econometrically feasible approach for consistently estimating an incomplete demand system currently exists in this context, and the hurdles that must be overcome are substantial. This reality suggests that, barring significant econometric innovations, the only empirically viable and theoretically consistent RUM-based approach to recovering demand parameters for a subset of goods entering preferences with micro data is complete demand system estimation.

The paper begins by developing a simple analytical model of consumer behavior to highlight the critical role virtual prices play in applied demand analysis when corner solutions are present. The model is used to suggest: 1) the potential biases arising from the common empirical practice of estimating an incomplete demand system as a function of income and all market prices; 2) the significant and complex hurdles that must be overcome for consistent estimation; and 3) additional biases arising when using misspecified incomplete demand systems for welfare measurement.

A Monte Carlo experiment is subsequently used to investigate whether the difficulties raised with the standard approach to estimating incomplete demand systems in the existing outdoor recreation literature imply significantly biased welfare estimates. A five-good complete demand system is developed and calibrated using parameter estimates, descriptive statistics, and empirical results reported in Phaneuf (1999). Five hundred simulated data sets are generated from the model and used to estimate an incomplete demand model for a single good that employs observed market prices in place of the behaviorally relevant virtual prices for the remaining goods. A comparison of welfare estimates from the incomplete demand system specification, with estimates generated from the assumed model for two alternative policy scenarios, suggests that the bias introduced by using market prices in place of virtual prices is relatively small (less than 4 percent) for small price changes regardless of the correlation structure among the good's own price and substitute prices. However, for scenarios involving large price changes such as the elimination of a good, the bias can be as large as 35 percent if the good's own price is strongly correlated with substitute prices.

At present, econometric techniques for addressing the substantial challenges of consistent incomplete demand system estimation have not been developed and thus remain important but challenging areas for future applied research. Therefore, one of this paper's main implications is that the only fully consistent option for consistently accounting for interior and corner solutions in applied demand analysis is to estimate complete demand systems.

## Analytical Model

This section uses a simple analytical model of consumer behavior to clarify the difficulties arising from the improper use of market prices in applied demand analysis when corner solutions are present. In what follows, the implications of corner solutions for estimating and calculating welfare measures from incomplete demand systems are emphasized. Unless otherwise noted, however, these implications apply equally to complete demand systems (Pudney 1989 and Phaneuf, Kling, and Herriges 1998) although consistent estimation and welfare calculation techniques have been developed to address these difficulties in the latter context. A maintained assumption throughout the section is that consumer preferences for a set of $M + N$ ($M > 0$, $N > 1$) goods can be represented by a smooth, continuously differentiable, strictly increasing, and strictly quasi-concave utility function,

$$U(\boldsymbol{x},\boldsymbol{z},z_N), \tag{1}$$

where $\boldsymbol{x}$, $\boldsymbol{z}$, and $z_N$ are an $M$-dimensional vector, an ($N$-1)-dimensional vector, and a scalar of goods, respectively. In traditional RUM-based econometric models of consumer choice, individual utility will also depend on a set of estimable parameters, $\boldsymbol{\Omega}$, and unobserved heterogeneity, $\boldsymbol{\varepsilon}$, known to the individual but unobserved and random from the analyst's perspective. For compactness these arguments are suppressed in this section. The partitioning of goods into $\boldsymbol{x}$ and $\boldsymbol{z}$ subgroups reflects the analyst's interest in the welfare implications of price or access changes for the goods in $\boldsymbol{x}$ alone and/or a lack of consumption data for $\boldsymbol{z}$. To allow for corner solutions, all goods in $\boldsymbol{x}$ and $\boldsymbol{z}$ are assumed nonessential, but to minimize excessive notation, $z_N$ is assumed essential, i.e.,

$$\boldsymbol{x} \geq 0,\ \boldsymbol{z} \geq 0,\ \&\ z_N > 0. \tag{2}$$

The individual behaves as if she maximizes (1) with respect to her budget constraint,

$$\boldsymbol{p}^{\mathsf{T}}\boldsymbol{x} + \boldsymbol{q}^{\mathsf{T}}\boldsymbol{z} + z_N = y, \tag{3}$$

where $\boldsymbol{p}$, $\boldsymbol{q}$, and $y$ are exogenous prices and income normalized by the price of $z_N$. The optimal consumer demand functions can be solved for by maximizing the following Lagrangian:

$$L = U(\boldsymbol{x},\boldsymbol{z},z_N) + \lambda(y - \boldsymbol{p}^{\mathsf{T}}\boldsymbol{x} - \boldsymbol{q}^{\mathsf{T}}\boldsymbol{z} - z_N) + \boldsymbol{\delta}^{\mathsf{T}}\boldsymbol{x} + \boldsymbol{\mu}^{\mathsf{T}}\boldsymbol{z},$$

where $\lambda$, $\boldsymbol{\delta}$, and $\boldsymbol{\mu}$ are a scalar and two vectors of Lagrange multipliers. Because a strictly increasing utility function implies budget exhaustion, the implied Kuhn-Tucker conditions, in addition to equations (2) and (3) above, are:

$$\frac{\partial U(\boldsymbol{x},\boldsymbol{z},z_N)/\partial x_i}{\lambda} = p_i - \delta_i/\lambda,\ i = 1,\ldots,\ M, \tag{4}$$

$$\frac{\partial U(\boldsymbol{x},\boldsymbol{z},z_N)/\partial z_j}{\lambda} = q_j - \mu_j/\lambda,\ j = 1,\ldots,\ N-1, \tag{5}$$

$$\delta^{\mathsf{T}}\boldsymbol{x} = 0,\ \mu^{\mathsf{T}}\boldsymbol{z} = \boldsymbol{0}, \tag{6}$$

where $\lambda$ is the marginal utility of income.

