A METHODOLOGICAL PERSPECTIVE ON VALUING WATER: A COMPARISON BETWEEN LINEAR PROGRAMMING AND STOCHASTIC DYNAMIC PROGRAMMING

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Information on the value of water in use is a prerequisite for the efficient allocation, utilization, trading and transfer thereof. These issues are becoming very important for South Africa. Linear programming (LP) and stochastic dynamic programming (SDP) are two techniques that can be applied to value water. Based on a simulated irrigation farming situation downstream of the Vanderkloof Dam wherein farmers hold a capacity share (CS) this paper draws a comparison between the marginal value products (MVP's) obtained from applying LP and SDP simultaneously. Linear programming is used to optimize water use on the farm during the immediate season while SDP is used to optimize the use of water in storage in the farmers capacity share (CS) in the Vanderkloof Dam through time. Emphasis is placed on the interpretation of the results which are presented graphically.

1. INTRODUCTION

With the freshwater supply of the globe more or less fixed and an expanding demand due to increasing growth in human population and economic activities, water is increasingly becoming scarcer. A critical stage for South Africa is predicted in about 20 years time when demand will exceed supply. With a matured water economy it is of paramount importance that water should be used as efficiently as possible. A critical ingredient to achieve efficient water use would be information on the value of water in alternative uses. With the irrigation sector being the largest user of water in South Africa (it uses about 54% of the total freshwater, Backeberg & Odendaal, 1998) achieving efficient water use in this sector could make a major contribution to alleviate the water scarcity problem of the country.

This paper focuses on a very relevant issue, namely determining the value in use of different dimensions of irrigation water. This is achieved by applying Linear Programming (LP) and Stochastic Dynamic Programming (SDP)simultaneously on a simulated irrigation farming situation down stream

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of the Vanderkloof Dam with the farmer holding an assumed capacity share (CS) in the Vanderkloof Dam. LP is used to optimize water use on the farm during the immediate season while SDP is used to optimize the optimal use of water in storage in the farmers CS in the Vanderkloof Dam through time, taking inflow probabilities into account (Dudley, Reklis & Burt, 1976 and Dudley & Hearn, 1993):

2. METHODOLOGY

2.1 Derivation of the marginal value product with linear programming

A 75ha irrigation farm with a water quota of 11 000 cubic meter per hectare per year is supplied with water from the Ramah canal downstream of the Vanderkloof Dam. The farmer considers a combination of five crops, maize, groundnuts, lucerne, wheat and green peas. For each crop, per hectare gross margins were derived from a previous study (Viljoen, Symington & Botha, 1992) for three levels of water application; low, medium and high, with water demand specified on a monthly basis for each crop. This information was used in a linear programming formulation and optimal gross margins were obtained for the farm by varying parametrically the quantity of water available per summer and winter season. By arranging the results (farm gross margins for different, progressively higher levels of water availability per season) and fitting regression equations to the data, total value product functions were obtained separately for the summer and winter seasons (see Figure 1). The first derivatives of these total value product functions are the marginal value products (MVP's) of water for the two seasons. The MVP tells the farmer what he can afford to pay for an additional unit of water in the immediate season.

2.2 Derivation of marginal value product with stochastic dynamic programming

The water that the farmer uses on his 75ha farm is obtained from his CS in the Vanderkloof Dam. His CS is determined by an assumed institutional arrangement which determines the users rights in terms of percentage of the dam's inflow and a percentage of the dam's active storage space, in which to store his share of the dam's inflow for use through time as he desires. The size of CS and inflow shares (IS) for the base case (BC) scenario were chosen so that the maximum contents of the CS is just sufficient to maximize the farm gross margin (obtained from the TVP functions of the LP simulation) in the season of the highest demand, which is the summer season in this case.

