



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
<http://ageconsearch.umn.edu>
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*



Title:

The Social Discount Rate under Intertemporal Risk Aversion and Ambiguity

Author:

[Traeger, Christian](#)

Publication Date:

12-01-2009

Publication Info:

UC Berkeley, Department of Agricultural and Resource Economics, UCB

Permalink:

<http://www.escholarship.org/uc/item/2w614303>

Citation:

Traeger, Christian. (2009). The Social Discount Rate under Intertemporal Risk Aversion and Ambiguity. UC Berkeley: Department of Agricultural and Resource Economics, UCB. CUDARE Working Paper No. 1092. Retrieved from: <http://www.escholarship.org/uc/item/2w614303>

Keywords:

ambiguity, discounting, expected utility, intertemporal substitutability, intertemporal risk aversion, recursive utility, risk aversion, social discount rate, uncertainty

Local Identifier:

CUDARE Working Paper No. 1092

Abstract:

<p>The social discount rate crucially determines optimal mitigation policies. This paper examines two shortcomings of the recent debate and the models on climate change assessment. First, removing an implicit assumption of (intertemporal) risk neutrality reduces the growth effect in social discounting and significantly amplifies the importance of risk and correlation. Second, debate and models largely overlook the difference in attitude with respect to risk and with respect to non-risk uncertainty. The paper derives the resulting changes of the risk-free and the stochastic social discount rate and points out the importance of even thin tailed uncertainty for climate change evaluation.</p>



The Social Discount Rate under Intertemporal Risk Aversion and Ambiguity*

First version: December 2008 – This version: December 2009

Christian P. Traeger

Department of Agricultural and Resource Economics,
University of California at Berkeley

Abstract: The social discount rate crucially determines optimal mitigation policies. This paper examines two shortcomings of the recent debate and the models on climate change assessment. First, removing an implicit assumption of (intertemporal) risk neutrality reduces the growth effect in social discounting and significantly amplifies the importance of risk and correlation. Second, debate and models largely overlook the difference in attitude with respect to risk and with respect to non-risk uncertainty. The paper derives the resulting changes of the risk-free and the stochastic social discount rate and points out the importance of even thin tailed uncertainty for climate change evaluation.

JEL Codes: D61, D81, D90, H43, Q00, Q54

Keywords: ambiguity, discounting, expected utility, intertemporal substitutability, intertemporal risk aversion, recursive utility, risk aversion, social discount rate, uncertainty

Correspondence:

Department of Agricultural and Resource Economics
207 Giannini Hall #3310
University of California
Berkeley, CA 94720-3310
E-mail: traeger@berkeley.edu

*I am particularly grateful to Larry Karp and to participants of the EAERE 2009 and the EEA 2009 meetings for valuable comments.

1 Introduction

This paper points out how the modeling of uncertainty attitude should be improved in cost benefit analysis and the integrated assessment of climate change and relates the suggested extension to the recent debate on the social discount rate for mitigation and adaptation projects. Current assessment models, if taking uncertainty into account at all, build on the intertemporally additive expected utility standard model. It is well known that this standard model has serious limitations in explaining behavior under risk. In particular, it cannot explain the observed premium decision makers are willing to pay for avoiding risk (equity premium puzzle, riskless rate puzzle). Against this background, it is questionable how reliable current integrated assessment models are in designing optimal investment policies in the face of large climate change related uncertainties. This paper provides a simple analysis of how important risk and more general uncertainty become for the evaluation of climate change and other long-term environmental investment projects when risk and uncertainty attitude are modeled more seriously.

The paper links these risk and uncertainty effects to the social discount rate, which has been identified as the major driving force behind differences in integrated assessments of climate change (Stern 2007, Nordhaus 2007, Weitzman 2007, Weitzman 2009, Dasgupta 2009, Heal 2009). A low social discount rate of 1.4%, as chosen in the Stern (2007) review of climate change, implies quick and strong mitigation recommendations, including a carbon tax for the first commitment period (the upcoming decade) of \$360 as opposed to \$35 per ton C for a more standard social discount rate of 5.5% (Nordhaus 2007). The large difference between the social discount rate suggested by Stern's team and the more common rate proposed by Nordhaus roots mostly in a normative as opposed to a descriptive perspective on social discounting. This paper follows a descriptive approach but shows that a more elaborate model of uncertainty attitude yields values for the social discount rate for mitigation and adaptation projects that are closer to the rate suggested by Stern (2007) than to the one of Nordhaus (2007). From this perspective, the paper reduces some of the conflict of normative versus descriptive decision making simply by improving the descriptive side.¹

¹In that sense the paper is complementary to Dasgupta (2009) arguing that a more coherent argument for intergenerational consumption smoothing would increase the normatively founded social discount rate.

Coming from a similar motivation as this paper, Weitzman (2007, 2009) argues that a low social discount rate can be defended when incorporating uncertainty comprehensively into climate change assessment. Weitzman reaches this conclusion by following a Bayesian approach to modeling structural uncertainty that delivers a fat-tailed posterior over damages (for critical discussions of this approach, see in particular Pindyck (2009), Nordhaus (2009), and Horowitz & Lange (2009)). Instead of following Weitzman's path of augmenting *uncertainty*, I follow the decision theoretic development of models that treat uncertainty *attitude* more comprehensively. In a stylized two-period model I derive analytic solutions for the social discount rate incorporating intertemporal risk aversion and ambiguity aversion. In the standard intertemporally additive expected utility model, risk only has a negligible effect on social discounting unless fat-tailed distributions are introduced. In contrast, intertemporal risk aversion and ambiguity aversion deliver significant corrections of the social discount rate even with thin-tailed distributions. Intertemporal risk aversion also increases the importance of taking into account the correlation between the stochastic payoff of a mitigation or adaptation project and the baseline risk of the economy. In contrast, for a small project, ambiguity aversion only affects ambiguity over the baseline and not over correlation.

In the classical "Ramsey (1928) formulation" the social discount rate comprises pure time preference and a term that expresses the change in marginal consumption appreciation caused by a combination of growth and the propensity to smooth consumption over time. These two terms are the ones most hotly debated over in the past years (Nordhaus 2007, Dasgupta 2009, Heal 2009).² This paper focuses on the effects of risk and uncertainty. The first extension considers the generalized isoelastic model proposed by Weil (1990) and Epstein & Zin (1989) to solve the equity premium puzzle. The model disentangles a decision maker's aversion to risk from her aversion to smooth consumption over time. Traeger (2007*e*) characterized the difference between these two preference characteristics axiomatically, pointing out that it is a measure of risk aversion itself. In contrast to the Arrow Pratt measure of risk aversion, this new measure, which was tagged intertemporal risk aversion, extends in a straightforward manner to multicommodity settings. In this paper, I give a

²Heal (2009) also points out a different adjustment stemming from limited substitutability between goods under unbalanced growth as it can prevail for produced versus environmental goods (Guesnerie 2004, Hoel & Sterner 2007, Traeger 2007*d*, Sterner & Persson 2008) and notes the importance of hard uncertainty addressed here.

simplified axiomatic characterization of intertemporal risk aversion for the setting described above. The axiom carves out the intuition of intertemporal risk aversion and shows why the widely employed standard model implicitly assumes risk neutrality. I use estimates of the generalized isoelastic model by Vissing-Jørgensen & Attanasio (2003) to quantify the contribution. I show that under intertemporal risk aversion, the (negative) risk effects in the social discount rate cancel out the (positive) growth effect already for risks that are several orders of magnitude smaller than in models neglecting intertemporal risk aversion.

The second extension addresses that probability distributions describing future climate change impacts are generally not (uniquely) given. I adopt a recent model of smooth ambiguity by Klibanoff, Marinacci & Mukerji (2005,2009) that distinguishes between risk and more general uncertainty by employing first and second order probabilities. The model conveniently resembles a standard Bayesian approach (also used by Weitzman 2009). However, the key difference is that Klibanoff et al.'s model captures different degrees of aversion for risk and for ambiguous uncertainty. I show that ambiguity aversion and ambiguity over the growth rate create a very similar effect for social discounting as created by intertemporal risk aversion in the context of risk. A conceptual contribution of this paper is merging the concepts of intertemporal risk aversion (or disentangling Arrow Pratt risk aversion from intertemporal substitutability) with the smooth ambiguity model. The only other working paper currently engaged in merging the two approaches is Ju & Miao (2009) in an asset pricing framework. The way I merge the two models helps to develop a better intuition of smooth ambiguity aversion by relating and contrasting it with intertemporal risk aversion.

Apart from Weitzman (2007,2009), the work of Ha-Duong & Treich (2004) and Gierlinger & Gollier (2008) most closely relate to this paper. Ha-Duong & Treich (2004) is a stylized 4 period, two-states-of-the-world numerical integrated assessment model building on the generalized isoelastic preference framework. Ha-Duong & Treich (2004) find that risk aversion and aversion to intertemporal substitution generally have opposite effects on optimal emission control. They conclude that, in consequence, setting risk aversion equal to (the inverse of) intertemporal substitution in the standard models would underestimate the effect of risk aversion. In light of the current paper, Ha-Duong & Treich's (2004) conclusion can be reformulated as stating that intertemporal risk neutrality underestimates the effects of risk aversion. Gierlinger & Gollier (2008) analyze the social discount rate in Klibanoff et al.'s (2005,

2009) smooth ambiguity framework. While the authors derive a special case of my result in Proposition 4, the two papers are largely complementary. Gierlinger & Gollier (2008) focus on the term structure of the risk-free social discount rate in an equilibrium asset pricing context. Moreover, the authors show (in a context without intertemporal risk aversion) how particular probability distributions and ambiguity attitudes (which lie outside of the scope of my analysis) can increase rather than decrease a risk-free social discount rate. In contrast, my model captures intertemporal risk aversion and merges it with Klibanoff et al.'s (2005,2009) ambiguity framework. Moreover, my focus is on the social discount rate of a mitigation or adaptation project with a stochastic payoff that is correlated with the baseline uncertainty of the economy rather than on the risk-free rate describing a certain transfer into the future. Finally, several economists have pointed to the low observed riskless rate of interest making a point in favor of Stern's (2007) choice for the social discount rate. From this perspective on choosing the discount rate, the paper provides a formal model that not only explains a lower observed risk-free interest rate but provides the tools for a more sophisticated extrapolation from observed benchmarks in asset markets to stochastic climate projects by capturing differences in correlation and types of uncertainty.

Section 2 provides an overview over the basic setting of this paper and over the recent debate on the social discount rate. Section 3 introduces the concept of intertemporal risk aversion and calculates the corresponding risk-free social discount rate. Section 4 derives the stochastic social discount rate capturing intertemporal cost benefit pricing for a mitigation or adaptation project with stochastic payoffs correlated to baseline risk. Section 5 incorporates smooth ambiguity aversion and applies it to second order uncertainty over expected growth and over the correlation between project payoff and baseline uncertainty. Section 6 concludes.

2 Discounting the Future

Following the Stern (2007) review of climate change, few economic parameters have been as hotly debated over the last years as the different contributions to the social discount rate. The social discount rate characterizes in a convenient way how the value of consumption develops over time. It turns out that differing assumptions in social discounting explain the major differences between most integrated assessments of climate change and mitigation policies (Plambeck, Hope & Anderson 1997, Nordhaus

2007, Weitzman 2007). The various contributions to the social discount rate have been considered and discussed extensively in the recent debate, among others, by Nordhaus (2007), Weitzman (2007), Weitzman (2009), and Dasgupta (2009). A different adjustment that will not be addressed in this paper stems from limited substitutability between goods under unbalanced growth as it can prevail for produced versus environmental goods (Guesnerie 2004, Hoel & Sterner 2007, Traeger 2007*d*, Sterner & Persson 2008). This section is an introduction to the debate. I start by laying out the basic formal setup. In the remainder of this paper, I will add intertemporal risk aversion, stochastic transfers, and ambiguous uncertainty.

