Does Excluding Cross-commodity Interactions Matter? Beef and Lamb in Australia

Siobhan K. Dent[*], Roley R. Piggott[+], Christopher J. O'Donnell[+] and Garry R. Griffith[#+1]

Abstract

Australian broadacre agriculture is typified by strong cross-commodity relationships, where sheep and cattle grazing enterprises compete for pasture and both compete with wheat and other crops for land. Further, some commodities produced by multi-product farms are also used in the production of final products that are substitutes in demand, such as beef and lamb. Economic analyses of the beef market, for example, should also include consideration of the market for the related product, lamb. In this paper the effects of exogenous demand and supply shifters on the wholesale-farm price ratio in two closely related industries, beef and lamb, are examined in a Gardner type of model and the effects of excluding consideration of the cross-commodity interactions is measured. Parameter values were chosen based on previous empirical estimates and the judgement of the authors. Due to uncertainty about many of these parameters, a stochastic approach to sensitivity analysis was adopted where an appropriate probability distribution for each of the unknown parameters was chosen and 5,000 values from each of these distributions were drawn to construct estimated frequency distributions of results. It was found that the value for the general equilibrium elasticity of the beef wholesale to farm price ratio with respect to an exogenous demand shift was slightly larger with lamb cross-commodity effects imposed than without. Therefore the inclusion of cross-commodity relationships increases the responsiveness of the beef/cattle price ratio to the demand shift, but only slightly. The results further suggest that the demand and supply cross-price effects are additive rather than counteracting. Therefore incorrectly excluding the cross-commodity effects may result in large errors.

Keywords

Marketing margins, beef, lamb, general equilibrium elasticities

1. Introduction

In supply, Australian broadacre agriculture is typified by strong cross-commodity relationships, since "for the most part the production of agricultural commodities (is derived) from multi-product farms which are characterised by complementary- and substitution-type relationships in resource use and enterprise combination" (Ockwell 1990, p.32). For example, sheep and cattle grazing enterprises compete for pasture and both compete with wheat and other crops for land, yet pasture and cropping are complementary land uses in modern cropping rotation systems. Some commodities produced by multi-product farms are also used in the production of final products that are substitutes in demand. For example, various breeds of beef cattle and sheep are

[1] [*]Business Strategy Unit, Queensland Department of Primary Industries, Brisbane; [+]Graduate School of Agricultural and Resource Economics, University of New England, Armidale; and [#] NSW Agriculture Beef Industry Centre, University of New England, Armidale. This paper is based on Morrow (2001), supervised by Piggott, O'Donnell and Griffith.

used to produce beef and lamb, respectively, which are close substitutes in domestic demand. Under these circumstances, if there was interest in analysing empirically an economic issue in the beef market, an appropriate structural model should include the beef market and the market for the related product, lamb. Only in such a structure would the prices of the related products and inputs be determined endogenously and the full cross-commodity effects be properly measured.

Three recent examples from Australian broadacre agriculture illustrate the need for such an approach. First, the Australian sheep flock fell from 174 million in 1990 to 120 million in 2001 (ABARE, 2002), a contraction of more than 30 per cent. This was due primarily to the termination of the Reserve Price Scheme for wool in the early-1990s and the subsequent significant declines in the price of greasy wool that more accurately reflected weakened world demand. With such a contraction of the sheep industry it is plausible that grazing resources were allocated elsewhere, perhaps into the Australian beef industry. In fact, the number of beef cattle in Australia rose from 22.3 million in 1990 to 25.4 million in 2001, an expansion of some 14 per cent (ABARE, 2002). So the decline in sheep numbers resulting from the exogenous fall in wool demand effected wool and lamb industry supply and beef industry supply. Second, the demand for Australian beef declined during the mid-1990s when high levels of the pesticide Helix were found in beef that was exported to Asia. Such contamination reduced both the domestic and export demand for beef and, as beef and lamb are substitutes in demand and supply, the effects of the initial reduction in beef demand would have had flow-on effects to the lamb industry. Third, in 1999, the US announced that it would increase import tariffs on Australian lamb. While only temporary, such a rise in tariffs increased the price of Australian lamb in the US and therefore reduced the quantity exported to the US. Due to the presence of cross-commodity relationships, the initial decline in lamb demand effected related industries such as the beef industry.

There are several models of Australian broadacre agriculture which account for the types of cross-commodity relationships mentioned above. Vere *et al.* (2000) developed a quarterly econometric model of the Australian grazing industries which has a primary focus on cross-commodity linkages within the beef, lamb, mutton and wool industries. However this model is large and data intensive and becoming more difficult to keep updated. There are numerous multi-product profit function and/or cost function studies published (see Hill 2001 and the review by Griffith *et al*. 2001b), but again these typically require quite detailed time series cross-sections of farm survey data for their estimation. ABARE and some of the larger consulting companies have synthetic in-house models that include cross-commodity relationships, but access to such models is limited and/or costly. None of the freely available, published industry models based on the simpler equilibrium displacement modelling approach (for example, Zhao *et al.* 2000, Hill *et al.* 1996, Hill *et al*. 2001) have included cross-commodity relationships on both the demand and supply sides of the market.

To address this gap, the broad aims of this paper are (a) to examine the effects of exogenous demand and supply shifters, like those outlined above, on the wholesale-farm price ratio in two closely related industries, beef and lamb, and (b) to measure the effect of excluding consideration of the cross-commodity interactions, in a relatively easy to implement Gardner type of model.

2. The Structural Model

Gardner (1975) used a one-product, two-input model to '…generate quantifiable predictions about how various shifts in the demand for and the supply of food will affect the retail-farm price ratio.' (p.399). Gardner's model allowed substitution to occur between inputs, and the basic model was later extended to a two-product, two-input case (Gardner 1987, pp.129-137) to examine the impact of exogenous shifts when there is substitution between products. However it was assumed that the same inputs are used in the production of both products and the focus of the model was on studying the impact of price supports on the endogenous prices and quantities within the model. Here, the aim is to extend Gardner's model to make more general provision for the cross-commodity relationships in marketing margin analysis, when each of the two products related in demand are produced from different inputs as would seem appropriate in the case of beef and lamb in Australia. For example, the farm input in the case of beef production is live cattle and the farm input in the case of lamb production is sheep. It is also reasonable to assume that the nature of the aggregated 'marketing input' is different between these two products. Further, in the Australian context, the demand for both beef and lamb is an aggregate of domestic and export demand, but in different proportions.

The reason for focusing on the wholesale-farm price ratio (instead of the traditional retail-farm price ratio) is that a significant proportion of Australian beef and lamb is consumed in export markets (ABARE 2002). More particularly, in considering the retail demand for an Australian product in another country, one would have to take into account the fact that the Australian product has, as a substitute, the retail product supplied from all other sources (domestic supply and other exporting countries). These substitution relationships could be stronger than the substitution between Australian beef and Australian lamb in these export markets. A structural model that accommodated all the substitution relationships occurring at the retail level would need to be very large. A way to overcome this is to focus on the demand for beef and lamb at the 'abattoir door,' or the 'wholesale' level. The wholesale-level demand for both beef and lamb will be derived from the retail level demands for those products in domestic and export markets.

