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Utility-derived Supply Function of Sheep Milk: The Case of Etoloakarnania, Greece

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Abstract: Sheep farming is an important agricultural activity in Greece, since it contributes highly in the country's gross agricultural production value. Recently, sheep milk production received further attention because of the increased demand for feta cheese and also because of the excessive price level suffered by the consumers, in contrast with the prices paid at the farm level. In this study, we suggest the use of multicriteria analysis to estimate the supply response of sheep milk to price. The study focuses in the Prefecture of Etoloakarnania, located in Western Greece, where sheep farming is a common and traditional activity. A non-interactive technique is used to elicit farmers' individual utility functions which are then optimized parametrically subject to technico-economic constraints, to estimate the supply function of sheep milk. Detailed data from selected farms, representing different farm types and management strategies, have been used in the analysis. The results indicate that the multicriteria model reflects the actual operation of the farms more accurately than the gross margin maximization model and therefore leads to a more robust estimation of the milk supply.

Keywords: Sheep-farming, multi-criteria, utility function, milk supply

1. Introduction

Milk supply and its response to price changes has been the object of a number of economic studies^[1,2,3,4]. The majority of these studies focus on the production of cow milk while the estimation of the supply response to price is achieved through econometric approaches. Unlike other developed countries, the production of sheep milk in Greece is equally important as the production of cow milk^[5]. Sheep farming is one of the most important agricultural activities in the country since it constitutes main or side activity for a large number of farms^[6]. Greek sheep farms aim at the production of both milk and meat, but over 60% of their total gross revenue comes from milk^[7,8,9]. Recently, the sheep farming activity has received further attention because of the excessive demand for feta cheese which consists mainly of sheep milk.

The purpose of this study is to estimate the supply response of sheep milk to price through the use of mathematical programming. Specifically, a mixed integer programming model that incorporates detailed technico-economic characteristics of the sheep farms is used to simulate their operation. Linear programming models are commonly used to capture livestock farmers' decision making process^[10,11,12,13,14]. The common characteristic of these models is that they aim to maximize gross margin assuming that this is the only objective of farmers. But the structure of the sheep farming activity in Greece indicates that this assumption is rather unrealistic.

The nature of the sheep farming activity and its ability to profitably utilize less fertile soil has caused its expansion in many agricultural areas of Greece, and traditionally its concentration in isolated and less favored areas. In these areas the prevailing farm type is the small, extensive, family farm. According to the N.S.S.G.^[6] almost 63% of the Greek sheep farms have a number of sheep less than 50. Furthermore, almost 85% of the Greek sheep farms are extensive and have low invested capital^[15]. Apart from sheep farming found in mountainous and less favored areas, more intensive and modern farms have appeared, recently, especially in lowland areas. The different production systems identified in the country have different technical and economic characteristics and achieve different levels of productivity^[16].

This high degree of diversification implies different management strategies developed according to farmers' individual preferences and combination of goals. The multiple goals of farmers and the development of different management styles and strategies has been the object of many studies^[17,18,19,20,21,22,23]. These studies indicate that farm level models that incorporate multiple goals can be

more effective and can assist policy makers in developing more efficient and targeted policy measures and adjusting the existing policy regime accordingly^[24].

Thus, in this study a farm level model that incorporates multiple goals is built to replace the traditional single objective model. In most multi-criteria studies the elicitation of the individual utility function is accomplished through the implementation of interactive techniques. But the use of interactive techniques comes with many problems and often yields ambiguous results^[25,26]. To overcome interaction problems we have used a non interactive technique to elicit farmers' individual utility functions, proposed by Sumpsi et al.^[26] and further extended by Amador et al.,^[27]. The individual utility functions are then optimized parametrically, subject to the technico-economic constraints of the farms to estimate the supply response of sheep milk to price. Kazakçi et al.^[28] minimize maximum regret instead of maximizing gross margin for better approximation of supply response curves of energy crops in France, while a number of studies use multi-criteria analysis for the estimation of the demand for irrigation water since it leads to a more accurate reflection of the actual operation of the farms and therefore to a more robust estimation of supply response^[29,30,31].

For the purpose of this paper detailed data from selected farms, representing different farm types have been used. The study focuses in the Prefecture of Etoloakarnania, where sheep farming is a well known and traditional activity. Results of our analysis support the point of view expressed in previous studies regarding the usefulness of the methodology to researchers and policy makers.

In the following section the methodology, used in this analysis, is described. Section 3 presents the case study and the model specification. Finally, the last two sections contain the results of the analysis and some concluding remarks.

2. Methodology

The methodology used for the estimation of the milk supply function, in this study, can be analyzed in three distinct parts. First, for each of the selected farms, a mixed integer programming model that reflects its operation is built. The techno-economic constraints and decision variables are defined according to the data collected from the selected farms. Secondly, the set of farmers' goals to be used in the analysis is determined and the multi-criteria technique is applied to elicit the individual utility function of each farmer. Then, third, the estimated utility function is optimized parametrically (various price levels) and the individual (disaggregated) supply function for each farmer is extracted. Finally, the total supply function of sheep milk is estimated, using the number of farms represented by each farm type.