2.1 Setting

The simplest framework to convey the message of this paper is the “certain \times uncertain” setting, two periods with uncertainty only in the future period. I denote first period consumption by $x_1 \in X$, and uncertain second period consumption is represented by a probability measure p over X . Formally, let X be a compact metric space and P be the space of Borel probability measures on X . I will refer to the “standard model” as the modeling framework where a decision maker evaluates utility separately for every period and for every state of the world by a function u and then sums it over states and over time,

$$U^s(x_1, p) = u(x_1) + \beta E_p u(x_2) , \quad (1)$$

where β is the utility discount factor representing pure time preference. Preferences are assumed to be isoelastic. This choice is based on three reasons. First, a constant intertemporal elasticity of substitution in the standard model is a ubiquitous assumption in the social discounting debate. Second, isoelastic preferences in combination with normally distributed uncertainty make the model analytically tractable. Third, for the case of intertemporal risk aversion, isoelasticity will allow me to use estimates of intertemporal risk aversion based on the generalized isoelastic model by Vissing-Jørgensen & Attanasio (2003). I analyze a model with one aggregate commodity so that utility within a period from certain consumption is described by $u(x) = \frac{x^\rho}{\rho}$ with $\rho \leq 1, \rho \neq 0$.

The decision maker faces a stochastic growth scenario. Given some x_1 , the model assumes that the consumption growth rate $g = \ln \frac{x_2}{x_1}$ is normally distributed with $g \sim N(\mu, \sigma^2)$. The *risk-free social discount rate* is $r = \ln \frac{dx_2}{dx_1} |_{\bar{U}}$ characterizing a

marginal certain trade-off between the future (dx_2) and the present (dx_1) that leaves overall welfare unchanged. The pure rate of time preference is $\delta = -\ln \beta$, and the consumption elasticity of marginal utility is $\eta = 1 - \rho$.

2.2 Social Discounting in the Standard Model

The equation characterizing the social discount rate in a standard setting underlying most of the debate on climate change and discounting is

$$r = \delta + \eta\mu - \eta^2 \frac{\sigma^2}{2} . \quad (2)$$

It is an extension of the classical Ramsey (1928) formula by making growth stochastic. The right-hand side of equation (2) characterizes the individual components of the certainty equivalent social discount rate.³ The first term is the rate of pure time preference δ , which captures impatience and is also known as the utility discount rate. The second term expresses devaluation of future consumption caused by the combination of growth and decreasing marginal utility. The parameter η characterizes the percentage decrease in marginal utility from a percentage increase of consumption. It captures aversion to fluctuations over time and, in the standard model, also with respect to risk. Together with the expected growth rate μ , the term $\eta\mu$ characterizes the decrease of marginal utility from future consumption because of growth. Most of the debate concentrates on the first two terms because the risk term generally turns out to be negligible. It characterizes the effect of Arrow-Pratt risk aversion on the certainty equivalent discount rate. The parameter σ captures the standard deviation of the growth rate. For the annual discount rate, the parameters δ , μ , and σ are in the order of percent, while η is in the unit order. Therefore, σ^2 easily makes the third term 10 – 100 times smaller than the other two and risk can be neglected in social discounting. Be aware that σ characterizes risk in the sense of volatility. The frequently met usage of the term risk in the climate change debate, incorporating a reduced expected value as a consequence of possible catastrophic events, would partly be captured by the second term of the social discount rate.

³In a complete market without distortions the social discount rate equals the real rate of interest (Ramsey equation). However, incomplete markets, distortions, and long time horizons generally imply that the social discount rate cannot be observed easily on the market. Moreover, individuals can have differing valuation in their political role, e.g. valuing future generations' welfare, from the preferences observed on a market where they optimize individual utility. For a closer discussion see Hepburn (2006).

The parameter choices of Stern (2007) can be approximated by $\delta = 0.1\%$, $\eta = 1$, and $\mu = 1.3\%$ delivering $r = 1.4\%$ under certainty. While Stern's team clearly argues for a normative dimension of these choices, the majority of integrated assessment modelers refuses such a standpoint.⁴ For this second group, Nordhaus, creator of the widespread open-source integrated assessment model DICE is somewhat representative, adhering to a strictly positive perspective. His parameter choices in the recent version of DICE-2007 (Nordhaus 2008) are $\delta = 1.5\%$, $\eta = 2$, and $\mu = 2\%$ ⁵ delivering $r = 5.5\%$ (again under certainty). In the introduction, I pointed out the factor 10 difference in the optimal carbon tax implied under Stern's and under Nordhaus's discounting assumptions. Introducing uncertainty with a standard deviation of $\sigma = 2\%$ results in an adjustment of the risk-free rate by 0.02% in the case of Stern and 0.08% in the case of Nordhaus, both negligible. A standard deviation of $\sigma = 2\%$ is used by Weitzman (2009) to approximate the volatility of economic growth without climate change and possible catastrophic risks. The next section continues the discussion of the effect of uncertainty in the face of climate change.

3 Intertemporal Risk Aversion

The intertemporally additive expected utility framework (standard model) used in the discussion above implicitly assumes that a decision maker's aversion to risk coincides with his aversion to intertemporal variation. Epstein & Zin (1989) and Weil (1990) derive an alternative setting in which these two a priori quite different characteristics of preference can be disentangled. This section motivates their setting from a slightly different perspective by introducing the concept of intertemporal risk aversion developed in Traeger (2007*e*). In particular, I point out that the standard model contains an implicit assumption of risk neutrality driving the discounting results.

3.1 Representing Welfare

The curvature of the utility function u in equation (1) captures both aversion to risk and aversion to intertemporal variation. A priori, however, risk aversion and

⁴Moreover, Dasgupta (2008) points out that, from a normative perspective, an egalitarian choice of $\delta = 0.1\%$ should also call for a higher propensity of intergenerational consumption smoothing $\eta > 1$.

⁵The growth rate is endogenous in the DICE model and has been reconstructed from Nordhaus (2007, 694).

the propensity to smooth consumption over time are two distinct concepts. Welfare can also be characterized starting out with two independent function for these two distinct preference characteristics resulting in the form

$$U(x_1, p) = u(x_1) + \beta f^{-1} [E_p f \circ u(x_2)] . \quad (3)$$

Opposed to a widespread believe, equation (3) – not equation (1) – is also the general representation of preferences satisfying the von Neumann & Morgenstern (1944) axioms, additive separability over time, time consistency, and (finite time) stationarity (Traeger 2007c).⁶ Here, the concavity of u captures the aversion to intertemporal consumption variation and the curvature of f describes intertemporal risk aversion. The representation gives an interpretation of intertemporal risk aversion as aversion with respect to utility gains and losses, whereby utility is a measure for liking a particular state of the world that derives from the willingness to trade certain states of the world (or consumption bundles) over time. Note that the curvature of f is a one-dimensional risk measure even in a multi-commodity world.⁷

3.2 Characterizing Intertemporal Risk Aversion

The interpretation of intertemporal risk aversion as aversion to utility gains and losses depends on the preference representation (3). It is insightful to give a representation-independent and preference-based definition. The general characterization for recursive multiperiod settings with stationary and non-stationary preferences is found in Traeger (2007c) and Traeger (2007e). However, these definitions require at least two uncertain periods – a context in which the recursive nature of the representation makes it slightly more complicated than needed for the current analysis. Here, I give a simplified version for the stationary certain \times uncertain setting, which, however, requires the absence of pure time preference.⁸ Let \succeq characterize preferences on

⁶Note that, in general, preferences represented by equation (3) cannot be represented by an evaluation function of the form $U^s(x_1, p) = u_1(x_1) + E_p u_2(x_2)$.

⁷See Kihlstrom & Mirman (1974) for the complications that arise when trying to extend the Arrow Pratt risk measures to a multi-commodity setting. Even more interestingly, measures of intertemporal risk aversion can be applied straightforwardly to contexts frequently met in environmental economics where impacts (e.g. on an ecosystem) do not have a natural cardinal scale.

⁸I abandon pure time preference for the sake of simplicity in the characterization only. The interested reader can verify that this step does not change the intuition of the axiom given in its general form in the two cited papers above. Obviously, I keep pure time preference when discussing discount rates.

$X \times P$ that are representable by equation (3) with $\beta = 1$. A decision maker is called (weakly)⁹ intertemporal risk averse, if and only if, for all $x^*, x_1, x_2 \in X$

$$(x^*, x^*) \sim (x_1, x_2) \quad \Rightarrow \quad (x^*, x^*) \succeq (x^*, (\tfrac{1}{2}, x_1; \tfrac{1}{2}, x_2)) , \quad (4)$$

where the term $(\frac{1}{2}, x_1; \frac{1}{2}, x_2)$ characterizes a fair coin flip returning either x_1 or x_2 . The premise in equation (4) states that a decision maker is indifferent between a certain constant consumption path delivering the same outcome x^* in both periods and another certain consumption path that delivers outcome x_1 in the first and outcome x_2 in the second period. For example, x_1 can be an inferior outcome with respect to x^* . Then, x_2 is a superior outcome with respect to x^* . On the right-hand side of equation (4), the decision maker receives x^* in the first period, independent of his choice. For the second period, he has a choice between the certain outcome x^* or a lottery that returns with equal probability either the superior or the inferior outcome. The decision maker is called (weakly) intertemporal risk averse if he prefers the certain outcome x^* in the second period over the lottery.¹⁰ A decision maker is defined as (weakly) intertemporal risk loving if the preference relation \succeq in equation (4) is replaced by \preceq . He is defined to be risk neutral if he is both intertemporal risk loving and intertemporal risk averse (relation \succeq in equation 4 is replaced by \sim). The following proposition shows that intertemporal risk aversion is reflected in the concavity of the function f in representation (3).¹¹

Proposition 1: Let preferences over $X \times P$ be represented by equation (3) with a continuous function $u : X \rightarrow \mathbb{R}$ and a strictly increasing and continuous function $f : U \rightarrow \mathbb{R}$, where $U = u(X)$ and $\beta = 1$.

a) The corresponding decision maker is (weakly) intertemporal risk averse [lov-

⁹The strong notion would involve the additional requirement $(x^*, x_1) \not\sim (x^*, x_2)$ in the premise and a strict preference in the implication.

¹⁰Let me point out that the lottery on the right-hand side of equation (4) will either make the decision maker better off or worse off than (x^*, x^*) , while, on the left-hand side, the decision maker knows that if he picks an inferior outcome for some period he certainly receives the superior outcome in the other.

Calling preferences satisfying equation (4) intertemporal risk averse is motivated by the facts that, first, the definition intrinsically builds on intertemporal trade-offs and, second, Normandin & St-Amour (1998, 268) make the point that the conventional Arrow Pratt measure of risk aversion is an atemporal concept.

¹¹Recasting the proposition for a strictly decreasing continuous function $f : U \rightarrow \mathbb{R}$ turns concavity in statement a) into convexity [and convexity into concavity]. Replacing the definition of intertemporal risk aversion by its strict version given in footnote 9 switches concavity to strict concavity in the statement.

ing], if and only if, the function f is concave [convex].

b) The corresponding decision maker is intertemporal risk neutral, if and only if, there exist $a, b \in \mathbb{R}$ such that $f(z) = az + b$. An intertemporal risk neutral decision maker maximizes intertemporally additive expected utility (equation 1).