In the model described below, each industry combines a farm product (live cattle in the case of beef and live sheep in the case of lamb) and an aggregate marketing input (different between the two industries) to produce a final product (beef or lamb.) The model has twelve endogenous variables ($C, S, B, L, X, Y, P_C, P_S, P_B, P_L, P_X$, *and* P_Y) and there are assumed to be three exogenous variables (N, T *and* W).[2] The model and notation are as follows:

The beef industry

(1) $B = \alpha(C, X)$ (beef production function)

(2) $B = \beta(P_B, P_L, N)$ (beef demand function)

(3) $P_C = P_B \cdot \alpha_C$ (cattle demand function)

(4) $P_X = P_B \cdot \alpha_X$ (beef marketing input demand function)

(5) $C = \delta(P_C, P_S)$ (cattle supply function)

(6) $X = \gamma(P_X)$ (beef marketing input supply function)

---

[2] The model could be expanded to include other exogenous demand and supply shifters but, for convenience, only three shifters have been included here.

The lamb industry

(7)	$L = \varphi(S, Y)$	(lamb production function)
(8)	$L = \kappa(P_B, P_{L,} T)$	(lamb demand function)
(9)	$P_S = P_L \cdot \varphi_S$	(sheep demand function)
(10)	$P_Y = P_L \cdot \varphi_Y$	(lamb marketing input demand function)
(11)	$S = \lambda(P_{C,} P_S, W)$	(sheep supply function)
(12)	$Y = \nu(P_Y)$	(lamb marketing input supply function)

Where
C = quantity of cattle;
S = quantity of sheep;
B = quantity of beef;
L = quantity of lamb;
X = quantity of the marketing input in beef production;
Y = quantity of the marketing input in lamb production;
P_C = price of cattle;
P_S = price of sheep;
P_B = price of beef;
P_L = price of lamb;
P_X = price of the marketing input in beef production;
P_Y = price of the marketing input in lamb production;
N = demand shifter for beef (e.g. helix contamination);
T = demand shifter for lamb (e.g. increased US lamb tariff);
W = supply shifter for sheep (e.g flock size reduction due to RPS termination);
α_C = partial derivative of B with respect to C;
α_X = partial derivative of B with respect to X;
φ_S = partial derivative of L with respect to S; and
φ_Y = partial derivative of L with respect to Y.

It is assumed that the production functions for both lamb and beef exhibit constant returns to scale. It is also assumed that the markets for all inputs and outputs are competitive and that firms in both industries will exhibit profit-maximising behaviour. Firms will therefore purchase inputs up to the point at which the value of marginal product is equal to the price of the input. The demand for both beef and lamb is a function of both the price of beef and the price of lamb. Similarly the supply of both sheep and cattle is a function of both the price of sheep and the price of cattle. Thus, beef and lamb are substitutes in demand and cattle and sheep are substitutes in supply. It is assumed that there are no substitution relationships in the supply of the marketing inputs, so the marketing input supply relations are functions only of their own-prices (P_X or P_Y).

Following the approach used by Gardner (1975), the structural model described by equations (1)-(12) can be expressed in proportionate change form, as follows:

The beef industry

(13)	$E_B = K_C E_C + K_X E_X$	(beef production function)
(14)	$E_B = \eta_{B,B} EP_B + \eta_{B,L} EP_L + \eta_N E_N$	(beef demand function)

(15) $EP_C = -(K_X/\sigma_B)E_C+(K_X/\sigma_B)E_X + EP_B$ (cattle demand function)
(16) $EP_X= -(K_C/\sigma_B)E_X+(K_C/\sigma_B)E_C+EP_B$ (beef marketing input demand function)
(17) $E_C = \varepsilon_{CC}\, EP_C + \varepsilon_{CS}\, EP_S$ (cattle supply function)
(18) $E_X = \varepsilon_{XX}\, EP_X$ (beef marketing input supply function)

The lamb industry
(19) $E_L = K_S E_S + K_Y E_Y$ (lamb production function)
(20) $E_L = \eta_{L,B}\, EP_B + \eta_{L,L}\, EP_L + \eta_T\, E_T$ (lamb demand function)
(21) $EP_S = -(K_Y/\sigma_L)E_S + (K_Y/\sigma_L)E_Y+ EP_L$ (sheep demand function)
(22) $EP_Y = -(K_S/\sigma_L)E_Y+(K_S/\sigma_L)E_S+EP_L$ (lamb marketing input demand function)
(23) $E_S = \varepsilon_{SC}\, EP_C + \varepsilon_{SS}\, EP_S + \varepsilon_W\, E_W$ (sheep supply function)
(24) $E_Y = \varepsilon_{YY}\, EP_Y$ (lamb marketing input supply function)

Where
η_{BB} = own-price elasticity of demand for beef (negative);
η_{LL} = own-price elasticity of demand for lamb (negative);
η_{BL} = cross-price elasticity of demand for beef with respect to the price of lamb (positive);
η_{LB} = cross-price elasticity of demand for lamb with respect to the price of beef (positive);
ε_{CC} = own-price elasticity of supply of cattle (positive);
ε_{SS} = own-price elasticity of supply of sheep (positive);
ε_{CS} = cross-price elasticity of cattle supply with respect to the price of sheep (negative);
ε_{SC} = cross-price elasticity of sheep supply with respect to the price of cattle (negative);
ε_{XX} = supply elasticity of beef marketing input (positive);
ε_{YY} = supply elasticity of lamb marketing input (positive);
η_N = demand elasticity of beef demand with respect to the exogenous shifter N (negative);
η_T = demand elasticity of lamb demand with respect to the exogenous shifter T (negative);
ε_W = supply elasticity of sheep with respect to the exogenous shifter W (positive);
σ_B = elasticity of substitution between inputs in the beef industry;
σ_L = elasticity of substitution between inputs in the lamb industry;
K_C = cattle factor share;
K_S = sheep factor share;
K_X = beef marketing input factor share;
K_Y = lamb marketing input factor share; and
EP_i, E_i = proportionate change in the ith variable.[3]

This set of twelve equations can be solved to obtain solutions for the twelve endogenous E_i and EP_i variables, as typically done in EDM work (see for example Zhao *et al*. 2000). However, as shown by Zhao *et al.* (2001), it is quite difficult to correctly measure economic surplus changes when there are multiple sources of feedback, as here.

Alternatively, the system can be solved for 'general' equilibrium elasticities (i.e., the ratio of a proportionate change in an endogenous variable to the proportionate change in an exogenous

---

[3] The term E_i in this study and in Gardner (1987) denotes proportionate change, while in Gardner (1975) it denotes a general equilibrium elasticity.

variable). These 'general equilibrium' elasticities are 'partial' in the sense that when one exogenous shifter changes, the others are held constant (Gardner 1975, p. 400, footnote 1.)

Method of Calculating Results

The concern of this study is with the effects that exogenous shifts in demand or supply have on the wholesale-farm price ratio when cross-commodity effects are included or excluded. To illustrate these effects, three separate exogenous shifts have been assumed as shown in equations (2), (8) and (11). Each of these exogenous events is assumed to occur separately.

When the model is solved, a 12×3 matrix of general equilibrium elasticities is generated, where the first column contains the twelve general equilibrium elasticities associated with a negative beef demand shift (*N*), the second column contains the elasticities associated with a negative lamb demand shift (*L*), and the third column contains the elasticities associated with a negative sheep supply shift (*W*). The steps involved in transforming the model in proportionate change form (equations 13 to 24), to the matrix system that produces the general equilibrium elasticities, are outlined in the Appendix.

