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Supply Chain and Network Performance: Metrics for Profitability, Productivity, and Efficiency

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Abstract

The architecture of the firm involves determination of a boundary that encompasses the functions managed by the firm. The past decade has seen substantial reorganization of firms where vertical or horizontal integration has been unbundled into weaker forms of collaborations including value chains and networks. This observation has forced a re-conceptualization of the boundaries of the firm to incorporate such collaborations. These collaborations are virtual and highly dynamic. They emerge and persist when two conditions are met. First, they must enable generation of greater value than might be attained through independent operation and anonymous transactions through markets. Second, the resulting growth must be shared with members in a way that retains their participation. Each of these conditions can be verified only if performance of the collaboration can be established. This paper recognizes the need for such "metrics of performance". While conceptual approaches have been studied in the management literature, this paper considers from theoretic perspectives these issues and derives measures of the performance of the overall collaboration as well as of the participating enterprises. The paper presents a framework that can be applied to both vertical and horizontal collaborations as found in supply chains and networks. The paper offers suggestions on empirical methods for estimation of measures derived.

Keywords: *Networks, Collaboration, metrics, productivity, efficiency*

1. Introduction

Three trends are widely cited as having fundamentally changed the business environment during the 1990's. These changes were so fundamental that they induced substantial opportunity for change in the organization of private enterprise. The trends were: 1) dramatic technological change affecting information acquisition, analysis, and dissemination for control of complex systems, 2) institutional change that led to substantial reduction in transaction costs enabling new methods of coordination, and 3) technological change in industrial processes that enabled flexibility of product composition and enhanced responsiveness to changes in the external environment. Weaver (2008) presents a review of these forces of change and their implications for innovation in the food system. Together, these trends facilitated a substantial change in the organization of enterprise. They enhanced the feasibility of voluntary cross-enterprise coordination allowing access to the net benefits of vertical integration or horizontal coordination without joint asset ownership. At the same time, by radically shifting the historical economics of vertical and horizontal integration, these trends induced a substantial wave of "unbundling" of peripheral functions to specialist enterprises. Finally, they enabled enterprises to shift from "push" supply to consumer-oriented "pull" strategies. Competition forged these possibilities into imperatives. By enabling rapid response to consumer demands for highly differentiated goods, these trends supported a substantial augmentation of welfare.

The food system has not been exempt from these forces or their imperatives for change. Of interest in this paper is the measurement of performance of the new virtual organization of enterprise that continues to emerge in this new environment. In particular, these trends have induced changes in the internal organization of firms, see e.g. Brynjolfsson and Hitt (2000), as well as substantial re-specification of the boundaries enterprises. Within the parlance of the economics of efficiency and productivity analysis, the trends fundamentally changed what we think of as a decision management unit (DMU).

Economists have traditionally focused on technological change that has implications for cost reduction for operations managed by an enterprise. This perspective is consistent with the competitive, autonomous firm that is push-oriented. In this case, cost-reducing technical change is the firm's only option for managing profit margins over time. In notable contrast, competition imposed an imperative that enterprises adapt in three ways to these forces of change. First, enterprises have faced an imperative for strategic re-orientation toward the consumer. Second, enterprises have been challenged to pursue strategic organizational change to specialize and often reduce the scale of their fixed asset position, see Powell (1990). Third, these newly specialized enterprises have by competition been driven to engage in collaborations with both peer enterprises and vertically aligned partners. The result has been the emergence of a new fabric of highly interdependent "relational enterprises" that define a new paradigm for private enterprise relative to the old paradigm of independent, silo enterprise coordinated through anonymous market transactions. From the perspective of transaction co-ordination, these strategic organizational changes have defined new means of co-ordination and methods of integrating activities, decisions, and economic performance across enterprises. Together, these changes have defined new organizational approaches to co-ordination of economic transactions through integration of enterprise functions in new ways to redefine what is known in the economics literature as the boundaries of the firm, see Coase (1937).

This paper considers from theoretic perspectives these issues and derives measures of the performance of the overall collaboration as well as of the participating DMUs. The paper considers both vertical and horizontal collaborations as found in supply chains and networks. The paper offers suggestions on empirical methods for estimation of measures derived. To sharpen focus, DEA methods are given greatest attention. In so doing, the paper clarifies the role in efficiency estimation that DMUs engaged in collaboration must play. While traditional methods would choose a group of DMUs facing a common technology, we examine the role that partnering DMUs engaged in collaboration can play and how traditional approaches should be modified.