2.1. Mixed-integer livestock farm detailed model

Optimization models taking into account interrelationships like resource and agronomic constraints as well as synergies and competition among activities, usually select the most profitable activity plan and have been extensively used in agriculture. They allow for a techno-economic representation of production units (farms) containing a priori information on technology, fixed production factors, resource and agronomic constraints, production quotas and set aside regulations, along with explicit expression of physical linkages between activities.

Livestock mathematical programming models are in general more complicated than arable cropping ones. They include a large number of decision variables and resource, agronomic and policy constraints^[4,12,14]. The model used in this analysis uses similar decision variables and constraints, though it is in fact a mixed integer programming model, since some variables are constrained to receive only integer numbers. These variables refer to the number of ewes. The mixed integer programming models are commonly used, when livestock, crop-livestock and aquaculture farms are studied^[32,33].

2.2 Non interactive multi-criteria methodology

Multi-criteria approaches mainly goal programming and multi objective programming are most common in agricultural studies^[34,35,36,37]. In the majority of these multi-criteria approaches, the goals incorporated in the model and the weights attached to them are elicited through an interactive process with the farmer^[38,39,40]. This interaction with the farmer and the self reporting of goals comes with many mishaps, since farmers often find it difficult to define their goals and articulate them^[25]. Another problem associated with this interactive process is that individuals feel uncomfortable when asked about their goals or are often influenced by the presence of the researcher and adjust their answers to

what they feel the researcher wants to hear. The above problems denote the need to employ a different method to determine farmers' objectives in multi-criteria studies.

In this study, we apply a well-known non-interactive methodology to elicit the utility function of each farmer^[26]. The basic characteristic of this methodology is that the farmer's actual and observed behavior is used for the determination of the objectives and their relative importance. Assume that:

- x = vector of decision variables (see appendix)
- F = feasible set (see appendix)
- $f_i(x)$ = mathematical expression of the i -th objective (equations 6-10 in section 3)
- w_i = weight measuring relative importance attached to the i -th objective
- f_i^* = ideal or anchor value achieved by the i -th objective
- f_{*i} = anti-ideal or nadir value achieved by the i -th objective
- f_i = observed value achieved by the i -th objective
- f_{ij} = value achieved by the i -th objective when the j -th objective is optimized
- n_i = negative deviation (underachievement of the i -th objective with respect to a given target)
- p_i = positive deviation (overachievement of the i -th objective with respect to a given target)

The first step of the methodology involves the definition of an initial set of objectives $f_1(x), \dots, f_i(x), \dots, f_q(x)$. The researcher can define this initial set of goals according to previous research and relative literature or through preliminary interviews with the farmers. In the second step, each objective is optimized separately over the feasible set. At each of the optimal solutions the value of each objective is calculated and the pay off matrix is determined^[26]. Thus, the first entry of the pay-off matrix is obtained by:

$$Max f_1(x), \text{ subject to } x \in F \quad (1)$$

since $f_1^* = f_{11}$. The other entries of the first column of the matrix are obtained by substituting the optimum vector of the decision variables in the rest $q-1$ objectives. The entries of the rest of the columns are obtained accordingly. In general, the entry f_{ij} is acquired by maximizing $f_j(x)$ subject to $x \in F$ and substituting the corresponding optimum vector x^* in the objective function $f_i(x)$.

The elements of the pay off matrix and the observed (actual) values for each objective are then used to build the following system of q equations. This system of equations is used to determine the weights attached to each objective:

$$\begin{aligned} \sum_{j=1}^q w_j f_{ij} &= f_i & i = 1, 2, \dots, q \\ \sum_{j=1}^q w_j &= 1 \end{aligned} \quad (2)$$

The non negative solution generated by this system of equations represents the set of weights to be attached to the objectives so that the actual behavior of the farmer can be reproduced (f_1, f_2, \dots, f_q). Usually the above system of equations has no exact solution and thus the best solution has to be alternatively approximated.

To minimize the corresponding deviations from the observed values, the entire series of L metrics can be used. In our analysis, we have used the L_1 criterion that aims at the minimization of the sum of positive and negative deviational variables.^[26,27] The L_1 criterion assumes a separable and additive form for the utility function. Alternatively, the L_∞ criterion according to which the maximum deviation D is minimized can be used^[41]. Both criteria are commonly used in agricultural studies, partly because they can be managed through an LP specification. The L_∞ criterion corresponds to a Tchebycheff utility function that implies a complementary relationship between objectives^[27]. Nevertheless, in this

first attempt to explore the behavior of sheep farmers in Greece we use the L_1 criterion and assume the separable and additive utility function (equation 4), often used in agricultural studies^[26,42].

To solve the minimization problem (minimization of the sum of positive and negative deviational variables) we use the weighted goal programming technique^[43,41,26]. The formulation of the weighted goal programming technique is shown below:

$$\text{Min} \sum_{i=1}^q \frac{(n_i + p_i)}{f_i}$$

subject to:

$$\begin{aligned} \sum_{j=1}^q w_j f_{ij} + n_i - p_i &= f_i \quad i = 1, 2, \dots, q \\ \sum_{j=1}^q w_j &= 1 \end{aligned} \quad (3)$$

As mentioned above the L_1 criterion corresponds to a separable and additive utility function. The form of the utility function is shown below:

$$u = \sum_{i=1}^q \frac{w_i}{k_i} f_i(x) \quad (4)$$

k_i is a normalizing factor (for example: $k_i = f_i^* - f_{i*}$). It is essential to use the normalizing factor, to avoid overestimating the weights of goals with high absolute values in the utility function, when goals used in the analysis are measured in different units^[40,26,44].