A measure of relative intertemporal risk aversion is defined analogously to the Arrow-Pratt measure:

$$\text{RIRA}(z) = -\frac{f''(z)}{f'(z)} |z| .$$

The measure $\text{RIRA}(z)$ depends on the choice of zero in the definition of the utility function u . This normalization-dependence is the analog to e.g. the wealth level dependence of the Arrow Pratt measure of relative risk aversion.¹² Note that positivity of RIRA indicates intertemporal risk aversion independently of whether f is increasing and concave or decreasing and convex (see footnote 11).¹³ In the isoelastic setting where $u(x) = \frac{x^\rho}{\rho}$ the parameter $\eta = -\frac{u''}{u'}x$ is constant and, now, only characterizes the propensity to smooth consumption over time. Furthermore, isoelasticity implies a functional form

$$f(z) = (\rho z)^{\frac{\alpha}{\rho}} \tag{5}$$

where the parameters are chosen to reproduce Epstein & Zin's (1989, 1990) and Weil's (1990) generalized isoelastic model. Here relative Arrow Pratt risk aversion corresponds to $\text{RRA} = 1 - \alpha$ and intertemporal risk aversion is

$$\text{RIRA} = \begin{cases} 1 - \frac{\alpha}{\rho} & \text{if } \rho > 0 \\ \frac{\alpha}{\rho} - 1 & \text{if } \rho < 0 . \end{cases}$$

Then, equation (3) turns into

$$U(x_1, p) = \frac{x_1^\rho}{\rho} + \beta \frac{1}{\rho} [\mathbb{E}_p x_2^\alpha]^{\frac{\rho}{\alpha}} . \tag{6}$$

¹²I.e. in the standard model, the Arrow Pratt measure of relative risk aversion depends on what is considered the $x = 0$ level. For example, whether or not breathing fresh air is part of consumption or whether human capital is part of wealth changes the Arrow Pratt coefficient.

¹³In both cases $-\frac{f''}{f'}$ is positive. Moreover, measuring utility in negative units as in the isoelastic case for $\rho < 0$ makes z negative. Therefore, the definition of relative risk aversion has to employ the absolute of the variable z (Traeger 2007b).

Equation (6) is the ‘certain \times uncertain’ version of Epstein & Zin’s (1989, 1991) infinite horizon recursive utility model.¹⁴ Traeger (2007e) further elaborates that f (equivalently, RIRA) can be interpreted as a measure for the difference between Arrow Pratt risk aversion and the willingness to smooth over time.

This isoelastic special case of equation (3) is the setting used in the literature to disentangle risk attitude from the propensity to smooth consumption over time. A recent estimate of the corresponding preference parameters is Vissing-Jørgensen & Attanasio (2003), who build on Campbell’s (1996) approach of log-linearizing the Euler equations. For the risk aversion parameter, the authors propose $\text{RRA} \in [5, 10]$ for what they consider realistic assumptions.¹⁵ Vissing-Jørgensen & Attanasio (2003) single out the pair $\eta = \frac{2}{3}$ and $\text{RRA} = 9.5$ as a best guess in its ability to match the observed riskless rate and the equity premium. In an overview of different estimates of the above preference parameters (not all based on the simultaneous approach taken in the generalized isoelastic model) Giuliano & Turnovsky (2003) suggest $\eta > 1$ and $\text{RRA} > 2$. All of these papers reject the standard model with its underlying assumption that $\alpha = \rho \Leftrightarrow \text{RRA} = \eta$.¹⁶ The precise estimation of these preference characteristics remains a challenge for econometric analysis, and might also have to be extended beyond the isoelastic special case. For the present discussion, I will use Vissing-Jørgensen & Attanasio’s (2003) best guess of $\eta = \frac{2}{3}$ and $\text{RRA} = 9.5$. It implies a coefficient of relative intertemporal risk aversion $\text{RIRA} = 26.5$ (and furthermore $\eta^2 = \frac{4}{9}$ and $|1 - \eta^2| = \frac{5}{9}$). Subsequently, I will employ two parameter sets based on Giuliano & Turnovsky’s (2003) survey for a “sensitivity check”.

¹⁴In a multiperiod framework equation (6) translates into the recursion

$$U(x_{t-1}, p_t) = \frac{x_{t-1}^\rho}{\rho} + \beta \frac{1}{\rho} \left[\mathbb{E}_{p_t} (\rho U(x_t, p_{t+1}))^\alpha \right]^\frac{\rho}{\alpha}, \quad (\star)$$

To obtain the normalization used by Epstein & Zin (1989, 1991), multiply equation (\star) by $(1 - \beta)\rho$ and take both sides to the power of $\frac{1}{\rho}$. Define $U^*(x_{t-1}, p_t) = ((1 - \beta)\rho U(x_{t-1}, p_t))^\frac{1}{\rho}$. Expressing the resulting transformation of equation (\star) in terms of U^* delivers their version

$$U^*(x_{t-1}, p_t) = \left((1 - \beta)x_{t-1}^\rho + \beta [\mathbb{E}_{p_t} (U^*(x_t, p_{t+1}))^\alpha]^\frac{\rho}{\alpha} \right)^\frac{1}{\rho}.$$

¹⁵The authors have to make assumptions about the covariance of consumption growth and stock returns, the share of stocks in the financial wealth portfolio, the properties of the expected returns to human capital, and the share of human capital in overall wealth.

¹⁶The only empirical analysis of the generalized isoelastic model I am aware of that does not reject the intertemporally additive expected utility special case is Normandin & St-Amour (1998).

3.3 The Social Discount Rate Under Intertemporal Risk Aversion

Enriching the standard framework by intertemporal risk aversion yields the following modification of the social discount rate.

Proposition 2: The certainty equivalent social discount rate in the isoelastic setting with intertemporal risk aversion is

$$r = \delta + \eta\mu - \eta^2 \frac{\sigma^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma^2}{2}. \quad (7)$$

For a decision maker with positive intertemporal risk aversion $\text{RIRA} > 0$, the present value of an additional consumption unit in a risky future is higher. In consequence, an intertemporal risk averse decision maker is willing to invest in certain projects with a relatively lower productivity than a decision maker who bases his decision on the standard model.

In equation (7) the parameter η only reflects aversion to intertemporal fluctuations. Therefore, the term $\eta^2 \frac{\sigma^2}{2}$ should be interpreted as the cost of expected fluctuations triggered by the aversion to non-smooth intertemporal consumption paths. I will still refer to the expression as “the standard risk term”, as it is the only expression capturing risk in an analysis based on the standard model (corresponding to $\text{RIRA} = 0$ in equation 7). For my parametric best guess based on Vissing-Jørgensen & Attanasio (2003), the importance of the intertemporal risk aversion term in the social discount rate in relation to the standard risk term is represented by the ratio

$$\frac{\text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma^2}{2}}{\eta^2 \frac{\sigma^2}{2}} \approx 33.$$

A factor of 33 easily brings the importance of risk back into the social discount rate. Note that, because of the slightly lower $\eta = \frac{2}{3}$, the standard risk term in (7) is even lower than in the examples discussed in section 2.2. However, the effect of intertemporal risk aversion is significantly larger. For a numerical example of the terms in equation (7) I take again an expected growth rate of $\mu = 2\%$ and a standard deviation of $\sigma = 2\%$ as in section 2.2. This results in a growth effect of $\eta\mu = 1.3\%$, a standard risk effect of 0.01% , and an intertemporal risk aversion effect of 0.3% . For example,

with a pure rate of time preference of $\rho = 1.5\%$, the risk-free social discount rate becomes $r = 2.5\%$ instead of $r = 2.8\%$ without intertemporal risk aversion and instead of $r = 5.5\%$ proposed by Nordhaus (2007). Obviously, there are two effects reducing the discount rate. First, there is a direct effect of intertemporal risk aversion reducing the discount rate by .3%. However, the larger reduction from 5.5% to 2.8% is due to an indirect effect. By allowing for intertemporal risk aversion, observed risk attitude is no longer attributed to the intertemporal consumption smoothing coefficient η . Therefore, the disentanglement of risk attitude and intertemporal substitutability in the generalized isoelastic model reduces the η estimate by Vissing-Jørgensen & Attanasio (2003) and, thus, the growth effect in the social discount rate. In an entangled approach to estimating observed preferences, part of what is risk aversion is falsely attributed to the aversion to intertemporal fluctuations η , overestimating the growth effect and the social discount rate.

In the face of climate change baseline risk is likely to increase. If I increase the standard deviation of the yearly growth rate to $\sigma = 4\%$, the risk effects become 0.04% (standard risk term) and 1.2% (intertemporal risk aversion term). Because of the nonlinearity in the risk terms, the risk effect almost cancels out the growth effect. While $\sigma = 4\%$ might be high for a period of one year, with time horizons typical to climate change mitigation projects the importance of risk increases significantly.¹⁷ Let me extend the distance between present and future to 50 years. In this 50 year scenario, I ask the question how much risk is needed in terms of the standard deviation of a thin-tailed normal distribution so that risk effects crowd out the growth effect in the social discount rate. Keeping the expected growth rate at 2% per year, I ask the question when $r_{50} = 50\delta$ implying $\eta 50\mu \stackrel{!}{=} \eta^2 \frac{\sigma^2}{2} + \text{RIRA } |1 - \eta^2| \frac{\sigma^2}{2}$. Solving the latter equation results in

$$\sigma = \left(\frac{1}{2} \left(\eta + \frac{|1 - \eta^2|}{\eta} \text{RIRA} \right) \right)^{-\frac{1}{2}}. \quad (8)$$

Equation (8) yields $\sigma = 0.3$ for the parameter values based on Vissing-Jørgensen & Attanasio's (2003) best guess. This standard deviation implies a probability of $p^* = 0.0004$ that climate change (or anything else) causes society to be worse off in 50 years than it is today. That is a rather small probability for the thin left tail of a

¹⁷This point relies on the assumption that climate change based risk is not an independent event in every year but highly correlated over time. This assumption seems reasonable enough to interpret the results below as a first order analysis. However, it does not replace a detailed multi-period analysis for differing stochastic processes and, in particular, an analysis including learning.

Table 1 summarizes the numerical discount rates for the different settings.

	η	RRA	RIRA	$\sigma = 2\%$						$\sigma = 4\%$					
				grow	sra	ira	tra	sdr	sdr*	grow	sra	ira	tra	sdr	sdr*
N	2	2	0	4	0.1	0	0.1	5.5	5.4	4	0.3	0	0.3	5.2	5
VA	$\frac{2}{3}$	9.5	27	1.3	0	0.3	0.3	2.5	2.3	1.3	0	1.2	1.2	1.6	0.9
S1	2	9.5	7.5	4	0.1	0.5	0.5	5	4.8	4	0.3	1.8	2.1	3.4	3
S2	2	5	3	4	0.1	0.2	0.3	5.2	5.1	4	0.3	0.7	1	4.5	4.3

Notes: Growth rate is 2% per year. Parameters are η =consumption elasticity of marginal utility; RRA=relative Arrow Pratt risk aversion; RIRA=relative intertemporal risk aversion; σ =standard deviation of yearly growth rate. Discounting effects are measured in percent and abbreviated grow= $\eta\mu$ =growth effect; sra= $\eta^2\sigma^2/2$ =standard risk aversion effect; **ira**=RIRA $|1 - \eta^2| \sigma^2/2$ =intertemporal risk aversion effect; tra=sra+ira=total risk aversion effect; **sdr**=1.5%+grow-sra-ira=social discount rate, where 1.5% is the pure rate of time preference; sdr*=sdr reduced by the correlation effect for $\kappa = -.5$ discussed in the next section. Numbers are rounded to two significant digits and to one decimal digit. The settings are ‘N’ based on Nordhaus; ‘VA’ based on Vissing-Jørgensen & Attanasio’s parameter estimates; sensitivity scenarios ‘S1’ and ‘S2’.

distribution capturing the current standard of living and below. However, it suffices to cancel out the growth effect in the social discount rate. This probability contrasts sharply with a probability of $p^* = .16$ that would be needed in the standard model with Nordhaus’s specifications for risk effects canceling out the growth effect.

While Vissing-Jørgensen & Attanasio’s (2003) estimate corresponds to the case where $\eta < 1$, a literature survey by Giuliano & Turnovsky (2003) proposes $\eta > 1$ along with values RRA > 2 . Building on Giuliano & Turnovsky (2003), let me exchange Vissing-Jørgensen & Attanasio’s (2003) value of $\eta = \frac{2}{3}$ by Nordhaus’s (2008) value of $\eta = 2$. This choice eliminates the indirect effect of intertemporal risk aversion but increases the multiplier $|1 - \eta^2|$ in the direct effect. In the first sensitivity check (S1), I keep RRA = 9.5 – a specification also in the range suggested by Giuliano & Turnovsky (2003). This parameter constellation reduces the coefficient of relative intertemporal risk aversion to RIRA = 7.5. In the one-year scenario with $\mu = \sigma = 2\%$, the growth effect grows back to Nordhaus’s 4% and standard and intertemporal risk aversion together cut it back by .5%. Increasing the variance to $\sigma = 4\%$ increases the negative risk effect to 2.1% – roughly half of the growth effect. In the 50 year scenario, a standard deviation of $\sigma = .39$ is needed for completely crowding out the growth effect. This standard deviation implies a probability of being worse off in 50 years than we are today of approximately $p^* = 0.5\%$.