2. Observations and interpretation

Under the old, commodity paradigm, inflexibility of technology meant that investment in specific equipment and technologies was required for production, implying that substantial risk was associated with product differentiation. At the same time, high information and communication costs implied high search costs that forced firms to standardize products to reduce such costs as a condition of competitiveness. With standardized products, sales could be coordinated through markets where buyer and seller are anonymous. The welfare cost of this supply-oriented "push" strategy is that consumers are forced to consume generic, standardized products rather than specific, differentiated products that might provide increased utility. In this paradigm, the producing enterprise had little direct relationship with the consumer.

In contrast, a new paradigm has been defined over the past decade by information and communication technological change and a shift to flexible production technologies, traditional push-oriented enterprises have been enabled to collect, analyze, and adapt to market data rapidly. As this became feasible, the potential for strategic re-orientation of the enterprise from supply-orientation to a consumer-orientation (from a "push" orientation to a "pull" orientation) became an imperative of competition. Not only could revenue be dramatically increased as consumers were offered their most highly valued product attributes, but costs of supply could also be dramatically reduced through use of information to ensure that the right product was available at the right place, at the right time, and at the right price. Inventories could be dramatically reduced and became seen as evidence of error in forecasting, a cost borne as a primitive method of coping with uncertain markets.

With respect to strategic organizational change, this new economic playing field has defined significant opportunities to enhance productivity, efficiency, profitability, and economic growth through strategic organizational change of the functions managed within and across firms. In particular, this opportunity for re-organization has allowed firms to shed subsidiary functions to specialized firms (see e.g. Birch (1987), Baker and Hubbard (2003)) leaving the firm itself both more specialized and in many cases smaller in scale. Opportunity of these organizational changes followed from substantial reduction in transactions costs offered by new information and communication technologies. In particular, under the old paradigm when transactions costs for external acquisition and management of services and functions were high, such services and functions were integrated into the firm. Given substantially reduced transaction costs associated with the new paradigm, enterprises have been enabled to shed (outsource) many of functions. This has resulted in substantial gains in productivity and cost reductions associated with specialization. Further, the shedding activities has spawned demand for wholly new service organizations that through specialization can deliver service levels not possible when such functions were embedded within diversified firms.

Having shed functions through strategic organizational change, newly specialized firms have been enabled by information and communication technical changes to procure outsourced services through relational rather than market mechanisms. These relational mechanisms have taken many forms from informal to formal. They have involved, on the one hand, what might be distinguished as "outsourcing" that is coordinated through traditional command-control implemented with contracts. On other hand, organizational change has often moved toward collaborations across enterprises involving interdependence such as merged operations and functions, planning, problem-solving, or strategy. While such functions might be available through market mechanisms, this alternative method of coordination defined here as "cross-enterprise collaboration" (CEC) has often been found to offer increased specificity of service or product quality and reduced search costs and risk.

These collaborations have spanned both vertically and within horizontal alliances across peer enterprises to efficiently optimize value generated for the consumer. Through collaboration, management of outsourced functions has often proven both feasible and more effective than management of similar functions as peripheral functions within an enterprise. Across peer enterprises, new found specialization has demanded new ways of accessing peripheral services once integrated in the firm. These demands have often spawned horizontal collaboration across peer enterprises to establish shared services such as R&D, marketing, or accounting. In the vertical dimension, strategic re-orientation has been implemented by anchoring within "value chains" through vertical collaborative alliances and relationships that connect the series of firms from raw resource to final goods and the consumer.

Through these collaborative organizational forms, the walls of the traditional firm have been peeled open, leaving a new virtual enterprise strategically linked to the consumer. Firms at any position along these vertical chains or within horizontal networks are able to strategically re-orient toward the consumer despite their position. Thus, through vertical collaboration a metal castings enterprise, isolated as a silo under the old paradigm, can re-orient to be responsive to changing consumer preferences for automobile functions. Through these vertical collaborations, firms substitute information for inventories and poorly fitting or obsolete products.