After estimating the farmer's individual utility function, we maximize it subject to the constraint set (see appendix) and the results of the maximization are compared to the actual values of the q goals. This way the ability of the utility function to accurately reproduce farmers' behavior is checked and the model is validated. Namely, the following mathematical programming problem is solved:

$$\text{Max} \sum_{i=1}^q \frac{w_i}{k_i} f_i(x)$$

Subject to:

$$\begin{aligned} f_i(x) + n_i - p_i &= f_i \quad i = 1, 2, \dots, q \\ \mathbf{x} &\in \mathbf{F} \end{aligned} \quad (5)$$

If the estimated function gives results for each goal close to the actual values then it is considered the utility function that is consistent with the preferences of the farmer. On the other hand if the above utility function cannot reproduce farmer's behavior, other forms of the utility function should be examined^[26,27]. But it should be noted that the utility function has to represent the actual situation accurately, not only against alternative objectives, but also against decision variables.

2.3. Parametric optimization to estimate supply response at the farm and the sector level

The microeconomic concepts of supply curve and opportunity cost could be approximated in a satisfactory way by using mathematical programming models, called supply models, based on a representation of farming systems. Thanks to supply models, it is possible to correctly estimate these costs by taking into account heterogeneity and finally to aggregate them in order to obtain raw material supply for industry. It is postulated that the farmers choose among crop and animal activities so as to maximize the agricultural income or gross margin. Variables take their values in a limited feasible area defined by a system of institutional, technical and agronomic constraints. To estimate the individual supply function for each farmer the above optimization problem can be solved for various levels of milk price. Moreover, the total supply function can be estimated by aggregating the individual supply functions, taking into account the total number of farms in the area under study represented by the

farms used in the analysis. Similar methodology has been used by Gómez-Limón & Riesgo^[30] for the estimation of the demand for irrigation water in Andalusia and by Sourie^[45] and Kazakçi et al^[28] for the estimation of supply of energy biomass in the French arable sector.

3. Case study

3.1. Data

In this analysis we aim at the estimation of milk supply function in the Prefecture of Etoloakarnania, located in Western Greece. The Prefecture of Etoloakarnania produces 7% of the total sheep milk in Greece and gathers almost 9% of the total number of Greek sheep farms^[5]. Sheep farming is a common and traditional activity in the area. The majority of farms have a small herd, which indicates that sheep farming is often a part time or side activity. Specifically 42% of the farms have a number of sheep less than 50, while less than 9% of the farms have a number of sheep larger than 200.

Thus, the estimation of the milk supply function of the area is achieved through the use of technico-economic data from three sheep farms with different herd size and milk production. Other differences amongst the selected farms –which are more or less linked to the herd size- are the amount of farm produced forage and concentrates, the labor requirements and the breeding system (extensive or intensive). The selection of farms with different sizes indicates that our analysis will be laid out in groups of farmers, leading to a more precise estimation of milk supply. This is essential in a multi-criteria analysis since previous studies indicate that the goals of farmers can differ between large and smaller farms^[46,47]. In the case of sheep farming in Greece, where 63% of the farms have a small livestock, it is necessary to study these farms along with the larger farms and stress any differences between them.

For the above reasons, the first selected farm is a large and commercial farm. It produces part of the forage and concentrates it uses and has an annual milk yield of 135 kg/ewe. According to the number of sheep, this farm represents 764 farmers in the area under study^[48]. The second farm has a middle size flock (80 ewes), it is located in lowland area and has a lower yield while it produces alfalfa and corn not only to cover the needs of the livestock activity but also for sale. Although this farm is a commercial farm, and the owner is a full time farmer, it has a different production orientation than the large farm, since it aims at the production of feedstock and not only in the production of milk. According to the N.P.A.G. ^[48] there are about 4379 farmers in the area with a herd size of 50-200 sheep. The third farm is a small scale farm, representing only a part-time activity for the owner. The part-time farmer produces no feedstock and aims only at a supplementary income from sheep farming. This farm represents 3750 farmers in the area under study (less than 50 sheep). It should be mentioned that the gathered data refers to the year 2004-2005 (annual data) that was a typical year for Greek agriculture.

3.2. Model specification

The estimation of the individual supply functions supposes the construction of a linear programming model that can reflect the characteristics and constraints of each of the three farms accurately. The model used in the analysis, has also been used in previous work^[49] and has undergone a slight modification. This modification involves an extra constraint on the percentage of energy requirements satisfied from concentrates, which varies between farms. The model is adjusted according to the specific characteristics of each farm. The main difference of the multi-criteria model among the three farms is the different objective function (utility function). The other parts of the model (decision variables and constraints) are adapted to the specific farm features. In its basic form the model consists of 144 decision variables and 95 constraints that cover both animal and crop activities of the farms (see appendix).