Stacking the deck further against the effects of intertemporal risk aversion, I reduce the Arrow Pratt coefficient to RRA = 5 in a second sensitivity check (S2), implying

a further reduction of intertemporal risk aversion to $RIRA = 3$. In the one year scenario with $\mu = \sigma = 2\%$, the risk terms cut back less than .3% of the 4% growth term. In the one-year $\sigma = 4\%$ scenario the risk terms cut back on the social discount rate by a little more than 1% – about a quarter of the growth effect.¹⁸ The precise estimation of the parameters η and RRA, respectively RIRA, remains a challenge, but the values calculated here can be taken as a clear indicator that we miss an important contribution to the social discount rate by neglecting intertemporal risk aversion in the intertemporally additive expected utility standard model.

4 Stochastic Projects

The previous section derived an expression for the risk-free social discount rate under intertemporal risk aversion. However, climate change projects are themselves stochastic – not only in their physical payoff but, in particular, with respect to the value of their mitigation or adaptation services. Once stochasticity of the project is introduced, its correlation to the baseline risk becomes crucial for its value. Weitzman (2007) points out that the standard approach in cost benefit analysis based on Lind (1982) is to assume full correlation of a project with the economic baseline risk. He continues to argue that there is little reason to assume that the payoff of climate change related projects would be correlated with the baseline of the economy. The major areas impacted by climate change would be “ ‘outdoor’ aspects (broadly defined) like agriculture, coastal recreational areas, and natural landscapes” which are little correlated to technological progress. Moreover, some of these impacts directly affect utility rather than production. Various economists used this or related arguments to promote the use of the risk free rate of return as a proxy for the social discount rate for climate change projects. Indeed, this section will show that for a marginal investment into a stochastic project the uncorrelated discount rate is the same as the risk-free rate.

However, climate change induces risks of sea level rise, floods, droughts, spreading diseases, biodiversity loss, and other sometimes irreversible impacts. Thereby, climate change related uncertainties become a significant part of societies baseline risk affecting non-produced output and direct arguments of the welfare function (here

¹⁸Here, the standard risk effect with .18 is of similar order as the intertemporal risk aversion effect with .41 – a consequence of the relatively high aversion to intertemporal substitution and the low coefficient of intertemporal risk aversion.

all summarized as ‘consumption’). But mitigation and adaptation projects, such as building a dam or developing drought resistant crops, pay out most in the states of the world where climate change turns out to be most serious. Thus, these projects are negatively correlated with at least part of the baseline risk. In this section I derive the social discount rate for projects that are stochastic and show how correlation affects the rate.

Formalizing a stochastic social discount rate, the decision maker no longer trades a marginal unit dx_1 of her current certain consumption x_1 against a certain marginal unit dx_2 of her overall uncertain future consumption x_2 . Instead, she now trades a marginal current unit dx_1 against a marginal part $d\epsilon$ of a stochastic project y with expected unit payoff, i.e. $Ey = 1$. In general, the project y will be correlated with baseline uncertainty. The intertemporal trade off that leaves overall welfare constant is

$$0 = \frac{d}{dx_1} u(x_1) dx_1 + \beta \frac{d}{d\epsilon} f^{-1} [E_{p(x_2, y)} f \circ u(x_2 + \epsilon y)] \Big|_{\epsilon=0} d\epsilon. \quad (9)$$

This welfare indifferent trade-off allows me to define the stochastic social discount rate by $r = \ln \frac{d\epsilon}{-dx_1}$. I briefly comment on this extension of the standard derivation of a risk-free social discount rate. First, in the standard derivation of a risk-free social discount rate, the stochastic project y in equation (9) is replaced by its expected value 1. Then, a reduction dx_1 in current consumption is compensated by an increase in future consumption from x_2 to $x_2 + \epsilon$ evaluated for an infinitesimal $d\epsilon$ at the point $\epsilon = 0$.¹⁹ Second, marginality in the trade-off defining the social discount rate plays the same role as it does in any other economic price concept. The analytic formula for the social discount rate to be derived characterizes (in rates) the present value willingness to pay for a marginal unit of an adaptation or mitigation project with expected unit payoffs in the future. As an additional input it takes into consideration correlation to baseline risk. Third, I formalize a trade-off between a marginal current unit and the first marginal part of a finite stochastic unit project y . Modeling an infinitesimal share of a non-marginal unit project rather than a marginal project itself is important. It is well known that risk effects are second order effects. Therefore, stochasticity effects of an infinitesimal project would vanish. Fourth, observe that the derivation does not

¹⁹In this case the formula above reduces to a more precise notation of what is commonly written as $\frac{d}{dx_2} \dots E_p \dots u(x_2)$ – the difference being that the above notation makes explicit that (for $y = 1$) the decision maker trades a certain unit (ϵ or dx_2) while having an uncertain baseline x_2 . Observe that also the first period derivative in equation (9) can be rewritten as $\frac{d}{d\epsilon_1} u(x_1 + \epsilon_1 y_1) \Big|_{\epsilon_1=0, y_1=1} d\epsilon_1$.

rely on an optimal allocation of an adaptation-mitigation-portfolio – an assumption that I would consider inadequate in the present context.

I assume that $\ln y$ and the growth rate g are jointly normally distributed with expected values μ_y and μ_g , standard deviations σ_y and σ_g , and correlation κ .²⁰

Proposition 3: The stochastic social discount rate in the isoelastic setting with intertemporal risk aversion is

$$r = \delta + \eta\mu_g - \eta^2 \frac{\sigma_g^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma_g^2}{2} + \eta\kappa \sigma_g \sigma_y + |1 - \eta| \text{RIRA} \kappa \sigma_g \sigma_y. \quad (10)$$

The second line distinguishes the stochastic social discount rate from its risk-free relative of the previous section. In the case of certainty about the project this second line vanishes ($\sigma_y = 0$). The same is true if the risk of the project and the baseline scenario are uncorrelated ($\kappa = 0$). Note that the social discount rate models the value of the first marginal trade of current consumption against a stochastic project that adds to the baseline.²¹ Therefore, risk aversion with respect to the marginal project itself is a second order effect that does not find its way into the social discount rate. Stochasticity of the small project only contributes through its interaction with baseline uncertainty. The second term in the second line of equation (10) is what distinguishes the correlation contribution in a model including intertemporal risk aversion from the correlation contribution in the standard model. In order to assess the magnitude of the correlation contributions I use the same scenarios as before. For simplicity, I set the standard deviation $\sigma_y = \sigma_g$. Of course, $\sigma_y \gg \sigma_g$ is not unreasonable and would further decrease the discount rate for a project that is negatively correlated with the overall risk of the economy.

In the Nordhaus setting the standard multiplier of the correlation coefficient takes on the value $\eta\sigma_g\sigma_y = .1\%$ (.3%) for $\sigma_g = \sigma_y = 2\%$ (4%). Admitting intertemporal

²⁰Note that μ_y and σ_y are the moments of $\ln y$ rather than y . However, the condition $Ey = 1$ implies $\mu_y = -\frac{\sigma_y^2}{2}$. Making use of this constraint it is $\text{Var}(y) = e^{\sigma_y^2} \approx \sigma_y^2 + \frac{\sigma_y^4}{2}$ so that, in the percentage range, σ_y approximates well the standard deviation of the project y itself. I will also refer to κ as the correlation between the project and the baseline even though, more precisely, it is the correlation between $\ln y$ and the growth rate $g = \ln \frac{x_2}{x_1}$.

²¹Marginality here plays the same role as it plays in any other economic price concept. The derived analytic formula for the social discount rate characterizes (in rates) the present value willingness to pay for a marginal unit of an adaptation and mitigation project. As an additional input it takes into consideration correlation to baseline risk. A non-marginal increase in the adoption of mitigation and adaptation projects changes the baseline and, thus, the valuation of the last unit of the project and of further projects.

risk aversion and employing once more Vissing-Jørgensen & Attanasio's (2003) best guess reduces the standard multiplier of the correlation coefficient to $\eta\sigma_g\sigma_y = .0\%$ (.1%) but adds an intertemporal risk aversion multiplier of $|1 - \eta| \text{ RIRA } \sigma_g\sigma_y = .4\%$ (1.4%) for $\sigma_g = \sigma_y = 2\%$ (4%). Thus, in the case of intertemporal risk aversion and a (negative) correlation of the mitigation or adaptation project of $\kappa = -.5$ with baseline uncertainty would further reduce the social discount rate to 2.3% (0.9%) in the $\sigma_g = \sigma_y = 2\%$ (4%) scenario (with a pure time preference of $\delta = 1.5\%$). The reduction increases linearly in $-\kappa$. A brief look at the same sensitivity scenarios examined in the previous section gives the following picture. Increasing η to the value used by Nordhaus of 2%, and keeping $\text{RRA} = 9.5$, yields an overall correlation multiplier of .4% (.8%) for $\sigma_g = \sigma_y = 2\%$ (4%) with intertemporal risk aversion contribution four fifth of the effect. Further decreasing intertemporal risk aversion by reducing $\text{RRA} = 5$ yields an overall correlation multiplier of .2% (.4%) for $\sigma_g = \sigma_y = 2\%$ (4%) with intertemporal risk aversion contributing two thirds (see also sdr^* in Table 3.3).

The 50 year scenario is more interesting in the climate policy context. Once more, I ask the question for which standard deviation $\sigma = \sigma_g = \sigma_y$ the risk effects cancel out the growth effect. Formally, this requirement $r_{50} = 50\delta$ implies $\eta 50\mu \stackrel{!}{=} \eta^2 \frac{\sigma^2}{2} + \text{RIRA } |1 - \eta^2| \frac{\sigma^2}{2} - \eta\kappa \sigma_g\sigma_y - |1 - \eta| \text{ RIRA } \kappa \sigma_g\sigma_y$. Solving the latter equation for $\mu = 2\%$ results in

$$\sigma = \left(\frac{1}{2} \left(\eta + \frac{|1 - \eta^2|}{\eta} \text{RIRA} \right) - \kappa \left(1 + \frac{|1 - \eta|}{\eta} \text{RIRA} \right) \right)^{-\frac{1}{2}}.$$

Table 4 summarizes the numerical results for the different scenarios and for differing degrees of correlation. Note that $\kappa = 0$ makes the setting equivalent to that of the preceding section with a risk-free social discount rate. Moreover, the table translates the standard deviations into the probabilities p^* that society will be worse off in 50 years than it is today. With a correlation coefficient $\kappa = -.5$ a probability of $p^* = 0.000009$ is sufficient to make the risk terms cancel the growth effect, yielding a social discount rate that is equivalent to pure time preference. This is true in a setting with thin tails and contrasts sharply with a probability of $p^* = .11$ that is needed in the Nordhaus setting employing the standard model to produce the same result.

Table 2 determines the risk that makes the risk related reductions in the social discount rate equivalent in magnitude to the growth term in a 50 year scenario. For these values, the social discount rate is equal to pure time preference.

	$\kappa = 0$				$\kappa = -.5$				$\kappa = -1$			
	N	VA	S1	S2	N	VA	S1	S2	N	VA	S1	S2
η	2	2/3	2	2	2	2/3	2	2	2	2/3	2	2
RIRA	0	27	7.5	3	0	27	7.5	3	0	27	7.5	3
σ in %	100	30	39	56	82	23	30	42	71	20	33	47
p^* in %	16	0.04	0.50	3.6	11	0.0009	0.13	1.7	7.9	0.00002	0.04	0.82

Notes: $\sigma = \sigma_y = \sigma_g$ =standard deviation; p^* =probability of being worse off in 50 years than today; κ =correlation coefficient between project and baseline risk. The $\kappa = 0$ case is equivalent to the risk-free social discount rate. The settings are ‘N’ based on Nordhaus; ‘VA’ based on Vissing-Joergensen & Attanasio’s parameter estimates; sensitivity scenarios ‘S1’ and ‘S2’.

