ANALYSIS OF RANKED ORDERED DATA WHEN THERE ARE COMPLETE AND PARTIAL RANKINGS

Mahesh Pandit, Louisiana State University Agricultural Center

Krishna P. Paudel, Louisiana State University Agricultural Center

Michael A. Dunn, Louisiana State University Agricultural Center

Corresponding Author

Krishna P Paudel, Associate Professor
Department of Agricultural Economics and Agribusiness
Louisiana State University Agricultural Center
Baton Rouge, LA 70803
Phone: (225) 578 7363
Fax: 225 578 2716
Email: Kpaudel@agcenter.lsu.edu

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Abstract

Many methods are available to analyze rank ordered data. We used a spectral density analysis to identify most preferred options of Formosan Subterranean Termites (FST) control as ranked by Louisiana homeowners. Respondents are asked to rank four termite control methods form the most preferred option to the least preferred option. Spectral analysis of both complete and partial ranked data indicates that the most preferred termite control choice is a relatively cheap ($0.13 per square foot) option of a Liquid treatment.

Keywords: Complete and partial ranked data, Formosan Subterranean Termite, rank ordered data, spectral analysis

JEL Classifications: C65

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Spectral analysis (or Fourier analysis) is a very useful tool to analyze time-series data. This paper shows that spectral analysis is an equally powerful tool that can be applied in a non-time series subject in economics such as to analyze preference ranking data. Ranked data are frequently seen in categorical forms with each category reflecting the preference intensity. A number of questions are asked to participants to rank items according to their order of preference. For example, opinions about some objects often ranked as strongly agree, agree, disagree, and strongly disagree. In research, rank order data are often coded as consecutive integer from one to the number of category to their degree of preference, but the number does not represent their distance. When intensity of preference is presented, economists try to find the factors affecting these rankings but leave which of the option provided is the most preferred option. This paper attempts to fill this void in economics literature using a case of Formosan Subterranean Termite (FST) (*Coptotermes formosanus* Shiraki) control options ranked by Louisiana homeowners.

A group of respondents may rank their choices as first, second, third, fourth preference and so on. Existing research discusses the ranking of preference data in two cases: when there are complete rankings and when there are partial rankings. A comprehensive review of both complete and partial data is available in both Diaconis (1988) and Critchlow (1985). If respondents rank all items available in a survey, it represents a complete ranking or full ranking. However, some of the respondents in a survey only rank a few items and leave others. In such a case, we do not know the ranks of remaining candidates in the survey. This type of incomplete ranking represents a partial ranking. Hence, a partial ranking is a preference list of r items out of n items. Generally, the way of analyzing the data is to remove partial ranking and estimate the result using complete data and analyze only the subset of complete rankings (Murphy and Martin, 2003). This type of practice decreases sample size by removing partial ranking observations, which can result in a significant decrease of estimation accuracy (Busse et al., 2007).

In this paper, we focus on the preference ranking of Formosan Subterranean Termite control methods by Louisiana homeowners. Several states in the U.S. suffer significantly from FST damage. In order to control FST, researchers and government agencies are attempting to identify the most preferred option of FST control so that economic damage can be minimized. Additionally, if needed, a subsidized treatment program can be implemented based on preferences ranking information. We collected data using contingent ranking method to find alternative FST control options. As discussed above, we have both full and partial rankings in the data set. We will discuss complete details of data features in the data section. A partial analysis of complete ranking is available (Paudel et al., 2009). In this paper, we will expand the analysis of complete ranking and extend the analysis with partial ranking data.

Relevant Literature

There exist several approaches to analyze rank order data. A few examples include nonparametric analysis of unbalance paired-comparison of ranked data (Andrew and David, 1990). Andrew and David compared simple and nonparametric methods of analyzing unbalanced ranked data to an existing method of rank analysis for unbalanced data. Busse et. al. (2007) used a cluster analysis of heterogeneous rank data and found that the parameter estimation improved when incomplete ranking data were included in the inference process. Another way of analyzing ranked data is to use a Markov Chain Monte Carlo Technique (Eriksson, 2006). In addition, Lebanon and Mao (2008) improve analysis of partial ranking data using nonparametric methodology and derive a computationally efficient procedure which is also suitable when there are large numbers of items to be ranked. In addition, Fagin et al. (2006) provided a broad image of methodology to compare partial ranking using several metrics. Although all of the above methods are computationally efficient to analyze, they are not easy to conceptualize in different dimensions. Thompson (1993) applies a generalized permutation polytopes and exploratory graphical method for ranked data. The author presents an exploratory graphical method to display frequency distribution for fully and partially ranked data. In addition, Kidwell and Lebanon (2008) build an approach for the visualization of ranking data for large n, which is easy to use and computationally efficient to use too. An alternative way of investigating preference ranked data is completely randomized factorial design (Scheirer et al., 1976). However, this procedure as an extension of the Kruskal-Wallis rank test allows for the calculation of interaction effects and linear contrasts. Paudel et al. (2007) applied exploded logit and ordered probit models to identify the most preferred Formosan termite control method in Louisiana. A new way, generalized spectral decomposition by Diaconis (1988) and Diaconis (1989), is a very useful methodology to analyze full and partial rank preference data. Lawson and Orrison (2002) used these ideas to detect hidden coalition in the vote of nine judges of the United States Supreme Court.

