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GROWTH DIAGNOSTICS AND A MULTISECTOR RAMSEY MODEL: THE CASE OF BRAZIL

CRISTINA VINYES
TERRY ROE

ECONOMIC DEVELOPMENT CENTER
Department of Economics, Minneapolis
Department of Applied Economics, St. Paul

UNIVERSITY OF MINNESOTA

Growth diagnostics and a Multisector Ramsey model: the case of Brazil

Cristina Vinyes & Terry Roe*†

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Abstract

Disenchantment with the Washington Consensus has led to an emphasis on growth diagnostics. In the case of Brazil, the literature suggests three main factors impeding growth: low domestic savings, a shortage of skilled workers, and lack of investment in the country's transportation infrastructure. The unique contribution of this study is to show the inter-temporal implications of relaxing these constraints. We fit a multi-sector Ramsey model to Brazilian data, validate its fit to times data, and provide empirical insights into the economy's structural transformation to long-run equilibrium. Then, the sensitivity of these results to relaxing each of these three constraints is investigated in a manner that yields the same long-run level of well-being. Analytical concepts adapted from static trade theory are used to provide a detailed explanation of how the economy responds in transition growth to the relaxation of these impediments. Addressing these factors clearly benefits the economy, but they do not launch the economy on a substantially higher growth path.

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Key words: economic growth, Ramsey, growth diagnostics

*PhD Candidate, Applied Economics Department, Unniversity of Minnesota and Professor, Applied Economics Department, University of Minnesota, respectively.

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1 Introduction

A World Bank review (2005) of economic reform experience finds many countries, and Brazil in particular, experienced less growth in the 1990s than prior decades despite the dismantling of state-led populist and protectionist policy regimes, while many high growth countries, like China and India - though they increased their reliance on market forces - pursued highly unconventional policies. Disenchantment with the traditional guidelines provided by the Washington Consensus has led to an emphasis on growth diagnostics as a first step to guide the focus of policy makers on the most daunting constraints to growth. Rodrik (2006) suggests that growth strategies are likely to differ according to domestic opportunities and constraints. Therefore, policy reforms need to be country specific, selective and focus on the binding constraints to growth rather than on a laundry-list à la the Consensus (Rodrik, 2006).

The relatively recent literature in this area, such as Hausmann et al. (HRV from now on) (2005) and Blyde et al. (2008), has identified possible impediments to growth for the case of Brazil, but a model of the economy has not been developed to more fully explore their possible effects on structural transformation and growth. This paper contributes to this literature by fitting a multi-sector Ramsey model to Brazilian data, validating its fit over a period of years, and then focuses on the possible constraints identified by HRV and Blyde et al. to empiricially assess their effects on transition growth to long-run equilibrium. This analysis is used to explain the economic forces causing differences in the evolution of sectoral shares in GDP, and the evolution of factor payments to skill and unskilled labor, remuneration to capital stock and rental incpwomes from the agricultural sector.

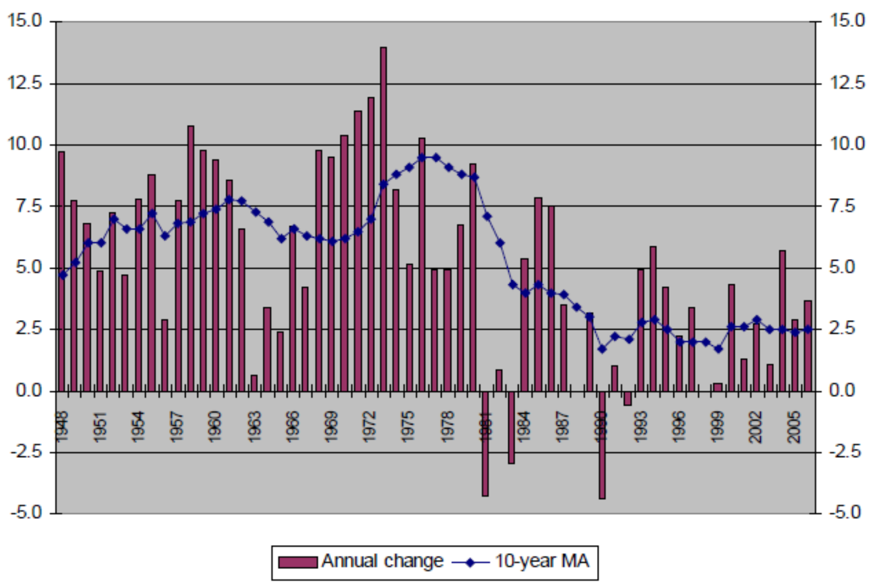
The paper is organized as follows. First, the following section provides the context and rational for the choice of the impediments to growth of the Brazilian economy. These include the cost of domestic financial intervention, the shortage of skilled workers, and the infrastructural costs imposed on the economy from the lack of investment in transport infrastructure. A snapshot of the economy is also provided in a way which helps motivate our specification of the multi-sector Ramsey model. Second, the model is presented where intra and inter-temporal equilibrium is characterized and special features of the model are highlighted. Third, we briefly discuss our growth accounting exercise, the fitting the model to data, and the numerical methods employed to solve the model. In the appendix, we add a validation exercise to assess the models fit to the data both backward and forward from the point the model is fit to the economy's transition path, there it is shown that the model fits the data surprisingly well, and we find that the country's potential to increase real income per capita from transition growth is indeed limited.

We then analyze the three possible constraints to growth by adjusting model parameters, solving and comparing results to the base model. This analysis is performed in a way that interventions to relax the constraints to growth yield the same long-run level of utility of the representative household. In this way, the representative household is, in principle, indifferent to the magnitude or source of the shock chosen. We also report the results from a simultaneous relaxation of each of the three constraints, which shows a greater effect in GDP per worker than the sum of individual reforms. In each case we find that transition growth accelerates to a higher long-run equilibrium so that indeed, the three impediments analyzed constrain growth. However, they do not launch the economy on a substantially higher growth path, so it seems that other more fundamental constraints appear to remain that serve to lower the economy's productivity of its evolving capital stock.

2 Background

During the period 1930-80, Brazil ranked among the countries with highest growth in GDP per capita in the world. During most of this period, she pursued import substitution - industrialization policies. The effects of the second petroleum price shock and a sharp increase in international interest rates in the 1980s led to historically low rates of growth in real GDP, and annual inflation rates of over 200 percent during the 1990s (figure 1). Since 1990, Brazil experienced much lower and more volatile growth, with its long-term annual growth in real GDP (ten-year moving average) fluctuating in the 2 to 3 percent range, well below the 6 to 10 percent range that prevailed in1950-80. Government reacted to this down turn by embarking on many reforms appearing on the Washington Consensus list, including trade liberalization and changes in fiscal and social policies. Nevertheless, these reforms have not returned the country to her previous long-run trend rates of growth, nor to growth rates attained by other Latin American countries. (Blyde et. al., 2008). For example, her average annual rates of growth in real GDP per capita of 1.68 percent over the 1990-99 period, and 0.08 during the 2000-06 period pale in comparison to that of Chile (3.1 and 4.7 percent) and Mexico (1.8 and 1.7 percent), respectively.

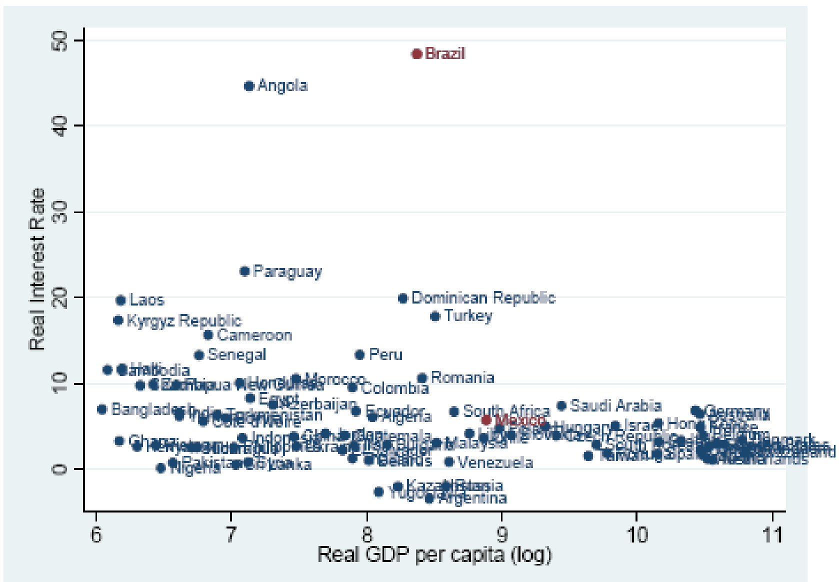
Figure1: Real GDP growth in percent, 1950-2005



HRV, argue that Brazil has eliminated some policies distorting her economy, but she has not addressed the binding constraints. Following Rodrik's growth diagnostic approach, HRV suggest two key constraints that slow the rate of capital deepening, and affect the productivity of labor. They conclude that a low rate of domestic savings (17 percent of GDP compared to 30 percent for countries in East Asia) and a low stock of human capital (only 20 percent of the labor force is skilled) are likely major causes of the country's poor growth performance.

The first constraint may seem surprising since the country has the highest real interest rates (returns on capital) in the world (figure 2). But according to HRV, the relatively low rate of domestic savings reflect disincentives to save that are linked to the high tax burden on capital income (taxing financial transactions). The high taxes and low savings reflect a high level of current spending and transfers. For example, social security expenditures stand at 8.5 percent of GDP, but the transfers are mostly spent on middle class public sector employees (Rodrik 2005).

Figure 2: Real Interest Rates vs GDP per capita, 2005.



Source: EIU. Real interest rate is taken as the lending interest rate minus inflation (change in consumer prices).

At the same time, the country faces a highly inelastic supply of external savings due to the fact that the country's external debt burden is relatively high (debt service ratio is 45 percent of Brazil's exports of goods and services, ranked the highest of any other country by WDI) . Hence lack of growth in the country's stock of capital precludes the capital deepening effects that lead to growth in output and factor payments.

Turning to the second concern, HRV point out that the country's stock of skilled labor is in relatively short supply compared to other countries of the region. High returns to education together with a low level of human capital is suggestive that the supply of skilled labor is an impediment to growth (HRV, 2005). The data presented in Figure 3, shows Brazil to be an outlier in the context of other Latin American countries for the period 1996-1997.

Figure 3: Returns to education and years of schooling.

Source: Years of schooling of population age 25 and over are taken from the Barro-Lee dataset. Returns to education are taken from Menezes-Filho (2001).

Following HRV's growth diagnostics, Blyde et al. (2008) adds a third binding constraint, inadequate transportation infrastructure. They show that the slowdown in economic growth coincided with a significant drop in the pace of expansion in infrastructure stock and that political reforms (over the 1990s) clearly failed to reverse this process, except for telecommunications (table 1). The most significant slowdown occurred in the expansion of the road network. In 2006, the National Confederation of Transport (CNT) assessed the quality of paved roads in Brazil, and classifying 25 percent as good, 38 percent as inadequate and 37 percent as very bad.