5 Ambiguity Aversion and Second Order Uncertainty

A different shortcoming of the intertemporal expected utility standard model currently used in the social discounting debate is its assumption that the uncertainty over tomorrow can be described by a unique probability measure. In many real world applications these probability distributions (or “risks”) are unknown. This is particularly true for uncertainties we face as consequences of greenhouse gas emissions and climate change. The decision-theoretic literature has developed different frameworks to capture these situations. One way to characterize non-risk uncertainty is by extending the concept of probabilities to more general set functions called “capacities”. These set functions weigh possible events but are not necessarily additive in the union of disjoint events. Because of this non-additivity, the standard measure integral has to be changed for the more general Choquet integral for calculating expected utility, giving rise to the name “Choquet expected utility”. A second approach defines an evaluation functional that expresses beliefs in the form of sets of probability distributions rather than unique probability distributions. The first and simplest such representation goes back to Gilboa & Schmeidler (1989). Here a decision maker evaluates a scenario by taking expected values with respect to every probability distribution deemed possible and then identifies the scenario with the minimal expected value in this set.²² A more general representation of this type is given by Ghirardato,

²²Hansen & Sargent (2001) give conditions under which this approach is equivalent to what is known as robust control or model uncertainty, which again has overlapping representations with the model of constant absolute intertemporal risk aversion presented in Traeger (2007a).

Maccheroni & Marinacci (2004), Maccheroni, Marinacci & Rustichini (2006*a*), and, in an intertemporal framework, Maccheroni, Marinacci & Rustichini (2006*b*). There are several equivalence results between the Choquet approach and that of multiple priors as well as rank dependent utility theory where a decision maker uses distorted probabilities in an expected utility approach increasing the weights given to small probability events. Axiomatically, all of these models relax the independence axiom in one way or another.

5.1 A Setting with Ambiguity and Ambiguity Attitude

In this paper, I focus on a recent representation result by Klibanoff et al. (2005) and, in an intertemporal setting, Klibanoff, Marinacci & Mukerji (2009). The authors model non-risk uncertainty (ambiguity) as second order probability distributions – i.e. probabilities over probabilities. Most importantly, they introduce a different attitude for evaluating second order uncertainty as compared to first risk. For the purpose of my paper, this model has two advantages over the other approaches cited above. First, the model is time consistent, making it suitable not only for a descriptive but also for a normative decision framework. Second, the setting is very close to a standard Bayesian setting with parameter uncertainty. That makes the model more accessible to a general audience and, moreover, allows me to briefly relate to Weitzman (2009). In contrast to the standard Bayesian model, the ambiguity model evaluates parameter uncertainty (second order uncertainty) with a different degree of aversion than the risk (first order uncertainty) that prevails for a given distribution (with a given parameter). Translated into the simplified setting of this paper, Klibanoff et al.’s (2005, 2009), generally recursive, evaluation of the future can be written as

$$V(x_1, p, \mu) = u(x_1) + \beta \Phi^{-1} \left\{ \int_{\Theta} \Phi [E_{p_{\theta}} u(x_2)] d\mu(\theta) \right\} .$$

For a given parameter θ , the probability measure p_{θ} on X denotes first order or “objective” probabilities. However, these are not known uniquely and depend on a parameter θ that is unknown and subjective. The probability measure μ denotes the prior over the parameter $\theta \in \Theta$.²³ In Klibanoff et al.’s setting, the utility function u

²³In Klibanoff et al.’s (2009) axiomatization of the model the parameter space Θ is finite, while in my application I will make it continuous. Note moreover that Klibanoff et al. (2005, 2009) setting features acts rather than probability measures on the outcome space.

corresponds to the utility function of the standard model. It jointly captures aversion to intertemporal substitutability and “objective” or first order risk. The function Φ captures additional aversion with respect to second order uncertainty which is called *ambiguity aversion*. Note that, for Φ linear, the model collapses to the standard Bayesian model. The coefficient describing relative ambiguity aversion can be defined as

$$\text{RAA} = \frac{\Phi''(z)}{\Phi'(z)} |z| .$$

In this paper, I combine Klibanoff et al.’s model of ambiguity aversion with my model of intertemporal risk aversion leading to a welfare representation of the form

$$V(x_1, p, \mu) = u(x_1) + \beta \Phi^{-1} \left\{ \int_{\Theta} \Phi \left[f^{-1} E_{p_{\theta}(x_2)} f \circ u(x_2) \right] d\mu(\theta) \right\} .$$

Now u characterizes aversion to intertemporal substitution only, f characterizes intertemporal risk aversion, and Φ characterizes ambiguity aversion.²⁴ In this representation ambiguity aversion characterizes attitude with respect second order uncertainty similar to the way that intertemporal risk aversion characterizes attitude with respect to first order risk. This result will also be reflected in the expression for the social discount rates. Obviously, this parallel only arises in the threefold disentanglement of this paper and not in Klibanoff et al. (2005, 2009) and improves the intuition of ambiguity aversion.

To enable an analytic derivation of the social discount rate, I will once more revert to the isoelastic setting, implying in addition to the earlier assumptions of section 2.1 and equation (5) that $\Phi(z) = (\rho z)^{\varphi}$, which yields a coefficient of relative ambiguity aversion

$$\text{RAA} = \begin{cases} 1 - \varphi & \text{if } \rho > 0 \\ \varphi - 1 & \text{if } \rho < 0 . \end{cases}$$

5.2 The Social Discount Rate and Ambiguity about Growth

Weitzman (2009) recently argued that in the context of climate change the parameters

²⁴In an alternative representation, I could apply the inverse of the function f characterizing intertemporal risk aversion in front of Φ^{-1} instead of its current position where it acts on the expected value operator. Then, the same preferences are represented with a different function Φ that would characterize only “access aversion” to ambiguity as opposed intertemporal risk aversion.

of the distribution governing the growth process might not be known. Like Weitzman, I adopt a Bayesian setting to capture such a form of second order uncertainty. While Weitzman sticks with the standard risk evaluation model underlying equation (2), in contrast, I introduce ambiguity attitude as formulated by Klibanoff et al. (2005, 2009) and explained in section 5.1 (as well as intertemporal risk aversion). Taking the simplest example of Bayesian second order uncertainty, I assume that *expected* growth is itself a normally distributed parameter θ with expectation μ and variance τ^2 . Formally, that is $E(g|\theta) \sim N(\theta, \sigma^2)$ and $\theta \sim N(\mu, \tau^2)$, preserving the interpretation of μ as characterizing the overall expectation of the growth trend. The special case of Proposition 4 for RIRA = 0 and $\kappa = 0$ has independently been derived by Gierlinger & Gollier (2008).

Proposition 4: The stochastic social discount rate in the isoelastic setting with intertemporal risk aversion and ambiguity about expected growth is

$$\begin{aligned} r = & \delta + \eta\mu_g - \eta^2 \frac{\sigma_g^2 + \tau^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma_g^2}{2} \\ & + \eta \kappa \sigma_g \sigma_y + |1 - \eta| \text{RIRA} \kappa \sigma_g \sigma_y \\ & - \text{RAA} \left| 1 - \eta^2 \right| \frac{\tau^2}{2} . \end{aligned} \quad (11)$$

The first two terms on the right hand side reflect, once more, the discount rate as in the standard Ramsey equation under certainty. The third term $-\eta^2 \frac{\sigma_g^2 + \tau^2}{2}$ reflects the well-known extension for risk. Note that the overall variance of the growth process is now $\sigma^2 + \tau^2$ as I added an additional layer of uncertainty characterized by the second order variance τ^2 . The second line gives the corrections if the project is stochastic and remains as in the previous section. The third line characterizes the new effect on intertemporal value development that is due to ambiguity aversion. The term is proportional to second order variance τ^2 , relative ambiguity aversion RAA, and the term $|1 - \eta^2|$, which was already encountered in the correction of the social discount rate for intertemporal risk aversion. In fact, the contribution of ambiguity aversion is formally equivalent to the contribution of intertemporal risk aversion, replacing first by second order variance and RIRA by RAA. If a decision maker is more averse to ambiguity than he is to risk, then the discount rate reduction for second order variance (ambiguity) will be higher than the reduction based on first order variance (risk) discussed in detail in section 3.3. In general, an ambiguity averse decision maker will employ a lower (risk-free or stochastic) discount rate when the baseline scenario

is ambiguous. He is willing to invest in a certain or stochastic project with relatively lower productivity than is a decision maker who is ambiguity neutral or just faces (first order) risk.

Relating my result to Weitzman (2009), I ignore everything but the first three terms for the moment. The only difference between these remaining terms of equation (11) and the standard equation (2) is the additional variance τ in the third term on the right hand side (standard risk term). It is a straightforward consequence of making the growth process more uncertain by introducing a prior (second order uncertainty) over some parameter of the growth process. In the case of the normal distributions adopted here, the variance simply adds up. From the given example, it is difficult to see how adding a Bayesian prior would bring the standard risk term back into the order of magnitude needed to compare to the other characterizing terms of the social discount rate. Instead of a doubling, a factor of 10 – 100 is needed. The only way to reach this result is by sufficiently increasing the variance of the prior. Effectively, this is what Weitzman (2009) does in deriving what he calls a dismal theorem. He introduces a fat tailed (improper) prior whose moments do not exist. Consequently, the risk-free social discount rate in equation (11) goes to minus infinity implying an infinite willingness to transfer (certain) consumption into the future. Weitzman limits this willingness by the value of a (or society's) statistical life.²⁵ Instead of augmenting uncertainty, the above proposition introduces ambiguity aversion, i.e. the term $\text{RAA} |1 - \eta^2| \frac{\tau^2}{2}$, into social discounting, reflecting experimental evidence that economic agents tend to be more afraid of unknown probabilities than they are of known probabilities (most famously, Ellsberg 1961). Unfortunately, I am not yet aware of estimates for the parameter RAA in the Klibanoff et al. model. However, the formal similarity of the ambiguity aversion effect to the direct effect of intertemporal risk aversion gives a feeling for the magnitude by which a given degree of relative ambiguity aversion changes the social discount rate.

²⁵Note that Weitzman (2009) puts the prior on the variance σ rather than on the expected value of growth. He loosely relates the uncertainty to climate sensitivity. The above is a significantly simplified, but insightful, perspective on Weitzman's approach – abstracting from learning.

5.3 The Social Discount Rate and Uncertainty about Correlation

In section 4 I explained how opinions vary whether climate change related projects result in payoffs that are positively, negatively or not at all correlated to baseline risk. The standard approach in cost benefit analysis assumes full correlation of a project with the economic baseline risk. Weitzman (2007) argues that major areas impacted by climate change are little correlated to technological progress some of the impacts directly affect utility rather than production. Therefore he concludes that the correlation should be small. I made the point that climate change starts to become a serious part of society's baseline risk. Moreover, mitigation and adaptation projects pay out most in states of the world where climate change turns out to be more serious. Thus, these projects are negatively correlated with part of the baseline risk.

In this subsection, I introduce uncertainty about correlation. As the opposite extreme of knowing correlation perfectly, I assume an ignorant prior over the correlation coefficient, which still permits an analytic solution. In particular, I am interested the difference between complete ignorance about the correlation and the assumption of an uncorrelated transfer. I assume that the correlation κ between $\ln y$ and g (see section 4) is uniformly distributed between $[-1, 1]$.