Consider the producer's interest in having product forms that satisfy specific consumer needs. Each enterprise along the vertical chain of processing, transformation, and shipping that takes a product from initial processor to consumer has a different interest in responding to consumer signals concerning product form and attributes. The challenge is to coordinate these enterprises to respond to the consumer signals and share the value harvested. The recent paradigm shift has enabled this coordination. The paradigm revolution of strategic re-orientation and re-organization of private enterprise continues to permeate the economy providing substantial competitive advantage to firms that participate in value chains, and disadvantaging firms and enterprises that continue to pursue commodity-oriented "push" strategies.

While considerable attention has been paid by the management literature to so-called "metrics of performance", this literature has not taken on the question of measurement of efficiency and related productivity aspects of the performance of the new virtual enterprises resulting from simple outsourcing to the highly relational organizational structures I call "cross-enterprise collaborations". Of particular interest is to assess the implications of these new types of DMUs for efficiency and productivity analysis and to consider new measures capable of supporting a consideration of the sources of efficiency and productivity results they generate. More specifically, it is of interest to establish empirical means of identifying the relative contributions of particular DMUs engaged in collaborations, as well as the overall performance of the collaboration.

Not surprisingly, past literature offers the foundation and starting point for this assessment. Within this context, considerable literature has attempted to break open the "black box" conceptualization of a DMU implicit in early efficiency and productivity analysis. Within the nonparametric literature, Fare and Whittaker (1995) considered the internal structure of functions within a DMU based on a reference set of DMUs. Färe and Grosskopf (1996a) considered the problem for a simple fixed path network where private good product allocation along vertical stages is fixed, see Figure 1. They consider a simple vertical network of sequentially linked nodes and allow for external inputs, intermediate outputs used as inputs, and final output. Later, Färe and Grosskopf (1999, 2000) allowed for allocation between intermediate use and final output to be endogenous. Of interest for them was measurement of sub-DMU (SU) as well as network efficiency and productivity. In both cases, they modeled a disaggregation of the input-output process of a particular DMU, and evaluated its performance against an identically structured set of peers. Lothgren and Tambour (1999) adopted the Färe and Grosskopf (1996a) organizational structure and recognized that the interdependence implicit in a vertical series of SUs implies that output flows must be consistent with the input flows between transacting SUs. Further, they noted selection of weighting for reference DMUs should be consistent with the observed flow structure. Castelli, et al. (2001) considered the relative efficiency of SUs within a DMU and defined a set of DEA-like performance measures relative to 1) the set of homogeneous SUs across a set of DMUs, 2) the set of heterogeneous SUs spanning all SUs operated in the set of DMUs under consideration, and 3) the set of SUs that produce output necessary a particular output. Castelli, et al. (2004) shows that consideration of the internal structure of SUs within a DMU leads to different relative performance estimates. Lewis and Sexton (2004) consider use of DEA to gauge the relative contributions of SUs to overall DMU performance and incorporate returns-to-scale. Further, they expand the reference set for a particular DMU to include hypothetical SUs identified from like SUs in other DMUs. Further discussion of these contributions follows.

3. The conceptual framework

To consider the performance of the new virtual enterprises spawned by outsourcing and cross-enterprise collaboration, we first articulate the salient features of the organization of these enterprises. Next, we specify an activity analysis form for the new underlying "production" technology implied by these organizational forms. Here, it is important to note that we speak of "production" as production of value. Thus, we deviate from conventional vocabulary and extend it to consider the "technologies" of procurement, marketing, transactions, and management.

Salient features of virtual integration

Traditionally, the boundaries of the firm have been set through consideration of asset ownership as a sufficient condition for identifying the frontier of the "reach" of command-control management. Within the context of virtually integrated enterprises, it is now clear that asset ownership is not a reasonable basis for resolving these boundaries. Instead, we focus on the boundaries of the virtual enterprise as encompassing the set of enterprises that have joint interests that are managed. In the case of outsourcing, the joint interests in product specification are regulated through explicit rules and incentives established either by a dominant agent or gaming among more equally powerful agents. In this sense, outsourcing is only a short-step away from command-control based on asset ownership. Importantly, however, outsourcing allows for alternative scale as well as specialization economies.