Equations (2) through (6) implicitly define the optimal consumption bundle, $[\boldsymbol{x}^*, \boldsymbol{z}^*, z_N]$, as well as the Lagrange multipliers, $\lambda^* > 0$, $\delta^* \geq 0$, and $\mu^* \geq 0$. Inserting these optimal values into (4) and (5) allows one to define the Marshallian "virtual" prices (Neary and Roberts 1980) for each of the ($M$+$N$–1) goods:

(7)
$$\xi^*_{x_i} = \frac{\partial U(\boldsymbol{x}^*, \boldsymbol{z}^*, z^*_N) / \partial x_i}{\lambda^*} = p_i - \delta^*_i / \lambda^*, i = 1,..., M,$$

(8 )
$$\xi^*_{z_j} = \frac{\partial U(\boldsymbol{x}^*, \boldsymbol{z}^*, z^*_N) / \partial z_j}{\lambda^*} = q_j - \mu^*_j / \lambda^*, j = 1,..., N-1.$$

The structure of equations (7) and (8) suggest several points. $\xi^*_{x_i}$ and $\xi^*_{z_j}$ $(\forall i,j)$ are simultaneously determined with $[\boldsymbol{x}^*, \boldsymbol{z}^*, z_N]$ and functions of the same exogenous factors—income, other good's prices, preference parameters, and unobserved heterogeneity. Unlike $[\boldsymbol{x}^*, \boldsymbol{z}^*, z_N]$, however, they are unobservable from the analyst's perspective when they diverge from their corresponding market prices. From (6), these divergences occur whenever $\delta^*_i$ or $\mu^*_j$ $(\forall i,j)$ are strictly positive, i.e., whenever $\boldsymbol{x}^*_i$ or $\boldsymbol{z}^*_j$ $(\forall i,j)$ equal zero.

Using virtual prices, one can recast the consumer's problem as the maximization of (1) with respect to the following notional budget constraint:

(9)
$$\boldsymbol{\xi}^{*\mathsf{T}}_{\boldsymbol{x}} \boldsymbol{x} + \boldsymbol{\xi}^{*\mathsf{T}}_{\boldsymbol{z}} \boldsymbol{z} + z_N = y,$$

where $\xi^*_x$ and $\xi^*_z$ are $M$ and ($N$-1)-dimensional vectors of quasi-fixed virtual prices implied by (7) and (8), respectively. Approaching the constrained optimization problem in this way suggests that optimal demand can be written as functions of all virtual prices and income, i.e.,

(10)
$$\boldsymbol{x}^* = \boldsymbol{x}(\boldsymbol{\xi}^*_{\boldsymbol{x}}, \boldsymbol{\xi}^*_{\boldsymbol{z}}, y),$$

(11)
$$\boldsymbol{z}^* = \boldsymbol{z}(\boldsymbol{\xi}^*_{\boldsymbol{x}}, \boldsymbol{\xi}^*_{\boldsymbol{z}}, y).$$

With the analytical model derived above as background, it is possible to critically assess the incomplete demand system specifications that have been suggested or used in the outdoor recreation literature (e.g., Gum and Martin 1975, Hof and King 1982, Caulkins, Bishop, and Bouwes 1985, Rosenthal 1987, Kling 1989, Smith 1993, Ozuna and Gomez 1994, Gurmu and Trivedi 1996, Eom and Larson 2006, Phaneuf, Carbone, and Herriges 2009). For concreteness, a commonly used empirical specification—the single equation censored regression model (i.e., $M$ = 1) where consumption is modeled as a linear and additive function of its own price, $N$–1 other goods' prices, and income—is considered, i.e.,

(12)
$$x^*_1 = \begin{cases} \alpha_1 + \beta_{11} p_1 + \sum_{i=1}^{N-1} \phi_{1i} q_i + \rho_1 y + \varepsilon_1 \ \textbf{if} \\ \qquad \varepsilon_1 > \alpha_1 + \beta_{11} p_1 + \sum_{i=1}^{N-1} \phi_{1i} q_i + \rho_1 y \\ 0 \ \text{otherwise} \end{cases}$$

where $\alpha_1, \beta_{11}, \phi_{1i} (\forall i)$ and $\rho_1$ are estimable structural parameters and $\varepsilon_1$ captures unobserved heterogeneity. In general, estimating equation (12) within the censored regression framework will not produce consistent parameter estimates. As suggested by the structure of equations (7) and (10), the estimates will only be consistent if interior solutions for the $N$–1 goods are present for all individuals in the target population. A priori, one cannot determine the direction or magnitude of bias in the estimated parameters because the analyst is in essence employing a fundamentally misspecified demand model.

A comparison of equations (10) and (12) might suggest a relatively simple approach to consistently estimate the demand parameters for $x_1$—estimate (12) with $\xi^*_z$ replacing $\boldsymbol{q}$ throughout. Such a strategy is problematic for at least two reasons. Recall that virtual prices are latent and probabilistic from the analyst's perspective, implying that using $\xi^*_z$ in (12) is an unobserved random variable. With strong if not heroic assumptions about the structure of demand and the distribution of unobserved heterogeneity, however, the analyst could construct unbiased predictions for $\xi_z^*$ that can be used in (12). Even if generating these regressors is possible, exploiting them in demand estimation would imply a complex multivariate errors-in-variables problem. And to the degree that the estimation equation is nonlinear or nonadditive in virtual prices, the difficulty of the errors-in-variables problem would be even more pronounced.