Proposition 5: The stochastic social discount rate in the isoelastic setting with intertemporal risk aversion and a uniform prior over correlation is

$$r = \delta + \eta\mu_g - \eta^2 \frac{\sigma_g^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma_g^2}{2} - \ln \left[\frac{\sinh \{ \eta \sigma_g \sigma_y + |1 - \eta| \text{RIRA} \sigma_g \sigma_y \}}{\eta \sigma_g \sigma_y + |1 - \eta| \text{RIRA} \sigma_g \sigma_y} \right].$$

The terms in the first line resemble the risk-free social discount rate under intertemporal risk aversion derived in section 3.3. The second line captures the effect of uncertainty about the project and its correlation with the baseline growth. This additional component is of the form $h(z) = \ln \left[\frac{\sinh\{z\}}{z} \right]$, non-negative, and always reduces the discount rate as long as $z = (\eta + |1 - \eta| \text{RIRA}) \sigma_g \sigma_y \neq 0$ – a condition satisfied when the project and the baseline are stochastic and preferences do not simultaneously satisfy $\eta = 0$ and $\text{RIRA} = 0$. The function h can be expanded into

$h(z) = \frac{z^2}{6} - \frac{z^4}{180} + \frac{z^6}{2835} + O[z^7]$, where the first term already gives a good approximation for the magnitude relevant for the yearly discount rate. Here, section 4 finds that z was below one percent in all scenarios making $h(z)$ negligible. Note that the expression does not involve ambiguity aversion. While ambiguity aversion with respect to the baseline (section 5.1) is a first order effect, ambiguity with respect to the interaction of the project and the baseline becomes a second order effect not reflected in the social discount rate describing a marginal change.

Only in the 50 year scenario can ignorance about correlation deliver a significant difference from being uncorrelated. I use the same simplified 50 year scenario as in the preceding sections. I start by assuming a probability that society will be worse off in 50 years than today of $p^* = 0.001$. Then, in the scenario based on Vissing-Jørgensen & Attanasio's (2003) parameter estimates (VA) ignorance over correlation would reduce the average discount rate from an uncorrelated 1.3% to 1.2%. In the first sensitivity scenario (S1), where $RRA = 9.5$, it would reduce the average discount rate from 2.7% to 2.6%. The differences in the second sensitivity scenario (average rate of 4.1%) and in Nordhaus's scenario (average rate of 5.1%) are negligible. The difference between the assumptions of ignorance over correlation and not being correlated grow as the risk increases. For $p^* = 0.005$ ignorance as opposed to being uncorrelated reduces the average rate from 0.6% to 0.4% in the VA scenario, and from 1.5% to 1.3% in the S1 scenario, still leaving the last digit unchanged in the (S2) (average rate of 3.5%) and the (N) (average rate of 4.9%) scenarios.

6 Conclusions

The economics of climate change and many other fields in environmental and resource economics are largely an economics of long time horizons and uncertainty. The recent discussion on climate economics and policy following the Stern review has put a spotlight on a particularly important aspect of intertemporal evaluation: the social discount rate. The discussion is framed almost exclusively in a standard intertemporally additive expected utility setting. I pointed out the limitations of this standard model and analyzed four contributions relevant to the economics of climate change that are omitted in this framework. Three of these effects can be analyzed by abandoning an implicit assumption of risk neutrality that equates Arrow-Pratt risk aversion with the aversion to intertemporal consumption change. First, decoupling

Arrow-Pratt risk aversion from intertemporal substitutability lowers the growth effect in the social discount rate. This is an immediate consequence of the empirical finding that the aversion to intertemporal consumption smoothing η is overestimated when the parameter simultaneously has to capture the (generally stronger) attitude with respect to risk. Second, decoupling these two a priori independent preference parameters also removes an implicit assumption of (intertemporal) risk neutrality. I characterized intertemporal risk aversion axiomatically and have shown that a term proportional to the coefficient of intertemporal risk aversion further reduces the risk-free social discount rate. The third contribution is for a stochastic project, where payoffs are correlated to the economic baseline. Here, intertemporal risk aversion significantly increases the correlation effect in the social discount rate. It is argued in the literature that climate related projects depend on climate risk, which would largely be uncorrelated with the economic baseline risk. I suggest that climate change becomes a major economic risk over the coming decades and, thus, adaptation and mitigation projects that pay off most in states of the world where climate change turns out worst may actually be negatively correlated to the economic baseline risk. I contrast the cases of no correlation, a given correlation, and an evaluation where the decision maker is completely ignorant about the correlation between the project and economic baseline growth. While complete ignorance about correlation makes the social discount rate smaller than in the uncorrelated case, the magnitude of the effect is comparatively low and only becomes significant for long time horizons.

I have quantified a best guess of how these three effects change an observation-based social discount rate. I compared these rates to those generated with the standard model based on the parameterization suggested by Nordhaus (2007) and with two other model specifications. Moreover, I have calculated the probability of the event that society will be worse off in 50 years than today that is needed to make the (negative) risk effects in the social discount rate for a 50 year project cancel out the (positive) growth effect - implying that discounting comes down to pure time preference. These probabilities are a hundred up to a million times smaller in the model factoring in intertemporal risk aversion. I conclude that risk is of first order importance to social discounting in the context of climate change as well in the context of thin tailed probability distributions as soon as risk attitude is modeled comprehensively.

The fourth correction to the social discount rate stems from aversion to ambiguity.

Experimental evidence shows that decision makers are more averse to uncertainty in situations where uncertainty cannot be specified as risk. In the context of climate change, these situations of ambiguity (or hard uncertainty) are ubiquitous. I use a smooth ambiguity model to capture this distinction in uncertainty and in uncertainty attitude. Moreover, I merge this model with the model of intertemporal risk aversion. I point out the similarities between ambiguity attitude and intertemporal risk aversion, in general, and derive that ambiguity aversion has an analogous influence on the social discount rate as does intertemporal risk aversion. The analytic derivations in this paper and their quantitative assessment can serve as a rule of thumb of how the introduced effects change the social discount rate in the cost benefit analysis of climate related projects as well as in other applications where time and uncertainty play a similar role. Moreover, the modeling framework suggests itself for a less stylized numerical implementation in integrated assessment models of climate change.

Appendix

Proof of Proposition 1: a) *Sufficiency* of axiom (4): The premise of axiom (4) translates with $\beta = 1$ into the representation (3) as

$$\begin{aligned} (x^*, x^*) &\sim (x_1, x_2) \\ \Leftrightarrow u(x^*) + u(x^*) &= u(x_1) + u(x_2) \\ \Leftrightarrow u(x^*) &= \frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) \end{aligned} \quad (12)$$

Writing the implication of the axiom in terms of representation (3) yields

$$\begin{aligned} (x^*, x^*) &\succ (x^*, \frac{1}{2}x_1 + \frac{1}{2}x_2) \\ \Leftrightarrow u(x^*) &\geq f^{-1} \left(\frac{1}{2}f \circ u(x_1) + \frac{1}{2}f \circ u(x_2) \right). \end{aligned} \quad (13)$$

Combining equations (12) and (13) returns

$$\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) \geq f^{-1} \left(\frac{1}{2}f \circ u(x_1) + \frac{1}{2}f \circ u(x_2) \right), \quad (14)$$

which for an increasing [decreasing] version of f is equivalent to

$$\Leftrightarrow f \left(\frac{1}{2}u(x_1) + \frac{1}{2}u(x_2) \right) > [<] \frac{1}{2}f \circ u(x_1) + \frac{1}{2}f \circ u(x_2).$$

Defining $z_i = u(x_i)$, the equation becomes

$$\Leftrightarrow f \left(\frac{1}{2}z_1 + \frac{1}{2}z_2 \right) \geq [\leq] \frac{1}{2}f(z_1) + \frac{1}{2}f(z_2). \quad (15)$$

Because preferences are assumed to be representable in the form (3), there exists a certainty equivalent x^* to all lotteries $\frac{1}{2}x_1 + \frac{1}{2}x_2$ with $x_1, x_2 \in X$. Taking x^* to be the certainty equivalent, the premise and, thus, equation (15) have to hold for all $z_1, z_2 \in u(X)$. Therefore, f has to be concave [convex] on $U(x)$ (Hardy, Littlewood & Polya 1964, 75).

Necessity of axiom (4): The necessity is seen to hold by going backward through the proof of sufficiency above. Strict concavity [convexity] of f with f increasing [decreasing] implies that equation (15) and, thus, equation (14) have to hold for $z_1, z_2 \in u(X)$. The premise corresponding to (12) guarantees that equation (14) implies equation (13) which yields the implication in condition (4). Replacing \succeq by \preceq

and \geq by \leq in the proof above implies that the decision maker is intertemporal risk averse, if and only if, f is convex [for an increasing version of f and concave for f decreasing].

b) The decision maker is intertemporal risk neutral, if and only if, f is concave and convex on $u(X)$, which is equivalent to f being linear.²⁶ However, a linear function f cancels out in representation (3) and makes it identical to the intertemporally additive expected utility standard representation (1).

Proof of Proposition 2: The first step of the proof calculates the marginal value of an additional certain unit of consumption in the second period (dx_2) in terms of first period consumption (dx_1). This value derives from the marginal trade-off that leaves welfare unchanged.

$$\begin{aligned}
 U(x_1, p) &= \frac{x_1^\rho}{\rho} + \beta \frac{1}{\rho} [E_p x_2^\alpha]^\frac{\rho}{\alpha} \\
 \Rightarrow dV(x_1, p) &= x_1^{\rho-1} dx_1 + \beta \frac{1}{\alpha} [E_p x_2^\alpha]^\frac{\rho}{\alpha}-1 E_p \alpha x_2^{\alpha-1} dx_2 \stackrel{!}{=} 0 \\
 \Rightarrow x_1^{\rho-1} dx_1 &= -\beta [E_p x_2^\alpha]^\frac{\rho}{\alpha}-1 E_p x_2^{\alpha-1} dx_2 \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta \left[E_p \left(\frac{x_2}{x_1} \right)^\alpha \right]^\frac{\rho}{\alpha}-1 E_p \left(\frac{x_2}{x_1} \right)^{\alpha-1} \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta \left[E_p e^{\alpha \ln \frac{x_2}{x_1}} \right]^\frac{\rho}{\alpha}-1 E_p e^{(\alpha-1) \ln \frac{x_2}{x_1}} \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta \left[e^{\alpha \mu + \alpha^2 \frac{\sigma^2}{2}} \right]^\frac{\rho}{\alpha}-1 e^{(\alpha-1)\mu + (1-\alpha)^2 \frac{\sigma^2}{2}} \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta e^{\rho\mu + \alpha\rho\frac{\sigma^2}{2} - \alpha\mu - \alpha^2\frac{\sigma^2}{2}} e^{(\alpha-1)\mu + (1-\alpha)^2\frac{\sigma^2}{2}} \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta e^{(\rho-1)\mu + (\alpha\rho+1-2\alpha)\frac{\sigma^2}{2}} \\
 \Rightarrow \frac{dx_1}{dx_2} &= -\beta e^{(\rho-1)\mu + (\alpha\rho+1-2\alpha)\frac{\sigma^2}{2}}.
 \end{aligned}$$

The second step translates the relation into rates by defining the social discount rate $r = -\ln \frac{dx_1}{-dx_2} \left(= -\ln \frac{dx_2}{-dx_1} |_{\bar{U}} \right)$, the rate of pure time preference $\delta = -\ln \beta$, and

²⁶Alternatively use \sim and $=$ instead of \succeq and \geq in part a) and use Aczél (1966, 46).