Note that in this formulation, the focus is on the relative responsiveness of the endogenous variables to any changes in the exogenous variables.

Generally only four of these elasticities are of interest. These are the general equilibrium elasticities for the price of beef and the price of cattle (used to calculate the general equilibrium elasticity for the beef/cattle price ratio), and the general equilibrium elasticities for the price of lamb and the price of sheep (used to calculate the general equilibrium elasticity for the lamb/sheep price ratio). Space limitations prevent the reporting of other general equilibrium elasticities but they could be used in other studies that focus on alternative measures of the marketing margin, such as the 'farmer's share' of the consumer food dollar (see Gardner 1975, p.405).

3. Model Implementation

Implementation of the model requires various Marshallian demand and supply elasticities, elasticities of input substitution and input shares. These values were chosen based on previous empirical estimates (as reviewed in Griffith *et al.* 2001a,b) and the judgement of the authors.

Demand Elasticities

Previously empirically-estimated demand elasticities reviewed in Griffith *et al*. (2001a) were used to represent the demand elasticities for beef and lamb.

Because the demand for beef and lamb is, in each case, an aggregate of domestic demand and export demand, the demand elasticities for both goods are a quantity weighted average of the domestic and export demand elasticities. Australia exports approximately 60 per cent of its beef production, so the weight attached to the export elasticity is 0.6 and the weight attached to the domestic elasticity is 0.4. Similarly, Australia exports approximately 25 per cent of its lamb production, so the weight attached to the export elasticity is 0.25 and the weight attached to the domestic elasticity is 0.75 (ABARE, 2002).

In the most recent studies cited, the estimated domestic demand elasticities measure responsiveness at the retail level, whereas the export elasticities measure the demand response at the farm gate. To ensure that both of the elasticities are measuring demand response at the same level, the reported demand elasticities have been adjusted to reflect the demand for beef and lamb at the wholesale level.

It has been assumed that the extent of input substitution in the wholesale to retail transformation is limited and that the retail demand elasticity exceeds (in absolute terms) the elasticity of substitution. Therefore, the retail domestic demand elasticity will be more elastic than the wholesale domestic demand elasticity. Likewise, it is assumed that the wholesale demand elasticity has an absolute value that is greater than the elasticity of substitution between inputs in the farm gate to wholesale transformation. Then, because farm gate demand is derived from wholesale demand, the farm-gate export demand elasticity will be less responsive than the wholesale export elasticity.

Based on this reasoning the estimated domestic elasticities have to be adjusted downward and the estimated export elasticities have to be adjusted upward, by an arbitrarily chosen factor of 20 per cent. It was also assumed that the domestic cross-price response is more elastic than the export cross-commodity response. This seems reasonable as it is thought that the main substitute for Australian beef in export markets is domestically produced beef or beef imported from other countries, not Australian lamb.

For simplicity, the export cross-price elasticities were obtained by halving the adjusted domestic cross-price demand elasticities.

Making the above adjustments results in a set of domestic and export demand elasticities that represent the wholesale demand for beef and lamb. Using these elasticity values the quantity weighted average demand elasticities for beef and lamb were calculated and are given in Table 1. These estimated elasticity values are technically ‘short-run’ elasticities and represent the demand response to a price change that would occur over as short as three months or as long as 12 months. However there is little evidence of any long-run dynamics in the demand for meat and there should be little difference between these estimates and any hypothetical long-run estimates. One other point to note is that the demand elasticities used here could be considered to be relatively larger than those used in other similar studies.

Supply Elasticities

Previously empirically-estimated supply elasticities were used to represent the supply elasticities for cattle and sheep (Table 2). Two common methods used to estimate supply elasticities are econometric modelling and programming methods. Typically estimates from programming models are higher than estimates from econometric models. Own-price supply elasticity estimates were obtained by averaging the results from both methods in a study undertaken by Hall, Fraser and Purtill (1988). This study also estimated cross-price elasticities using programming methods only and these estimates are shown in Table 2. While the supply elasticities used here are similar in magnitude to those used in other similar studies, they are quite inelastic and much smaller than the demand elasticities.

These supply elasticities are 'medium term' supply elasticities in that they represent the supply response over a five year adjustment period. It is thought that this length of time is sufficient for the majority of adjustment to occur in the beef and lamb industries. It is considered that the demand and supply elasticities both measure responsiveness over approximately the same length of run.

It was of some concern that these estimates were done in 1988 as it is thought that the supply response of agriculture in general may have changed since then. Some recent trends may have increased supply response (increased use of purchased inputs), while some may have decreased supply response (increase in farm size and accompanying contraction in product mix). It is difficult therefore, to qualitatively argue whether the supply response of beef and lamb has increased or decreased over time, and the 1988 supply elasticity estimates are not adjusted.

Determining the appropriate marketing input supply elasticity is difficult. There have been no empirical studies undertaken on the responsiveness of marketing input supply in the context used in this study. However it is thought that the supply of such services would be quite elastic since, unlike the more specialised nature of agricultural inputs, marketing inputs tend to be unspecialised in usage. This implies that the supply of marketing inputs will be highly price responsive. Consequently, quite high own-price elasticity values have been chosen. There exists no intuitive reason why the supply response of marketing services should be greater in one industry than the other and, therefore, a supply elasticity of 15 has been chosen for the marketing inputs used in both industries.

The Elasticity of Substitution and Input Shares

The elasticity of substitution between inputs in beef (lamb) production measures the extent of substitution between cattle (sheep) and the beef marketing input (sheep marketing input) in the transformation of the farm product into the wholesale product. Wohlgenant (1989) reported values for the elasticity of substitution between inputs used in the production of agricultural products that range from zero to one. More specifically he reported an elasticity of substitution between inputs in the production of beef equal to 0.72. Intuitively there exists no reason to assume that the extent of input substitution is greater in one industry than the other. Therefore the value used for the elasticity of substitution between inputs in both the industries is 0.5.

Values are also required for the shares of the inputs. Theory stipulates that a factor share value must be positive and the sum of the shares is one. All of the factor shares have been given a value of 0.5, since no reason could be found to suggest why they would differ between the two industries.

Sensitivity Analysis

Since many of the required parameter values required for implementing the model are uncertain, a stochastic approach was adopted following Zhao *et al*. (2000). Using the econometric computer program, SHAZAM, the analysis involved choosing an appropriate probability distribution for each of the unknown parameters and drawing 5000 values from each of these distributions to construct estimated frequency distributions of results. The subsequent distributions were then used to make various qualifying statements about the response of the wholesale-farm price ratio in the Australian beef and lamb industries to exogenous shocks.

The Choice of Probability Distributions

Economic theory provides some information concerning the expected signs and size of each of the unknown parameters. Such information can be used to assign appropriate probability distributions to each of the parameters.

The domestic and export own-price demand elasticities and cross-price supply elasticities are expected to have negative signs. The appropriate distribution will therefore assign a zero probability for positive values. A normal distribution truncated at zero from above was chosen as an appropriate distribution for these elasticities. Conversely, the domestic cross-price demand elasticities, the own-price supply elasticities and the elasticity of substitution between inputs in the beef and in the lamb industry are expected to have positive signs. A normal distribution truncated at zero from below was chosen for these parameters. The distributions for the export cross-price demand elasticities were derived from the truncated distributions stipulated for the domestic cross-price demand elasticities by halving the values of the domestic elasticities.