A second set of complications arises from the fact that demand and virtual prices are simultaneously determined. As a result, $\xi_z^*$ and $\varepsilon_1$ would be correlated, implying that the (predicted) virtual prices would require instruments. These instruments would have to come from outside the set of exogenous factors that directly enter the demand system (prices, income, as well as any demographic shifters). With enough assumptions, it may be possible to predict virtual prices and account for the simultaneity and errors-in-variables problems, but the consistency of the estimated parameters, however, would hinge on several assumptions that are highly suspect if not implausible. Thus, it seems reasonable to conclude that recovering credible parameter estimates from a single- or multiple-equation incomplete demand system is extremely if not prohibitively difficult when corner solutions are present.

In addition to the estimation problems associated with (12), further difficulties arise with welfare measurement. Returning to the single equation context, consider a policy scenario involving a price increase from $\boldsymbol{p}_1'$ to $\boldsymbol{p}_1''$. If income effects are absent, it is well known that the Marshallian and Hicksian demands for $x_1$ are equivalent and the integral from $\boldsymbol{p}_1'$ to $\boldsymbol{p}_1''$ of either represents the Hicksian compensating variation (*CV*), i.e.,

$$CV = \int_{p_1'}^{p_1''} \left( \alpha_1 + \beta_{11} p_1 + \sum_{i=1}^{N-1} \phi_{1i} \xi_{zi}^* + \varepsilon_1 \right) dp_1, \tag{13}$$

assuming $x_1 > 0$ is strictly positive before and after the price change. It is important to recognize once again that the virtual prices for the ($N$–1) other goods are endogenous functions of $p_1$. As a result, the structure of (13) suggests that as $p_1$ rises from $\boldsymbol{p}_1'$ to $\boldsymbol{p}_1''$, these virtual prices may endogenously change. If the analyst naively uses the demand specification in (12) to construct welfare measures, however, she will miss these feedback linkages between $p_1$ and the other goods' virtual prices.

There is a final difficulty with using (12) when estimating either the total Hicksian value of $x_1$ or a relatively large price change that drives the demand for $x_1$ to zero for some individuals in the relevant population. In these situations, the Hicksian consumer surplus is defined as the integral from the current market price to the Hicksian "choke" price, $\boldsymbol{p}_1^c(\cdot)$, i.e., the price that drives Hicksian demand for $x_1$ to zero. It is important to recognize that $\boldsymbol{p}_1^c(\cdot)$ is endogenously determined along with the Hicksian demands and virtual prices. Therefore, the choke price implied by (12) that ignores this simultaneity may be very different from the choke price that does not. When this occurs, the bounds of the integral that define the Hicksian consumer surplus will not be properly specified and the integrity of welfare estimates will be further compromised.

Collectively, what these potential sources of bias suggest is that, unless the analyst knows the structure of the virtual price functions for the remaining goods, consistent welfare measurement is not feasible even when consistent estimation of the structural demand parameters is. This finding casts further doubt on the usefulness of the incomplete demand system framework for applied demand analysis with disaggregate consumption data. As noted above, consistent econometric and welfare calculation techniques have been developed to account for corner solutions in the context of complete demand systems (Wales and Woodland 1983, Lee and Pitt 1986, von Haefen, Phaneuf, and Parsons 2004, von Haefen 2007). Thus, the only theoretically consistent approach for recovering welfare measures affecting a subset of goods' prices at present is through the complete demand system framework. More will be said on this topic in the paper's conclusion.

Although the above discussion suggests a number of potential biases with the standard approach to estimating and constructing welfare measures from incomplete demand systems, their combined effects are uncertain. It is possible, albeit unlikely, that they do not substantially influence derived welfare measures. In the next section, a Monte

Carlo experiment is used to illustrate that the net bias may in fact be substantial for large price changes when own and substitute good prices are highly correlated. This finding is of course conditional on the assumed structure of consumer preferences, the judgments made in calibrating the simulation experiment, and the policy scenarios considered. Nevertheless, it illustrates the critical role of virtual price functions in the construction of welfare measures from incomplete demand systems

**Monte Carlo Experiment**

The Monte Carlo experiment employs the following quasi-linear demand system specification considered by Bockstael, Hanemann, and Strand (1986):

$$x_i^* = \alpha_i + \sum_{k=1}^{M} \beta_{ik} \xi_{x_k}^* + \sum_{l=1}^{N-1} \phi_{il} \xi_{z_l}^* + \varepsilon_i, \; i = 1, \ldots, M \tag{14}$$

$$z_j^* = a_j + \sum_{k=1}^{M} b_{jk} \xi_{x_k}^* + \sum_{l=1}^{N-1} c_{jl} \xi_{z_l}^* + \varepsilon_j, \; j = 1, \ldots, N-1 \tag{15}$$

where $z_N^*$ is strictly positive by assumption. The corresponding indirect utility function can be written compactly using vector notation as:

$$V(\cdot) = y - (\alpha + \varepsilon)^{\mathsf{T}} \xi^* - \tfrac{1}{2} \xi^{*\mathsf{T}} \mathbf{\Phi} \xi^*, \tag{16}$$

where $\boldsymbol{\alpha}$ and $\mathbf{\Phi}$ are a vector and matrix of all constant terms and (Hicksian) own and cross price effects in (14) and (15), respectively; $\varepsilon$ is the ($N$+$M$-1)-dimensional vector of unobserved determinants of choice; and $\xi^*$ is the concatenation of the $\xi_x^*$ and $\xi_z^*$ vectors of the endogenous virtual price functions. The above specification is homogenous of degree zero in prices and income and satisfies the adding-up condition. To insure economic consistency, $\mathbf{\Phi}$ must be symmetric (i.e., $\mathbf{\Phi} = \mathbf{\Phi}^{\mathsf{T}}$) and negative semi-definite (the eigenvalues of $\mathbf{\Phi}$ must be non-positive).