$\eta = 1 - \rho$ ($= \frac{1}{\sigma}$). Further below, I make use of the relation $1 = \frac{1-\eta}{\rho}$.

$$\Rightarrow r = \delta + (1 - \rho)\mu - (\alpha(\rho - 1) + 1 - \alpha)\frac{\sigma^2}{2} \quad (16)$$

$$\Rightarrow r = \delta + \eta\mu - \eta^2\frac{\sigma^2}{2} + (\eta^2 + \alpha(\eta + 1) - 1)\frac{\sigma^2}{2}$$

$$\Rightarrow r = \delta + \eta\mu - \eta^2\frac{\sigma^2}{2} + (\eta^2 + \frac{\alpha}{\rho}(1 - \eta)(\eta + 1) - 1)\frac{\sigma^2}{2}$$

$$\Rightarrow r = \delta + \eta\mu - \eta^2\frac{\sigma^2}{2} + (\eta^2 + \frac{\alpha}{\rho}(1 - \eta^2) - 1)\frac{\sigma^2}{2}$$

$$\Rightarrow r = \delta + \eta\mu - \eta^2\frac{\sigma^2}{2} - (1 - \frac{\alpha}{\rho})(1 - \eta^2)\frac{\sigma^2}{2}$$

$$\Rightarrow r = \delta + \eta\mu - \eta^2\frac{\sigma^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma^2}{2}. \quad (17)$$

□

Proof of Proposition 3: For the isoelastic specification and with the definition

$$U_2(\epsilon) = f^{-1} \left[\mathbb{E}_{p(x_2, y)} f \circ u(x_2 + \epsilon y) \right] = \frac{1}{\rho} \left[\mathbb{E}_{p(x_2, y)} (x_2 + \epsilon y)^\alpha \right]^{\frac{\rho}{\alpha}}$$

equation (9) translates into

$$x_1^{\rho-1} dx_1 + \beta \left. \frac{d}{d\epsilon} U_2(\epsilon) \right|_{\epsilon=0} d\epsilon \stackrel{!}{=} 0 \quad (18)$$

In order to calculate $\left. \frac{d}{d\epsilon} U_2(\epsilon) \right|_{\epsilon=0} d\epsilon$ the following definition is useful.

$$V_\epsilon(a, b) = \mathbb{E}_{p(x_2, y)} (x_2 + \epsilon y)^a y^b. \quad (19)$$

Then

$$\begin{aligned} \left. \frac{d}{d\epsilon} U_2(\epsilon) \right|_{\epsilon=0} &= \frac{1}{\alpha} V_\epsilon(\alpha, 0)^{\frac{\rho}{\alpha}-1} \alpha V_\epsilon(\alpha - 1, 1) \Big|_{\epsilon=0} \\ &= V_0(\alpha, 0)^{\frac{\rho}{\alpha}-1} V_0(\alpha - 1, 1) \end{aligned} \quad (20)$$

where equality between the first and the second line follows from Lebesgue's dominated convergence theorem. Analogously to step 1 in the proof of Proposition 2, I

calculate with $z = \ln y$

$$\begin{aligned} V_0(\alpha, 0) &= x_1^\alpha \mathbb{E}_{p(x_2, y)} \left(\frac{x_2}{x_1} \right)^\alpha = x_1^\alpha \mathbb{E}_{p(g, z)} e^{\alpha g} \\ &= x_1^\alpha \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\alpha g} \frac{e^{-\frac{1}{2(1-\kappa^2)} \left[\left(\frac{g-\mu_g}{\sigma_g} \right)^2 + \left(\frac{z-\mu_y}{\sigma_y} \right)^2 - 2\kappa \left(\frac{g-\mu_g}{\sigma_g} \right) \left(\frac{z-\mu_y}{\sigma_y} \right) \right]}}{2\pi\sigma_g\sigma_y\sqrt{1-\rho^2}} dg dz \end{aligned} \quad (21)$$

$$= x_1^\alpha e^{\alpha\mu_g + \alpha^2 \frac{\sigma_g^2}{2}}. \quad (22)$$

Similarly,

$$\begin{aligned} V_0(\alpha - 1, 1) &= x_1^{\alpha-1} \mathbb{E}_{p(x, y)} \left(\frac{x_2}{x_1} \right)^{\alpha-1} y = x_1^{\alpha-1} \mathbb{E}_{p(g, z)} e^{(\alpha-1)g+z} \\ &= x_1^{\alpha-1} e^{-(1-\alpha) \left[\mu_g - (1-\alpha) \frac{\sigma_g^2}{2} + \kappa\sigma_g\sigma_y \right] + \mu_y + \frac{\sigma_y^2}{2}} \end{aligned}$$

so that

$$\begin{aligned} \left. \frac{d}{d\epsilon} U_2(\epsilon) \right|_{\epsilon=0} &= x_1^{\rho-\alpha+\alpha-1} e^{(\alpha\mu_g + \alpha^2 \frac{\sigma_g^2}{2})(\frac{\rho}{\alpha}-1)} e^{-(1-\alpha) \left[\mu_g - (1-\alpha) \frac{\sigma_g^2}{2} + \kappa\sigma_g\sigma_y \right] + \mu_y + \frac{\sigma_y^2}{2}} \\ &= x_1^{\rho-1} e^{(\rho-1)\mu_g + [\alpha(\rho-1) + (1-\alpha)] \frac{\sigma_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}}. \end{aligned} \quad (23)$$

Substituting the result into equation (18) and solving for the discount rate yields

$$\begin{aligned} r = \ln \frac{d\epsilon}{-dx_1} &= \delta + (1-\rho)\mu_g - [\alpha(\rho-1) + 1-\alpha] \frac{\sigma^2}{2} \\ &\quad + (1-\alpha)\kappa\sigma_g\sigma_y - \left(\mu_y + \frac{\sigma_y^2}{2} \right). \end{aligned}$$

The first line corresponds to equation (16) and, thus, equation (17), yielding the risk-free discount rate under intertemporal risk aversion. Moreover, the random variable y was assumed to yield an expected value (project payoff) of unity, which implies

$$\mathbb{E}_{p(x, y)} y = e^{\mu_y + \frac{\sigma_y^2}{2}} \stackrel{!}{=} 1 \quad \Rightarrow \quad \mu_y + \frac{\sigma_y^2}{2} = 0,$$

eliminating the last bracket. Finally, $1-\alpha$ has to be expressed in terms of η (capturing the effects of the standard model) and RIRA (capturing the additional effects of intertemporal risk aversion). I find for $\rho > 0$ that

$$1-\alpha = 1 - (1-\eta)(1-\text{RIRA}) = \eta + (1-\eta)\text{RIRA}$$

and for $\rho < 0$ that

$$1 - \alpha = 1 - (1 - \eta)(1 + \text{RIRA}) = \eta - (1 - \eta) \text{RIRA} .$$

In both cases this yields

$$1 - \alpha = \eta + |1 - \eta| \text{RIRA} , \quad (24)$$

which gives rise to the form stated in the proposition. \square

Proof of Proposition 4:

Define for the isoelastic specification

$$\begin{aligned} U_2^a(\epsilon) &= \Phi^{-1} \left\{ \int_{\Theta} \Phi [f^{-1} E_{p_{\theta}(x_2, y)} f \circ u(x_2 + \epsilon y)] d\mu(\theta) \right\} \\ &= \frac{1}{\rho} \left\{ \int_{\Theta} [E_{p_{\theta}(x_2, y)}(x_2 + \epsilon y)^{\alpha}]^{\frac{\rho}{\alpha} \varphi} d\mu(\theta) \right\}^{\frac{1}{\varphi}} . \end{aligned}$$

I have to solve once more the equation

$$dV(x_1, p, \mu) = x_1^{\rho-1} dx_1 + \beta \frac{d}{d\epsilon} U_2^a(\epsilon) \Big|_{\epsilon=0} d\epsilon \stackrel{!}{=} 0 \quad (25)$$

for $\ln \frac{d\epsilon}{-dx_1}$. Making use again of the definition

$$V_{\epsilon}(a, b) = E_{p_{\theta}(x_2, y)}(x_2 + \epsilon y)^a y^b ,$$

where θ replaces μ_g in $p_{(x, y)}$ of equations (19) and (21), I find

$$\begin{aligned} \frac{d}{d\epsilon} U_2^a(\epsilon) \Big|_{\epsilon=0} &= \frac{1}{\rho} \frac{1}{\varphi} \left\{ \int_{\Theta} V_{\epsilon}(\alpha, 0)^{\frac{\rho}{\alpha} \varphi} d\mu(\theta) \right\}^{\frac{1}{\varphi}-1} \\ &\quad \left\{ \int_{\Theta} \frac{\rho}{\alpha} \varphi V_{\epsilon}(\alpha, 0)^{\frac{\rho}{\alpha} \varphi - 1} \alpha V_{\epsilon}(\alpha - 1, 1) d\mu(\theta) \right\} \Big|_{\epsilon=0} \\ &= \left\{ \int_{\Theta} V_0(\alpha, 0)^{\frac{\rho \varphi}{\alpha}} d\mu(\theta) \right\}^{\frac{1}{\varphi}-1} \\ &\quad \int_{\Theta} V_0(\alpha, 0)^{\frac{\rho \varphi}{\alpha} - 1} V_0(\alpha - 1, 1) d\mu(\theta) . \end{aligned} \quad (26)$$

With the help of equation (22), the $\{\cdot\}$ expression calculates to

$$\begin{aligned} \int_{\Theta} x_1^{\alpha(\frac{\rho\varphi}{\alpha})} e^{(\alpha\theta + \alpha^2 \frac{\sigma_g^2}{2})(\frac{\rho\varphi}{\alpha})} d\mu(\theta) &= x_1^{\rho\varphi} e^{\rho\varphi\alpha \frac{\sigma_g^2}{2}} \int_{\Theta} e^{\rho\varphi\theta} \frac{e^{-\frac{1}{2}(\frac{\theta - \mu_g}{\sigma_g})^2}}{\sqrt{2\pi}\sigma_g} d\theta \\ &= x_1^{\rho\varphi} e^{\rho\varphi\alpha \frac{\sigma_g^2}{2}} e^{\rho\varphi\mu_g + \rho^2\varphi^2 \frac{\tau_g^2}{2}}. \end{aligned}$$

Acknowledging the equality of equations (20) and (23) and their similarity to the second integrand in equation (26) (for $\rho \leftrightarrow \rho\varphi$), this second integral becomes

$$\begin{aligned} &\int_{\Theta} V_0(\alpha, 0)^{\frac{\rho\varphi}{\alpha}-1} V_0(\alpha-1, 1) d\mu(\theta) \\ &= \int_{\Theta} x_1^{\rho\varphi-1} e^{(\rho\varphi-1)\theta + [\alpha(\rho\varphi-1) + (1-\alpha)] \frac{\sigma_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}} d\mu(\theta) \\ &= x_1^{\rho\varphi-1} e^{[\alpha(\rho\varphi-1) + (1-\alpha)] \frac{\sigma_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}} \int_{\Theta} e^{(\rho\varphi-1)\theta} d\mu(\theta) \\ &= x_1^{\rho\varphi-1} e^{[\alpha(\rho\varphi-1) + (1-\alpha)] \frac{\sigma_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}} e^{(\rho\varphi-1)\mu_g + (\rho\varphi-1)^2 \frac{\tau_g^2}{2}}. \end{aligned}$$

Substituting these results back into equation (26) delivers

$$\begin{aligned} \left. \frac{d}{d\epsilon} U_2^a(\epsilon) \right|_{\epsilon=0} &= x_1^{\rho\varphi(\frac{1}{\varphi}-1)} e^{(\rho\varphi\alpha \frac{\sigma_g^2}{2})(\frac{1}{\varphi}-1)} e^{(\rho\varphi\mu_g + \rho^2\varphi^2 \frac{\tau_g^2}{2})(\frac{1}{\varphi}-1)} \\ &\quad x_1^{\rho\varphi-1} e^{[\alpha(\rho\varphi-1) + (1-\alpha)] \frac{\sigma_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}} e^{(\rho\varphi-1)\mu_g + (\rho\varphi-1)^2 \frac{\tau_g^2}{2}} \\ &= x_1^{\rho-1} e^{[\alpha(\rho-1) + (1-\alpha)] \frac{\sigma_g^2}{2} + (\rho-1)\mu_g + [\rho\varphi(\rho-1) + 1 - \rho\varphi] \frac{\tau_g^2}{2} - (1-\alpha)\kappa\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}}. \end{aligned}$$

Substituting this result into equation (25) and solving for $r = \ln \frac{d\epsilon}{-dx_1}$ yields analogously to the proof of Proposition 3 the discount rate

$$\begin{aligned} r &= \delta + \eta\mu_g - \eta^2 \frac{\sigma_g^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma_g^2}{2} + \eta\kappa\sigma_g\sigma_y \\ &\quad + |1 - \eta| \text{RIRA} \kappa\sigma_g\sigma_y - [1 - 2\rho\varphi + \rho^2\varphi] \frac{\tau_g^2}{2}. \end{aligned}$$