There are three sets of decision variables included in the model. The first set involves the production of fodder and concentrates (mainly alfalfa and corn), the use of pastureland (area of different kinds of pastureland engaged by the farm) and the monthly consumption of in-farm produced or purchased forage and concentrates. The next set involves monthly family and hired labor engaged in crop and animal activities. The last set of decision variables involves the animal activities of the farm and the area engaged in the production of crops for sale (not consumption in the farm). It should be noted that there are four animal activities incorporated in the model, namely the production of lambs that are sold after weaning or three months after birth (rearing) and ewes that are premium eligible or not (previous CAP regime).

The constraint matrix includes land constraints (total own land, irrigated land, available pastureland e.t.c.), the monthly distribution of produced fodder and concentrates, monthly nutrient requirements (dry matter, NEL¹, digestible nitrogen), monthly labor requirements of all activities and policy constraints (number of premium eligible ewes). For the estimation of the nutrient requirements of the flock the methodology described by Zervas et al.^[50] has been used. The mathematical expression of the constraint matrix and the decision variables are presented in the appendix.

3.3. Initial set of goals

Five tentative goals are used in this analysis. The first goal is the maximization of the total gross margin which is considered the main economic goal of farmers and therefore is widely used in decision making models^[35,37,47]. But Greek farmers often place more value on keeping their expenses (mainly variable cost) low, than on making maximum profit. For this reason we have also included the minimization of variable cost at the initial set of goals, following a number of studies (for example: Piech & Rehman^[35]). The third goal refers to the minimization of family labour. This goal is strongly linked to the farmer's attempt to increase his leisure time. The importance of this goal is stressed in a number of studies of farmers' goals^[39,47].

The fourth goal refers to the minimization of all purchased feed and is linked mainly with the increasing concern about the quality and hygiene of forage and other concentrates and rather secondly to maintain expenses at a low level. Farmers often prefer to feed their livestock with forage and concentrates produced in the farm. This attempt is evident in farmers that consume part of their products, or aim to produce and promote quality products. The last goal is the minimization of the cost of foreign labour^[35,37]. This is a major concern of farms that attempt to utilise family labour to increase farm income. But this is not the only reason, since hired labour is not always abundant. Consequently, farmers may need to restrict the size of the livestock so as to depend only on family labour. The five goals used in this analysis and their mathematical expressions are given below (see the appendix for the indices, parameters and decision variables notation):

1. Maximization of gross margin (in euros)

$$f(1) = \text{Max} \left[\sum_{ti} gr_marc_{ti,sales} \cdot crop_{ti,sales} + \sum_r \sum_a gr_mara_{a,r} \cdot anim_{a,r} - \sum_g rqcw_g \cdot gland_g - \sum_t \sum_{fi} rqcw_{fi,t} \cdot feed_{fi,t} - \sum_t \sum_{fs} rqcw_{fs,t} \cdot feed_{fs,t} - \sum_r \sum_a rqcw_{a,t} \cdot anim_{a,r} - \sum_{ti} rqcw_{ti} \cdot crop_{ti,"con,sales"} \right] \quad (6)$$

2. Minimization of the variable cost (in euros)

$$f(2) = \text{Min} \left[\sum_g rqcw_{t,g} \cdot gland_g + \sum_t \sum_{fi} rqcw_{fi,t} \cdot feed_{fi,t} + \sum_t \sum_{fs} rqcw_{fs,t} \cdot feed_{fs,t} + \sum_r \sum_a rqcw_{a,t} \cdot anim_{a,r} + \sum_{ti} rqcw_{ti} \cdot crop_{ti,"con,sales"} + \sum_t \sum_{li} lab_{l,hire,t} \cdot w_{l,hire} \right] \quad (7)$$

3. Minimization of the family labour (in hours)

$$f(3) = \text{Min} \sum_l \sum_t lab_{l,own,t} \quad (8)$$

4. Minimization of the amount purchased forage and concentrates (in MJ)²

¹ Net Energy of Lactation (Mj)

² The variable $feed_{fi,t}$ refers to kilograms of purchased fodder and concentrates of various types, with different nutritional and energy value. Therefore minimising the sum of all purchased fodder and concentrates would lead to the substitution of low nutritional value crops (used in larger amount) with high nutritional value crops (used in smaller amount). To avoid this mistake we use the parameter $y_{fs,energy}$ as a normalizing factor. This means that the 4th goal expresses the "purchased energy" measured in Mj.

$$f(4) = \text{Min} \sum_{fs} \sum_t y_{fs,energy} feed_{fs,t} \quad (9)$$

5. Minimization of hired labor (in hours)

$$f(5) = \text{Min} \sum_l \sum_t lab_{l,hire,t} \quad (10)$$

4. Results of the analysis

4.1. Utility functions

In order to build the multicriteria model for each of the farms we use the methodology described in a previous section for the elicitation of the individual utility function. The first step of the analysis is to obtain the Pay-off matrix for each of the farms and apply the L_1 criterion. This way we estimate the weights attached to each of the initial goals. For the large farm the analysis indicates that the farmer aims at maximizing gross margin with a weight of 37%. But mainly the farmer aims at minimizing hired labor (52%), since the farm actually has high labor requirements, especially for grazing. The weight of the minimization of purchased forage is low but non negligible (11%). The other two of the initial goals receive zero weight, as far as the large farm is concerned. Using these weights and equation 4, we can estimate the utility function of the farmer:

$$U_1 = 0,37 * f_1 / 15682 - 0,11 * f_4 / 1446487 - 0,52 * f_5 / 41630 \quad (11)$$