The truncated distribution for each parameter is derived from a normal distribution which has a mean, μ, and a standard deviation, s. In the instance of a normal distribution that is truncated from above all positive values drawn from the normal distribution are discarded, whereas if the distribution is truncated from below all negative values are discarded. For each parameter (i.e. elasticity) the values for μ and s were chosen such that, once truncated, the resulting distribution had a mean that would equal the base parameter value and 95 per cent of the observations would fall within a specified range. For example, a normal distribution truncated from above with a mean of approximately –1 and a range of 0 to –1.5 was specified for the domestic own-price demand elasticity for beef. This distribution was derived from a standard normal distribution which had a mean (μ) = -1.0 and a standard deviation (s) = 0.31.

The ranges and means for the demand and supply elasticities were derived so as to ensure that certain economic relationships would hold. These relationships were that the absolute value of an own-price demand elasticity should exceed that of the related cross-price demand elasticity and that an own-price supply elasticity should exceed the absolute value of the related cross-price supply elasticity.

A uniform probability distribution implies that the parameter concerned has an equal probability of taking any value in a specified range. Such a distribution can be used when little information is known about the behaviour of a particular parameter, and it was chosen to represent the behaviour of the marketing input supply elasticities and the factor share parameters. It is known that the own-price supply response of marketing inputs will be positive and highly elastic. Therefore a uniform distribution was chosen with a range of 10 to 20 (and a mean of 15). It is also known that the input shares lie between 0 and 1, so a uniform distribution was chosen with a range of 0 to 1 (and a mean of 0.5). As input shares sum to one in a perfectly competitive industry, it is possible to generate observations for marketing input shares from the observations on the agricultural input shares. For example, once an observation on cattle shares is derived, a value for the beef marketing input share can be derived as 1.0 minus the cattle share.

To ensure that related parameters (e.g. η_{BL} and η_{LB}) do not vary greatly from each other, observations on related parameters were drawn jointly in such a way as to ensure the correlations between the parameters was 0.7. A summary of the information relating to the assumed distributions is provided in Table 3.

Since the focus of attention is on the effects of including or excluding cross-commodity relations, the general equilibrium elasticities were calculated twice for each of the 5000 draws of parameter values. In the first calculation, all the parameter values from the draw were used. In the second calculation, the same values as in the first were used except that the cross-price elasticities were assumed to be zero, so excluding cross-commodity impacts. In a sense, using the same own-price elasticities in each case is an 'unfair' representation of reality. If, in the real world, cross effects are present then the own-price elasticities will differ from those which exist when the cross effects are not present. Hence, the simulations should be interpreted as indicating the consequences of simply ignoring the cross-price effects that are known to be present.

4. Results

Replication of Gardner's Results Using US Parameter Values

As a validation exercise, Gardner's one-product model results were replicated. To do this it was assumed that, for a demand shift, Gardner's product *x* represents beef and, therefore, Gardner's assumed parameter values apply to the beef industry. To extend the Gardner model to the two-product case, elasticity values have to be assumed for the lamb industry and for the cross-price demand and supply elasticities. These elasticities are hypothetical values that are consistent with economic theory and Gardner's choice of parameter values. If the cross-commodity relationships between the two related industries are ignored by setting the cross-price elasticities to zero, a beef demand shift in this two-product model produces identical results to those obtained by Gardner's one-product model (Table 4).

The inclusion of cross-commodity effects changes the values of $G_{PB/PC,N}$ generally in the direction of making the general equilibrium elasticities larger. The percentage differences in the size of the general equilibrium elasticities are generally significant, in the order of 20-30 per cent. Therefore, not surprisingly, omitting these relationships from a model when they do exist will result in incorrect estimates of the general equilibrium elasticities.

Gardner shows that the general equilibrium elasticity, $G_{PB/PC,N}$ is negative for $\varepsilon_{CC}<\varepsilon_{XX}$, positive for $\varepsilon_{CC}>\varepsilon_{XX}$ and zero for $\varepsilon_{CC}=\varepsilon_{XX}$. The first two relationships carry through to the case where the cross-commodity effects are included, but not the third. Therefore, contrary to the implications of Gardner's results, a fixed percentage rule will not be viable when $\varepsilon_{CC}=\varepsilon_{XX}$ since competitive forces will require the markup to change whenever demand shifts if the markets are to clear[4].

[4] An attempt was made to show analytically why these results differ when the cross-commodity effects are included and when they are ignored. The attempt was made using the computer program Mathematica©. However the extent of simplification possible in the absence of severe constraints on

Results Using Australian Parameter Values

Reverting to the Australian model described above, the behaviour of the beef and lamb wholesale-farm price ratios were examined, both with and without cross-commodity effects, when three separate exogenous shifts occurred. The three shifts were a negative beef demand shift (e.g. Helix contamination scare), a negative lamb demand shift (e.g. an increase in import lamb tariffs by the US) and a negative sheep supply shift (e.g. a liquidation of the national sheep flock in response to the termination of the Reserve Price Scheme for wool). A summary of the results is provided in Table 5. The results concerning the negative beef demand shift are discussed in detail in the following section.

A Negative Shift in Beef Demand

Consider a one per cent decline in the demand for beef caused by an event such as the Helix contamination scare (an increase in N). Note that the elasticity of beef demand with respect to the demand shifter N (η_N) will be negative. From the simulation undertaken, the initial leftward shift of the demand function for beef will result in the general equilibrium elasticity values outlined in the first line of Table 5. The values obtained for $G_{PB/PC,N}$ are positive irrespective of whether the cross-commodity effects are included or excluded. The value for $G_{PB/PC,N}$ with cross-commodity effects is slightly larger than the value for $G_{PB/PC,N}$ without cross-commodity effects. Therefore the inclusion of cross-commodity relationships increases the responsiveness of the beef:cattle price ratio to the demand shift, but only slightly in this case.

The positive sign for the general equilibrium elasticity $G_{PB/PC,N}$ is opposite in sign to the same general equilibrium elasticity reported by Gardner for the single-product model. This is consistent for two reasons. Firstly, the value for η_N used by Gardner is positive, whereas in this study the value for η_N is negative. Secondly, it was found above that including the cross-commodity effects did not alter the sign of the general equilibrium elasticity, although this was not proven algebraically. These two observations suggest that in the context of the current simulation, the general equilibrium elasticity will be positive irrespective of whether the cross-commodity effects are included or excluded.

When the cross-commodity relationships between the beef and lamb industries are ignored, a negative shift in the demand for beef does not affect the lamb wholesale-farm price ratio (a zero value for $G_{PL/PS,N}$). When the cross-commodity effects are included, however, the negative demand shift does affect the lamb wholesale-farm price ratio. The value for $G_{PL/PS,N}$ is also positive, indicating that the negative demand shift for beef (resulting from an increase in N) will not only increase the beef wholesale-farm price ratio, but will also increase the lamb wholesale-farm price ratio, but less so than in the case of the beef wholesale-farm price ratio.