Recently, Pedrotti et al. (2006) used generalized spectral analysis to find preference for cars. As the theory of spectral analysis, they use first order and second order effects to compare preference for different cars.

The Theory of Spectral Analysis

We applied a spectral density analysis to find the most preferred control method for FST control as ranked by homeowners in Louisiana. We briefly outline a general theory of spectral analysis applicable for rank order data. Most of the materials used in this section are borrowed from Diaconis (1988) and Diaconis (1989).

Let us suppose we have n types of FST control methods denoted by i, $i = 1,2,\ldots,n$. Let $\pi(i)$ denotes the rank given to ith method. This type of data can be represented using permutation. A permutation π is a bijective function π: {1,2…n}→{1,…,n} associated with each item i∈{1,…,n} a rank π(i) ∈ {1,…,n} (Critchlow, 1985). Hence, the number of rankers choosing ranking preference π form a data set which is denoted by $f(\pi)$ can be expressed as

$$f(\pi) = \begin{pmatrix} 1 & 2 & \ldots & n \\ \pi(1) & \pi(2) & \ldots & \pi(n) \end{pmatrix}$$

If we are ranking n items, the permutation of the number of items gives the sample size of the data for complete ranking. And for partial ranking, let q denote the number of ranking option out of n, then the sample size for the partial ranking with each $q \leq n$ is given by $q!\binom{n}{q}$. Suppose there are four methods of controlling FST. Then, there are 4! (=24) complete rankings as shown in Table 1. The sample size for q=1 is 4 and q=2 is 12 as shown in Table 2. From the theory of Group we can represent it as a symmetric permutation group and denote it by S_n. Let G be a finite group operating transitively on X. And L(x) or M is the space of all functions on X with values in R. This is a vector space on which G acts linearly as a group transformation $[gf(x) = f(g^{-1}x)]$. Then, if M is the vector space of function defined on π, then subspace of N of V is invariant with respect to $G(S_n)$ if for every $f \in N$ and every $g \in S$,which implies that $gf \in N$. V decomposes into a direct sum of invariant irreducible subspace, as follows.

$\mathrm{M} = V_0 \oplus \mathrm{V}_1 \oplus \ldots \oplus \mathrm{V}_\mathrm{k}$

In other words, every function $f \in V$ may be written uniquely as a sum

$f = f_0 + \cdots + f_k$ Where $fi \in Vi$ and $gf_i \in V_i$ for all $g \in S$

Let $f(x)$ be a set (the number of times x appears in the sample), the spectral analysis is the projection of f onto the invariant subspaces and the approximation of f by as many pieces as required to give a reasonable fit. Sometimes rank data are not complete; therefore, we have to set up for partially ranked data as described by Diaconis (1988). For the decomposition, let $\lambda = (\lambda_1, \ldots, \lambda_r)$ be a partition of n, where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_r$ and $\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_r = n$. Then the Young's rules give the appropriate irreducible subspace in the decomposition of M^λ, where M^λ represents a partially ordered data "in configuration λ." To illustrate, $M^{(n-m,m)}$ is the data vector of the form "Pick the best m of n". The decomposition of $M^{(n-m,m)}$ gives spectral decomposition of the frequencies. According to this rule

$$M^{(n-m,m)} = S^{(n)} \oplus S^{(n-1,1)} \oplus S^{(n-2,2)} \oplus \ldots \oplus S^{(n-m,m)}$$

And dimensions of $S^{(n-m,m)} = \binom{n}{m} - \binom{n}{m-1}$

And the subspaces of $S^{(n-m,m)}$ have the following interpretations:

$S^{(n)}$ - The grand mean or number of people in the sample

$S^{(n-1,1)}$ - The effect of item i, $1 \leq i \leq n$

$S^{(n-2,2)}$ - The effect of items $\{i, j\}$ adjusted for the effect of i and j.