Cross-enterprise collaboration (CEC) involves integration of management functions of coordination (planning, problem-solving), motivation (incentive design and implementation through performance monitoring and analysis), as well as of processes (production and marketing functions). Integration of processes can result from direct integration, e.g. quality control, product design, process design, cross-docking. Alternatively, processes can be integrated through joint dependence on shared asset services (physical capital, financial, human capital (knowledge, know-how, etc.). Through alignment of goals and objectives, enterprises can focus their supplier set while reducing product verification and search costs. While enterprise merger would result in merger of all functions, processes, and assets, CEC only integrates selective elements of enterprise function and does not integrate asset ownership.

Three types of CEC are often cited in the literature: 1) asset service sharing, e.g. information-sharing; 2) tactical integration, e.g. joint modification of traditional processes such as quality control, accounting, process planning and forecasting; and 3) strategic integration, e.g. joint design of decision-making structure and extent of decentralization, operational rule and incentive design, functional specialization and composition, planning, problem-solving, performance monitoring. A substantial management literature considers these possibilities.

However, of interest here is to identify the implications of these characteristics of virtual enterprise for performance measurement. From this perspective, two types of implications are of interest: 1) implications for the boundaries of the DMU and the reference set relevant for analysis of performance and 2) implications for the types of products and services involved in the processes that create value. With respect to the first, it is clear that the nature and structure of interaction of SUs constituting a virtual DMU (VDMU) deserve attention. Further, the VDMU will be composed of DMUs defined by asset ownership, so the nature and structure of their interaction must be considered. With respect to types of products and services, the perspective must be expanded beyond simple private goods with single attributes.

Transactions

With respect to the nature of SUs, consider first the question of transactions. As outsourcing and CEC offer dramatically different modes of transactions, it is logical to consider their introduction into the model. Traditionally, we have not considered transactions in performance evaluation done with DEA or SFA. Castelli et al. (2004) suggest the intuitive approach of specifying transactions management as accomplished by a SU when transactions are productive. Given that clear objectives of relational transactions are the reduction of transactions costs and enhancement of value creating processes, it is of interest to incorporate transactions in models that attempt to gauge performance of VDMUs. A further aspect of transactions is the nature of dependency across supplier SUs. Here, VDMUs typically reflect multiple sourcing being replaced by more focused sourcing where risk is managed through parallel process organization, see Figure 1, rather than through diversification.

From the perspective of boundaries of enterprise, CECs appear to shift the boundaries to encompass collaborating DMUs while outsourcing appears to leave the boundaries of DMUs in place by changing the nature of transaction operations.

Inter-enterprise effects

Within a vertical chain of processes perhaps embedded within a DMU, inter-enterprise effects can be represented as intermediate goods flowing across functions of SUs. The motivation for collaboration across DMUsoften follows from the existence of inter-enterprise effects that require management that can not be attained through market mechanisms and for which it may be undesirable to manage through asset integration and command-control mechanisms. These inter-enterprise effects can be positive or negative, desirable or undesirable. These could also involve what Lewis and Sexton call “reverse quantities” where productivity declines with their increase.

In many cases, these inter-enterprise effects may be controllable or mitigated by the source or receiving SU. In others, they may be nondiscretionary from the perspective of the source SU, and weakly disposable from the perspective of the receiving SU.

Joint resources

An important distinguishing characteristic of CECs is the possibility of a productive role for joint resources. Service flows from these resources could be proprietary, or private in nature forcing competitive consumption across DMUs or SUs encompassed by a VDMU. These private service flows could be allocatable or nondiscretionary. In addition to private flows, service flows could be quasi-public in nature.

Joint operations

Clearly, an important aspect of CEC performance is the role of joint operations and services. Central among such services are those of management including strategy development and implementation, problem identification and solution, and forecasting and planning. Quality control is often cited as a prime example. Rather than each SU implementing independent QC processes, CEC organizational forms offer the possibility of joint operation of QC and alignment of standards, etc.

Implications of these salient features of collaborative organizations for performance evaluation can now be broadly summarized motivating the need to consider the performance of: 1) subsets of SUs within a collaboration, 2) transactions, and 3) forms of collaboration.