Additive versus Counteracting Cross-Commodity Effects

---

parameter values was insufficient to allow one to make analytical statements about the extent to which the general equilibrium elasticities differed. However, it is expected that the results of the sensitivity analysis will at least provide some evidence on the signs of the general equilibrium elasticities.

From Table 4 it is clear that the inclusion of cross-commodity effects increases the responsiveness of the beef and lamb wholesale-farm price ratios with respect to each of the exogenous shifters, although only slightly. So it is possible that the cross-demand and cross-supply effects are counteracting each other to some extent. If this is the case then excluding the cross-commodity relations might be a small 'empirical sin' whilst remaining a conceptual error.

To discover whether the supply and demand cross-price effects are offsetting each other, each of the exogenous shifts were modelled when only the demand cross-price effects were included and when only the supply cross-price effects were included. If the resulting general equilibrium elasticity values are greater than those obtained when the cross-commodity effects were both included it can be said that the two effects are offsetting one another. However if the values lay between those obtained by including and excluding both the cross-price effects, the demand and supply cross-price effects are not offsetting each other and are additive.
Table 6 contains the values for $G_{PB/PC}$ and $G_{PL/PS}$ with respect to a negative beef demand shift, when various cross-price effects are considered. The values obtained for the general equilibrium elasticities when only one of the cross-commodity effects were included are the same (to 3 decimal places) as the general equilibrium elasticity when both effects were excluded, but are smaller than the value obtained when both effects were included. This suggests that the demand and supply cross-price effects are additive rather than counteracting. Therefore incorrectly excluding the cross-commodity effects may result in large errors. A similar situation was found in relation to the negative lamb demand shift and the negative sheep supply shift, although the results are not presented here.

Sensitivity Analysis

Five thousand observations were generated on the general equilibrium elasticities $G_{PB/PC,N}$, $G_{PL/PS,T}$, and $G_{PL/PS,W}$ with and without cross-commodity effects. Five thousand observations were also generated on $G_{PL/PS,N}$, $G_{PB/PC,T}$ and $G_{PB/PC,W}$ when cross-commodity effects were included. A frequency distribution was then estimated for each of the general equilibrium elasticities. The summary statistics of each distribution are summarized in Table 7.

The probability distributions for each of the *unknown parameters* were specified such that the mean of each distribution was approximately equal to the base value of the parameters used in the simulations reported above. Whilst one might expect that the means of the distributions for the general equilibrium elasticities would approximate the values for the general equilibrium elasticities from the base value simulation (reported in Table 5 and as the base in Table 7), their means exceed the base values reported for the elasticities. For example, in the initial simulations reported in Table 5, the value for $G_{PB/PC,N}$ when cross-commodity effects were included is 0.099, whereas the mean value of the distribution of $G_{PB/PC,N}$ when cross-commodity effects were included is 0.179, almost 100 per cent greater than the base value. The differences between the means of the distributions of the general equilibrium elasticities and the base values of the general equilibrium elasticities are partly due to the fact that the distributions are only estimated distributions and partly due to the fact that the these distributions are asymmetric. The means of the distributions would more closely approximate the base values if the distributions were symmetric. When the distributions of each of the general equilibrium elasticities are plotted, it becomes apparent that the distributions are skewed to the right, owing to the fact that many of the distributions for the unknown parameters are truncated.

The mean of a random variable is a weighted average of the values taken by the random variable, where the weights are respective probabilities. As the distributions of concern are rightward skewed the means are higher than the most commonly occurring value, the mode. It is thought that the mode of each distribution for the general equilibrium elasticities may be closer to the base values for the general equilibrium elasticities. The mode value reported for each general equilibrium elasticity (Table 7) is calculated using the estimated distribution for each elasticity. The estimated frequency distribution is obtained by using the relative frequencies of observations falling into a set of mutually exclusive and exhaustive cells. The mode is simply calculated as the mid point of the cell with the highest frequency.

In all cases the estimated mode of the distributions are closer to the base value than the mean. For example, the mode of the distribution for $G_{PB/PC,N}$ with cross-commodity effects is 0.069. This value is approximately 40 per cent less than the base value for $G_{PB/PC,N}$ (0.099) but is closer than the mean which was approximately 100 per cent greater. The true mode of the distribution may be closer to the base value, but as the mode is simply estimated as the mid-point of the cell with the highest relative frequency, it is subject to estimation error.

To obtain a better idea of the range of each unknown parameter, a Highest Probability Density (HPD) interval was constructed for each distribution. This interval represents the smallest range in which ninety-five per cent of the observations on the general equilibrium elasticity lie. This interval is calculated by: (1) sorting the observations from lowest to highest; (2) calculating all the intervals which contain 95 per cent of observations (e.g.: observation 1 to observation 1+0.95N; observation 2 to observation 2+0.95N); and (3) choosing the smallest of these intervals. The HPD intervals for each unknown parameter are reported in Table 7 and from this table it is apparent that the base parameter values all lie within the estimated HPD intervals.

In Table 5 it was found that a negative beef demand shift (an increase in *N*) caused the beef wholesale-farm price ratio to increase, and that the increase was greater when cross-commodity effects were included. Additionally, it was found that when the cross- commodity effects were included the negative beef demand shift also caused the lamb wholesale-farm price ratio to increase, although to a lesser extent.

The estimated frequency distributions for $G_{PB/PC,N}$ in the absence of cross-commodity effects, for $G_{PB/PC,N}$ in the presence of cross-commodity effects and for $G_{PL/PS,N}$ in the presence of cross-commodity effects (not shown here) all suggest each of the general equilibrium elasticities are positive (all 5000 draws on these elasticities were positive). This confirms the earlier finding that a negative beef demand shift will increase the beef wholesale-farm price ratio regardless of whether the cross-commodity effects are included or excluded, and the lamb wholesale-farm price ratio in the presence of cross-commodity effects.

As the general equilibrium elasticities for the beef wholesale-farm price ratio are always positive it appears that the cross-commodity effects are not strong enough to override the events that were occurring in the beef industry when the cross-commodity effects were ignored.

5. Conclusions

The principal objective of this study was to determine whether the inclusion of a related industry and associated cross-commodity effects altered the behaviour of the wholesale-farm price ratio in a Gardner type model under conditions of perfect competition. The motivation for doing so is the fact that cross-commodity relationships in demand and supply are an important feature of Australian broadacre agriculture.

The study focused on the relationships between the Australian beef and lamb industries, since in recent times several external events have occurred that have affected the demand and supply of beef and lamb.

The results generated in the study suggest that the inclusion of cross-commodity relationships does alter the effect that each of the exogenous shifters has on the wholesale-farm price ratio. More specifically, it was found that the inclusion of cross-commodity relationships does not alter the directional influence an exogenous shifter has on the wholesale-farm price ratio, but does slightly increase the responsiveness of the wholesale-farm price ratio to an exogenous shifter.

The fact that the empirical differences between the general equilibrium elasticities when cross-commodity effects are included and excluded are small is not a justification for ignoring these cross-commodity effects. These differences are dependent on the choice of values for the endogenous parameters and the elasticities of demand and supply with respect to the exogenous shifters. In particular, as noted above, the differences in percentage changes between Tables 4 and 5 can be explained largely by the relatively larger demand elasticities and the relatively smaller supply elasticities assumed for the Australian model compared with the US model. Therefore, the empirical differences shown in Table 5 could be increased by choosing different distributions for the unknown parameter values.