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$S^{(n-k,k)}$ -The effect of a subset of k items adjusted for lower order effects.

The procedural details to decompose higher dimension are available in Diaconis (1988). Here, we are only interested on first order i.e. $\lambda = (n-1, 1)$ and second order unordered λ = (n-2, 2) effects. Hence, If X is the set $\{1, 2, \ldots, n\}$ and $f(x)$ is the number of people choosing x, then the decomposition with their dimensions are:

first order - $\underset{n}{\mathrm{L(x)}} = \underset{1}{S^{(n)}} \oplus \underset{n-1}{S^{(n-1,1)}}$

Second order - $\underset{n}{\mathrm{L(x)}} = \underset{1}{S^{(n)}} \oplus \underset{n-1}{S^{(n-1,1)}} \oplus \underset{n(n-3)/2}{2S^{(n-2,2)}} \oplus \underset{(n-1)(n-2)/2}{2S^{(n-2,1,1)}}$

As mentioned previously, the spectral analysis is the projection of f onto the invariant subspaces. This type of projection is also called isotopic projection. Many researchers use spectral analysis in time series data where the dimensions are lower and easy for computation. However, ranked order data have higher dimensions compared to time series data, so we cannot find the orthogonal basis to compute projection in the isotopic subspace easily. Mallows (1957) provides an approach to deal with such difficulty. This paper uses his approach to compute both first and second order analysis. We use inner product to compute the final projection of the data by using the following mathematical expression.

$\langle f_1 | f_2 \rangle = \sum_{\pi} f_1(\pi) f_2(\pi)$

First order analysis

The space V1 is the set of constant function that is the average frequency of the data, so it has one dimension. The space V2 is the space of first order function evaluated using Mallows' approach. Therefore, consider a function

$$\pi \to \delta_{i\pi(j)} \begin{matrix} = 1 \; if \; \pi(j) = 1 \\ = 0 \; Otherwise \end{matrix}$$

Where i is the control method and j is the rank given to the control method.

The first order function becomes

$\sum_{i,j} a_{ij}\delta_{i\pi(j)}$. In order to get direct sum decomposition, the coefficients should satisfy the following condition.

$\sum_{i,j} a_{ij} = 0$.

If we consider our data set, it will consist of 3 dimensional space projected into 3 dimensional space, so it projects a 9 dimensional space, which can be shown using hook-length formulae following Young's rule as presented in Table 4.

Second order analysis

Second order analysis consists of ranking a pair of control methods to a pair of rank methods. For example, someone can choose first and second methods in third and fourth or fourth and third ranking positions. The position of rank can be considered in ordered or unordered position. Therefore, there are two types of second order functions. Again following Mallow's approach, let

$$\pi \rightarrow \delta_{\{ii'\}\{\pi(j)\pi(j')\}} \begin{matrix} = 1 \; if \; \{\pi(j)\pi(j')\} = \{i, i'\} \\ = 0 \; Otherwise \end{matrix}$$

Then, The general, unordered, second order function will be of the following form.

$$\sum_{i,i',j,j'} a_{ii',jj'} \delta_{\{ii'\}\{\pi(j)\pi(j')\}}$$

Where, $a_{ii',jj'}$ are chosen so that V3 is orthogonal to V1⊕V2. In this case, the order does not matter and it has 2 dimensional space each with 2 dimensional projection so, the total 4 dimensional space . In a similar way we can find the higher order function, which is beyond the objective of this paper.

Data

Data were collected by means of a survey of homeowners regarding their preference of Formosan FST control options in Louisiana. FST are invasive species of termites which is currently present in more than 13 states in the U.S. It has been found that the damage by the species is so severe that infested houses become uninhabitable if not controlled in time. Damage estimates due to FST infestations reach approximately one billion dollars per year.

In this study, four FST control options were provided for each individual homeowner to rank from the most preferred choice to the least preferred choice. The FST control choices provided are i. No control option: cost \$0/square foot, ii. Liquid treatment option: cost \$0.13/square foot, iii. Bait treatment option: \$0.43/ square foot, iv. Liquid+Bait treatment option: \$0.56/square foot. Individuals ranked these options as their first, second, third and fourth most preferred option to control FST. There were a total of 972 observations obtained from the survey. Out of those, only 747 respondents provided complete rankings, which are shown in Table 1. Remaining respondents provided partial rankings as shown in Table 2. The entries of columns of Table 1 shows the control method ranked in the given permutation. Thus, 1234 respondents ranked No control option method as their first preferred method, Liquid treatment as their second preferred option, Bait treatment as their third preferred option, and Liquid+Bait treatment as their fourth preferred option. Zero or blanks in Table 2 indicate respondents did not rank all control options. For example, 42 homeowners ranked first control method first and left others unranked.