To calibrate (14) and (15), the Monte Carlo experiment employs parameter estimates, descriptive statistics, and empirical results reported in Phaneuf (1999). Table 1 documents the assumptions and procedures used to fit Phaneuf's homothetic Indirect Translog model for four Wisconsin outdoor recreation sites to the above framework. These four goods and a Hicksian composite are partitioned into disjoint sets such that only the demand for a single good is explicitly modeled (and, by implication, $N = 1$ and $M = 4$). A potentially significant piece of information for this application, not reported in Phaneuf, is the correlation structure among implicit prices (i.e., travel costs) for the four sites. In related work, Kling (1989) identified the importance of the correlation structure among prices for estimation and welfare calculation in an incomplete demand system framework but did not consider the added complexity raised by corner solutions. To evaluate the sensitivity of derived welfare measures to alternative correlation structures, five alternative correlation specifications are developed. Because all prices are assumed to be log-normally distributed, the five specifications assume that the correlation coefficient between $\ln p_1$ and $\ln q_j$ equals $r$ for $j = 1,2,3$, and that the correlation coefficient between $\ln q_i$ and $\ln q_j$ equals zero, $\forall_i$, $j = 1,2,3$; $i \neq j$. Across the five specifications, $r$ ranges from -0.5 to +0.5 in 0.25 increments.

Table 2 includes descriptive statistics from the calibrated model. They suggest that the simulated sample's average prices and participation rates match the observed behavior in the Wisconsin data set reasonably well. Table 2 also suggests that the induced correlations between ($p_1$, $q_j$), $j = 1,2,3$, resulting from the assumed correlations between ($\ln p_1$, $\ln q_j$), $j = 1,2,3$, range from roughly -0.30 to +0.40 across the five specifications.

These five calibrated models were then used to generate estimates of the sample's unconditional expected Hicksian consumer surplus arising from a \$20 price increase and the loss of the first good. These estimates were generated by a simulation algorithm with 500 replications and are used to benchmark the subsequent analysis. Table 3 outlines the main components of the algorithm. All components were coded in Stata 10.0 (Stata Data Analysis and Statistical Software, StataCorp LP, College Station, Texas). A copy of the source code can be obtained from the author upon request.

To evaluate the welfare implications of incomplete demand systems specified as functions of market prices when corner solutions are present,

**Table 1. Specification of Monte Carlo Experiment**

- 500 individuals and 5 goods (including Hicksian composite), i.e., $M = 1$ and $N = 4$.
- Prices (i.e., travel costs):

  Log-normally distributed for $(p_1, q_1, q_2, q_3)$. The means and variances reported in Phaneuf's Table I are used to specify the location and scale parameters separately for each of the four goods. Because Phaneuf does not report the correlation matrix among the four goods' prices, five alternative correlation structures are considered. Each of these correlation structures assumes that $Corr\,(\ln p_1, \ln q_j) = r$ for $j = 1,2,3$, and $Corr\,(\ln q_i, \ln q_j) = 0$ for $\forall i, j = 1,2,3$, $i \neq j$. For each of the five alternative cases, $r$ equals -0.5, -0.25, 0, 0.25, and 0.5. For each case, the generated prices are held fixed throughout the analysis. Hicksian composite good's price, $q_4$, is normalized to one.

- The unobserved determinants of choice vector, ε:
  - Normally distributed with means of zero, standard errors of 2, 7.5, 10, and 20 for goods $x_1, z_1, z_2$, and $z_3$, respectively, and no correlations.
- Structural parameter values for the quasi-linear demand specification:
  - $\boldsymbol{\alpha}$ vector and $\boldsymbol{\Phi}$ matrix were specified using results reported in Phaneuf (1999) as follows:

| | $\alpha_1$ | $\beta_{11}$ | $\phi_{11}$ | $\phi_{12}$ | $\phi_{13}$ |
|---|---|---|---|---|---|
| $x_1$ (North Lake Michigan) | 5 | -0.0860 | 0.0279 | -0.0136 | 0.0599 |
| | $a_i$ | $b_{i1}$ | $c_{i1}$ | $c_{i2}$ | $c_{i3}$ |
| $z_1$ (South Lake Michigan) | 5.5 | 0.0279 | -0.0986 | -0.0135 | 0.0455 |
| $z_2$ (Green Bay) | -4.5 | -0.0136 | -0.0135 | -0.0347 | 0.0444 |
| $z_3$ (Lake Superior) | 1 | 0.0599 | 0.0455 | 0.0444 | -0.1063 |

  - The eigenvalues for the Slutsky matrix are -2.373e-4, -2.571e-2, -0.1208, and -0.1785.
  - The Hicksian own price effects ($b_{11}, c_{11}, c_{22}, c_{33}$) were generated using the parameter estimates reported in Phaneuf's Tables III and IV, the mean travel cost and trip data reported in Table I, and the following formulas:

$$\beta_{11} = -\frac{\bar{x}_1^*}{\bar{p}_1} + \frac{b_{11}^P \bar{y}}{\bar{p}_1^2} + \frac{(\bar{x}_1^*)^2}{\bar{y}} \quad \& \quad p_{ii} = -\frac{\bar{z}_i^*}{\bar{q}_i} + \frac{b_{ii}^P \bar{y}}{\bar{q}_i^2} + \frac{(\bar{z}_i^*)^2}{\bar{y}}, \; i = 1,2,3$$

  where $\bar{p}_i$ and $\bar{q}_i$, $\bar{x}_1^*$ and $\bar{z}_i^*$, $\bar{y}$, and $b_{\bullet}^P$ are sample mean prices, quantities, expenditures on recreation, and parameter estimates reported in Phaneuf.