There are several limitations to this analysis that should be recognised. First, the two-product model assumes that an agricultural input is combined with a bundle of marketing inputs to produce a final product. There are many marketing inputs used in the production of beef and lamb and they have been aggregated into just a single marketing input variable for each industry. Such aggregation may cause difficulties if the prices of the aggregated components do not move together. However, if the relative prices of the components remain constant, such aggregation should not cause any analytical difficulties (Gardner 1975, p401). The wholesale products, namely beef and lamb, are also an aggregation of a number of wholesale cuts of meat. However it is thought that the relative prices of different cuts of beef and lamb remain fairly constant and therefore aggregation should not pose a problem.

Second, in the modelling approach used in this study, unknown demand and supply functions, and unknown production functions, have been approximated by functions that are linear in Marshallian elasticities and proportionate changes. The accuracy of these approximations will depend on the true nature of the underlying functions. This is discussed in various places in Gardner (1987). However these approximations will be accurate for constant elasticity functions and if the proportionate changes are measured as log differentials (Zhao *et al* 1997). Generally speaking, the approximations are more accurate the smaller the percentage change in the exogenous variable under consideration.

Finally the research reported in this study is strictly 'comparative static' and does not attempt to examine mathematically the adjustment paths between successive equilibria.

In terms of further research, the analysis reported here could be extended to look at other measures of the 'marketing margin', such as the elasticity of price transmission (the ratio of the percentage change in the farm and product price) associated with various exogenous shifts. Examination of the latter would be useful given the frequency with which farmers complain about saleyard livestock prices not being reflected in the prices for retail meat products. This would be particularly useful given the recent concern with the competitive structure of the food marketing chain (Griffith 2000, Piggott *et al*. 2000).

6. References

ABARE (2002), *Commodity Statistical Bulletin 2002*, Australian Bureau of Agricultural and Resource Economics, Canberra Australia.

Gardner, B.L. (1975), 'The Farm-Retail Spread in a Competitive Food Industry,' *American Journal of Agricultural Economics*, August 1975, 399-409.

Gardner, B.L. (1987), *The Economics of Agricultural Policies*, Macmillan Publishing Company, New York.

Griffith, G.R. (2000), "Competition in the food marketing chain", *Australian Journal of Agricultural and Resource Economics* 44(3), 333-367.

Griffith, G., I'Anson, K., Hill, D., Lubett, R. and D. Vere (2001a), *Previous Demand Elasticity Estimates for Australian Meat Products*, Economics Research Report No. 5, NSW Agriculture, Orange, January.

Griffith, G., I'Anson, K., Hill, D. and D. Vere (2001b), *Previous Supply Elasticity Estimates for Australian Broadacre Agriculture*, Economics Research Report No. 6, NSW Agriculture, Armidale, August.

Hall, N., Fraser, L., & Purtill, A. (1988), 'Supply Response in Broadacre Agriculture,' *Review of Marketing and Agricultural Economics* **56** (3), 361-372.

Hill, D.J., Piggott, R.R. and G.R. Griffith (1996), "Profitability of incremental expenditure on fibre promotion", *Australian Journal of Agricultural Economics* 40(3), 151-174.

Hill, Debbie, O'Donnell, Chris, Piggott, Roley and Garry Griffith (2001), *The Dual Approach to Research Evaluation: A Simplified Empirical Illustration*, Working Paper Series, Number 39, Impact Assessment Program, ACIAR, Canberra.

Hill, D., Piggott, R.R. and G.R. Griffith (2001), "Profitability of incremental generic promotion expenditure of Australian dairy products", *Agricultural Economics* 26(3), 249-262 .

Morrow, S.K. (2001), Marketing Margin Behaviour in the Presence of Cross-Commodity relationships: The Case of Australian Beef and Lamb, unpublished B.Ag.Ec. (Hons) dissertation, University of New England, Armidale.

Ockwell, A. (1990), 'The economic structure of Australian agriculture', in Williams, D.B. (ed.), *Agriculture in the Australian Economy,* 3rd ed., Sydney University Press, 27-49

Piggott, Roley, Griffith, Garry, and Nightingale, John (2000), *Market Power in the Australian Food Chain: Towards a Research Agenda, Final Report to the Rural Industries Research and Development Corporation on Project UNE-64A*, RIRDC Publication No. 00/150, Rural Industries Research and Development Corporation, Canberra, October.

Wohlgenant, M.K. (1989), 'Demand for Farm Output in a Complete System of Demand Functions,' *American Journal of Agricultural Economics*, **71**(2), 241-252.

Wohlgenant, M.K & Haidacher, R.C. (1989),'Retail to Farm Linkage for a Complete Demand System of Food Commodities,' *USDA Technical Bulletin*, No. 1775.

Vere, D.T., Griffith, G.R. and R.E. Jones (2000), *The Specification, Estimation and Validation of a Quarterly Structural Econometric Model of the Australian Grazing Industries*, CRC for Weed Management Systems, Technical Series No. 5, University of Adelaide, Adelaide.

Zhao, X., Mullen, J.D. and Griffith, G.R. (1997), "Functional forms, exogenous shifts and economic surplus changes", *American Journal of Agricultural Economics* 79(4), 1243-1251.

Zhao, X., Mullen, J.D., Griffith, G.R., Griffiths, W.E. and R.R Piggott (2000), *An Equilibrium Displacement Model of the Australian Beef Industry*, Economics Research Report No. 4, NSW Agriculture, Orange, December.

Zhao, Xueyan, Mullen, John and Griffith, Garry (2001), "Some practical issues in economic surplus measurement in multi-market models", paper presented to 45th Annual Australian Agricultural and Resource Economics Society Conference, Adelaide, 22-25 January.

Table 1: Chosen Base Demand Elasticities

Industry	Domestic Own-Price	Export Own-Price	Weighted Own-Price	Domestic Cross-Price	Export Cross-Price	Weighted Cross-Price
Beef	-1	-12	**-7.6**	0.4	0.2	0.3
Lamb	-1.2	-7.2	-3	0.6	0.3	0.5

*Source***:** Derived from estimates reported in Griffith *et al*. (2001a) and after adjustments as described in the text.