Results

Complete ranked data were analyzed first which is then followed with the analysis of partially ranked data. The percentage of respondents ranking preference i in position j is shown in Table 6. This table indicates that 52.2% of respondents preferred the Liquid control method as the first choice and 55.7% of respondent favored Bait as their second choice.

Complete Ranking

Let M be the space of all real valued functions on the symmetric group S4. This vector space decomposes uniquely into the direct sum of five subspaces. These are shown with their dimensions in Table 3. V1 is the set of constant functions with one dimension. Second space V2 is the set of functions whose sum is zero with 9 dimensional space and orthogonal to V1. Similarly, V3 is the second order unordered effect with 4 dimensional space and orthogonal to V1+V2. The result of first order spectral analysis is shown Table 5. First order data decomposed in two parts, for first ranking, $f^{(3,1)}$ is found by projecting $f_0^{(3,1)}$ onto $M_0^{(3,1)}$ and $f_1^{(3,1)}$ is found by projecting $f^{(3,1)}$ onto $M_1^{(3,1)}$. This gives the following decomposition:

Where:

$$f^{(3,1)} = \begin{pmatrix} 142 \\ 400 \\ 93 \\ 112 \end{pmatrix} f_0^{(3,1)} = \begin{pmatrix} \frac{747}{4} \\ \frac{747}{4} \\ \frac{747}{4} \\ \frac{747}{4} \end{pmatrix} f_1^{(3,1)} = \begin{pmatrix} -\frac{179}{4} \\ \frac{853}{4} \\ -\frac{375}{4} \\ -\frac{299}{4} \end{pmatrix}$$

The largest number, 213 in first the column, indicates Liquid control option received the most votes as respondents' first choice of control option. The largest number in the second column, 213, shows that Bait received the most votes as the second most favorable control option. The Liquid plus Bait treatment option is the third choice and 337 (what is 337) is in fourth choice. This means that respondent homeowners want to control FST using some form of control measure.

The result of the second order analysis is shown in Table 7. This is second order unordered effects. The second order unordered effect decomposes in invariant subspace with 1 dimensional subspace M0, 3 dimensional first order effect space M1, which indicates the number of items an option chosen in the given preference Finally, 2 dimensional second order unordered effect space M2 gives pairs of control options, which is purely a second order effect. In particular, the decomposition for the pair of No Control and Liquid Control is illustrated below. The vector $f^{(2,2)}$, the number homeowners who favor No control option and Liquid control option as first or second preference, uniquely can be written as sum of $f_0^{(2,2)}, f_1^{(2,2)}, f_2^{(2,2)}$, and they are orthogonal to each other.

$$||f^{(2,2)}||^2 = ||f_0^{(2,2)}||^2 + ||f_1^{(2,2)}||^2 + ||f_2^{(2,2)}||^2$$

$$188301 \quad 93001.5 \quad 66678.5 \quad 28621$$

$$\begin{pmatrix} 180 \\ 12 \\ 10 \\ 370 \\ 46 \\ 129 \end{pmatrix} = \begin{pmatrix} 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \\ 124.5 \end{pmatrix} + \begin{pmatrix} 25.5 \\ -17 \\ -180 \\ 180 \\ 17 \\ 25.5 \end{pmatrix} + \begin{pmatrix} 30 \\ -95.5 \\ 65.5 \\ 65.5 \\ -95.5 \\ 30 \end{pmatrix}$$

$$f^{(2,2)} = f_0^{(2,2)} + f_1^{(2,2)} + f_2^{(2,2)}$$

Details of all pair choices are shown in Table 7. Each pair can be chosen as 6 easily interpreted functions. Geometrically, the function projects to 36 points in a 4 dimensional space. This means there are only four independent values in the table consisting 36 values. It is easy to interpret second order unordered effects when there are more choices (greater than four, see Diacons,1989). Since we have only four dimensional second order decomposition this gives some equal values as shown in Table 7. The largest value 141 indicates that there is huge effect between corresponding method and rank preference. For example, there is a great effect between methods No Control and Bait in ranking (1,3) and (2,4). For pairs of methods like Liquid and Liquid+ Bait, there is an opposite effect: Every homeowner likes both or hates both, because the row entry begins and ends (-,-) with the same value.

Partial Ranking

The subspace decomposition, dimensions, sum of squares and first and second ordered projections are illustrated in Table 8. There are 131 respondents who only ranked q=1 of the 4 control methods. Thus, $f(i)$ the number of homeowners who ranked control option I first. The space of all such functions is denoted by $M^{(4,1)}$. There are two invariant subspaces in the isotopic decomposition with constant function and first order function (summing zero). These are denoted by $S^5, S^{4,1}$.