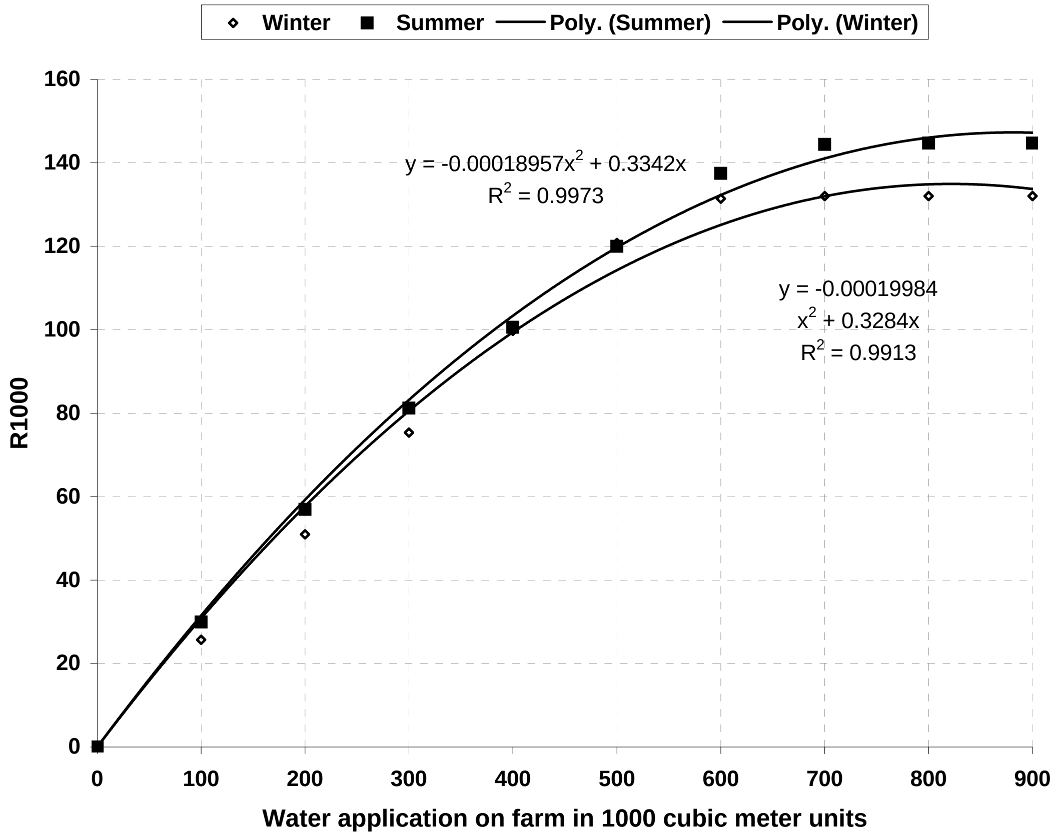

Figure 1: Total value product (farm gross margin) functions for winter and summer season based on LP simulation

By inputting the TVP functions for the summer and winter seasons (obtained from LP) as well as inflow shares for 19 summer and 19 winter seasons (calculated from actual stream inflow data for the Vanderkloof Dam) into the SDP model it was possible to derive the expected present value of MVP's for water. The MVP's in this case tells us what a farmer with a specified CS could afford to pay for an extra unit of water in storage in his capacity share for use any time in the future (or the value to the farmer of the marginal unit of water given up by him). By varying the sizes of the capacity and inflow shares the impact on MVP's and optimal water use decisions are investigated.

The optimal water use decisions from the SDP-model are then used in a simulation model to derive yet another set of MVP's. In this case the MVP's show the value of a marginal unit of a CS and/or IS size, where this marginal value is expressed in terms of average annual gross margin.

3. EMPIRICAL RESULTS

3.1 MVP's from LP simulation for inter season decisions

Figure 2 shows the MVP's for the summer and winter seasons derived from the TVP curves (Figure 1), based on LP simulation. These MVP curves must be seen as average curves, which differs from the stepwise demand curves normally obtained from LP simulation (Hazel & Norton, 1986). Because the TVP curves are quadratic in nature, the MVP curves are linear. The MVP curves for the summer and winter season have nearly the same slope with the summer curve having a slightly higher intercept.