After reaching 5.4 percent of GDP in 1971-80 (measured in constant 1980 prices), the rate of infrastructure investment dropped by a third in the following decade, and then by an additional 50 percent by the mid-1990s (see table 1). This contraction in infrastructure investment reflected the retrenchment in public investment, including both the government and its state controlled companies and the failure of the privatization cum regulatory reform to reverse this decline.

Table 1: Investment breakdown (as percent of GDP, in constant 1980 prices)

Year	1971-1980	1981-1989	1990-94	1995-96	1997-98	1999	2000
Total	23.5	18	14.9	17	16.4	16.1	16.5
Residential Buiding	4.95	4.71	4.03	3.99	4.24	3.97	3.6
Petroleum	0.95	0.88	0.39	0.35	0.36	0.45	0.51
Public Sector (excludes Transport)	3	1.43	1.86	1.65	1.68	1.1	1.2
Infrastructure	5.42	3.62	2.16	1.79	2.77	2.7	2.58
Electricity	2.13	1.47	0.85	0.52	0.79	0.77	0.67
Telecommunication	0.8	0.43	0.5	0.66	0.98	1.17	1.07
Transport	2.03	1.48	0.69	0.48	0.68	0.56	0.63
Sanitation	0.46	0.24	0.07	0.13	0.32	0.2	0.21
Others	9.18	7.36	6.46	9.22	7.35	7.88	8.61

Source: Bielschowsky (2002: 25-29)

According to an analysis by Calderon and Serven (2003), 35 percent of the increase in the gap of GDP per worker between Brazil and East Asia since the early 1980s resulted from the country's slower accumulation of infrastructure capital. In another study (Calderón and Servén, 2004), they estimate that if the stocks and quality of Brazilian infrastructure rose to the level of Costa Rica, Brazil's annual GDP growth rate would rise by 2.9 percentage points.

Calderón and Serves (2004) and Ferreira and Nacimento (2005) show that these three factors appear to impede growth, but their approach fails to provide insights into the structural dynamics of this process. The interest here is to understand the forces of structural transformation across multiple sectors of an economy in the process of capital deepening and on how the relaxing of possible constraints to growth are likely to play out over time. Important to obtaining these insights is the extent to which the economy is disaggregated into sectors, and the forward and backward market linkages that infrastructure may cause on the costs of intermediate inputs, as emphasized by Hirschman (1985), and most recently by Jones(2007). Hirschman introduces intermediate inputs into a Solow economy and shows the multiplier effects of a shock to productivity far exceed the same shock without intermediates. Another possible disaggregation is to account for capital stock as a composite of the output of all sectors of the economy. Roe et al. (2010) show that this modification is important because the data indicate that a country's capital stock is comprised of outputs from virtually every sector of an economy. In transition growth, prices evolve in a way that can affect the price of capital stock differently than if a single sector is assumed to be the sole supplier.

A snapshot of the economy, aggregated to the level of the model presented in the next section, appears in table 2. The model has capitalized on available input-output data from the ERS, and from the Global Trade Analysis Project (GTAP) database, and are organized into a social accounting matrix (SAM) for the year 2001. Briefly, agriculture accounts for 8 percent of the country's GDP, employs about 20 percent of the work force, but only accounts for 5 percent of total wage payments. The dominant share of this labor is unskilled which is defined according to ILO's occupation 4–9 (tradespersons, clerks, salespersons, machine operators, laborers, and farm workers) and skill workers are those in ILO occupations 1–3 (managers, professionals, and para-professionals).

Brazil has the third most advanced industrial sector in The Americas, accounting for 44 percent of the country's GDP, and employs 45 percent of the work force, About 34 percent of workers are classified as unskilled and 11 percent skilled. The country spends 90 percent of the income, and therefore saves 10 percent. The country's service sector accounts for the remaining 47 percent of GDP. This sector employs about 50 percent of the total labor force, where approximately half is skilled and the other half unskilled (table 4). Labor is the main source of factor income with 36 percent of total factor income accruing to unskilled labor and 21 percent to skilled labor. Capital accounts for the next major share of income, equalling about 47 percent with most of the reminder of domestic income accruing to land.

Intermediate inputs account for 64 percent of manufactures gross output, and 60, 59 and 29 percent of gross output for agriculture, transportation and service sector, respectively (table 3). While own output accounts for the largest share of each sector's gross output, (except in the case of transportation), the service sector has the second largest share as an intermediate in each sector's gross output. In table 3 the relative sector factor intensities are also reported, showing that agriculture is capital intensive, transportation is unskilled labor intensive and the service sector is skill labor intensive. Factor intensities are important because they affect the evolution of firm level costs in transition growth and consequently differences in sectoral output.

Table 2: Social accounting matrix, Brazil in millions of 2001 USD.

		Expenditures															
		1. Activities				2. Commodoties				3. Factors				4. Institu	5. Cap	6. Trade	7. Total
Receipts:		Industry	Agri.	Transpoı	Services	Industry	Agri.	Transport	Services	K	Lu	Ls	T	HH	KA	WT	TT
1. Activities	Ind.					275312										0	275312
	Ag.						101504									11983	113487
	Trans.							27611								0	27611
	Serv.								471850							0	471850
2. Commod	Ind.	123632	11478	8667	41846									73728	27944		287295
	Ag.	4205	38292	125	7748									50843	291		101504
	Trans.	4763	2174	3628	5408									11639	0		27611
	Serv.	42657	16103	3994	82450									264762	61883		471850
3. Factors	K	51007	26493	1771	131177												210448
	Lu	42165	13309	7928	115121												178523
	Ls	6884	1391	1499	88099												97872
	T	0	4248	0	0												4248
4. Institutioı	HH									210448	178523	97872	4248				491091
5. Cap. Acc	KA													90119			90119
6. Trade	WT					11983	0	0	0								11983
7. Total	TT	275312	113487	27611	471850	287295	101504	27611	471850	210448	178523	97872	4248	491091	90119	11983	

Source: GTAP and ERS (USDA), data in millions of 2001 USD

Table 3: Intermediate inputs usage and relative sector factor intensities

	Industry	Ag.	Transport	Services
Industry[1]	0.449	0.101	0.314	0.089
Ag.	0.015	0.337	0.005	0.016
Transport	0.017	0.019	0.131	0.011
Services	0.155	0.142	0.145	0.175
K intens[2]	1.040	1.398	0.188	0.645
Lu intens	0.728	0.414	2.425	0.525
Ls intens	0.074	0.032	0.155	0.358

[1] Intermediate in real values divided by the gross output value

[2] Relative sector factor Intensities

3 The model

The consumed goods are: manufactures (Q_m), agriculture (Q_a), transportation (Q_t)and service sector (Q_s), they are also employed as intermediates. Households are endowed with the economy's resources which include skill labor (L_s), unskilled labor (L_u) and capital (K),that evolve over time, and land which remains constant H_a. Each of the four goods is produced by perfectly competitive firms. Firms employ primary and intermediate factors of production. The manufactured and agricultural goods are traded internationally while the transportation and service good is only traded in the domestic economy. Any surplus or deficit of the traded goods is exported or imported at the given world prices. The manufactured, the agriculture and the service good are reinvested to increase the economy's stock of capital. The two types of labor and capital are employed in the production of all four goods. Land is specific to agriculture and it is traded among firms within each of these sectors so that clearing of the land market yields a positive rental rate each period. Labor services are not traded internationally and domestic residents own the entire stock of capital. Households exchange labor $L = L_s + L_u$, capital and land for wages w_u (unskilled),and w_s (skilled), capital rents r, and land rents π_a per-unit returns.

3.1 Behavior of households and firms

3.1.1 Households

Households are represented by the infinitely-lived Ramsey consumer that receives utility $u : \mathbb{R}^4_{++} \rightarrow \mathbb{R}_+$ from the sequence $\{q_m, q_a, q_t, q_s\}_{t\in[0,\infty)}$ expressed as a weighted sum of all future flows of utility

$$\int_{t\in[0,\infty)} u(q_m, q_a, q_t, q_s)e^{(n-\rho)t}dt$$

where $u(q)$ is increasing and strictly concave in q, is everywhere continuous, and twice differentiable, and it is homothetic. The number of households members are assumed to be proportional to the number of workers, to grow at the exogenously given positive rate n. $L(t) = e^{(n)t}L(0)$ and the discount future consumption at the rate $\rho > 0$.

The household's intra-temporal problem is to choose (q_m, q_a, q_t, q_s) to minimize the cost ϵ of composite consumption q

$$\epsilon = \mathcal{E}\left(p_m, p_a, p_t, p_s\right) q \equiv \min_{q_m, q_a, q_t, q_s} \left\{ \sum_{j=q_m, q_a, q_t, q_s} p_j q_j \mid q \leq u(q_m, q_a, q_t, q_s) \right\}$$

where (p_m, p_a, p_t, p_s) are the respective prices of each good. At each instant in time, $\mathcal{E}\left(p_m, p_a, p_t, p_s\right)$ represents the price (cost) of aggregate consumption q. Shephard's lemma gives the Hicksian demand,

$$q_j = q^j \left(p_m, p_a, p_t, p_s\right) q \quad j = m, a, t, s$$

which is homogeneous of degree zero in prices p_j.

Unspent income accumulates as an asset for future consumption, and capital and loans are assumed to be perfect substitutes. Then, the representative household's budget constraint is given by

$$\dot{k} = \frac{1}{p_k}(w_u l_u + w_s l_s + k r^k (1-\tau) + \pi_a H_a - \mathcal{E}\left(p_m, p_a, p_t, p_s\right) q) - k\left(n+\delta\right) + T$$

where $\tau \geq 0$ is a tax on capital. At the end of each period the tax is returned to the household as a lump sum transfer T, p_k is the price of a unit of capital stock k, which is derived below. This constraint is derived based on the assumption that the noarbitrage condition between the returns received by households from loans and returns to capital is satisfied, as well as the noarbitrage condition between loans and the price of land in the agricultural sector, i.e.

$$r = \frac{r^k(1-\tau)}{p_k} - \delta + \frac{\dot{p}_k}{p_k} = \frac{\pi_a}{p_{ha}} + \frac{\dot{p}_{ha}}{p_{ha}}$$

In this case, throughout transition growth, capital and land prices adjust so that agents have no incentive to exchange one asset for another.