Table 8 illustrates that the first order coefficient is highest for the Liquid control method. Therefore, homeowners prefer the Liquid method. The value for Liquid + Bait is -27.7, which is considered to be a "hate vote" because it has the highest negative value. This indicates that homeowners do not want both Liquid + Bait control methods applied in FST deterrence. This result coincides with the complete ranking result.

Conclusions

A generalized spectral analysis method is applied to identify the preference of Louisiana homeowners for four FST control option. The first and second order analysis shows that the Liquid control method is the most preferred option to control FST termite. The results are consistent in both partial and complete ranking cases.

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Table 1. Preference of Formosan Subterranean Termites (FST) control option in Louisiana – Complete Ranking

	Ranking(π)				Number
Combinations	First	second	Third	Fourth	
1	1	2	3	4	123
2	1	2	4	3	1
3	1	3	2	4	6
4	1	3	4	2	4
5	1	4	2	3	1
6	1	4	3	2	7
7	2	1	3	4	55
8	2	1	4	3	1
9	2	3	1	4	15
10	2	3	4	1	305
11	2	4	3	1	24
12	2	4	1	3	0
13	3	1	2	4	1
14	3	1	4	2	1
15	3	2	1	4	2
16	3	2	4	1	48
17	3	4	1	2	2
18	3	4	2	1	39
19	4	1	2	3	2
20	4	1	3	2	0
21	4	2	1	3	2
22	4	2	3	1	20
23	4	3	1	2	0
24	4	3	2	1	88
Total					747

Table 2. Preference of Formosan subterranean termites (FST) control options in Louisiana.

q=1		q=2	
Partial Ranking	No. of votes cast on this type	Partial Ranking	No. of Votes cast of this type
1000	42	1200	1
100	67	1020	0
10	17	1002	0
1	5	2100	0
		2010	0
		2001	0
		120	0
		102	0
		210	0
		201	0
		12	0
		21	0

Table 3. Decomposition of the regular representation.

$M^{11111} = S^{4} \oplus 3S^{3,1} \oplus 2\, S^{2,2} \oplus 3S^{2,1,1} \oplus S^{1,1,1,1}$

M	=	V1	⊕	V2	⊕	V3	⊕	V4	⊕	V5
Dim 24		1		9		4		9		1

Table 4. Young Tableaux for subspaces.

	Sub Space		Young Tableaux notation		
Constant	[4]	V1	4 3 2 1		
First Order	[3,1]	V2	4 2 1 3	4 3 1 2	4 3 2 1
Second Order (Unordered)	[2,2]	V3	4 3 2 1	4 2 3 1	
Second Order (Ordered)	[2,1,1]	V4	4 3 2 1	4 1 3 2	4 2 3 1
	[1,1,1,1]	V5	4 3 2 1		

Table 5. First order effects - complete ranking.

	Rank			
Method	1	2	3	4
No Control	-45	-127	-166	337
Liquid	213	9	-50	-173
Bait	-94	231	42	-180
Liquid +Bait	-75	-114	173	15

Table 6. Percentage of respondents ranking preference i in position j.

	Rank			
Method	1	2	3	4
No Control	22.8	7.98	2.8	70.2
Liquid	52.2	26.5	18.3	1.9
Bait	12.6	55.7	30.7	0.9
Liquid +Bait	12.5	9.8	48.2	27

Table 7. Second order, unordered effects.

	Rank					
Method	1,2	1,3	1,4	2,3	2,4	3,4
No Control and Liquid	30	-79	49	49	-79	30
No Control and Bait	-96	141	-45	-45	141	-96
No Control and Liquid+Bait	66	-62	-4	-4	-62	66
Liquid and Bait	66	-62	-4	-4	-62	66
Liquid and Liquid+Bait	-96	141	-45	-45	141	-96
Bait and Liquid+Bait	30	-79	49	49	-79	30

Table 8. Spectral analysis for q=1, n=131.

	$M^{3,1}$	$= s^4$	$\oplus S^{3,1}$
Dim	4	1	3
$SS/4$		1073	570

Option	Projection
No Control	9.25
Liquid	34.25
Bait	-15.75
Liquid +Bait	-27.75

Table 9. Spectral analysis for q=2, n=1.

	$M^{2,1,1}=$	$s^4\oplus$	$2S^{3,1}\oplus$	$s^{2,2}\oplus$	$S^{2,1,1}$
Dim	12	1	6	2	3
$SS/4$					