Organization of collaboration

While efficiency and productivity analysis traditionally follows the neoclassical focus on the firm as a blackbox, it is of interest here to consider the organizational forms taken on by collaborations. While outsourcing has often been cited as involving only market-based transactions to coordinate across DMUs, it is clear that outsourcing most often involves managed relationships guided by a common understanding of core business values, expectations, and goals as well as pre-agreed plans or rules for operation of the relationship, management of problems, and sharing of returns. Plans and rules can be formalized into contracts, or simple verbal agreements that emerge either from dominant agent specifications or negotiation across agents. In the dominant agent case, the yardstick competition literature is relevant to these forms of collaboration.
To proceed, it is useful to distinguish key structural architectures for collaboration to provide a basis for examination of how that organizational form might impact efficiency and productivity. We distinguish three process types for SUs: transactions (procurement, marketing), transformation, and management. However, while differing in operational function, we argue they can be described in a similar fashion at a conceptual level. In each case, we define these processes as being operated by sub-DMUs at a *network node*. While transactions will be conceptualized as not transforming the physical properties of a product or service, their transformation of property rights will be noted as a transformation of quality attributes of the product.

4. The Model

To proceed, we extend notation used in past literature, e.g. Castelli et al. (2004). Define $u \in U$ as the set of homogenous DMUs. We define a DMU u as a decision-making unit, that is a unit that coordinates decisions across SUs, not necessarily holding their assets. We define $d \in D$ as the set of all SUs contained within U, and $d \in D(u) \subset D$ as the set of SUs in DMU u. To allow for specification of various organizations of networks of SUs, we define $l=1,..L$ as an index of the horizontal layer to which a SU d belongs, and $s=1,...S$ as an index of the vertical stage to which a SU belongs. To allow for strictly parallel chains, we use $r=1,...R$ to indicate a replicate of a particular layer. Thus, each uniquely configured layer can be assigned an arbitrary layer number. Based on the stage index, we define $d(s) \subset d$ as the set of SUs operating at stage s, and $d(l) \subset d$ as the set of SUs operating on layer l.
To identify each SU, we use a superscript to indicate position of the SU, and subscripts to indicate origin and destination. We note that each d is uniquely identified by the double (s,l). To economize on this notation, where possible we use a superscript d to indicate an arbitrary SU. We distinguish a particular SU using the notation d_j, or d_k. We note the position d_j as (s_j, l_j) and use only the indexes in sub- and super-scripting other variables. Finally, to consider performance across SUs, we define $\hat{D}$ as the set of all subsets of homogeneous SUs. We note $\hat{d} \in \hat{D}$ as a class (reference) group of homogeneous SUs. With this notation, we can define outsourcing as procurement by DMU u from $d \in D(u)$.
We define an $m \times 1$ vector y of outputs, indexed with typical element y_i. We define an $n \times 1$ vector x of variable inputs obtained externally from the class $\hat{d}$ of interest, indexed with typical element x_h and use a superscript to indicate the SU position. Similarly, we define two types of intermediate products. We use z subscripted with output indexes, e.g. z_i, as an intermediate output. To account for use and supply directions, we define z^j_k as a vector of intermediate output use by d_j originating from d_k. Similarly, a tilde is added to indicate intermediate supply from d_j to d_k, i.e. $\tilde{z}^j_k$. To indicate supply of final outputs, we use m_i to indicate output allocated as supply external to the class $\hat{d}$ of interest. Note this differs from the convention adopted by Färe

and Grosskopff who note both origin and destination. Summarizing, the *(n+2m) x 1* vector of inputs used by SU *d(s,l)* is [x^d, z^d], the *m x1* output vector is y^d , and the *m x 1* vector of intermediate output supplied to other SUs, $\tilde{z}^d$.
Within this notation, for private goods that are allocatable to external, peer, or forward stage uses, we assume that the allocation exhausts available supply, noting of course that inventories could be added to this notation with possible interesting complications. Thus, the following physical balance conditions are assumed to hold for each d_j :

(1) $$y^j = m^j + \tilde{z}^j \quad \forall\, d_j \in \hat{d}$$

(2) $$z^j \le \sum_{d \subset D} \tilde{z}_d^j$$

(1) clarifies that each output from layer l_j , stage s_j can be allocated externally with respect to the class $\hat{d}$, and across SUs at all forward stages and all layers. (2) requires that at node d_j use does not exceed available supply of intermediate good.