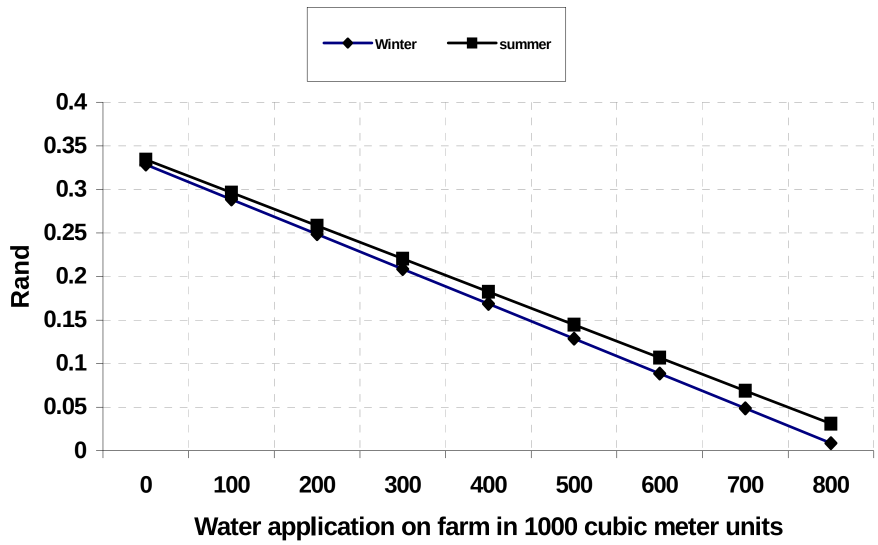

Figure 2: Marginal value product in Rand per cubic meter for winter and summer season derived from TVP functions based on LP simulations

For the first 1000 cubic meters of water the farmer can pay about as much as R0.33 per cubic meter (for either the summer and winter season) to use during the immediate seasons. If the farmer already has 600 000 cubic meters then according to the MVP's (shadow prices) he should not pay more than about R0.11 per cubic meter for summer water or more than R0.08 per cubic meter for winter water.

3.2 MVP from SDP for inter-year decisions

The following scenarios were investigated:

Scenario 1	-	Base case (BC) (100% of CS and inflow shares (IS))
Scenario 2	-	CS and IS are75% of BC
Scenario 3	-	CS and IS are 50% of BC
Scenario 4	-	CS and IS are 25% of BC
Scenario 5	-	CS same as BC but IS doubles
Scenario 6	-	CS same as BC but IS only 50% of BC
Scenario 7	-	CS is 50% of BC but IS are the same
Scenario 8	-	CS and IS are double that of BC

Where the MVP's obtained from LP reflect what a farmer can pay for marginal units of water during the immediate season, the MVP's obtained from SDP are once-off payment/values for marginal units of stored water to use any time in the future. The following interpretation can be given regarding the results of the different scenarios which are presented in graphical format in Figures 3 to 10.

Scenario 1 (base case) - Figure 3

- The MVP of summer season water at the start of the last summer in the planning horizon (1//1) declines linearly since, for the last season, the MVP is based solely on the LP output for the summer season which is a quadratic equation: therefore, the MVP is linear throughout.

- When many seasons remain in the planning horizon the summer MVP is the same as the MVP for the last summer in the planning horizon until the reservoir CS is 75% full (1//13). Then the value of adding extra water to the CS is greater when many years remain in the planning horizon, because the extra water can be used over those remaining years. Water saving for later seasons begins at 80% of CS contents.

- The MVP's of water at the start of the winter season is not responsive to the number of seasons remaining, as shown by the high similarity of the two winter MVP curves (1//2 and 1//14)

- The winter MVP's are a little below the summer MVP's until the CS is almost half full. That is, over the linear portion of the winter MVP's which come directly from the LP output. Then the winter MVP's exceed the summer MVP's due to water saving policies. This is because the probability of inflows is much less during the winter season, hence, the value of any water saved at the start of the winter is likely to be much greater than the value of water saved at the start of the summer season.

- The MVP's of the winter season exceed the summer MVP's when water use is optimal because the probability of inflows is much less during the winter season, hence, the value of any water saved at the start of the winter is likely to be much greater than the value of water saved at the start of the summer season.