Household's optimization problem of maximizing the discounted present value of utility subject to the budget constraint and a transversality condition, leads to the Eurler condition

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{1}{\theta}\left[\frac{r^k(1-\tau)}{p_k} - \delta - \rho - (1-\theta)\frac{\dot{p}_s}{p_s} + \frac{\dot{p}_k}{p_k}\right]$$

where inter-temporal elasticity $1/\theta \leq 1$. Since below our variables are expressed in effective labor units, it is convenient to express this condition in effective units as well, which results in the condition, for $\theta = 1$,

$$\frac{\dot{\hat{\epsilon}}}{\hat{\epsilon}} = \frac{r^k(1-\tau)}{p_k} - \delta - \rho - x + \frac{\dot{p}_k}{p_k}$$

3.1.2 Composite capital

Recognizing that a country's stock of capital is composed of more than manufactured goods, capital is modeled as a composite of the output of manufactures, agriculture and services, that are presumed to be combined in a least cost manner to produce a capital good. Following Roe et al (2010), the problem is

$$p_k = c^k\left(p_m, p_a, p_s\right) \equiv \min_{y_{mk}, y_{ak}, y_{sk}} \left(\sum_{j=m,a,s} p_j \hat{y}_{jk} \mid 1 \leq F\left(\hat{y}_{mk}, \hat{y}_{ak}, \hat{y}_{sk}\right) \right)$$

where for purposes here $F(\cdot)$ is a neoclassical Cobb-Douglas function that is CRS in its arguments. Thus, at each instant in time, we have the result that the total cost of capital provision from savings $\dot{\hat{k}} + \hat{k}\left(x+n+\delta\right)$ is

$$p_k \left[\dot{\hat{k}} + \hat{k}\left(x+n+\delta\right) \right]$$

where Shephard's lemma applied to $c^k\left(p_m, p_a, p_s\right)$ retrieves y_{mk}, y_{ak}, y_{sk}. Finally, note that

$$\frac{\dot{p}_k}{p_k} = \lambda_{sk} \frac{\dot{p}_s}{p_s}$$

where λ_{sk} is the cost share of y_{sk} in the total cost of producing an increment of capital stock in each t. Thus, the Euler condition can be restated as

$$\frac{\dot{\hat{\epsilon}}}{\hat{\epsilon}} = \frac{r^k(1-\tau)}{c^k\left(p_m, p_a, p_s\right)} - \delta - \rho - x + \lambda_{sk} \frac{\dot{p}_s}{p_s}$$

3.1.3 Firms

Firms in each sector are atomistic and identical. Firms producing manufactured, transportation and service goods employ technology $f^j : \mathbb{R}^7_{++} \rightarrow \mathbb{R}_+$ defined as $Min\left\{ f^j(A\left(t\right) l_{uj}, A\left(t\right) l_{sj}, k_j), \frac{\hat{y}_{mj}}{\sigma_{mj}}, \frac{\hat{y}_{aj}}{\sigma_{aj}}, \frac{\hat{y}_{tj}}{\sigma_{tj}}, \frac{\hat{y}_{sj}}{\sigma_{sj}} \right\} \quad j = m, t, s$ where $A\left(t\right) = e^{xt}$ and x is the Harrod rate of growth in effective labor services, and σ_{mj} are input-output coefficients.

The corresponding sectoral total cost functions, in units per effective worker, are given by:

$$TC_j = \left(C^j(\hat{w}_u, \hat{w}_s, r^k) + \sum_{i=m,a,t,s} \sigma_{ij} \right) \hat{y}_j \quad j = m, t, s.$$

Agricultural technologies is represented by the production functions $f^a : \mathbb{R}^8_{++} \rightarrow \mathbb{R}_+$, defined as

$$\min \left\{ f^a(A(t)\, l_{ua}, A(t)\, l_{sa}, k_a, \mathcal{B}(t)\, h_a), \frac{\hat{y}_{ma}}{\sigma_{ma}}, \frac{\hat{y}_{aa}}{\sigma_{aa}}, \frac{\hat{y}_{ta}}{\sigma_{ta}}, \frac{\hat{y}_{sa}}{\sigma_{sa}} \right\}$$

Given land H fixed, and given f^a is linearly homogeneous in all inputs, the sectoral aggregate technology, denoted $F^a(.)$, exhibits decreasing returns to scale in inputs l_{ua}, l_{sa}, k_a, h_a.

The value added function (in units per effective worker) is defined as

$$\pi^a(p_{va}, \hat{w}_u, \hat{w}_s, r^k)h_a \equiv \max_{l_{ua}, l_{sa}, k_a} \left\{ \begin{array}{c} p_{va}F^a(l_{ua}, l_{sa}, \hat{k}_a; h_a) - \\ \hat{w}_u l_{ua} - \hat{w}_s l_{sa} - \hat{k} r^k - \pi^a h_a \end{array} \right\}$$

where p_{va} is the value added price of output, and h_a is clearly specific to the sector, and hence, is not treated as a choice variable at the sector level. In equilibrium, the rental rate $\pi^a(p_{va}, \hat{w}_u, \hat{w}_s, r^k)$ is the rate that prevails in a competitive rental market for the fixed resource H_a. By Hotelling's lemma, this sector's partial equilibrium supply function in units per effective worker is given by

$$y^a(p_{va}, \hat{w}_u, \hat{w}_s, r^k)h_a = \pi^a_{pa}(p_{va}, \hat{w}_u, \hat{w}_s, r^k)h_a$$

A perfectly competitive land market among producers implies that the shadow price $\pi^a(p_a, w_u, w_s, r)$ is also the land rental price that causes the land market to clear among individual producers. Thus, firms in this sector make zero profits since, in equilibrium, the value of output is exhausted by payments to factors,

$$p_{va}\hat{y}_a = \hat{w}_u l_{ua} + \hat{w}_s l_{sa} + r^k \hat{k}_a + \pi^a(p_{va}, \hat{w}_u, \hat{w}_s, r^k)h_a$$

3.2 Equilibrium characterization

Restricting analysis to the case where all sectors are open, i.e. $Y_j > 0 \quad j = m, a, t, s$, a competitive equilibrium is defined by the positive prices $\{\hat{w}_u, \hat{w}_s, r^k, p_t, p_s\}_{t\in[0,\infty)}$of inputs and output, household consumption plans

$$\{q^*_m, q^*_a, q^*_t, q^*_s\}_{t\in[0,\infty)}$$

and production plans

$$\{\hat{y}_m^*, \hat{y}_a^*, \hat{y}_t^*, \hat{y}_s^*, \hat{k}_m^*, \hat{k}_a^*, \hat{k}_t^*, \hat{k}_s^*, l_{um}^*, l_{ua}^*, l_{ut}^*,$$
$$l_{us}^*, l_{sm}^*, l_{sa}^*, l_{st}^*, l_{ss}^*, \}_{t\in[0,\infty)}$$

given initial resource endowments $\{\hat{k}(0), L_u(0), L_s(0), H\}$ such that the discounted present value of household utility is maximized, firms maximize profit subject to their technology at each instant of time t, and markets clear for all inputs and the outputs. In addition, the no-arbitrage condition between the values of capital and land, and the transversality condition are satisfied.

3.2.1 Intra-temporal equilibrium

Given the endogenous sequence $\{\hat{k}, \hat{\epsilon}\}_{t\in[0,\infty)}$,intra-temporal equilibrium is given by the sequence of positive values $\{\hat{w}_u, \hat{w}_s, r^k, \hat{y}_m, \hat{y}_t, \hat{y}_s, p_t, p_c\}_{t\in[0,\infty)}$ satisfying the following eight equations for each t:

- Zero profit conditions in sector m, t, s

$$\begin{aligned} C^m\left(\hat{w}_u, \hat{w}_s, r^k\right) &= p^{vm}\left(p_t, p_s\right) \\ C^t\left(\hat{w}_u, \hat{w}_s, r^k\right) &= p^{vs}\left(p_t, p_s\right) \\ C^s\left(\hat{w}_u, \hat{w}_s, r^k\right) &= p^{vz}\left(p_t, p_s\right) \end{aligned}$$

where p_{vj} is the value added price of sector j, $p^{vj}\left(p_t, p_s\right) = p_j - \sum_i p_i\sigma_{ij}$, $i = m, a, t, s$, where we suppress the exogenous prices p_m, p_a in $p^{vj}\left(p_t, p_s\right)$,

- Unskilled labor market clearing,

$$\sum_{j\in m,t,s} C^j_{\hat{w}_u}\left(\hat{w}_u, \hat{w}_s, r^k\right)\hat{y}_j - \pi^a_{\hat{w}_u}\left(p^{va}\left(p_t, p_s\right), \hat{w}_u, \hat{w}_s, r^k\right)h_a = \ell_u$$

- Skilled labor market clearing,

$$\sum_{j\in m,t,s} C^j_{\hat{w}_s}\left(\hat{w}_u, \hat{w}_s, r^k\right)\hat{y}_j - \pi^a_{\hat{w}_s}\left(p^{va}\left(p_t, p_s\right), \hat{w}_u, \hat{w}_s, r^k\right)h_a = \ell_s$$

- Capital market clearing,

$$\sum_{j\in m,t,s} C^j_{r^k}\left(\hat{w}_u, \hat{w}_s, r^k\right)\hat{y}_j - \pi^a_{r^k}\left(p^{va}\left(p_t, p_s\right), \hat{w}_u, \hat{w}_s, r^k\right)h_a = \hat{k}$$

- Clearing of the domestic market for transportation good including the intermediate demands,

$$\frac{\lambda_t\hat{\epsilon}}{p_t} = y_t - \sum_{j\epsilon m,a,t,s}\sigma_{tj}\hat{y}_j$$

- Clearing of the domestic market for service home good including the intermediate demands,

$$\frac{\lambda_s\hat{\epsilon}}{p_s} = y_s - \sum_{j\epsilon m,a,t,s}\sigma_{sj}\hat{y}_j - \hat{y}_{sk}\left[\dot{\hat{k}} + \hat{k}\left(x+n+\delta\right)\right]$$

where ℓ_u is the unskilled share of the total work force which remains constant because both labor categories grow at the same rate.

Reduce dimensionality

Derivation of the equations of motion are facilitated by reducing the dimensionality of the system. From zero profit we obtain

$$\hat{w} = \tilde{w}^u\left(p_t, p_s\right) \equiv W^u(p_{vm}, p_{vt}, p_{vs})$$

$$\hat{w}_h = \tilde{w}_s\left(p_t, p_s\right) \equiv W^s(p_{vm}, p_{vt}, p_{vs})$$
$$r^k = \tilde{r}(p_t, p_s) \equiv R(p_{vm}, p_{vt}, p_{vs})$$

These equations are homogeneous of degree zero in prices, however, p_s and p_c are endogenous variables. Nevertheless, these equations can be substituted into the factor market clearing equations for $W^u\left(p_{vm}, p_{vt}, p_{vs}\right), W^s\left(p_{vm}, p_{vt}, p_{vs}\right)$ and $R\left(p_{vm}, p_{vt}, p_{vs}\right)$. Since the resulting equations are linear in output, we easily obtain

$$\hat{y}_j = \tilde{y}^j\left(p_t, p_s, \hat{k}\right),\ j = m, t, s$$

where again, we omit the exogenous variables to reduce clutter.