Using this notation, Figure 1 presents a set of possible network configurations that will serve as a basis for consideration. We distinguish the following types:

1) Vertically aligned series
2) Parallel series
3) Cross-layer sourced
4) Peer sourced
5) Shared service collaboration

By comparison, we note particular configurations considered in past literature. We note that organizations 1) - 3) could be coordinated through anonymous markets, asset integrated DMUs, or collaborative DMUs. To describe these networks graphically, we denote SUs as network nodes indicated with an open diamond, and external linkages and flows are indicated with an arrows.
We are now prepared to introduce a method to compute the efficiency performance of networks. Before doing so, we note past approaches in part to motivate our approach. Färe and Grosskopf focused on each SU, or node, computing intensity for each SU at that node in each DMU under consideration, and a radial reduction measure of technical efficiency of the each SU based on the resulting reference frontier. Underlying this approach is the notion that SUs in the same stage *s* are homogeneous and face a common frontier. Färe and Grosskopf also consider network efficiency as a radial reduction of exogenous inputs to the two nodes in their network, conditional on intermediate and final outputs lying on the virtual frontier. Lewis and Sexton (2004) extended Sexton and Lewis (2003) to consider efficiency at each node of a network. They recursively solve for efficiency at each stage and for the network as a whole by solving a DEA radial reduction at the most upstream SU and progressively moving downstream. With each move, the intermediate output flows that are on the frontier are used as inputs in the next downstream level. This ensures that at intermediate levels, computed efficiency is consistent with the frontiers at further upstream levels. They view the network as a DMU and network efficiency is computed as the efficient level of final output at the last downstream point relative to observed final output at that point. Both the Färe and Grosskopf and Lewis and Sexton approaches proceed with different reference groups and frontiers at each node. Thus, the final network efficiency is based on the reference group or frontier of the previous upstream SU, not on a joint consideration of the SU activities throughout the network. In this sense, these approaches fail to comprehensively assess network performance.

To proceed, we first note that the distinction between SU and DMU or network is important. From the perspective of a network, the set of SUs are heterogeneous across stages, though homogeneous across layers for any particular stage. For example, the class of SUs defined by $\hat{d}(s)$ contains all SUs at stage s across all layers in the network and for all DMUs u . By contrast, $\hat{d}(l)$ is the class of SUs at layer l across all stages. While $\hat{d}(s)$ contains a homogeneous set of SUs, $\hat{d}(l)$ does not. Lewis and Sexton compute frontiers for each $\hat{d}(s)$ progressively from $s=1$, back to $s=S$, using intensity weights from each stage to compute the efficient level of intermediate output that is carried into the next stage. Färe and Grosskopf pool all SUs into a class $\hat{d}(s,l)$ involving heterogeneity, though they recognize this heterogeneity by allowing for different intensity weights and frontiers for each class $\hat{d}(s)$.
To proceed, consider the following problem (3) for computation of the output-oriented efficiency for SU "0" in the class $\hat{d}(s)$, recalling $d^0=(s^0,l^0)$ for all u in $\hat{d}(s)$.

(3) $\quad max\ \phi(s)^0$

$$\sum_{s,l=1}^{L} \lambda^d x_h^d \le x_h^0 \quad h = 1,...n$$

$$\sum_{s,l=1}^{L} \lambda^d z_i^d \le z_i^0 \quad i = 1,....m$$

$$\sum_{s,l=1}^{L} \lambda^d y_i^d \ge \phi(s)^0\, y_i^0 \quad i = 1,....m$$

$$\sum_{s,l=1}^{L} \lambda^d \le 1, \quad \lambda^d \ge 0$$

where λ^d is a vector dimensioned by U, $d=(s,l)$, and RTS can be re-specified appropriately for particular settings. This problem could be augmented following Färe and Grosskopf(2000) with constraints to accommodate allocatable exogenous inputs.
While it clear that (3) considers only a subset of SUs involved in the network, it does consider a subset of homogeneous SUs. However, it is of interest to reconsider the appropriateness of doing so independently of other DUs in the network. In particular, the above technology is not necessarily *network feasible*.
Specifically, network feasibility must be ensured for the efficient levels of product flows between SUs. This physical balance between interacting SUs must be preserved by the choice of activity weights, λ^d. This requirement raises the need to ensure that flows involved in constraining physical balance equilibrium should be efficient flows based on estimated frontiers at corresponding node positions in the network. While Lewis and Sexton (2004) noted that the external and intermediate output levels used as inputs by a SU at any stage s must be consistent with efficient intermediate outputs from supplying stages, they did not extend their concern to physical balance constraints. In particular, we need to ensure the following physical balance constraints are met for the efficient levels of intermediate product flows throughout the network:

(4) $$\lambda^d z_i^d \le \sum_{s,l=1}^{L} \lambda^d \tilde{z}_i^d * \quad i=1,\dots.m \quad s=1,\dots S$$

(5) $$\lambda^j \tilde{z}_{i,k}^j * \le y_i^j - m^j - \sum_{\substack{s=1 \\ s\neq j}}^{S} \sum_{l=1}^{L} \lambda^k \tilde{z}_{i,d}^j * \quad i=1,\dots.m \quad \forall \quad d^j \in D$$

where the asterisk indicates the intermediate product supply to each SU is consistent with physical balance in the network. (4) requires that the use of intermediate products by each efficient SU is supplied by the sum of supplies of the intermediate products from efficient SUs $d \subset \hat{d}(s)$. Further, for each of these efficient SUs, supply allocated to intermediate supply is constrained by (5). (5) requires that the supply that SU *j* allocates to any SU *k* must be feasible given available supply from SU *j*.

The implication of adding (4) and (5) as constraints to problem (3) is that the intensity weights in (3) must be chosen conditional on those chosen for efficient SUs in all stages, not simply stage *s*. Proposition 1 follows directly.

Proposition 1. Efficiency of a SU relative to a reference set, e.g. $d \in \hat{d}(s)$ that is a subset of the set of SUs contained in a network, e.g. $\hat{d}(s) \subset D$, must be determined conditional on efficient allocations of intermediate product by SUs not contained in the subset.

Operationally, Proposition 1. implies problem (3), and all other such problems that examine efficiency of a subset of SUs, must be solved as a second step after the efficient intermediate product flows are determined for the entire set of SUs contained in the DMUs under study.

To proceed, we define problem (6) below for computation of the output-oriented *stage efficiencies* for DMU "0" in the class $\hat{d}$, the set of all SUs operating in the network and across all DMUs contained in *D*. To do so, we need to expand our notation slightly by adding a superscript to indicate the DMU with which the flow is associated.

(6) $$max \sum_{s=1}^{S} \phi^{s,0}$$

(7) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} x_h^{d,u} \le x_h^{s,0} \quad h=1,\dots n \quad s=1,\dots S$$

(8) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} z_i^{d,u} \le z_i^{s,0} \quad i=1,\dots.m \quad s=1,\dots S$$

(9) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} y_i^{d,u} \ge \phi^{s,0} y_i^{s,0} \quad i=1,\dots.m \quad s=1,\dots S$$

(10) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} \le 1, \quad \lambda^d \ge 0 \quad s=1,\dots S$$

(11) $$\lambda^{j,u} z_i^{j,u} \le \sum_{s,l=1}^{L} \lambda^{d,u} \tilde{z}_{i,j}^{d,u} \quad i = 1,....m \quad s = 1,...S \quad u = 1,...U$$

(12) $$\lambda^{j,u} \tilde{z}_{i,k}^{j,u} \le y_i^{j,u} - m^{j,u} - \sum_{\substack{s=1 \\ s \ne j}}^{S} \sum_{l=1}^{L} \lambda^{k,u} \tilde{z}_{i,d}^{j,u} \quad i = 1,....m \quad \forall \quad d^j \in D, u \in U$$

This problem can be viewed as a stacking of stage activities that define stage specific frontiers and measure efficiency through radial output expansion at each stage. This activity analytic model is augmented however by physical balance constraints (11) - (12) that require that efficient levels of intermediate product use defined by the intensity weights are consistent through out the network.

Define the solutions for λ^d to this problem as λ^d*. Given these solutions, problems such as (3) that consider efficiency of a subset of SUs can now be solved i) subject to constraints such as (4) and (5) and ii) conditional on network efficient flows of intermediate outputs based on λ^d*.

Next, consider *network efficiency*. Several approaches are possible for this task. Lewis and Sexton (2004) used a sequential, upstream method. Starting with the most downstream SU, at stage *S*, they determine efficiency $T_j(S)$ of SU $d(S,u^0)$ and compute the associated efficient use of intermediate goods, z^S. Next, they determine efficiency $T_j(S-1)$ of SU $d(S-1,u^0)$ and compute the associated efficient use of intermediate goods, z^{S-1}.conditional on z^S. They continue this protocol through the vertical chain until the most upstream SU is evaluated. Final network efficiency is evaluated by comparison of final outputs based on efficient input use with observed final outputs for the most downstream SU. However, this approach does not require physical flow balance throughout the network. Thus, in this paper, the following output-oriented measure of network efficiency is introduced. Two issues are of interest to resolve in the specification. First, we would like physical balance to be ensured for efficient virtual DMUs. Second, we would like to each stage technology to be homogeneous across DMUs, though allow for heterogeneity across SUs in the same layers.