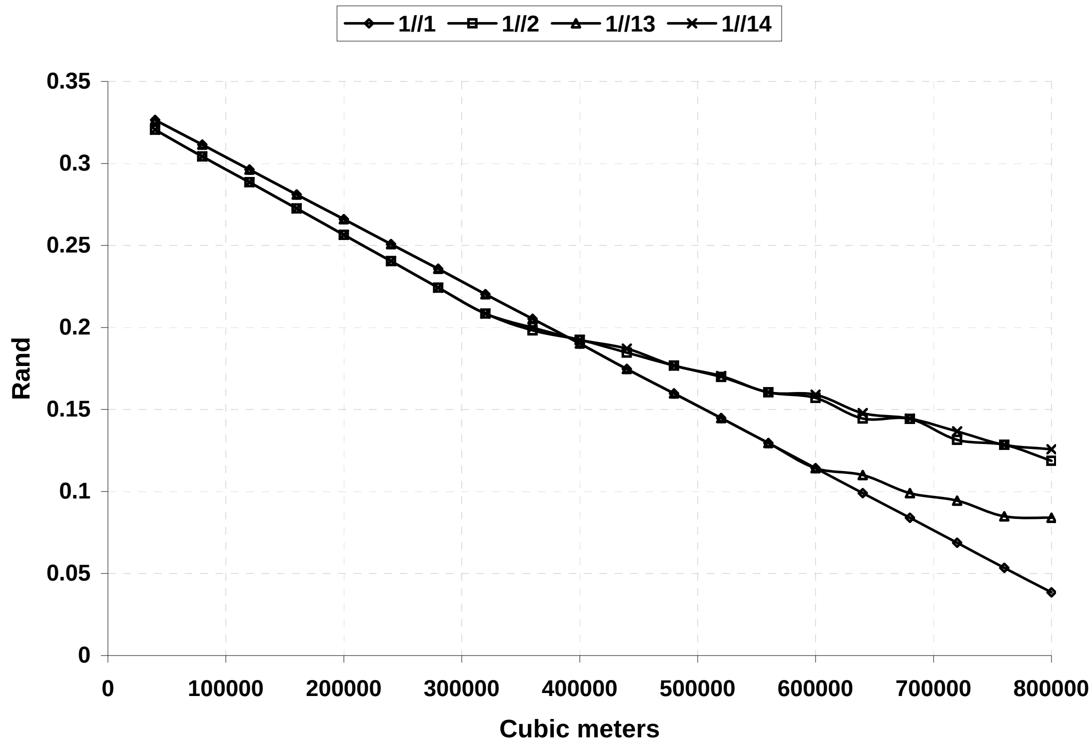

Figure 3: MVP in Rand per cubic meter of of CS contents for BC

Scenario 2 (CS and IS 75% of BC) - Figure 4

- Again the summer season MVP at the start of the last season in the planning horizon (2//1) comes directly from LP output. In this case the cut-off along the horizontal axis is at 600,000m^3.

- Water saving begins at less than 600,000 m^3 for the summer season (2//13), whereas it began at more than 600,000 m^3 for the summer season in the base case.

- Again the winter season MVP's are not responsive to the number of season left in the planning horizon, i.e. both winter cases are identical (2//2 and 2//14).

- Water saving during the winter begins at lower CS contents than in the base case, due to greater scarcity.

- Again the MVP's of winter season savings exceed those of the summer season.