For medium size farm, which is also commercial, the main attribute of the utility function is the maximization of the gross margin, since the weight attached to this objective is 55%. Another important attribute in the utility function of this farm is the minimization of purchased forage and concentrates, since one of the farm's main activities is the production of alfalfa and corn, not only for consumption but also for sale. The weight of this attribute is 0,39%. A smaller weight is given at the minimization of variable cost (6%). According to the estimated weights, the utility function for this farmer is shown below:

$$U_2 = 0,55 * f_1 / 4799 - 0,06 * f_2 / 3643 - 0,39 * f_4 / 4539 \quad (12)$$

Finally, as far as the small farm is concerned, the analysis indicates that the farmer aims not only at gross margin maximization but mainly at minimization of family labor. The weights attached next to these objectives are 23% and 77%, respectively. The weight attached next to the gross margin maximization is smaller than in the case of larger farms. On the other hand the minimization of family labor is included in the utility function of only the owner of the small farm, and it is given the higher weight. The reason for this is that the owner of the third farm is only a part time farmer. This pluriactive farmer probably needs to save on labor inputs so that he can invest time and effort in his off-farm activities. The estimated weights yield the utility function shown below:

$$U_3 = 0,23 * f_1 / 2209 - 0,77 * f_3 / 682 \quad (13)$$

4.2. Model validation

The utility functions, estimated above are next optimized (to the existing price level), subject to the model constraints to approximate farmers' behavior. It should be noted that, because of the small weight attached next to the gross margin maximization objective, an additional constraint has been used in the case of the small farm that does not allow the estimated gross margin to be less than 70% of the observed one. To allow for comparison, the traditional gross margin maximization objective function is also optimized. First, the predicted values of all objectives, according to both the traditional and the multicriteria model are compared^[27]. But in order to decide on the ability of the multi-criteria model to reproduce farmers' behavior, the decision variable space has to be taken into account as well. Tables 1-3 summarize the predicted values of the objectives and the decision variables for the farms. The observed values are included in the tables; while the last two columns contain the absolute deviations of the predicted values from the observed values, in the case of gross margin maximization

and the maximization of the estimated utility function. The total deviation from the observed behavior is also presented, while the last row contains the ratio of the deviations (total deviation in the case of the multi-criteria model/total deviation in the case of the traditional model)^[51]. The estimated utility function yields better results in all three farms. This means that the multi-criteria model can represent the behavior of farmers more accurately than the traditional gross margin maximization model.

Specifically, in the case of the first farm the suitability of the multi criteria model compared to the traditional model is transparent, especially when examining the values of objectives, where the relative fit index is 0,12 (Table 1). The traditional model fails to simulate the actual behavior especially in the case of the purchased forage and cost of hired labor.

Table 1. Observed and predicted values of the objectives and decision variables for the large farm

	Traditional model	Multi-criteria model	Observed values	Abs. deviation (Multi-criteria model)	Abs. deviation (Traditional model)
Values of objectives					
Gross margin (€)	41572	39057	36986	0.06	0.12
Variable cost (€)	60949	32068	31680	0.01	0.92
Family labour (h)	4843	4570	4843	0.06	0.00
Purchased feed (MJ)	786048	250753	324844	0.23	1.42
Hired labour (€)	19680	9011	7958	0.13	1.47
Total deviation				0.49	3.94
Relative fit					0.12
Decision variables					
3-month ewes	0	0	0		
Weaning ewes	380	237	262	0.10	0.45
Alfalfa produced*	72	50	40	0.25	0.79
Corn produced*	8	32	40	0.19	0.79
Total pastureland*	800	800	800	0.00	0.00
Other crops*	5	3	5	0.43	0.00
Total deviation				0.96	2.03
Relative fit					0.47

*Stremmas

Table 2. Predicted and observed values of the objectives and the decision variables for the middle farm

	Traditional model	Multi-criteria model	Observed values	Abs. deviation (Multi-criteria model)	Abs. deviation (Traditional model)
Values of objectives					
Gross margin (€)	21438	20398	20798	0.02	0.03
Variable cost (€)	7798	7504	8153	0.08	0.04
Family labour (h)	2756	2657	2274	0.17	0.21
Purchased feed (MJ)	0	0	0	0.00	0.00
Hired labour (€)	438	401	350	0.15	0.25
Total deviation				0.41	0.54
Relative fit					0.77
Decision variables					
Ewes	157	105	80	0.31	0.96
Alfalfa produced*	37	41	35	0.18	0.07
Corn produced*	29	25	31	0.20	0.08
Total pastureland*	15	15	15	0.00	0.00
Other crops*	9	9	9		
Total deviation				0.69	1.11
Relative fit					0.62

*Stremmas

As far as the basic decision variables are concerned, the number of ewes is better simulated in the multi-criteria model, although both models approximate the animal practice that the farm actually

maintains (sell lambs after weaning). Also the produced alfalfa and corn is better simulated using the multicriteria model. As for the middle farm, the multi-criteria model has an increased ability to reproduce farmer's behavior, compared to the traditional model as well, especially in the case of the number of ewes (Table 2).