The supply function for agriculture can be expressed in output price alone by substituting the rental rate equations into the partial equilibrium supply function.

$$\hat{y}_a = \tilde{y}^a\left(p_t, p_s\right) \equiv y^a\left(p^{va}, \hat{w}_u, \hat{w}_s, r^k\right) H_a \equiv \frac{\partial \pi^a (p^{va}\left(p_t, p_s\right), \hat{w}_u, \hat{w}_s, r^k) H_a}{p^{va}\left(p_t, p_s\right)}$$

Thus, the home good market clearing condition can be expressed as

$$\begin{aligned} \hat{\epsilon} \quad = \quad & \tilde{\epsilon}(p_t, p_s, \hat{k}) \equiv \frac{p_s}{\lambda_s}(\tilde{y}^s(p_t, p_s, \hat{k}) - \sum_{j \epsilon m,t,s} \sigma_{sj} \tilde{y}_j^s(p_t, p_s, \hat{k}) - \\ & \sigma_{sa} \tilde{y}_a^s(p_t, p_s) - \lambda_{sk} \hat{y}^{sk}\left(p_m, p_a, p_s\right) \left[\dot{\hat{k}} + \hat{k}\left(x + n + \delta\right)\right] \end{aligned}$$

At the same time, the transportation market clearing condition can be expressed also as

$$\hat{\epsilon} = \tilde{\epsilon}(p_t, p_s, \hat{k}) \equiv \frac{p_t}{\lambda_t}(\tilde{y}^t(p_t, p_s, \hat{k}) - \sum_{j \epsilon m,t,s} \sigma_{tj} \tilde{y}_j^s(p_t, p_s, \hat{k}) - \sigma_{ta} \tilde{y}_a^s(p_t, p_s)$$

3.2.2 Inter-temporal equilibrium

We have three differential equations that are linear in the "dot" variables $\dot{\hat{k}}, \dot{p}_t, \dot{p}_s$ that can be obtained from:

- Euler equation

$$\boldsymbol{E}\left(p_t, p_s, \dot{p}_s\right) \equiv \dot{\hat{\epsilon}} = \hat{\epsilon}\left(\frac{\tilde{r}(p_t, p_s)(1-\tau)}{c^k\left(p_m, p_a, p_s\right)} - \delta - \rho - x + \lambda_{sk}\frac{\dot{p}_s}{p_s}\right)$$

- Budget Constraint

$$\begin{aligned} \tilde{K}\left(p_t, p_s, \hat{k}\right) \equiv \dot{\hat{k}} = & \frac{1}{c^k\left(p_m, p_a, p_s\right)}[\tilde{w}^u\left(p_t, p_s\right)\ell_u + \tilde{w}^s\left(p_t, p_s\right)\left(\ell_s\right) + \\ & \hat{k}\tilde{r}(p_t, p_s) + \tilde{\pi}^a\left(p_t, p_s\right) h_a - \mathcal{E}\left(p_m, p_a, p_t, p_s\right) q] - \hat{k}\left(n + \delta + x\right) \end{aligned}$$

where

$$\tilde{\pi}^a\left(p_t, p_s\right) \equiv \pi^a(p^{va}\left(p_t, p_s\right)p, \tilde{w}^u\left(p_t, p_s\right), \tilde{w}^s\left(p_t, p_s\right), \tilde{r}(p_t, p_s))$$

- Home good market clearing

$$\hat{\epsilon} = \frac{p_s}{\lambda_s}[y_s - \sum_{j\epsilon m,t,s} \sigma_{sj}\tilde{y}^j\left(p_t, p_s, \hat{k}\right) - \sigma_{sa}\tilde{y}^a\left(p_t, p_s\right)$$

$$-\tilde{y}^{sk}\left(p_t, p_s\right)\left(\dot{\hat{k}} + \hat{k}\left(x+n+\delta\right)\right)]$$

- and transportation market clearing

$$\hat{\epsilon}=\frac{p_t}{\lambda_t}(\tilde{y}^t(p_t, p_s, \hat{k}) - \sum_{j\epsilon m,t,s} \sigma_{tj}\tilde{y}^s_j(p_t, p_s, \hat{k}) - \sigma_{ta}\tilde{y}^s_a(p_t, p_s)$$

These equations can be rearrange, with the use of a mathematical software, so that we have a differential equation for each dot variable. To solve for steady state, if such exists, we set these equations to zero, and solve for the endogenous variables $p_t^{ss}, p_s^{ss}, \hat{k}_t^{ss}$. The inter-temporal conditions permit solving for the remaining endogenous variables. To solve the model over time, we make use of the differential equations and the Time elimination methods (See Roe et al.). Given the sequence $\{p_t, p_s, \hat{k}\}_{t\in[0,\infty)}$, the dimensionality of the intra-temporal conditions permits the calculation of the remaining eight endogenous variables

$$\{\hat{w}_u, \hat{w}_s, r^k, y_m, y_t, y_s, p_t, p_s\}_{t\in[0,\infty)}$$

and consequently household consumption

$$\{q_m^*, q_a^*, q_t^*, q_s^*\}_{t\in[0,\infty)}$$

and resource allocations of capital and labor.

3.3 Comparative statics

The comparative static properties of the model are similar to properties discussed in the simpler model by Roe and Smith (2008), and are thus briefly stated without proof. The intra-temporal conditions admit to a comparative static analysis along the lines of the Stopler-Samuelson (Sto-Sa) and Rybcyznski (Ry) theorems of the static 2x2 model of a small open economy. The Sto-Sa property states that an increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduces the real return to the other factor (Feenstra, R. 2004). the Ry theorem, states that an increase in a factor endowment, will increase the output of the industry using it intensively, and decrease the output of the other industry (Feenstra, R. 2004).

The reduced form factor rental rate equations are homogenous of degree one in output prices, and hence Sto-Sa-like results can be obtained. That is, Sto-Sa-like in the sense that if the price of a final good that employs a factor intensively rises, it is not necessarily the case that the rental rate of this factor will rise in greater proportion to the increase in price.

The supply functions are homogeneous of degree zero in prices and of degree one in factor endowments. Consequently, Ry-like qualitative results can be obtained. When capital deepening occurs, as in the case of our empirical results, the unit cost of capital declines, while the marginal product of labor increases in all sectors, but relatively more so in sectors that are most capital intensive (agriculture in our case, see table 3). This sector then will experience a growth in output while the sector employing labor most intensively will experience a decline in output. However, the proportional change in output may be less than the proportional change in capital stock.

In the case of home good sectors where the domestic markets must clear, capital deepening along with growth in total factor income will cause an increase in the price of home goods that is necessary in order for these sectors to compete for resources that would otherwise be pulled into the traded good sectors. This effect can be viewed as have Sto-Sa-like affects on factor rental rates.

We rely on the well known properties of the indirect profit function $\pi^a(\cdot)$ to explain the supply and factor demand behavior of the sector employing a sector specific resource. Overall, this logic forms the basis of our explanation below of the model's solution showing the structural transformation as the economy approaches long-run equilibrium (for more details see Vinyes 2010).

3.4 Fitting the model to data

Growth accounting allows for the breakdown of observed growth in GDP into components associated with changes in factor inputs and in production technologies. This step is necessary to estimate the economy's rate of Harrod factor productivity growth ($x = 0.0014$), its capital stock (2,033,130 constant 2001 USD), labor force growth rate ($n = 0.0194$), and rate of capital depreciation ($\delta = 0.04$), all of which are parameters of the model. This step also provides insights to the economy's historical growth performance and it can be used to suggest "how far" the economy is from its long-run equilibrium. We can see from the growth accounting results, and the premise that the economy in the long-run behaves according to our Ramsey model, that Brazil in the year 2001 is not far from its long-run equilibrium (table 4). Therefore, we might expect a solution to our model to predict that the change in the economy's variables will be relatively small over the transition path.

Table 4: Contributions to GDP growth.

	Contribution at t<SS	Contribution at SS
Capital change Contribution	0.4932	0.4225
Labor change Contribution	0.3520	0.5316
Technical change Contribution	0.1548	0.0459

We assume Cobb-Douglas functional forms for all of the model's production functions, the household utility function as well as the composite capital function. Data reported in table 2 are used to estimate the parameters of these functions, including the input-output coefficients for intermediate inputs. Together, with the growth accounting data and the rate of work force growth, the model is fit to these data so as to reproduce the social accounting matrix values as though they reflect the economy at this point on the transition path for the year 2001 with initial conditions given by these data. To provide confidence that the model captures the key structural features of the Brazilian economy, a validation exercise is conducted. We solve the model backwards and forward from the 2001 base year, and contrast model forecast with time series data of total and sector GDP taken from WDI over the 1995- to 2005 period. This exercise suggests the model fits the data well. See appendix for a discussion of the results.

4 Base model results

4.1 An overview

The main results are presented in four tables; table 5 to table 9. The model predicts a GDP rate of growth of 2.48 percent for the year 2005 (in constant 2001 USD), which is close to the 2.3 percent reported in the WDI for that year and close to the average annual rate over the 2000-05 period of 2.56 percent. The economy approaches the mid-point to long run equilibrium in 2011, where the long-run rate of growth in GDP per worker is around 0.27 percent per annum in constant 2001 USD.

Gross Domestic Product per worker increases over the fifty year period 2001-51, from 5,297 to 6,606 constant 2001 USD (table 5, col. 1). This increase suggests the country's potential to increase households' real income from transition growth is limited with about 79 years required for the country to double GDP per capita, assuming the average annual rate of population growth remains at the 2001-06 average of 1.4 percent per annum. GDP doubles in 30 years. This result stands in contrast to that of other OECD countries where the doubling of income per capita ranges from 25 to 30 years.

The modeled economy's relatively weak potential to increase real income from transition growth is arguably embodied in the Brazilian data used to estimate the parameters of the economy's primitives, such as the parameters of sector technologies and household's incentives to save, which together, tend to constrain or limit the country's potential to grow from capital deepening. We return to this discussion later.

The source of income earnings are aggregated into labor (skill and unskilled), capital and land. Over the 2001-11 period, per worker earnings from unskilled and skilled labor increase each by about 4.8 percent, the stock of capital per worker increases by 8.7 percent. At the same time, land rents per farm worker increase by 6.24 percent (table 5). GDP growth rates are higher in the earlier periods of transition than in the long run, where GDP's rate of growth converges to the rate of factor productivity and work force growth, x+n.

Table 5: Evolution of GDP, factor earnings per worker and savings to GDP in constant 2001 USD, model results.