  Similarly, the Hicksian cross price effects were generated using the same information with the following formulas:

$$p_{1i} = b_{i1} = \frac{b_{1i}^P \bar{y}}{\bar{p}_1 \bar{q}_i} + \frac{\bar{x}_1^* \bar{z}_i^*}{\bar{y}}, \; i = 1,2,3 \quad \& \quad p_{ij} = \frac{b_{ij}^P \bar{y}}{\bar{q}_i \bar{q}_j} + \frac{\bar{z}_i^* \bar{z}_j^*}{\bar{y}}, \; i, j = 1,2,3; i \neq j$$

  The constant terms ($\alpha_1, a_1, a_2, a_3$) were chosen so that the predicted number of trips for each site matched the sample averages.

- Income is not specified explicitly because of the quasi-linear structure of preferences but is assumed to be sufficiently large such that $z_4 > 0$.

**Table 2. Demand Predictions, Average Prices, and Price Correlation Structures from Calibrated Models**

| | | Alternative Specifications | | | | | Reported Values in Phaneuf (1999) |
|---|---|---|---|---|---|---|---|
| | | (1) $r = -0.5$ | (2) $r = -0.25$ | (3) $r = 0$ | (4) $r = 0.25$ | (5) $r = 0.5$ | |
| $x_1$ | | | | | | | |
| | *Quantity Demanded* | 3.35 | 3.25 | 3.15 | 3.02 | 2.88 | 2.99 |
| | *% of Consumers with Strictly Positive Demands* | 38.9 | 40.4 | 41.2 | 41.6 | 41.1 | 52.4 |
| | *Baseline Price* | \$133.89 | \$126.54 | \$124.61 | \$127.79 | \$136.33 | \$123.97 |
| | | (143.20)[3] | (120.19) | (111.57) | (116.73) | (136.36) | (112.41) |
| $z_1$ | | | | | | | |
| | *Quantity Demanded* | 4.13 | 4.14 | 4.18 | 4.25 | 4.40 | 4.5 |
| | *% of Consumers with Strictly Positive Demands* | 40.7 | 41.1 | 41.5 | 42.1 | 42.7 | 46.9 |
| | *Baseline Price* [1, 2] | \$99.26 | \$99.26 | \$99.26 | \$99.26 | \$99.26 | \$93.04 |
| | | (98.08) | (98.08) | (98.08) | (98.08) | (98.08) | (101.75) |
| $z_2$ | *Quantity Demanded* | 1.49 | 1.49 | 1.49 | 1.50 | 1.54 | 1.25 |
| | *% of Consumers with Strictly Positive Demands* | 21.8 | 22.0 | 22.1 | 22.1 | 22.2 | 21.8 |
| | *Baseline Price* | \$138.42 | \$130.18 | \$127.92 | \$129.51 | \$137.24 | \$128.65 |
| | | (133.08) | (111.45) | (104.21) | (105.82) | (121.59) | (109.65) |
| $z_3$ | *Quantity Demanded* | 5.08 | 5.16 | 5.22 | 5.27 | 5.25 | 5.27 |
| | *% of Consumers with Strictly Positive Demands* | 33.5 | 34.2 | 34.5 | 34.7 | 34.2 | 29.5 |
| | *Baseline Price* | \$167.75 | \$157.93 | \$157.75 | 162.05 | \$180.87 | \$163.83 |
| | | (154.25) | (123.36) | (121.82) | (137.35) | (219.53) | (123.36) |
| $Corr(p_1, q_1)$ | | -0.3030 | -0.1981 | -0.0360 | 0.1567 | 0.3358 | NA |
| $Corr(p_1, q_2)$ | | -0.2954 | -0.2190 | -0.0803 | 0.1006 | 0.2931 | NA |
| $Corr(p_1, q_3)$ | | -0.2125 | -0.1177 | 0.0647 | 0.2738 | 0.4049 | NA |
| $Corr(q_1, q_2)$ | | 0.0347 | 0.0337 | 0.0306 | 0.0251 | 0.0154 | NA |
| $Corr(q_1, q_3)$ | | -0.0186 | -0.0169 | -0.0136 | -0.0132 | -0.0159 | NA |
| $Corr(q_2, q_3)$ | | -0.0782 | -0.0279 | -0.0316 | -0.0437 | -0.0468 | NA |

1 Descriptive statistics based on 500 simulations.
2 Recall that prices are fixed throughout for each of the 5 separate specifications.
3 Standard errors across the 500 simulated individuals are reported in parentheses.

**Table 3. Description of Simulation Algorithm for Constructing Hicksian Consumer Surpluses from the Specified Model**

1. Beginning with 1st individual, simulate $(\varepsilon_1, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3)$ using a pseudo random number generator.