Finally, as far as the small farm is concerned, the superiority of the multi-criteria model compared to the traditional model is transparent in both the objective and the decision variable space (Table 3).

Table 3. Predicted and observed values of the objectives and the decision variables for the small farm.

	Traditional model	Multi-criteria model	Observed values	Abs. deviation (Multi-criteria model)	Abs. deviation (Traditional model)
Values of objectives					
Gross margin (€)	4494	2292	3263	0.30	0.38
Variable cost (€)	5096	2055	3108	0.34	0.64
Family labour (h)	952	270	671	0.60	0.42
Purchased feed (MJ)	141594	53158	73567	0.28	0.92
Hired labour (€)	24	0	6	1.00	3.42
Total deviation				2.51	5.78
Relative fit					0.43
Decision variables					
3-month ewes	45	21	20	0.05	1.25
Weaning ewes	0	0	0		
Total pastureland*	23	26	23	0.13	0.00
Other crops*	3	0	3	1.00	0.00
Total deviation				1.18	1.25
Relative fit					0.94

*Stremmas

4.3. Milk supply functions

After validating the utility function for each farm we can move on to estimating the individual supply functions, by parametrizing the price of milk. The supply for the large farm is presented in Figure 1. The supply function estimated through the use of the traditional gross margin maximization model is also presented in the same figure. As we can observe, the supply function is less steep when the traditional model is used, which implies a higher elasticity, especially in the area of the current price level (0,8-1€/kgr). But if price falls lower than this level, then the response of the farmer is higher than that estimated using the traditional model.

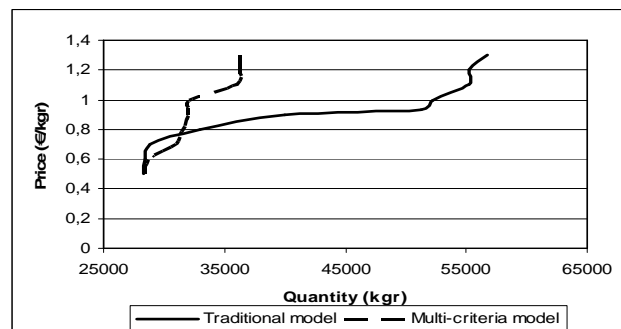


Figure 1. Milk supply of the large farm

In Figure 2, supply functions of the medium farm under the assumption of gross margin maximization and under the estimated utility function maximization are presented. As we can see the two functions do not look that different. This resemblance can be explained by the fact that gross margin maximization receives a high weight in the utility function of the farmer.

Nevertheless, as in the case of the first farm, the use of the utility function restricts the milk supply in lower levels and the supply shifts to the left. As mentioned in the case of the large farm, the elasticity of the alternative supply function is higher than that of the supply function estimated by the traditional model, in low price levels (in the range of 0.4 – 0.6 euro/kg).

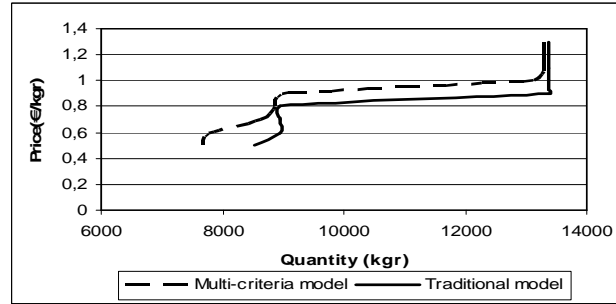


Figure 2. Milk supply of the middle farm

Finally, Figure 3 presents the individual supply functions for the small farm. The results indicate that the use of the traditional single objective model yields an inelastic supply function, at the milk price range examined. Under the assumption of gross margin maximization, the farm produces a large quantity of milk at all price levels. This result is rather unrealistic, since the actual milk produced is lower than 20% of what the traditional model suggests. On the other hand the multi-criteria model, yields a different form of the supply function. This supply function has a high elasticity, especially in the low price levels. In fact, the farmer, is willing to produce milk only if the price of milk is higher than 0,75€/kg.

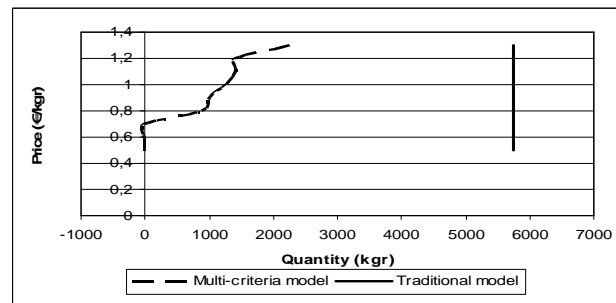


Figure 3. Milk supply of the small farm

The above analysis indicates that price changes affect the smaller farms more than the larger ones, especially in low price levels. Part time farmers will engage in the activity only if the price of milk is high enough. Ensuring the milk price level may lead not only in the income security of large farms but also in the continuing of the part time sheep farming activity.