	GDP/wkr	Period growth rate	K/wkr	U wage/wkru	S wage/wkrh	K earnings/wkr	Ag earnigns/farm wkr	Exp/wkr	Sav/GDP
1991	5296.6		19899.8	2354.0	5857.4	2269.6	600.7	3632.2	0.3142
2001	5790.6	9.33%	23606.8	2565.2	6398.9	2483.0	678.2	4115.4	0.2893
2011	6072.4	4.87%	25674.6	2685.8	6707.3	2604.9	720.5	4386.1	0.2777
2021	6254.9	3.00%	26897.9	2764.3	6907.2	2683.7	746.4	4552.2	0.2722
2031	6390.1	2.16%	27701.0	2823.0	7055.7	2742.0	764.6	4667.4	0.2696
2041	6503.1	1.77%	28299.4	2872.5	7180.0	2790.6	779.1	4758.1	0.2683
2051	6606.0	1.58%	28800.0	2917.6	7293.4	2834.9	791.9	4837.4	0.2677

Factor shares in GDP remain largely unchanged over time. Labor employment accounted for about 56.3 percent of total income in 2001 compared to 55.9 percent by 2051, while returns to capital assets remained virtually unchanged, accounting for 42.9 percent in 2001 and 43 percent in 2051. Land rental payments increased from 0.9 to 1.1 percent of GDP over the 2001-51 period.

The change in structure of the economy is relatively modest. Over the half-life period, industry and service's shares in GDP decline from 0.18 to 0.17 for industry and from 0.67 to 0.66 for services; the agricultural sector increases by 19 percent, albeit from a relatively small share of the total economy. The transportation component of services has a more modest increase of about 1.6 percent . (table 6, first four columns). Over the half-life period, households' expenditures per worker increase around 7 percent while savings to GDP decline by about 4 percent (table 5).

Corresponding to this transition are the changes in the share of the economy's labor force and capital employed in the four sectors (table 6). The share of the unskilled and skill labor force in the agriculture and service sectors rise, and declines in the industrial sector while remains constant in the transport sector.

Table 6: Change in the economy structure and resource allocation model results

	Sector Share in GDP				Unskilled Labor Share in				Skilled Labor Share in				Capital Share in			
Year	Industry	Ag.	Transport	Service	Industry	Ag.	Transport	Service	Industry	Ag.	Transport	Service	Industry	Ag.	Transport	Service
1991	0.203	0.109	0.020	0.676	0.205	0.054	0.032	0.528	0.013	0.002	0.002	0.162	0.258	0.111	0.007	0.624
2001	0.183	0.128	0.021	0.668	0.194	0.061	0.034	0.531	0.013	0.003	0.003	0.163	0.242	0.126	0.008	0.625
2011	0.174	0.138	0.021	0.664	0.188	0.065	0.035	0.532	0.012	0.003	0.003	0.163	0.234	0.133	0.008	0.625
2021	0.169	0.142	0.021	0.663	0.185	0.066	0.035	0.532	0.012	0.003	0.003	0.163	0.231	0.136	0.008	0.625
2031	0.167	0.145	0.021	0.662	0.184	0.067	0.036	0.533	0.012	0.003	0.003	0.163	0.229	0.138	0.008	0.625
2041	0.166	0.146	0.021	0.662	0.183	0.068	0.036	0.533	0.012	0.003	0.003	0.163	0.228	0.139	0.008	0.625
2051	0.165	0.146	0.021	0.661	0.183	0.068	0.036	0.533	0.012	0.003	0.003	0.163	0.228	0.139	0.008	0.625

4.2 Major economic forces of transition growth

The basic economics of these changes can be explained as follows. As capital deepening occurs, the most capital intensive sector of the economy experiences a Ry-like effect (in this case agriculture), which increases the productivity of labor in this sector relative to the labor intensive sectors of the economy (transportation and service for unskilled and skilled labor respectively). The rise in the productivity of labor in agriculture due to capital deepening, all else constant, causes this sector to increase the quantity of labor demanded which can only be obtained by an increase in the wage rate. Labor intensive sectors thus experience a rise in production costs in spite of the decline in the rental rate of capital.

The rise in factor income, since wages and the stock of capital have increased, augments the quantity demanded of all final goods, as well as the quantity of intermediate inputs employed in the capital intensive sector. In order for the domestic non-traded good markets to clear, the price of these goods must rise to compete for the resources that would otherwise be employed in the capital intensive sector (table 8). If a sector produces a traded good at a given world price, and the net effect of the rise in wages and decline in the capital rental rate cause a rise in the cost of production, labor or capital or both may depart the sector and the sector's demand for intermediate inputs may decline with the decline in its output (the industry sector in this case).

In the presence of capital deepening, the capital intensity of agriculture causes this sector to increase output relative to other sectors of the economy. Agricultural output grows in the range of 3.4 to 2.7 percent per annum over the half-life period due to the large contribution from growth in the employment of capital services (table 7). Service sector output growth is in the range of 2.5 to 2.2 percent, while the transportation sector's output growth is between 2.5 to 2.3 percent. Industrial sector output grows at the relatively constant and more modest rate of about 2 percent per year. Notice that the rate of growth in total gross output of all four sectors converge to their long-run growth rates from above (table 7).

Table 7: Growth in sector output and the percentage point contributions of prices, factors of production and technological change, model results.

Industry

Year	Gross Output	Value added price wrt p3	Value added price wrt p4	Capital Stock	Effective Labor
1991	0.0244	-0.0586	-0.0445	0.2019	-0.0745
2001	0.0218	-0.0277	-0.0206	0.1507	-0.0806
2011	0.0211	-0.0133	-0.0098	0.1280	-0.0839
2021	0.0209	-0.0064	-0.0047	0.1175	-0.0855
2031	0.0209	-0.0031	-0.0022	0.1125	-0.0863
2041	0.0209	-0.0015	-0.0011	0.1101	-0.0867
2051	0.0209	-0.0007	-0.0005	0.1090	-0.0869

Agriculture

Year	Gross Output	Value added price	Unsk. Wage effect	Skil. Wage effect	Interest rate Effect	Technical change effect
1991	0.0516	-0.0072	-0.0336	-0.0036	0.0752	0.0209
2001	0.0341	-0.0032	-0.0145	-0.0016	0.0325	0.0209
2011	0.0269	-0.0015	-0.0066	-0.0007	0.0149	0.0209
2021	0.0237	-0.0007	-0.0031	-0.0003	0.0070	0.0209
2031	0.0222	-0.0003	-0.0015	-0.0002	0.0033	0.0209
2041	0.0215	-0.0002	-0.0007	-0.0001	0.0016	0.0209
2051	0.0212	-0.0001	-0.0003	0.0000	0.0008	0.0209

Transportation

Year	Gross Output	Value added price wrt p3	Value added price wrt p4	Capital Stock	Effective Labor
1991	0.0323	0.8161	-0.0980	-1.3384	0.6527
2001	0.0258	0.3345	-0.0383	-0.8925	0.6221
2011	0.0231	0.1499	-0.0168	-0.7190	0.6090
2021	0.0219	0.0698	-0.0077	-0.6432	0.6030
2031	0.0214	0.0331	-0.0037	-0.6083	0.6002
2041	0.0211	0.0158	-0.0017	-0.5919	0.5989
2051	0.0210	0.0076	-0.0008	-0.5840	0.5982

Service

Year	Gross Output	Value added price wrt p3	Value added price wrt p4	Capital Stock	Effective Labor
1991	0.0304	-0.0090	0.0167	0.0035	0.0192
2001	0.0250	-0.0038	0.0071	0.0025	0.0192
2011	0.0227	-0.0017	0.0033	0.0020	0.0192
2021	0.0217	-0.0008	0.0015	0.0018	0.0192
2031	0.0213	-0.0004	0.0007	0.0017	0.0192
2041	0.0211	-0.0002	0.0003	0.0017	0.0192
2051	0.0209	-0.0001	0.0002	0.0017	0.0192

As the agriculture sector employs a larger share of capital services, about 12.6 percent (table 8, column 14), firms in the transportation and service sector compete for unskilled and skilled labor. As mentioned above, the market price for these sectors must increase for markets to clear. The real price of the transportation and services goods increase about 1.09 and 0.5 percent respectively, over the period (table 8). The increase is greater in the transportation sector than in the service sector because the agricultural sector uses unskilled labor more intensively than skilled labor, and therefore the transportation sector must increase its price by a larger amount than the service sector in order to compete for this resource.

Table 8: Evolution of prices, model results.

Year	Price of the Transport good	Price of the Service good	Value Added price of Industry	Agriculture	Transport	Service
1991	0.976	0.989	0.366	0.402	0.386	0.700
2001	1.000	1.000	0.363	0.400	0.405	0.709
2011	1.011	1.005	0.362	0.399	0.414	0.713
2021	1.016	1.007	0.362	0.399	0.418	0.715
2031	1.018	1.009	0.362	0.399	0.420	0.716
2041	1.019	1.009	0.362	0.399	0.421	0.716
2051	1.020	1.009	0.362	0.399	0.422	0.716

The increase in the price of the transportation good has multiple effects. First, on the sector's growth rate, which initially is in the range of 2.58 percent (table 7, column 1) and declines to 2.31 percent, as the price approaches long-run equilibrium. Second, the price rise has Sto-Sa like effects on factor rental rates, so the rate of return of the unskilled labor increases. This increase contributes to the increase in the unskilled wage rate as the share of workers employed in the transportation sector increase by 2.4 percent over the period (table 6, column 7).

Similarly, the increase in the price of the service good has multiple effects. First, the sector's growth rate is initially in the range of 2.5 percent (table 6, column 1) and declines to 2.2 percent, as the price approaches long-run equilibrium. Second, the price rise has Sto-Sa like effects, so the rate of return of the skilled labor increases, this contributes to the increase in the skilled wage rate as the share of workers employed in the service sector increase by 0.1 percent over the period (table 6, column 11).

Another effect of the rise in both prices is divided into the direct and the indirect effect on the internal terms of trade. The direct effect is the rise in price of the transportation and service good relative to unchanging prices of the industrial and agricultural goods provide producers an incentive to increase their production of transportation and services goods in spite of the rise in production costs. The indirect effect is the increase in the price of the transportation and services good employed as an intermediate factor of production in industry and agriculture. This indirect effect increases the cost of industrial sector production relative to agriculture since these intermediate inputs account for 17.2 percent of the industry gross value compared to 16.1 percent of agricultural gross value (table 3).

While the decline in terms of trade are small (table 8), owning to the fact that the country is close to its long-run equilibrium, the rise in the price of the transportation and service good helps to pull resources out of traded good production. An implication is that policies directed to increase efficiency of the home goods production will tend to release resources for the production of traded goods, with industry the major beneficiary due to its dependency on services as an intermediate input (table 3).

The competition for resources from the agricultural sector causes the share of the economy's labor force (unskilled and skilled) employed in manufacturing to decline by 2.9 percent and 3 percent respectively. However, due to the relatively high rate of growth of the labor force, 2.4 per cent per annum, the number of workers employed in the industrial sector increases. Similarly, while manufacture's share of capital service employed declines by 3.1 percent from 2001-2011 (table 6, column 13), the level of capital employed increases, i.e., there is a substitution of capital for labor such that the total capital employed in the sector increases.