2. Solve for the virtual prices ($\xi^0$) and demands ($z_1^0$, $\boldsymbol{x}^0$) for the baseline state.
   A. Begin by solving for the implied notional demands conditional on all market prices.
   B. If all demands are nonnegative, proceed to step 3.
   C. Otherwise, partition all commodities into groups with strictly negative (hereafter group 1) and nonnegative demands (group 2). For the goods with negative demands, solve for the virtual prices that drive their demands to zero [see Phaneuf (1999) for details].
   D. Test whether all calculated virtual prices are less than or equal to their market prices. If not, place the subset of goods with virtual prices greater than their market prices in group 2 and the remainder in group 1. For the goods in group 1, solve for the virtual prices that drive their demands to zero. Repeat until all goods in group 1 have virtual prices less than or equal to their market prices.
   E. Using the virtual prices constructed from step D and the market prices for the goods from group 2, recalculate the implied demands for all goods. Go to step B above.

3. Solve for the virtual prices ($\xi^1$) and demands ($z_1^1$, $\boldsymbol{x}^1$) for the changed state.
   A. Begin by solving for the implied notional demands conditional on all market prices.
   B. If all demands are nonnegative and the analyst is considering the $20 price increase scenario, proceed to step 4.
   C. Partition all commodities into groups with strictly negative (hereafter group 1) and nonnegative demands (group 2). For the loss of $x_1$ scenario, place $x_1$ in group 1 regardless. For the remaining goods with negative demands, solve for the virtual prices that drive their demands to zero [see Phaneuf (1999) for details].
   D. Test whether all calculated virtual prices are less than or equal to their market prices. If not, place the subset of goods with virtual prices greater than their market prices in group 2 and the remainder in group 1. For the site loss scenario, always place $x_1$ in group 1. For the goods in group 1, solve for the virtual prices that drive their demands to zero. Repeat until all goods in group 1 have virtual prices less than or equal to their market prices.
   E. Using the virtual prices constructed from step D and the market prices for the goods from group 2, recalculate the implied demands for all goods. Go to step B above.

4. Conditional on $(\varepsilon_1, \tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \tilde{\varepsilon}_3)$, the Hicksian consumer surplus is:

$$CS = (\alpha + \varepsilon)^{\mathrm{T}} (\xi^0 - \xi^1) + \tfrac{1}{2} (\xi^0)^{\mathrm{T}} \Phi(\xi^0) - \tfrac{1}{2} (\xi^1)^{\mathrm{T}} \Phi(\xi^1).$$

5. Repeat steps 1-4 500 times. The mean across the 500 simulations is an unbiased point estimate for the unconditional expectation for the individual's Hicksian consumer surplus.

6. Repeat steps 1-5 for all 500 simulated observations. The mean across the 500 mean Hicksian consumer surpluses is the unbiased estimate of the sample's unconditional expected Hicksian consumer surplus.

a second simulation algorithm was developed. As Table 4 describes, the demands for all goods were first simulated using the correctly specified structural model. The simulated demand for $x_1$ was then modeled as a linear function of the four goods' normalized prices and a constant term [i.e., the same structure as (15) except for market prices replacing virtual prices]. To account for the empirical regularity that some individuals in the simulated sample do not consume $x_1$, a censored regression model with a normal distribution for the unobserved determinants of choice was employed. The parameter estimates from the censored regression model were then used to construct welfare estimates for the three policies. These procedures were replicated 500 times and the results are reported in Table 5.

**Table 4. Description of Simulation Algorithm for Constructing Hicksian Consumer Surplus Estimates from the Misspecified Econometric Model**

1. Generate simulated demands for all four sites using the correctly specified model, the simulated prices, and steps 1 and 2 in Table 3.

2. Estimate censored regression model with the quantity demanded of $x_1$ as the dependent variable and market prices for all four goods as covariates, i.e.;

$$x_1^* = \begin{cases} \alpha_1 + \beta_{11} p_1 + \sum_{l=1}^{N-1} \phi_{1i} q_i + \varepsilon_1 & if f x_1^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_1 \sim \mathrm{N}(0, \sigma^2).$$

Save the coefficient $(a_1, b_{11}, c_{11}, c_{12}, c_{13})$ and scale ($s$) parameter estimates.

3. Beginning with the first individual, simulate $\varepsilon_1$ using a pseudo random number generator and the $\mathrm{N}(0, s^2)$ distribution. Using the equations from step 2 above with the estimated parameter values, calculate the demand for $x_1$ at $p_1^0$, $x_1^0$.

4. If $x_1^0 = 0$, the Hicksian consumer surplus for both policy scenarios is 0. Go to step 7.

5. If $x_1^0 > 0$, use the equations from step 2 above with the estimated parameter values, calculate the demand for $x_1$ at $p_1^1$, $x_1^1$. For the loss of $x_1$ scenario, $x_1^1$ must equal zero. Conditional on $x_1^1$, solve for the corresponding virtual price, i.e., the price that drives the consumer's demand to $x_1^1$:

$$\xi_1^1 = -\frac{1}{\hat{\beta}_{11}} \left[ a_1 + \sum_{l=1}^{N-1} c_{1l} q_l + \varepsilon_1 - x_1^1 \right]$$

6. For each simulation and policy scenario, the Hicksian consumer surplus now takes the form:

$$CS' = \begin{cases} \frac{1}{2}(x_1^0 - x_1^1)(p_1^0 - \xi_1^1) + x_1^1 (p_1^0 - \xi_1^1) & \text{if } x_1^0 > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Save simulated welfare estimate to an auxiliary data set.

7. Replicate steps 3-6 500 times. The mean across the 500 simulations serves as an estimate of the individual's unconditional expected Hicksian consumer surplus. Repeating this procedure for all 500 observations and averaging generates an estimate of the sample's unconditional expected Hicksian consumer surplus for the loss of $x_1$.