Before estimating the total milk supply of the area, it should be mentioned that the structure of the model we have used in this analysis allows farmers to fine-tune their milk supply by adjusting the number of sheep and not the adjustment of milk yield per ewe. As described in the appendix, this happens because the number of ewes is included as an endogenous variable in the model, while the milk yield is an exogenous variable. Although in practice the farmer can adjust both the number of sheep and milk yield per ewe, evidence from other studies indicate that the elasticity of milk supply is explained mainly from the flock size elasticity (see for example Rayner^[2]).

4.4. Aggregate milk supply

In the previous section we have used the farm specific utility functions to estimate the milk supply for each decision making unit. The next step of our analysis involves the aggregation of the individual supply to estimate the total milk supply for the area of Etoloakarnania. This is estimated by the weighted addition of the individual supply functions^[30]. The supply function estimated is presented in Figure 4 below. Figure 4 also presents the aggregate supply function that corresponds to the traditional, gross margin maximization model. The alternative supply function indicates a lower milk supply at all price levels. Using the traditional model to estimate the regional supply would lead to a serious and

unrealistic overestimation of this supply. Furthermore, the alternative supply function is less elastic than the traditional one in the prevailing price range (0,8-1€/kgr), but more elastic in low price levels. This means that the inclusion of multiple goals in our model smoothens the reaction of farmers to price changes since their behavior is also influenced by other motives (some among them may be irrational from the homo economicus point of view). The higher elasticity of the estimated supply function in the lower price levels is due to the behavior of small farm owners that only begin to produce if the price level is high enough.

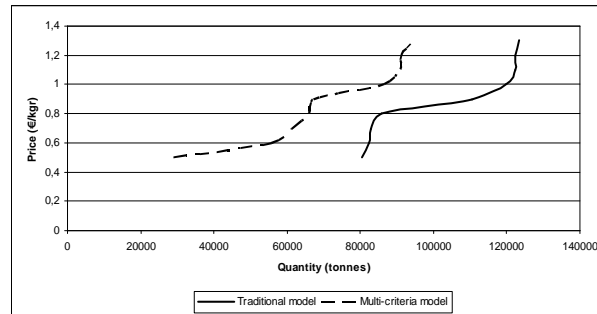


Figure 4. Aggregate milk supply

To conclude on the suitability of the estimated supply function, we compare the estimated supply with the actual observed value of milk supply of the Prefecture of Etoloakarnania. In 2004 the milk supply of the area was 48575 tones, while the price of milk was about 0,80-0,85€/kgr. The estimated supply function indicates that the supply should be 36% higher. This overestimation is mainly due to the high milk yield of the small farm used in the analysis (120 kgr/ewe) compared to the average milk yield (about 20% higher). If the milk yield was closer to the average then the estimation would be more accurate. On the other hand the supply function estimated using the traditional model yields a supply 75% higher than the actual one which is quite unrealistic.

5. Concluding remarks

In this analysis a multicriteria model is used to evaluate the supply function of sheep milk in the prefecture of Etoloakarnania. First a detailed whole farm model adapted to livestock is built that incorporates decision variables and constraints for all animal and crop activities. Then the individual utility functions are elicited through a non interactive methodology, so that the drawbacks of the interactive methods can be limited. The weights attached to the objectives of the farmers are estimated using the actual values of the objectives and the multi attribute utility function is then used to reproduce their behavior. By parametrising the milk price the individual supply functions are elicited and finally the total supply function is estimated as the weighted addition of the individual functions.

The first outcome of the analysis is that sheep farmers aim to achieve multiple goals, one of which is the maximization of gross margin. This objective is a more important attribute of the utility function of the larger and more commercial farms under study but the weight assigned to this objective is small in the cases of the less commercial part time farm. This farmer aims mainly at the minimization of family labor since he has other of farm activities to attend.

The analysis indicates that the performance of the mathematical model built to optimize the operation of a crop-livestock farm can improve through the use of multiple objectives. In this study this has been proven very useful since it leads to a more robust estimation of the milk supply function. The estimated supply function reveals that farmers are less responsive to price changes than the traditional gross margin maximization model suggests. Also, individual supply functions can be used to predict the reaction to price changes for different groups of farms, helping policy makers to design more affective and targeted measures. Similarly, the proposed methodology can be used to predict the impact of alternative policy measures on different farm types.

Finally it should be noted, that in this analysis we have used the additive form of the utility function, but the use and applicability of other forms of the utility function can also be investigated. This study is a first attempt to build a multi-criteria model to explain the behavior of livestock farmers, and study the milk supply response to price and therefore, further research is required. The existence of other objectives, such as minimization of risk, is another concept for future research.