The growth in labor supply has a positive effect on the transportation and service sectors' growth in output (table 7, column 5). The Ry-like effects of growth in capital stock affect the transportation sectors' production negatively due to the increased competition for agricultural sector's unskilled labor, whose marginal product rises with capital deepening (table 7, column 3). The growth in capital stock has a positive effect on the service sector, since it is the sector that absorbs the skilled labor that the industry sector releases as it contracts, and that the agriculture and transportation sectors cannot profitably absorb at the prevailing wage rates.

The rise in both wage rates have a dampening effect on the growth of the agriculture output, the unskilled and skilled wage effect is in the range of -3.4 to -1.45 percent and -0.36 to -0.16 percent per annum respectively, in the early stages of growth (table 7, column 3 and 4) while the decline in the cost of capital services has a positive effect in the range of 7.52 to 3.25 percent, for the same period. As the economy approaches its long-run equilibrium, the growth in agriculture's gross output approaches the steady state rate of growth from above and converges to 2.09 percent per annum (table 7). Given that the agricultural sector grows more rapidly than the manufacture sector, Brazil exports more agricultural products and imports more manufacture goods. As a result, a larger share of GDP is traded.

5 Constraints to growth

We next explore the potential to increase per capita income from transition growth exhibited by the base solution by considering, in sequence, three possible sources of constraints to growth according to HRV and Blyde et. al.. We consider a decrease in the cost of domestic financial intermediation, an increase in the share of skill labor, and an increase in productivity in the use of intermediate factors of production associated with the transportation sector of the economy. We analyze these constraints as though they were relaxed in 2001.

The magnitude of each shock is chosen to yield the same identical value of long-run household utility. In this way, each shock is neutral in the sense that in the long-run, the representative agent, while being made better off than the base solution, is indifferent between the three shocks considered.

5.1 Effects of a decrease in the cost of domestic financial intermediation

The net interest margin is reported to be 11.5 percent in Brazil (Caprio and Levine, 2001), which it is significantly higher than the averages for other

Latin American and upper middle income grouping of countries. In order to analyze the effect of this tax in domestic financial intermediation on economic growth, the net interest margin of the base model is reduced to the tax level of OECD countries, which it is reported to be 3.97 percent (Beck, 2000). Therefore, the domestic financial intermediation cost is reduced by 65 percent.

Table 9 reports the results of this simulation relative to the base solution. Lowering the cost of financial intermediation gives rise to a growth rate in GDP that is 4 percentage points higher than the base in 2011, and 1.75 percentage points higher in 2021 (table 9). In the long run, the economy converges to a higher level of GDP but the rate of growth converges to the same rate as the base, x+n. Consequently, growth rates are higher in the earlier periods of transition than the base. Also, from table 9 we can see that all primary factors of production contribute to growth.

Table 9: Effect of a domestic financial intermediation cost reduction relative to the baseline.

	GDP	Growth in GDP (Simulation - Base)	Uwage Payments	Swage Payments	Returns to K	Land Rents	Real Exp.
2001	1,000		0,999	1,001	1,000	1,000	0,933
2011	1,038	4,01%	1,036	1,038	1,039	1,052	0,988
2021	1,056	1,75%	1,052	1,055	1,057	1,075	1,014
2031	1,064	0,82%	1,060	1,064	1,066	1,086	1,026
2041	1,069	0,40%	1,064	1,068	1,070	1,092	1,032
2051	1,071	0,20%	1,066	1,070	1,072	1,094	1,036

A reduction of the cost of domestic financial intermediation decreases the gap between the interest rate that households receive and firms pay for capital. Now, households have an incentive to save more and therefore to forego real expenditure's relative to the base solution (table 9, column 6) in order to increase their holding of capital assets during the half-life to long-run equilibrium.

The effects on sector transformation vary by sector due to their differences in factor intensities, so the capital intensive sector (agriculture in this case) experiences a Ry-like effect, increasing output compared to the base line results, and its share in GDP raises 8 percent by 2011compared to the base as can be seen in table 9. At the same time, since the agricultural sector is the most capital intensive, capital deepening increases the remuneration to land in this sector (column 5) causing rents to rise by about 5.2 percent relative to the base solution in 2011, and by 9.4 percent in 2051. Agriculture's gross output rises significantly, from 12 percent per annum in 2011, and 23 percent of base values by 2051 (multiplying column one of table 9 and column 2 of table 10).

Capital deepening also increases the productivity of labor even more than in the base solution. Both types of labors' payments increase by the same amount, around 3.7 percent by 2001 and 6.8 percent by 2051 with respect to the base solution. Agriculture's increased competition for resources comes at the expense of the relatively labor intensive sectors, service and transportation, which experience a rise in production costs. The rise in factor income, since wages and the stock of capital have increased, augments the quantity demanded of all final goods, as well as the quantity of intermediate inputs employed in agriculture. In order for the domestic non-traded good markets to clear, the price of these goods must rise to compete for the resources that would otherwise be employed in the capital intensive sector. In this case, the sector that cannot increase its price (industry) is less able to compete for these resources relative to the base solution.

The rise of these prices have several effects as explained in the base line results; one of the effects is the incentives for service and transport sector firms to compete for resources so that markets clear.

Another effect is the change in the terms of trade. The terms of trade faced by the other sectors are more negative than base. Therefore the cost of the industrial sector production increased, and even more relative to agriculture since transportation and services account for 17.2 percent of the industry gross value as intermediate inputs compared to 16.1 percent of agricultural gross value (table 3).

The competition for resources from the agricultural sector causes the share of the economy's labor force (unskilled and skilled) employed in manufacturing to decline with respect to the base by 4 percent in 2001 and by 7 percent in the long run. The share of industry in GDP declines from the base line results around 6.4 percent in 2011 to 1 percent in 2051 (table 10) compared to the base.

Table 10: Effect of a domestic financial intermediation cost reduction for each sector relative to base.

	Sector Share in GDP				Unskilled Labor Share in				Skilled Labor Share in				Capital Share in			
Year	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service
2001	1.001	0.999	0.933	1.002	1.000	1.000	0.970	1.002	0.999	0.999	0.969	1.001	0.999	0.999	0.969	1.001
2011	0.937	1.080	0.940	0.996	0.966	1.066	0.997	1.004	0.963	1.063	0.995	1.002	0.962	1.063	0.994	1.001
2021	0.902	1.118	0.943	0.993	0.948	1.097	1.010	1.005	0.945	1.094	1.007	1.002	0.943	1.092	1.005	1.001
2031	0.883	1.136	0.945	0.992	0.939	1.112	1.016	1.006	0.936	1.108	1.012	1.003	0.934	1.106	1.010	1.001
2041	0.874	1.144	0.946	0.991	0.934	1.120	1.019	1.006	0.931	1.116	1.015	1.003	0.929	1.113	1.013	1.001
2051	0.869	1.149	0.946	0.991	0.932	1.123	1.020	1.006	0.929	1.119	1.016	1.003	0.927	1.117	1.014	1.001

As a result of these adjustments, the agriculture sector's share of unskilled and skill labor employed are 6.6 and 6.3 percent higher respectively, than that in the baseline solution for 2011 and 12.3 and 11.9 percent higher than the baseline in the long-run. Since the rental rate firms pay for capital is less than base at each t, this sector increases the use of capital by 6.3 percent above that of the base solution in year 2011, and reaching 11.7 percent by 2051.

The capital share in the transportation and service sector is less now compared to the baseline, but the trend is increasing over time. The share of labor employed in the both home-good sectors is higher than that of the base. We thus see the reduction in domestic cost of financial intermediation as causing the agricultural sector to compete for resources formerly employed in the industrial sector as opposed to those employed in the transportation or service sector (table 10).

5.2 Effects of an increase in the skill labor's share on the economy

The share of skill labor is 18.06 percent of the work force. A positive shock of 16.3 percent to this share (to 21 percent) yields the same long-run level of utility as the 65 percent shock to the tax on capital considered in the preceding simulation. The increase (decrease) in the skilled (unskilled) labor force is assumed to happen immediately in 2001, but the cost of educating and training is charged to the household over a period of 50 years. Thus it is assumed that the government borrows the necessary investment from abroad and remits payment on principle with interest over a fifty year period.

Table 11 reports the results of this simulation relative to the base solution. Increasing the supply of the skill labor force gives rise to a growth rate in GDP that is 1.2 percentage points higher than the base in 2011, and 1.1 percentage points higher in 2021 (table 11). From table 11 we see that unskilled wage payments and returns to capital are the primary factors of production that contribute to growth.

Table 11: Effect of an increase in total factor productivity on factor earnings relative to base.

	GDP	GDP growth	Uwage payments	Swage payments	Returns to K	Land rents	Real Exp.
2001	1,003		1,042	0,859	1,003	0,994	0,995
2011	1,011	1,178	1,050	0,866	1,012	1,009	1,007
2021	1,015	1,126	1,054	0,870	1,015	1,015	1,012
2031	1,017	1,081	1,056	0,871	1,017	1,019	1,015
2041	1,017	1,047	1,057	0,872	1,018	1,020	1,016
2051	1,018	1,025	1,057	0,872	1,018	1,021	1,017

Increasing the skill labor force amounts to increasing the supply of a relatively scarce resource and decreasing the supply of a relatively abundant resource (unskilled labor), so that the total bundle of labor services provided to the economy occur at a lower total resource cost, all else constant, at each instant in time. This change in factor abundance should increase the productivity of the country's stock of capital at each instant in time relative to the base solution.

Sectoral effects vary depending upon the share of total cost accounted for by skilled compared to unskilled workers. Consequently, the relatively high unskilled labor intensity of transportation sector is found to experience a higher evolution of its price relative to the base while the price of the service sector, where skilled labor accounts for a relatively large share of unit cost, experiences a lower evolution of its price compared to the base.

More specifically, skill wage payments per skill worker decrease with respect to the base solution by 4 percent while the unskilled wages per unskilled worker raise by 5 percent in 2011 (table 11). Due to the decline in the skill labor wage rate, the unit costs of the service sector (relative to base) falls, allowing the market for this good to clear at a lower price at each t relative to the base, while, as suggested above, the reverse applies to the transport sector. The magnitude of these changes is reported in table 12, where it can be seen that the skill labor and capital share in the transportation sector increases compared to the base. Since now there is less unskilled labor, the share of this resource for each sector is less than the base solution while the skill labor shares have increase for all sectors.

Table 12: Effect of an increase in labor productivity on output per worker for each sector relative to base.