8. Go to step 1 and repeat the above procedure 500 times.

**Table 5. Parameter Estimates from Misspecified Econometric Model**

| | Alternative Specifications | | | | | |
|---|---|---|---|---|---|---|
| | (1) $r = 0$-.5 | (2) $r = -0.25$ | (3) $r = 0$ | (4) $r = 0.25$ | (5) $r = 0.5$ | *Specified Values* |
| $a_{11}$ | 4.75[1] | 2.31 | 1.41 | 1.15 | 2.02 | 5 |
| | (2.13)[2] | (1.66) | (1.34) | (1.08) | (0.929) | |
| | [-1.99,11.93][3] | [-3.02,8.13] | [-3.03,5.27] | [-2.28,4.51] | [-1.09,4.55] | |
| $b_{11}$ | -8.63e-2 | -7.05e-2 | -6.32e-2 | -5.80e-2 | -5.11e-2 | -8.60e-2 |
| | (1.09e-2) | (8.41e-3) | (7.04e-3) | (6.47e-3) | (6.62e-3) | |
| | [-0.125,-5.68e-2] | [-0.102,-4.63e-2] | [-8.66e-2,-4.12e-2] | [-7.91e-2,-3.74e-2] | [-7.19e-2,-3.13e-2] | |
| $c_{11}$ | 7.75e-3 | 1.23e-2 | 1.46e-2 | 1.56e-2 | 1.45e-2 | 2.79e-2 |
| | (6.19e-3) | (5.38e-3) | (4.98e-3) | (4.82e-2) | (5.11e-3) | |
| | [-1.10e-2,2.41e-2] | [-2.90e-3,2.71e-2] | [9.44e-4,2.91e-2] | [1.31e-3,3.12e-2] | [-1.39e-3,2.94e-2] | |
| $c_{12}$ | -5.12e-3 | -2.24e-3 | -1.25e-3 | -1.51e-3 | -5.83e-3 | -1.36e-2 |
| | (4.31e-3) | (4.58e-3) | (4.59e-3) | (4.34e-3) | (4.41e-3) | |
| | [-2.18e-2,6.74e-3] | [-1.89e-2,1.04e-2] | [-1.80e-2,1.04e-2] | [-1.63e-2,9.77e-3] | [-2.22e-2,3.78e-3] | |
| $c_{13}$ | 9.05e-3 | 1.37e-2 | 1.55e-2 | 1.56e-2 | 9.83e-4 | 5.99e-2 |
| | (4.03e-3) | (4.46e-3) | (4.31e-3) | (3.93e-3) | (2.30e-3) | |
| | [-2.26e-3,2.17e-2] | [1.75e-3,2.60e-2] | [4.19e-3,2.63e-2] | [3.53e-3,2.52e-2] | [1.06e-3,1.55e-2] | |
| $s$ | 9.82 | 9.14 | 8.70 | 8.34 | 8.29 | 2 |
| | (0.503) | (0.439) | (0.407) | (0.397) | (0.402) | |
| | [8.31,11.49] | [7.93,10.53] | [7.66,10.14] | [7.34,9.71] | [7.03,9.65] | |

[1] Mean for the 500 simulations.
[2] Standard error for the 500 simulations.
[3] Range over the 500 simulations.

Table 5 suggests that using market prices in place of virtual prices can result in parameter estimates that are much larger than what would be expected from a correctly specified model estimated through maximum likelihood. In particular, the mean estimates of the own price effect, $b_{11}$, are biased towards zero for all specifications except (1) (when $r = -0.5$), while all estimates of cross price effects are consistently biased towards zero. Except for the specification (1), the estimates of the intercept term, $a_1$, are significantly biased downwards, and all standard error estimates ($s$) are consistently and significantly biased upwards.

Turning to the welfare estimates reported in Table 6, one finds that the mean percentage biases arising from using market prices in place of virtual prices are negligible for the \$20 price increase scenario. For all five correlation specifications, the percentage bias is less than 4 percent in absolute value. For the site loss scenario, however, the percentage biases vary substantially across the five specifications and are as large as 35.2 percent in absolute value. In general, the biases are smallest when prices are orthogonal and largest when prices are strongly correlated. These findings suggest that when: 1) corner solutions are prevalent, 2) prices are highly correlated, and 3) the analyst is attempting to evaluate the welfare implications of relatively large price changes, using market prices in place of behaviorally relevant virtual prices can result in substantially biased policy inference.

## Conclusion

This paper uses both an analytical model and a Monte Carlo experiment to illustrate how corner solutions raise difficulties for empirical applications of the incomplete demand system framework. The analytical model highlights the potential biases arising from the standard empirical practice of applying the incomplete demand system framework to disaggregate consumption data. These biases arise during the estimation of the structural parameters and the calculation of Hicksian welfare measures. For relatively small price changes, the results from the Monte Carlo experiment suggest that these biases may not significantly compromise the integrity of the derived welfare measures. However, when the analyst considers scenarios involving large price changes such as those arising from the elimination of a good, these biases may be substantial. These results cast doubt on the ability of the incomplete demand system framework to recover consistent welfare measures for site loss scenarios in the recreation demand context.

Although corner solutions raise similar difficulties for both complete and incomplete demand system applications, estimation and welfare calculation techniques to consistently handle them only exist in the complete demand system context (e.g., Wales and Woodland 1983, Lee and Pitt 1986, von Haefen, Phaneuf, and Parsons 2004). In the incomplete demand system framework, similar techniques have not been developed and, as emphasized above, raise formidable challenges. Thus the central implication of this paper is that, barring significant modeling innovations, the only fully consistent approach to recovering welfare measures for policies that impact the prices of a subset of goods entering preferences is through the complete demand system framework.