(13) $$max \ \phi^{N,0}$$

(14) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} x_h^{d,u} \le x_h^{s,0} \quad h = 1,...n \quad s = 1,...S$$

(15) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} z_i^{d,u} \le z_i^{s,0} \quad i = 1,....m \quad s = 1,...S$$

(16) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} y_i^{d,u} \ge \phi^{N,0} y_i^{s,0} \quad i = 1,....m \quad s = 1,...S$$

(17) $$\sum_{u=1}^{U} \sum_{s,l=1}^{L} \lambda^{d,u} \le 1, \quad \lambda^d \ge 0 \quad s = 1,...S$$

(18) $$\lambda^{j,u} z_i^{j,u} \le \sum_{s,l=1}^{L} \lambda^{d,u} \tilde{z}_{i,j}^{d,u} \quad i = 1,....m \quad s = 1,...S \quad u = 1,...U$$

$$\lambda^{j,u}\tilde{z}_{i,k}^{j,u} \le y_i^{j,u} - m^{j,u} - \sum_{\substack{s=1 \\ s\neq j}}^{S} \sum_{l=1}^{L} \lambda^{k,u}\tilde{z}_{i,d}^{j,u} \quad i = 1,\ldots m \quad \forall \quad d^j \in D, u \in U$$

(19)

where the input levels of exogenous and for intermediate outputs are restricted to their efficient levels by (18) and (19).

5. Conclusions

The goal of this paper was to consider the general problem of gauging efficiency of networks of inter-related or collaborating decision-making units that are subsidiary to more centralized decision-making. Thus, the concept of sub-DMU or SU was introduced and alternative organizations of SUs were identified. A detailed notation was introduced that elucidates two fundamental requirements of network efficiency analysis overlooked by previous work. First, network efficiency must allow for SUs in the same stage to face a common technology while not requiring such a restriction for SUs in the same layer. Second, intensity weights must be selected subject to physical flow constraints that span across all SUs in a DMU. This requires that a network-wide problem be defined even when efficiency of a subset of SUs within the DMUs is of interest.

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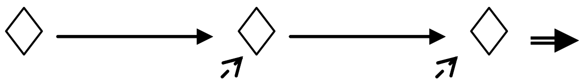
7. Figures

Figure 1. Network Configurations

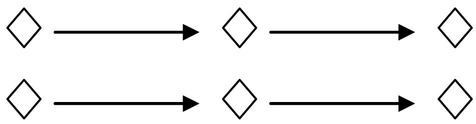
Each node is assumed to utilize external inputs, x^d indicated by a diagonal dotted stem arrow only in the first configuration. Final outputs, y^d, are noted by double stemmed arrow only in the first configuration and are assumed feasible at each node for all other configurations.

Single chain (*S=3,l=1,r=0)*

External inputs are assumed at each node, one final output (similar to Färe & Grosskopf (1995,1996)

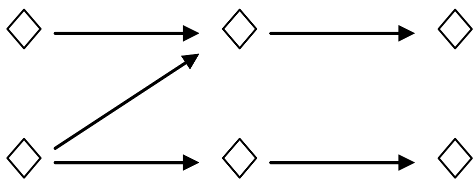


Parallel chains (*S=3,l=1, r=2*)



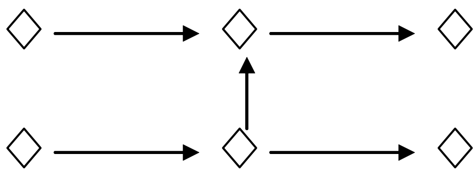
Cross-layer sourcing (*S=3,l=1,r=2)*

Similar to Färe & Grosskopf (1999, 2000), Castelli et al. (2004)



Peer sourcing (*S=3,L=1,R=2)*

Peer sourcing with heterogeneous layers (*S=3,L=2, R=2*)



Shared Service *(S=3,L=3,R=2)*