	Sector Share in GDP				Unskilled Labor Share in				Skilled Labor Share in				Capital Share in			
Year	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service
2001	1.033	0.957	0.991	1.021	0.992	0.937	0.973	0.956	1.203	1.137	1.180	1.160	1.031	0.973	1.011	0.993
2011	1.023	0.974	0.992	1.019	0.987	0.950	0.979	0.957	1.196	1.152	1.187	1.160	1.024	0.987	1.016	0.993
2021	1.018	0.982	0.992	1.019	0.984	0.956	0.981	0.957	1.193	1.159	1.189	1.160	1.021	0.993	1.019	0.994
2031	1.015	0.986	0.993	1.018	0.982	0.959	0.982	0.957	1.191	1.162	1.191	1.160	1.020	0.995	1.020	0.994
2041	1.014	0.987	0.993	1.018	0.982	0.960	0.983	0.957	1.190	1.164	1.191	1.160	1.019	0.997	1.020	0.994
2051	1.013	0.988	0.993	1.018	0.981	0.961	0.983	0.957	1.190	1.165	1.192	1.160	1.019	0.997	1.020	0.994

The change in skills tends to lower the cost of intermediates employed by all sectors, and makes the production of capital less costly compared to the baseline solution, and therefore the returns to capital rise (table 11). Now, the agricultural sector faces increased competition for resources employed in the service sector, as firms in this sector benefit from the Ry-like effect of an increase in skilled workers.

Gross output per worker in the agricultural sector declines by about 2 percent relative to the baseline in 2011 (multiplying sector share in GDP from table 12 by the first column of table 11). Thereafter, this difference decreases due to capital deepening, although output levels never reach their base values. For the same comparison, the service sector output increases by 3 percent of its base line levels, and maintains this difference over the long run. In the case of industry, the gross output per worker increases by 3.4 percent compared to the base in 2011.

The change in skilled labor raises total factor income (GDP), but not in a uniform manner over all factors. With respect to the base for each t, the share of returns to capital and land in GDP rises. Total labor payments also increase over the base, but their share in GDP remains virtually unchanged (as the increased of the unskilled share compensates the decrease of the skill's). Consequently, the rise in skilled labor has also increased the productivity of capital, thus raising its share in GDP (table 12).

5.3 Effects of an increase in the transportation sector productivity as an intermediate input on the economy

A 45 percent increase in efficiency in transport provision is required to yield the same long-run level of utility as a decrease of 65 percent yields in the decrease in financial cost. The increase in efficiency is assumed to happen immediately in 2001, but the cost of constructing new roads and improving their quality is charged to the household over a period of 50 years.

Table 13 reports the results of this simulation relative to the base solution. Increasing efficiency in transport provision gives rise to a growth rate in GDP that is 1.2 percentage points higher than the base in 2011, and 1.1 percentage points higher in 2021 (table 13). From table 13 we see all primary factors of production contribute to growth.

Table 13: Effect of an increase in transport sector productivity on factor earnings relative to base.

	GDP	GDP growth	Uwage payments	Swage payments	Returns to K	Land rents	Real Exp.
2001	1,019		1,014	1,022	1,023	1,014	1,012
2011	1,027	1,162	1,021	1,029	1,030	1,024	1,023
2021	1,030	1,118	1,024	1,033	1,034	1,028	1,028
2031	1,032	1,078	1,026	1,034	1,036	1,031	1,030
2041	1,033	1,046	1,026	1,035	1,037	1,032	1,032
2051	1,033	1,025	1,027	1,036	1,037	1,032	1,032

This change amounts to increasing the productivity of the transportation sector, so that the total bundle of transportation services provided to the economy occur at a lower total resource cost, all else constant, at each instant in time relative to the base solution. Sectoral effects vary depending upon the share of total cost accounted for by transportation as an intermediate input. Since transportation is made more efficient, it experiences a lower evolution of its price compared to the base, while the services sector is found to experience a higher evolution of its price relative to the base.

In spite of the rise in the price of services, the manufacturing and agriculture sectors experience less negative internal terms of trade in transition growth relative to the base. This increase in efficiency is "spread broadly" to gains in all factor payments (table 13), which in turn leads to increased expenditures and an increase in savings.

The improvement in transport infrastructure allows firms in other sectors to obtain the same level of transport services at lower cost thus causing some resources employed by transport firms in the base solution to be employed in other sectors of the economy. Since this sector is relatively unskilled labor intensive, a relatively larger amount of this resource is released to other sectorsat each instant in time. However, as the other sectors' output expand, relative to base, the demand for transport services also grows and exceeds the level demanded relative to the base (table 14).

Table 14: Effect of an increase in service sector productivity on output per worker for each sector relative to base.

	Sector Share in GDP				Unskilled Labor Share in				Skilled Labor Share in				Capital Share in			
Year	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service	Industry	Ag	T	Service
2001	1.013	0.979	1.051	0.991	1.012	1.005	0.738	1.012	1.004	0.997	0.732	1.004	1.003	0.996	0.732	1.003
2011	1.003	0.994	1.052	0.990	1.006	1.017	0.743	1.013	0.998	1.009	0.737	1.004	0.996	1.008	0.736	1.003
2021	0.998	1.001	1.052	0.989	1.003	1.023	0.745	1.013	0.994	1.015	0.739	1.004	0.993	1.013	0.738	1.003
2031	0.995	1.004	1.052	0.989	1.001	1.026	0.747	1.013	0.993	1.017	0.740	1.005	0.991	1.016	0.739	1.003
2041	0.994	1.005	1.052	0.989	1.001	1.027	0.747	1.013	0.992	1.019	0.741	1.005	0.991	1.017	0.740	1.003
2051	0.993	1.006	1.052	0.988	1.000	1.028	0.747	1.013	0.992	1.019	0.741	1.005	0.990	1.018	0.740	1.003

In early periods of transition, the reources released from the transport sector are mostly absorbed by the industrial sector. The effect on agriculture is non-monotonic. In the early periods of transition, agricultural firms experience an increase in cost from the rise in wages that dominate the cost of savings from a decline in the rental rate of capital. As the rise in wage payments moderate over time, the cost saving from the decline in the cost of capital dominate, relative to the base solution, causing output to rise above base line values. These adjustments cause a larger share of GDP to be traded (measured as the value of exports plus imports divided by GDP).

In terms of the value of output shares in GDP, relative to the base, manufacture's share rises by 1.3 percent initially, while agriculture's decreases by 3 percent. However, the transport sector's share in GDP rises substantially (by 5.2 percent in 2011) as these services are now provided more cheaply, stimulating growth in other sectors which in turn increase the demand for transport services. Growth in the output of the service sector is positively affected as well, but less so than the other sectors so that its share in GDP falls by 1 percent in 2001 and falls by two percent in the long run, in 2051.

5.4 Discussion

In conclusion, all three simulations raise GDP per worker compared to the base solution, and their effects on structural transformation and growth depends on relative factor intensities of each sector, yet they yield the same long-run level of wellbeing. The reduction in the cost of domestic financial intermediation has the biggest effect on GDP per worker in the long run (GDP is 7 percent higher than the base by 2051), while the increase in transportation sector productivity has the largest effect in the short run, increasing GDP per worker by 3 percent in 2011. However, our results suggest that neither of the three constraints, if relaxed individually, offer the potential for substantially increasing economic growth.

A fourth simulation was thus performed in which all three shocks were implemented in the initial period. The results showed a greater effect in GDP per worker than the sum of individual reforms, raising GDP per worker in the short run by 2.2 percent over the base line, and by 12.7 percent in the long run. GDP doubled in 30 years initially, now it doubles in 26 years. On a per worker basis, income doubled in 79 years initially as opposed to 68 years in the latter simulation.

In earlier periods, the industry and service sectors increase their shares in GDP compared to the base. In the long run, capital deepening effects benefit agriculture causing its long-run share in GDP to exceed that of the base solution. Consequently, agriculture also increases its share in factors of production relative to the base. The service sector's share of skilled workers exceeds base levels since this resource accounts for a large share of total costs. The lower costs per unit of transport caused by investment in transport infrastructure has the effect of lowering this sector's share of resources employed. Last, industry's share of skilled labor is also higher than base shares. Of course, capital to labor ratios are higher than base line ratios.

We conclude that while the factors considered impede growth, other factors remain that constrain the country's growth potential. An endogenous growth model providing insights into the resources and institutions affecting the countries rate of total factor productivity growth may offer potential for providing insights in this regard.

6 Conclusion

Disenchantment with the traditional guidelines provided by the Washington Consensus has led to an emphasis on growth diagnostics as a first step to guide the focus of policy makers on the most daunting constraints to growth. Rodrik (2006) suggests that growth strategies are likely to differ according to domestic opportunities and constraints. Hausmann et al. (2005) (HRV from now on), argue that Brazil has eliminated some policies distorting her economy, but she has not addressed the binding constraints. Following Rodrik's growth diagnostic approach, HRV suggest two key constraints that slow Brazil's rate of growth. They conclude that a low rate of domestic savings (17 percent of GDP compared to 30 percent for countries in East Asia) and a low stock of human capital (only 20 percent of the labor force is skilled) are likely major causes of the country's poor growth performance. Following HRV's growth diagnostics, Blyde et al. (2008) adds a third binding constraint, inadequate transportation infrastructure. They show that the slowdown in economic growth coincided with a significant drop in the pace of expansion in infrastructure stock and that political reforms (over the 1990s) clearly failed to reverse this process.

The unique contribution of this study is to analytically and empirically derive the inter-temporal implications of relaxing some of the constraints to growth that Brazil encounters. Our study fits to Brazilian data a Ramsey model with four sectors, depicting interconnections of the economy through various channels. The study also provides insights into what extent the structure of the economy changes over time, and whether such transition forces differ from the base solution given different initial conditions.

The baseline model result shows Gross Domestic Product per worker increases marginally over the fifty year period 2001-2051, from 5,297 to 6,606 constant 2001 USD. This increase suggests the country's potential to increase household's real income per capita from transition growth is somewhat limited requiring 79 years to double income, thus suggesting that impediments to growth are embodied in the data and hence in the model's parameters. The basic underlying economic forces of transition growth are such that the agricultural sector's share in GDP increase, while the industry sector's share decreases. Accompanying this transition is capital deepening in all sectors of the economy, rising wages and land rents. Land rent (or farm profits) rise as the cost of capital declines (a cost of production in agriculture). The rise in the prices of non-traded (service and transport) goods contributes to a decline in the internal terms of trade for the two internationally traded goods, thus inducing the non-traded good sectors of the economy to pull some resources from these sectors.

Three simulations are performed: decreasing the domestic cost of financial intermediation, increasing the skill labor share in the economy, and increasing productivity in the use of intermediate factors of production associated with transportation, which is modeled as one of the home goods. Second, the level of each shock is chosen such that each yields the same long-run (or steady-state) level of utility. In this way, the level of the shock studied is neutral in the sense that they lead to the same long-run level of wellbeing.