In practice, however, empirically estimating a complete demand system may not be possible due to data limitations, modeling complexities, or the inappropriateness of necessary separability or aggregation assumptions. In these cases, the analyst is left with a difficult decision of how best to proceed, and any choice can be criticized as ad hoc and inherently second best. One possible strategy is discussed here in closing. Instead of modeling consumer demand with traditional RUM-based microeconometric models that consistently account for interior and corner solutions, the analyst could instead model *expected* consumer demand. As discussed in von Haefen and Phaenuf (2003), this representative consumer approach is implicitly used in the count data demand literature (e.g., Ozuna and Gomez 1994, Gurmu and Trivedi 1996) where the behavioral restrictions implied by economic

**Table 6. Welfare Estimates**

| | Alternative Specifications | | | | |
|---|---|---|---|---|---|
| | (1) $r = -0.5$ | (2) $r = -0.25$ | (3) $r = 0$ | (4) $r = 0.25$ | (5) $r = 0.5$ |
| **$20 Increase in Price of Good 1** | | | | | |
| *Specified Model* | | | | | |
| *Welfare Estimate* | -$62.60[1] | -$60.31 | -$58.03 | -$55.23 | -$52.40 |
| *Misspecified Econometric Model* | | | | | |
| *Welfare Estimate* | -$60.13[1] | -$58.77 | -$56.95 | -$54.58 | -$52.50 |
| | (4.16)[2] | (3.88) | (3.69) | (3.54) | (3.53) |
| | [-$72.87,-$48.98][3] | [-$70.16,-$48.59] | [-$67.35,-$46.98] | [-$65.25,-$45.32] | [-64.17,-$42.71] |
| *Percentage Bias* | 3.9 | 2.6 | 1.9 | 1.2 | -0.2 |
| **Loss of Good 1** | | | | | |
| *Specified Model* | | | | | |
| *Welfare Estimate* | -$358.22 | -$306.80 | -$272.35 | -$241.33 | -$222.28 |
| *Misspecified Econometric Model* | | | | | |
| *Welfare Estimate* | -$260.29 | -$285.66 | -$290.15 | -$286.04 | -$300.91 |
| | (35.90) | (35.05) | (31.74) | (29.38) | (37.88) |
| | [-$398.69,-$173.71] | [-$422.09,-$194.03] | [-$393.21,-$211.27] | [-$374.61,-$197.20] | [-$446.00,-$212.29] |
| *Percentage Bias* | 27.3 | 6.9 | -6.5 | -18.5 | -35.4 |

[1] Mean for the 500 simulations.
[2] Standard error for the 500 simulations.
[3] Range over the 500 simulations.

theory are applied to the system of expected demands. Since expected demands are strictly positive, the difficulties raised by corner solutions are avoided. Whether such a representative consumer approach generates more accurate welfare measures relative to RUM-based incomplete demand system models is an empirical question for which the answer will likely vary from application to application. For example, Table 7 reports additional results from the Monte Carlo experiment that suggest log-linear representative consumer incomplete demand models, estimated via nonlinear least squares, generate average welfare estimates with slightly more bias than those reported in Table 6. By contrast, empirical results reported in von Haefen and Phaneuf suggest that welfare measures derived from representative consumer and traditional RUM-based microeconometric models can be similar in magnitude. In light of these conflicting results and the lack of theoretical guidance on how best to proceed, it is probably best in such second-best settings for analysts to consider alternative estimation strategies, such as the incomplete RUM-based and representative consumer demand system frameworks. The derived welfare measures from these models could then be used to identify a range of potential (albeit flawed) estimates that may nonetheless be sufficiently informative for policy purposes.

**Table 7. Welfare Estimates from Count Data Models**

| | Alternative Specifications | | | | |
|---|---|---|---|---|---|
| | (1) $r = -0.5$ | (2) $r = -0.25$ | (3) $r = 0$ | (4) $r = 0.25$ | (5) $r = 0.5$ |
| **$20 Increase in Price of Good 1** | | | | | |
| *Specified Model Welfare Estimate* | -$62.60 [1] | -$60.31 | -$58.03 | -$55.23 | -$52.40 |
| *Misspecified Log-Linear Econometric Model Welfare Estimate* | -$58.79 [1] | -$58.22 | -$57.08 | -$55.11 | -$53.08 |
| | (4.06) [2] | (3.89) | (3.75) | (3.66) | (3.60) |
| | [-$70.82,-$47.82] [3] | [-$68.61,-$47.73] | [-$67.60,-$46.88] | [-$66.59,-$45.13] | [-$65.01,-$43.21] |
| *Percentage Bias* | -6.1 | -3.5 | -1.6 | -0.2 | 1.3 |
| **Loss of Good 1** | | | | | |
| *Specified Model Welfare Estimate* | -$358.22 [1] | -$306.80 | -$272.35 | -$241.33 | -$222.28 |
| *Misspecified Log-Linear Econometric Model Welfare Estimate* | -$251.78 | -$289.97 | -$317.44 | -$331.60 | -$359.57 |
| | (32.63) | (37.61) | (41.87) | (43.92) | (53.78) |
| | [-$365,-$165] | [-$400,-$191] | [-$446,-$211] | [-$482,-$224] | [-$573,-$236] |
| *Percentage Bias* | -29.7 | -5.5 | 16.6 | 37.4 | 61.8 |

[1] Mean for the 500 simulations.
[2] Standard error for the 500 simulations.
[3] Range over the 500 simulations.

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