Table 2: Chosen Base Supply Elasticities

Input	Own-Price	Cross-Price
Cattle	0.5	-0.1
Sheep	0.4	-0.2

Source: Hall, Fraser & Purtill (1988)

Table 3: Summary of the Probability Distributions

Parameter[a]	Name[b]	Value	Source	Sign	Distribution [c]	Mean	'Range'	N(μ ,σ)
η_{BB}^{d}	Domestic o-p demand elasticity for beef	-1	Griffith et al (2001a)	(-)	NT from above	-1	$0 \rightarrow -1.5$*	N(-1, 0.31)
η_{LL}^{d}	Domestic o-p demand elasticity for lamb	-1.2	Griffith et al (2001a)	(-)	NT from above	-1.2	$0 \rightarrow -1.5$*	N(-1.2, 0.2)
η_{BL}^{d}	Domestic c-p demand elasticity for beef	0.4	Griffith et al (2001a) & author's judgement	(+)	NT from below	0.4	$0 \rightarrow 0.5$*	N(0.4 , 0.06)
η_{LB}^{d}	Domestic c-p demand elasticity for lamb	0.6	Griffith et al (2001a) & author's judgement	(+)	NT from below	0.6	Cor($\eta_{BL}^{D}, \eta_{LB}^{D}$)=0.7	N(0.6 , 0.24)
η_{BB}^{e}	Export o-p demand elasticity for beef	-12	Scobie et al (1979)	(-)	NT from above	-12	$0 \rightarrow 30$	N(17 , 21)
η_{LL}^{e}	Export o-p demand elasticity for lamb	-7.2	Scobie et al (1979)	(-)	NT from above	-7.2	$0 \rightarrow 25$	N(61 , 24)
η_{BL}^{c}	Export c-p demand elasticity for beef	0.2	Scobie et al (1979)	(+)	NT from below	0.2	0.5 η_{BL}^{D}	
η_{LB}^{c}	Export c-p demand elasticity for lamb	0.3	Scobie et al (1979)	(+)	NT from below	0.3	0.5 η_{LB}^{D}	
ε_{CC}	O-p supply elasticity of cattle	0.5	Average of econometric & LP estimates in Hall et al (1988)	(+)	NT from below	0.5	$0 \rightarrow 0.6$*	N(0.5 , 0.06)
ε_{SS}	O-p supply elasticity of sheep	0.4	Average of econometric & LP estimates in Hall et al (1988)	(+)	NT from below	0.4	Cor($\varepsilon_{CC}, \varepsilon_{SS}$)=0.7	N(0.38,0.24)
ε_{CS}	C-p supply elasticity of cattle	-0.1	Hall et al (1988)	(-)	NT from above	-0.1	$0 \rightarrow -0.4$*	N(0.8 ,-0.25)
ε_{SC}	C-p supply elasticity of sheep	-0.2	Hall et al (1988)	(-)	NT from above	-0.2	Cor($\varepsilon_{CS}, \varepsilon_{SC}$)=0.7	N(0.4,-0.3)
ε_{XX}	O-p supply elasticity of beef marketing inputs	15	Author's judgement	(+)	Uniform	15	$10 \rightarrow 20$	
ε_{YY}	O-p supply elasticity of lamb marketing inputs	15	Author's judgement	(+)	Uniform	15	$10 \rightarrow 20$	
K_C	Factor share of cattle	0.5	Author's judgement	(+)	Uniform	0.5	$0.4 \rightarrow 0.6$#	
K_X	Factor share of beef marketing inputs	0.5	Author's judgement	(+)	Uniform	0.5	$0.6 \rightarrow 0.4$#	
K_S	Factor share of sheep	0.5	Author's judgement	(+)	Uniform	0.5	$0.4 \rightarrow 0.6$#	
K_Y	Factor share of lamb marketing inputs	0.5	Author's judgement	(+)	Uniform	0.5	$0.6 \rightarrow 0.4$#	
σ_B	Elasticity of substitution in beef production	0.5	Commonly used value in literature	(+)	NT from below	0.5	1*	N(-0.001, 0.5)
σ_L	Elasticity of substitution in lamb production	0.5	Commonly used value in literature	(+)	NT from below	0.5	Cor(σ_B, σ_L)=0.7	N(0.001, 0.55)

[a] The superscript d represents a domestic elasticity, whereas the superscript e represents export elasticity[b] O-p represents own-price elasticity and c-p represents cross-price elasticity

[c] NT represents a normal distribution that has been truncated

* 95% of observations fall within the range

100% of observations fall within the range

Table 4: Values for $G_{PB/PC,N}$ with and without Cross-Commodity Effects, Using US Parameter Values

		Parameter	**Values**													
σ_B	ε_{CC}	ε_{XX}	η_{BB}	K_C	σ_L	ε_{SS}	ε_{YY}	η_{LL}	K_S	ε_{CS}	ε_{SC}	η_{BL}	η_{LB}	$G_{PB/PC,N}$ $	$G_{PB/PC,N}$ *$	**% Change**
0.5	1.0	2.0	-0.5	0.5	0.5	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**-0.158**	**-0.133**	**18.797**
0	1.0	2.0	-0.5	0.5	0	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**-0.224**	**-0.182**	**23.077**
0	1.5	2.0	-0.5	0.5	0	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**-0.084**	**-0.064**	**31.250**
0	2.0	2.0	-0.5	0.5	0	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**-0.010**	**0**	**#**
0	2.0	1.0	-0.5	0.5	0	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**0.177**	**0.182**	**-2.747**
0	1.0	2.0	-1.0	0.5	0	0.8	2.0	-1.5	0.6	-0.2	-0.4	0.3	0.4	**-0.172**	**-0.143**	**20.280**

* General equilibrium elasticities when all cross-price elasticities are set to zero and other parameters are set as the values shown in the corresponding row. They are the values as reported by Gardner (1975, Table 1, Column 6)

$ The value for η_N is assumed to be 1.

Not calculated because the base value is zero

Table 5: Values for $G_{PB/PC}$ and $G_{PL/PS}$ with respect to Three Exogenous Shifters, with and without Cross-Commodity Effects[e], Using Australian Parameter Values

	$G_{PB/PC}$			$G_{PL/PS}$		
Shift	**Cross Effects Included**	**Cross Effects Excluded**	**% Difference**	**Cross Effects Included**	**Cross Effects Excluded**	**% Difference**
Decrease in beef demand (Helix contamination)[a]	**0.099**	**0.098**	**1.020**	**0.023**	**0**	**[d]**
Decrease in lamb demand (US tariffs)[b]	**0.014**	**0**	**[d]**	**0.215**	**0.212**	**1.415**
Decrease in sheep supply (wool market collapse)[c]	**0.014**	**0**	**[d]**	**0.264**	**0.261**	**1.149**

[a] The value of η_N is set to –1.

[b] The value of η_T is set to –1.

[c] The value of ε_W is set to 1.

[d] Not calculated because the base value was zero.

[e] The set of parameter values used in the simulation assumes that the supply response of marketing inputs is more elastic than the supply response of agricultural inputs. This differs from Gardner's study, which considered this case as well as the opposite case.

Table 6: General Equilibrium Elasticities of the Wholesale-Farm Price Ratios with Respect to a Shift in Beef Demand, Using Australian Parameter Values

		Parameter	Values												
σ_B	ε_{CC}	ε_{XX}	η_{BB}	K_C	σ_L	ε_{SS}	ε_{YY}	η_{LL}	K_S	ε_{CS}	ε_{SC}	η_{BL}	η_{LB}	$G_{PB/PC,N}$	$G_{PL/PS,N}$
No cross-commodity effects included															
0.5	0.5	15	-7.6	0.5	0.5	0.4	15	-3	0.5	0	0	0	0	**0.098**	**0**
Only demand cross-commodity effects included															
0.5	0.5	15	-7.6	0.5	0.5	0.4	15	-3	0.5	0	0	0.3	0.5	**0.098**	**0.012**
Only supply cross-commodity effects included															
0.5	0.5	15	-7.6	0.5	0.5	0.4	15	-3	0.5	-0.1	-0.2	0	0	**0.098**	**0.011**
Both cross-commodity effects included															
0.5	0.5	15	-7.6	0.5	0.5	0.4	15	-3	0.5	-0.1	-0.2	0.3	0.5	**0.099**	**0.023**