Decreasing the domestic cost of financial intermediation increases the incentive for households to save which in turn increases the country's rate of capital deepening causing wages to rise. While capital deepening induces all sectors to increase output, the most capital intensive sectors tend to increase output over base line values to the largest degree. This Ry-like effect is dampened by the home good - relatively labor intensive - sector markets' response of increasing home good prices. Competition for resources causes the share of the agricultural sector in GDP to increase (since it is relatively capital intensive), while the share in service and transportation in GDP declines (as they are relatively labor intensive) and the industry sector share of GDP first increases but as capital deepening occurs it falls below the base values

The increase in the supply of the skill labor force contributes to the expansion of the service and industrial sector by increasing the share of the economy's resources that are allocated to these sectors. Also, higher skilled labor increases the productivity of capital and land as well, so that GDP growth increases by 3 percent in the short to medium run.

Investment in transport infrastructure decreases the unit cost of suppling intermediate inputs to all sectors. Therefore, firms supplying transport services experience a lower evolution of their output price for these services compared to the base. This increase in efficiency is "spread broadly" to gains in all factor payments and increases industry's share in GDP in earlier periods. Nevertheless, in the long run, we can see that the basic underlying economic forces of transition growth remain unchanged, the agricultural sectors' share in GDP increase as capital deepening comes about, pulling some resources out of the industry and service sectors.

A fourth simulation was performed to assess the combined effect of a simultaneous relaxation of the impediments to growth at the same level as assumed in the previous simulations. The results showed a greater effect in GDP per worker than the sum of individual reforms, raising GDP per worker in the short run by 2.2 percent over the base line, and by 12.7 percent in the long run. In conclusion, addressing any one of these constraint in isolation of the others, or simultaneously clearly benefits the economy, but it does not offer a panacea, i.e., it does not launch the economy on a substantially higher growth path.

7 Appendix

7.1 Confronting model forecasts to the data

Numerical measures of the model's forecast accuracy are reported in table 15. Pearson's correlation coefficient provides a linear measure of the correlation between the data and the forecast without accounting for differences in the level of the variables in the two series. As can be seen in table 15, the correlation in each sector and GDP is extremely high. Lin's (1989) concordance correlation measure is bounded between zero and unity, and accounts for discrepancies between the means of the two series. The result of this measure tells us that the Economy's agricultural sector has the lowest correlation. The mean absolute error is relatively lower for the GDP's forecast and pretty high for Agriculture. Theil's U statistic is unbounded from above with smaller values indicating a closer fit to the data. This measure also tends to show the predicted values for GDP to be lower than in the case of the other sectors, an agriculture is again relatively high. But overall, these measures show that the model's forecast accuracy is pretty high.

Table 15:Measures of the model's forecast accuracy, 1995-2005.

Measure	Economy GDP	Agriculture GDP	Industry GDP	Service GDP
Correlation Coefficient	0.9868	0.9858	0.9315	0.9891
Concordance Correlation Coefficient	0.0102	0.1036	0.0136	0.0816
Theil's U Statistic	0.0451	0.6457	0.4823	0.4001
Mean Absolute Error (%)	4.26	64.95	48.19	39.67

7.2 Validation results

The reluctance to validate structural models has been one of the main critiques leading to some skepticism as to their value in explaining and predicting economic events. Many early practitioners of applied general equilibrium models dismissed the need for validation. Kehoe (2003) suggests ex-post performance evaluations of applied general equilibrium models are essential if policy makers are to have confidence in the results produced by them.

Typically, the model provides the trajectory for a larger set of variables than are available in the form of time series data. We limit our comparison of model predictions to economy and sectoral GDP which we contrast with data taken from the WDI. Model predictions are indicated by the dashed line in Figure 4. The model is fit to the data such that the initial year is 2001 and then solved to provide predictions backward to 1995 and forward. The upper left-hand chart shows the model's forecast of GDP value added in terms of GDP's value in the base year 2001. The three remaining charts express sector value added in terms of the respective sector value in the base year 2001. The reason for this normalization is because of differences in GDP and sector's definitions.

Briefly, the problem is that the International Standard Industrial Classification (ISIC) defining the sectors in the WDI data are not the ideal classification needed for the model. For example, agriculture corresponds to ISIC divisions 1-5 in the WDI data. This division includes crop and animal agriculture, hunting and related service activities, forestry, fishing and mining of coal. Food products are included in manufacturing. Thus, the ISIC 1-5 aggregation for agriculture is not an appropriate characterization of the sector's output, if in the modeled economy, the sector's output is treated as a final good. The additional subsectors in agriculture cause the model's agriculture GDP to exceed that of the WDI by about 23 percent in the initial year 2001. Similar adjustments are made to the GDP, industrial and service sectors, which cause industrial sector GDP to exceed that of the WDI by 17 percent, while the service sector and GDP are underestimated by the model by 35 percent and 17 percent respectively.

In spite of the differences between the WDI and model's definition of sectoral GDP, the predicted values should show a similar trend to the data if the model captures some of the fundamental structural features of the economy. In this case, validation was done one two of the working models against data, and as it can be seen in Figures 4 to 7, the model predicts extremely well the trends in the data.

Figure 4: GDP

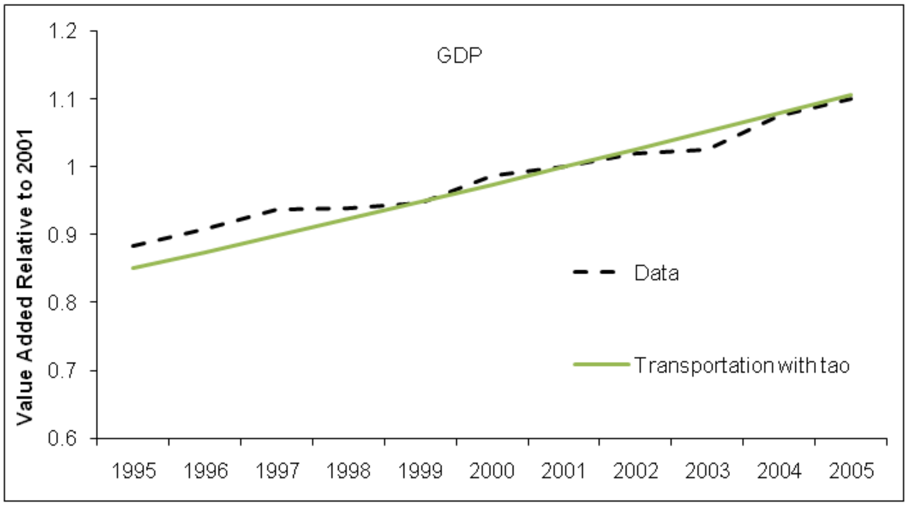


Figure 5: Industry Sector

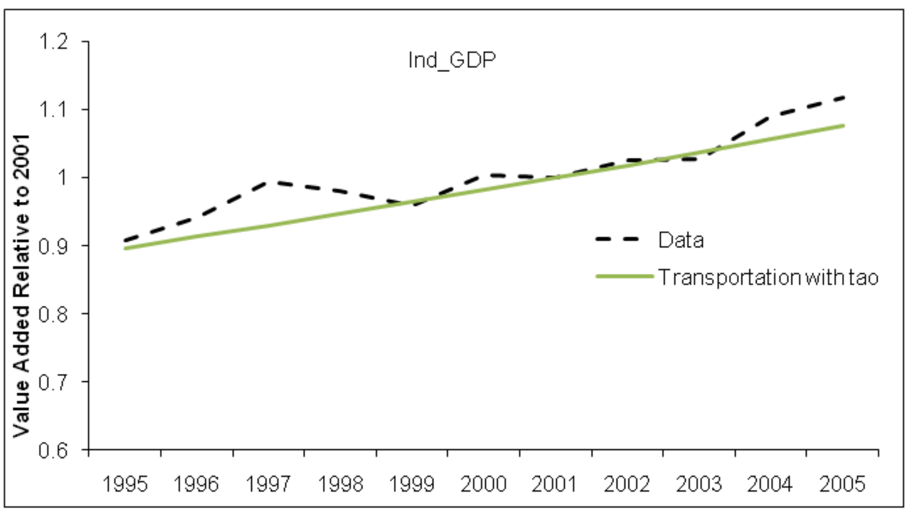


Figure 6: Agriculture

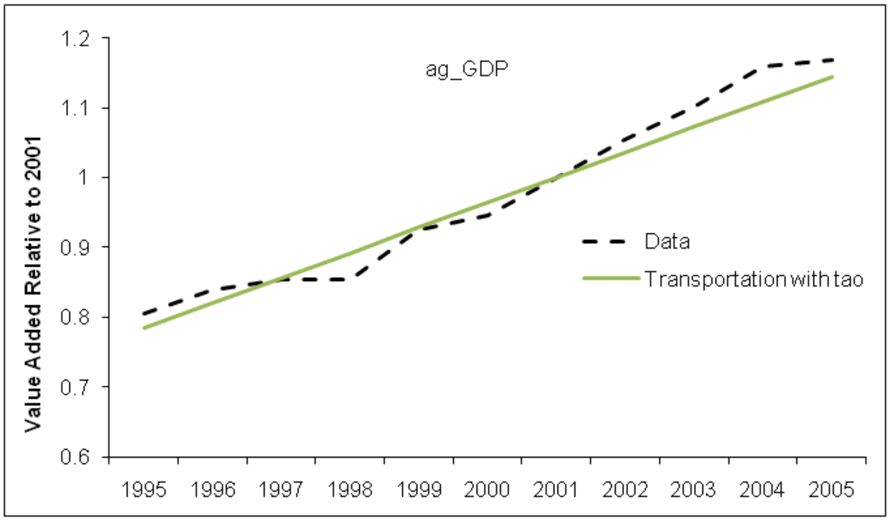
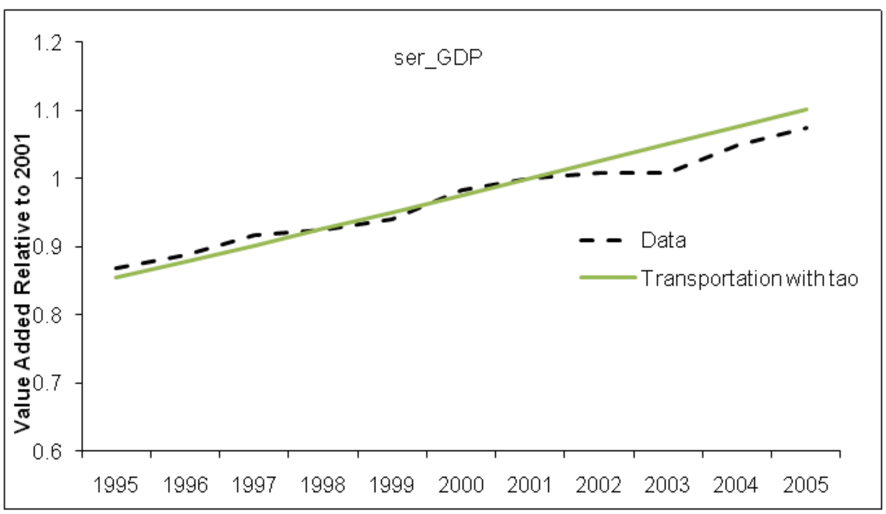


Figure 7: Services



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