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³ National Statistical Service of Greece

⁴ Hellenic Ministry of Rural Development and Food

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7. Appendix

Mathematical expression of the constraints and decision variables of the LP model:

<i>Indices:</i>	<i>ti</i>	cultivated crops (P = {corn, alfalfa, other})
	<i>fi</i>	cultivated fodder and concentrates (T = {corn, alfalfa})
	<i>fs</i>	purchased fodder and concentrates (N = {corn, alfalfa})
	<i>a</i>	animal activities (A = {sheep3, sheep-3})
	<i>r</i>	animal premiums (C= {elig, nelig})
	<i>m</i>	destination of produced fodder and concentrates (M = {con, sale})
	<i>l</i>	destination of labor (L = {crops, flock})
	<i>s</i>	origin of labor (S = {own, hire})
	<i>t</i>	month
	<i>g</i>	type of pastureland (G={rent, own, com})
	<i>u</i>	nutritional value (U={dry matter, nitrogen, energy})

Model parameters:

<i>Yield_{ti}</i>	crop yield (kg)
<i>y_gz_{t,u}</i>	nutritional value of pastureland per month (kg)
<i>y_{fi,u}</i>	nutritional value of produced forage and concentrates (kg)
<i>y_{fs,u}</i>	nutritional value of purchased forage and concentrates (kg)
<i>n_{a,t,u}</i>	monthly feed requirements (kg)
<i>n_{at,u}</i>	annual feed requirements (kgr)
<i>w_{t,s}</i>	wage (euros/hr)
<i>rclab_{ti,t}</i>	monthly labor requirements for crops (hr)
<i>ralab_{ti,t}</i>	monthly labor requirements for animal activities (hr)
<i>avail_{l,t}</i>	available family labor per month (hr)
<i>own_land</i>	available owned land (stremma ⁶)
<i>rent_land</i>	available pastureland for rent (stremma)
<i>irr_land</i>	irrigated land (stremma)
<i>graz_mun</i>	available communal pastureland (stremma)
<i>land</i>	total land (stremma)
<i>num_elig</i>	number of premium eligible ewes (number)
<i>gr_marc_{ti}</i>	gross margin of crops (gross revenue minus variable cost except labor) (€)
<i>gr_mara_{a,r}</i>	gross margin of animal activities (gross revenue minus all variable cost except labour and feed cost) (€)
<i>rqwgc</i>	variable cost required for pastureland (euro/stremma)
<i>rqwc_{ti}</i>	variable cost required for crops (euro/stremma)
<i>rqwc_a</i>	variable cost required for animal activities (euro/ewe)
<i>rqwc_{fi}</i>	monthly cost of produced fodder and concentrates (euro/kgr)
<i>rqwc_{fs}</i>	cost of purchased fodder and concentrates (euro/kgr)
<i>percent_energy</i>	percent of energy covered from concentrates

Decision variables

<i>crop_{fi,con}</i>	produced fodder and concentrates for consumption (kg)
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⁵ National Payment Agency of Greece (O.P.E.K.E.P.E.)

⁶ 1 Stremma = 0,1 Ha

$crop_{ti,sales}$	crops for sale (stremma)
$feed_{fs,t}$	monthly purchased fodder and concentrates (kg)
$feed_{fi,t}$	consumption of produced fodder and concentrates/month (kg)
$lab_{l,s,t}$	labor per month, destination and origin (hr)
$gland_g$	pastureland (stremma)
$anim_{a,r}$	ewe (number)

The mathematical expression of the constrain matrix is the following:

Distribution of produced feed crops:

$$yield_{fi} \cdot crop_{fi,con} = \sum_t feed_{fi,t} \quad \forall fi \in FI$$

Feed requirements:

$$\sum_g y_{gz,t,u} \cdot gland_g + \sum_{fi} y_{fi,u} \cdot feed_{fi,t} + \sum_{fs} y_{fs,u} \cdot feed_{fs,t} \geq \sum_r \sum_a n_{a,t,u} \cdot anim_{a,r}$$

$\forall t \in T, \forall u \in U$

Minimum annual energy requirements satisfied from concentrates:

$$y_{fi,energy} \cdot yield_{fi} \cdot crop_{fi,con} + \sum_t y_{fs,energy} \cdot feed_{fs,t} \geq percent_energy \cdot \sum_a \sum_r n_{a,t,energy} \cdot anim_{a,r}$$

$fs==corn, fi==corn$

Labor requirements for crops:

$$\sum_{ti} rclab_{ti,t} (crop_{ti,sales} + crop_{fi,con}) \leq \sum_s lab_{crops,s,t} \quad \forall t \in T$$

Available family labor:

$$lab_{l,own,t} \leq avail_{l,t} \quad \forall t \in T$$

Labor requirements of the flock:

$$\sum_a rlab_{a,t} anim_{a,r} \leq \sum_s lab_{l,s,t} \quad \forall t \in T$$

Available irrigated land:

$$\sum_{ti} (crop_{ti,sales} + crop_{fi,con}) \leq irr_land$$

Available own land:

$$\sum_{ti} (crop_{ti,sales} + crop_{fi,con}) + gland_{own} \leq land$$

Communal pasture land⁷

$$gland_{mun} \leq graz_mun$$

Available land for rental:

$$gland_{rent} \leq rent_land$$

Number of ewe rights:

$$\sum_a anim_{a,"elig"} \leq num_elig$$

⁷ Pastureland, property of the municipality, distributed among livestock farms according to their ewe rights. In exchange, livestock farms pay a small fee to the municipality.