Table 7: A Summary of the Sensitivity Analysis Results, Using Australian Parameter Values

	$G_{PB/PC}$								$G_{PL/PS}$							
	Cross Effects				No Cross Effects				Cross Effects				No Cross Effects			
	Base	Mean	Mode	HPD	Base	Mean	Mode	HPD	Base	Mean	Mode	HPD	Base	Mean	Mode	HPD
Beef demand shift (N)	0.099	0.179	0.069	0.020 → 0.475	0.098	0.169	0.069	0.024 → 0.442	0.023	0.062	0.025	0.001 → 0.188	0	0	0	0
Lamb demand shift (T)	0.014	0.034	0.015	0.002 → 0.099	0	0	0	0	0.215	0.327	0.316	0.072 → 0.644	0.212	0.314	0.171	0.081 → 0.612
Sheep supply shift (W)	0.014	0.035	0.015	0.001 → 0.102	0	0	0	0	0.264	0.382	0.278	0.124 → 0.708	0.261	0.368	0.263	0.131 → 0.677

Appendix

Equations 13 to 24 in matrix form is as follows:

$$\begin{bmatrix} 1 & 0 & -K_C & -K_X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -\eta_{BB} & -\eta_{BL} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_X/\sigma_B & -K_X/\sigma_B & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -K_C/\sigma_B & K_C/\sigma_B & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{CC} & 0 & -\varepsilon_{CS} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{XX} & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_S & -K_Y & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\eta_{LB} & -\eta_{LL} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_Y/\sigma_L & -K_Y/\sigma_L & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -K_S/\sigma_L & K_S/\sigma_L & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_{SC} & 0 & -\varepsilon_{SS} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{YY} \end{bmatrix} \begin{bmatrix} E_B \\ E_L \\ E_C \\ E_X \\ E_S \\ E_Y \\ E_{PB} \\ E_{PL} \\ E_{PC} \\ E_{PX} \\ E_{PS} \\ E_{PY} \end{bmatrix} = \begin{bmatrix} 0 \\ \eta_N E_N \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \eta_T E_T \\ 0 \\ 0 \\ \varepsilon_W E_W \\ 0 \end{bmatrix}$$

This system can be re-written as:

$$\begin{bmatrix} 1 & 0 & -K_C & -K_X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -\eta_{BB} & -\eta_{BL} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_X/\sigma_B & -K_X/\sigma_B & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -K_C/\sigma_B & K_C/\sigma_B & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{CC} & 0 & -\varepsilon_{CS} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{XX} & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_S & -K_Y & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\eta_{LB} & -\eta_{LL} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_Y/\sigma_L & -K_Y/\sigma_L & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -K_S/\sigma_L & K_S/\sigma_L & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_{SC} & 0 & -\varepsilon_{SS} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{YY} \end{bmatrix} \begin{bmatrix} E_B \\ E_L \\ E_C \\ E_X \\ E_S \\ E_Y \\ E_{PB} \\ E_{PL} \\ E_{PC} \\ E_{PX} \\ E_{PS} \\ E_{PY} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \eta_N & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \eta_T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_W \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_N \\ E_T \\ E_W \end{bmatrix}$$

Now if we post multiply both sides by the vector $\left[\dfrac{1}{EN}\ \dfrac{1}{ET}\ \dfrac{1}{EW}\right]$ we have:

$$\begin{bmatrix} 1 & 0 & -K_C & -K_X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -\eta_{BB} & -\eta_{BL} & 0 & 0 & 0 & 0 \\ 0 & 0 & K_X/\sigma_B & -K_X/\sigma_B & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -K_C/\sigma_B & K_C/\sigma_B & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{CC} & 0 & -\varepsilon_{CS} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{XX} & 0 & 0 \\ 0 & 1 & 0 & 0 & -K_S & -K_Y & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\eta_{LB} & -\eta_{LL} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_Y/\sigma_L & -K_Y/\sigma_L & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -K_S/\sigma_L & K_S/\sigma_L & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_{SC} & 0 & -\varepsilon_{SS} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{YY} \end{bmatrix} \begin{bmatrix} E_B/E_N & E_B/E_T & E_B/E_W \\ E_L/E_N & E_L/E_T & E_L/E_W \\ E_C/E_N & E_C/E_T & E_C/E_W \\ E_X/E_N & E_X/E_T & E_X/E_W \\ E_S/E_N & E_S/E_T & E_S/E_W \\ E_Y/E_N & E_Y/E_T & E_Y/E_W \\ E_{PB}/E_N & E_{PB}/E_T & E_{PB}/E_W \\ E_{PL}/E_N & E_{PL}/E_T & E_{PL}/E_W \\ E_{PC}/E_N & E_{PC}/E_T & E_{PC}/E_W \\ E_{PX}/E_N & E_{PX}/E_T & E_{PX}/E_W \\ E_{PS}/E_N & E_{PS}/E_T & E_{PS}/E_W \\ E_{PY}/E_N & E_{PY}/E_T & E_{PY}/E_W \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \eta_N & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \eta_T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_W \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The identity matrix on the RHS arises when the ratio of any two proportionate changes is set to zero to reflect the assumption that only one of the exogenous variables changes at a time.

Denoting E_B/E_N as $G_{B,N}$ etc, the solution for the system is given by:

$$
\begin{bmatrix}
1 & 0 & -K_C & -K_X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -\eta_{BB} & -\eta_{BL} & 0 & 0 & 0 & 0 \\
0 & 0 & K_X/\sigma_B & -K_X/\sigma_B & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -K_C/\sigma_B & K_C/\sigma_B & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{CC} & 0 & -\varepsilon_{CS} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{XX} & 0 & 0 \\
0 & 1 & 0 & 0 & -K_S & -K_Y & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -\eta_{LB} & -\eta_{LL} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & K_Y/\sigma_L & -K_Y/\sigma_L & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -K_S/\sigma_L & K_S/\sigma_L & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_{SC} & 0 & -\varepsilon_{SS} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{YY}
\end{bmatrix}^{-1}
\begin{bmatrix}
0 & 0 & 0 \\
\eta_N & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \eta_T & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \varepsilon_W \\
0 & 0 & 0
\end{bmatrix}
=
\begin{bmatrix}
G_{B,N} & G_{B,T} & G_{B,W} \\
G_{L,N} & G_{L,T} & G_{L,W} \\
G_{C,N} & G_{C,T} & G_{C,W} \\
G_{X,N} & G_{X,T} & G_{X,W} \\
G_{S,N} & G_{S,T} & G_{S,W} \\
G_{Y,N} & G_{Y,T} & G_{Y,W} \\
G_{PB,N} & G_{PB,T} & G_{PB,W} \\
G_{PL,N} & G_{PL,T} & G_{PL,W} \\
G_{PC,N} & G_{PC,T} & G_{PC,W} \\
G_{PX,N} & G_{PX,T} & G_{PX,W} \\
G_{PS,N} & G_{PS,T} & G_{PS,W} \\
G_{PY,N} & G_{PY,T} & G_{PY,W}
\end{bmatrix}
$$

$G_{B,N}$ therefore is the ratio of a proportionate change in the quantity of beef (E_B) to a proportionate change in the exogenous demand shift variable for beef (E_N).