The last term can be rearranged to the form

$$\begin{aligned}
 [1 - 2\rho\varphi + \rho^2\varphi]\frac{\tau_g^2}{2} &= [(1 - \varphi) + \varphi(1 - \rho) - \varphi\rho(1 - \rho)]\frac{\tau_g^2}{2} \\
 &= [(1 - \varphi) + (1 - \rho)^2 + (\varphi - 1)(1 - \rho)^2]\frac{\tau_g^2}{2} \\
 &= [\eta^2 + (1 - \varphi)(1 - \eta^2)]\frac{\tau_g^2}{2} = \eta^2\frac{\tau_g^2}{2} + \text{RAA} |1 - \eta^2| \frac{\tau_g^2}{2},
 \end{aligned}$$

completing the proof. \square

Proof of Proposition 5:

Up to equation (26) the proof is identical to that of Proposition 4. In the next step, in $V_0(\alpha, 0)^{\frac{\rho\varphi}{\alpha}}$ the ambiguity parameter θ replaces κ instead of μ_g . Thus the first integral in equation (26) becomes

$$\begin{aligned}
 \int_{\Theta} x_1^{\alpha(\frac{\rho\varphi}{\alpha})} e^{(\alpha\mu_g + \alpha^2\frac{\sigma_g^2}{2})(\frac{\rho\varphi}{\alpha})} d\mu(\theta) &= x_1^{\rho\varphi} e^{\rho\varphi\mu_g + \rho\varphi\alpha\frac{\sigma_g^2}{2}} \int_{-1}^1 \frac{1}{2} d\theta \\
 &= x_1^{\rho\varphi} e^{\rho\varphi\mu_g + \rho\varphi\alpha\frac{\sigma_g^2}{2}}.
 \end{aligned}$$

For the integrand of the second integral in equation (26), I find

$$V_0(\alpha - 1, 1) = x_1^{\alpha-1} e^{(\alpha-1)\mu_g + (\alpha-1)^2\frac{\sigma_g^2}{2} + (\alpha-1)\theta\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}}$$

delivering the integral

$$\begin{aligned}
 &\int_{\Theta} V_0(\alpha, 0)^{\frac{\rho\varphi}{\alpha}-1} V_0(\alpha - 1, 1) d\mu(\theta) \\
 &= \int_{\Theta} x_1^{\rho\varphi-\alpha} e^{\rho\varphi\mu_g + \rho\varphi\alpha\frac{\sigma_g^2}{2} - \alpha\mu_g - \alpha^2\frac{\sigma_g^2}{2}} \\
 &\quad x_1^{\alpha-1} e^{(\alpha-1)\mu_g + (\alpha-1)^2\frac{\sigma_g^2}{2} + (\alpha-1)\theta\sigma_g\sigma_y + \mu_y + \frac{\sigma_y^2}{2}} d\mu(\theta) \\
 &= x_1^{\rho\varphi-1} e^{(\rho\varphi-1)\mu_g + (\rho\varphi\alpha-2\alpha-1)\frac{\sigma_g^2}{2} + \mu_y + \frac{\sigma_y^2}{2}} \int_{-1}^1 e^{(\alpha-1)\theta\sigma_g\sigma_y} \frac{1}{2} d\theta \\
 &= x_1^{\rho\varphi-1} e^{(\rho\varphi-1)\mu_g + (\rho\varphi\alpha-2\alpha-1)\frac{\sigma_g^2}{2} + \mu_y + \frac{\sigma_y^2}{2}} \frac{\sinh[(\alpha-1)\sigma_g\sigma_y]}{(\alpha-1)\sigma_g\sigma_y}.
 \end{aligned}$$

Substituting these results back into equation (26) returns the second period welfare change in ϵ :

$$\begin{aligned} \left. \frac{d}{d\epsilon} U_2^a(\epsilon) \right|_{\epsilon=0} &= x_1^{\rho\varphi(\frac{1}{\varphi}-1)} e^{(\rho\varphi\mu_g + \rho\varphi\alpha\frac{\sigma_g^2}{2})(\frac{1}{\varphi}-1)} \\ &\quad x_1^{\rho\varphi-1} e^{(\rho\varphi-1)\mu_g + (\rho\varphi\alpha-2\alpha-1)\frac{\sigma_g^2}{2} + \mu_y + \frac{\sigma_y^2}{2}} \frac{\sinh[(\alpha-1)\sigma_g\sigma_y]}{(\alpha-1)\sigma_g\sigma_y} \\ &= x_1^{\rho-1} e^{(\rho-1)\mu_g + (\rho\alpha-2\alpha-1)\frac{\sigma_g^2}{2} + \mu_y + \frac{\sigma_y^2}{2}} \frac{\sinh[(\alpha-1)\sigma_g\sigma_y]}{(\alpha-1)\sigma_g\sigma_y}. \end{aligned}$$

Substituting this result into equation (25) and solving for $r = \ln \frac{d\epsilon}{-dx_1}$ yields analogously to the proof of Proposition 3 the discount rate

$$r = \delta + \eta\mu_g - \eta^2 \frac{\sigma_g^2}{2} - \text{RIRA} \left| 1 - \eta^2 \right| \frac{\sigma_g^2}{2} - \ln \left[\frac{\sinh[(\alpha-1)\sigma_g\sigma_y]}{(\alpha-1)\sigma_g\sigma_y} \right].$$

By symmetry of the hyperbolic sine, the sign of $(\alpha-1)$ can be flipped simultaneously in the numerator and the denominator. Using equation (24) to substitute for $(1-\alpha)$ then yields the result stated in the proposition. \square

References

- Aczél, J. (1966), *Lectures on Functional Equations and their Applications*, Academic Press, New York.
- Campbell, J. Y. (1996), ‘Understanding risk and return’, *The Journal of Political Economy* **104**(2), 298–345.
- Dasgupta, P. (2008), ‘Commentary: The stern review’s economics of climate change’. National Institute Economic Review 2007; 199; 4.
- Dasgupta, P. (2009), ‘Discounting climate change’, *Journal of Risk and Uncertainty* **37**, 141–169.
- Ellsberg, D. (1961), ‘Risk, ambiguity and the savage axioms’, *Quarterly Journal of Economics* **75**, 643–69.
- Epstein, L. G. & Zin, S. E. (1989), ‘Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework’, *Econometrica* **57**(4), 937–69.

- Ghirardato, P., Maccheroni, F. & Marinacci, M. (2004), ‘Differentiating ambiguity and ambiguity attitude’, *Journal of Economic Theory* **118**(2), 122–173.
- Gierlinger, J. & Gollier, C. (2008), ‘Socially efficient discounting under ambiguity aversion’, *Working Paper* .
- Gilboa, I. & Schmeidler, D. (1989), ‘Maxmin expected utility with non-unique prior’, *Journal of Mathematical Economics* **18**(2), 141–53.
- Giuliano, P. & Turnovsky, S. J. (2003), ‘Intertemporal substitution, risk aversion, and economic performance in a stochastically growing open economy’, *Journal of International Money and Finance* **22**(4), 529–556.
- Guesnerie, R. (2004), ‘Calcul économique et développement durable’, *Revue économique* **55**(3), 363–382.
- Ha-Duong, M. & Treich, N. (2004), ‘Risk aversion, intergenerational equity and climate change’, *Environmental and Resource Economics* **28**(2), 195–207.
- Hansen, L. P. & Sargent, T. J. (2001), ‘Robust control and model uncertainty’, *American Economic Review* **91**(2), 60–66.
- Hardy, G., Littlewood, J. & Polya, G. (1964), *Inequalities*, 2 edn, Cambridge University Press. first published 1934.
- Heal, G. (2009), ‘Climate economics: A meta-review and some suggestions for future research’, *Review of Environmental Economics and Policy* **3**(1), 4–21.
- Hepburn, C. (2006), Discounting climate change damages: Working notes for the Stern review, Working note.
- Hoel, M. & Sterner, T. (2007), ‘Discounting and relative prices’, *Climatic Change* **84**, 265–280.
- Horowitz, J. & Lange, A. (2009), ‘What’s wrong with infinity - a note on weitzman’s dismal theorem’, *Working Paper* .
- Ju, N. & Miao, J. (2009), Ambiguity, learning, and asset returns, MPRA Paper 14737, University Library of Munich, Germany.

- Kihlstrom, R. E. & Mirman, L. J. (1974), ‘Risk aversion with many commodities’, *Journal of Economic Theory* **8**(3), 361–88.
- Klibanoff, P., Marinacci, M. & Mukerji, S. (2005), ‘A smooth model of decision making under ambiguity’, *Econometrica* **73**(6), 1849–1892.
- Klibanoff, P., Marinacci, M. & Mukerji, S. (2009), ‘Recursive smooth ambiguity preferences’, *Journal of Economic Theory* **144**, 930–976.
- Lind, R., ed. (1982), *Discounting for Time and Risk in Energy Policy*, Resources for the Future, Washington, D.C.
- Maccheroni, F., Marinacci, M. & Rustichini, A. (2006a), ‘Ambiguity aversion, robustness, and the variational representation of preferences’, *Econometrica* **74**(6), 1447–1498.
- Maccheroni, F., Marinacci, M. & Rustichini, A. (2006b), ‘Dynamic variational preferences’, *Journal of Economic Theory* **128**(1), 4–44.
- Nordhaus, W. (2008), *A Question of Balance: Economic Modeling of Global Warming*, Yale University Press, New Haven. Online preprint: A Question of Balance: Weighing the Options on Global Warming Policies.
- Nordhaus, W. (2009), ‘An analysis of the dismal theorem’, *Working Paper* .
- Nordhaus, W. D. (2007), ‘A review of the Stern review on the economics of climate change’, *Journal of Economic Literature* **45**(3), 686–702.
- Normandin, M. & St-Amour, P. (1998), ‘Substitution, risk aversion, taste shocks and equity premia’, *Journal of Applied Econometrics* **13**(3), 265–281.
- Pindyck, R. S. (2009), ‘Uncertain outcomes and climate change policy’, *MIT Sloan Research Paper* (4742-09).
- Plambeck, E. L., Hope, C. & Anderson, J. (1997), ‘The Page95 model: Integrating the science and economics of global warming’, *Energy Economics* **19**, 77–101.
- Ramsey, F. P. (1928), ‘A mathematical theory of saving’, *The Economic Journal* **38**(152), 543–559.

- Stern, N., ed. (2007), *The Economics of Climate Change: The Stern Review*, Cambridge University Press, Cambridge.
- Sterner, T. & Persson, M. (2008), ‘An even sterner review: Introducing relative prices into the discounting debate’, *Review of Environmental Economics and Policy* **2**(1), 61–76.
- Traeger, C. (2007a), Disentangling risk aversion from intertemporal substitutability and the temporal resolution of uncertainty. Working Paper.
- Traeger, C. (2007b), The generalized isoelastic model for many commodities. Working Paper.
- Traeger, C. (2007c), Intertemporal risk aversion, stationarity and the rate of discount. Working Paper.
- Traeger, C. (2007d), ‘Sustainability, limited substitutability and non-constant social discount rates’. CUDARE Working Paper 1045.
- Traeger, C. (2007e), ‘Wouldn’t it be nice to know whether Robinson is risk averse?’. Working Paper.
- Vissing-Jørgensen, A. & Attanasio, O. P. (2003), ‘Stock-market participation, intertemporal substitution, and risk-aversion’, *The American Economic Review* **93**(2), 383–391.
- von Neumann, J. & Morgenstern, O. (1944), *Theory of Games and Economic Behaviour*, Princeton University Press, Princeton.
- Weil, P. (1990), ‘Unexpected utility in macroeconomics’, *The Quarterly Journal of Economics* **105**(1), 29–42.
- Weitzman, M. (2007), ‘A review of the Stern review on the economics of climate change’, *Journal of Economic Literature* **45**(3), 703–724.
- Weitzman, M. (2009), ‘On modeling and interpreting the economics of catastrophic climate change’, *The Review of Economics and Statistics* **91**(1), 1–19. 06.