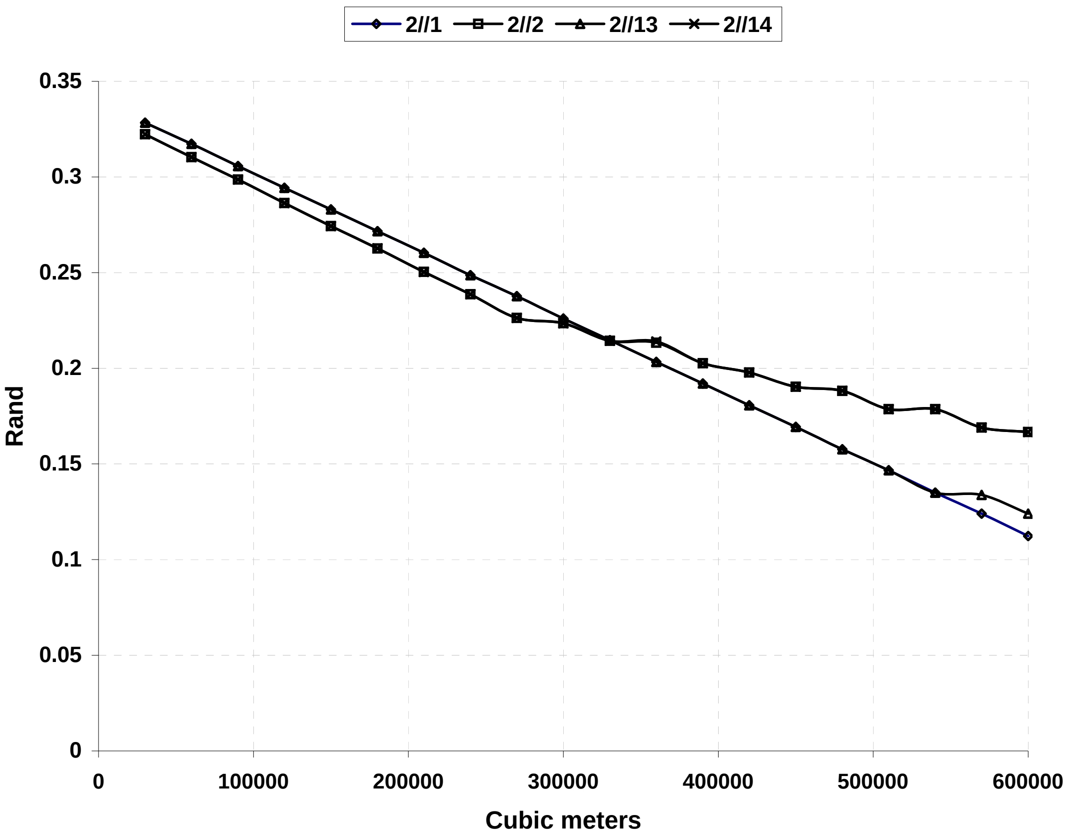

Figure 4: MVP in Rand per cubic meter of CS contents with CS and IS 75% of BC

Scenario 3 (CS and IS 50% of BC) - Figure 5

- The cut-off along the horizontal axis is at 400,000m^3

- No summer season water saving occurs regardless of the seasons left in the planning horizon.

- Again, the number of seasons remaining have no impact on winter MVP's.

- Winter water saving begins at lower CS contents than in previous cases, due to increasing water scarcity.

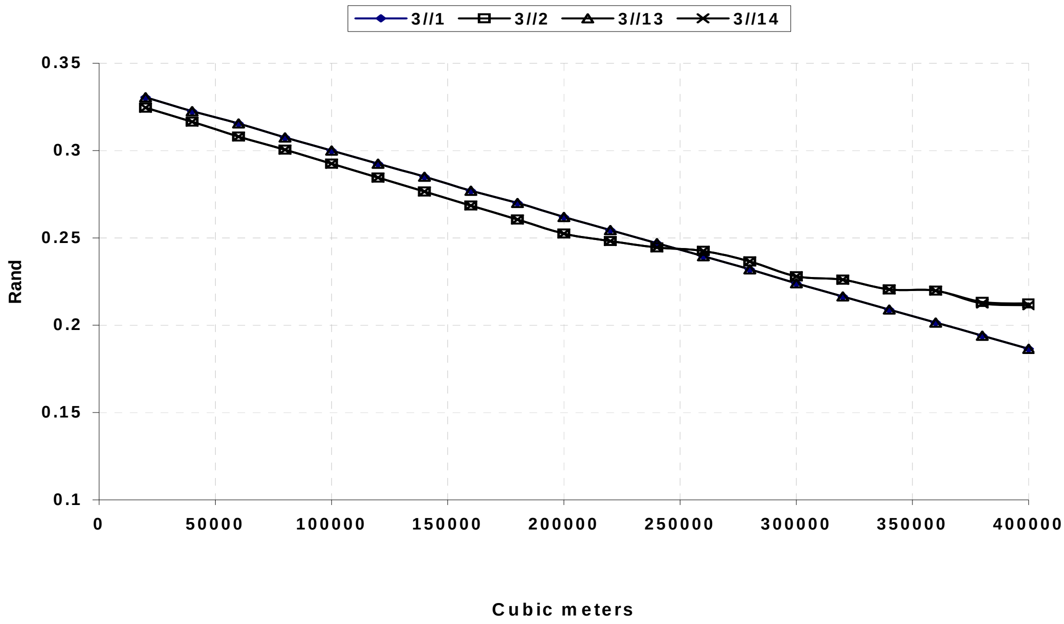

Figure 5: MVP in Rand per cubic meter of CS contents with Cs and IS 50% of BC

Scenario 4 (CS and IS 25% of BC) - Figure 6

- The cut-off along the horizontal axis is at 200 000m^3
- Winter water saving is less marked than before, but does begin at lower CS contents.

Scenario 5 (CS same as BC but IS doubles) - Figure 7

- The number of seasons left in the planning horizon makes no difference, due to higher inflows.
- No summer saving occurs, also due to higher probability of inflows.
- With water being more plentiful, winter water saving begins at higher levels of CS contents than in the base case, and has a lesser effect on MVP's.

Scenario 6 (CS same as BC but IS halves) - Figure 8

- Summer MVP is again the same as the LP MVP when only one season remains (6//1).

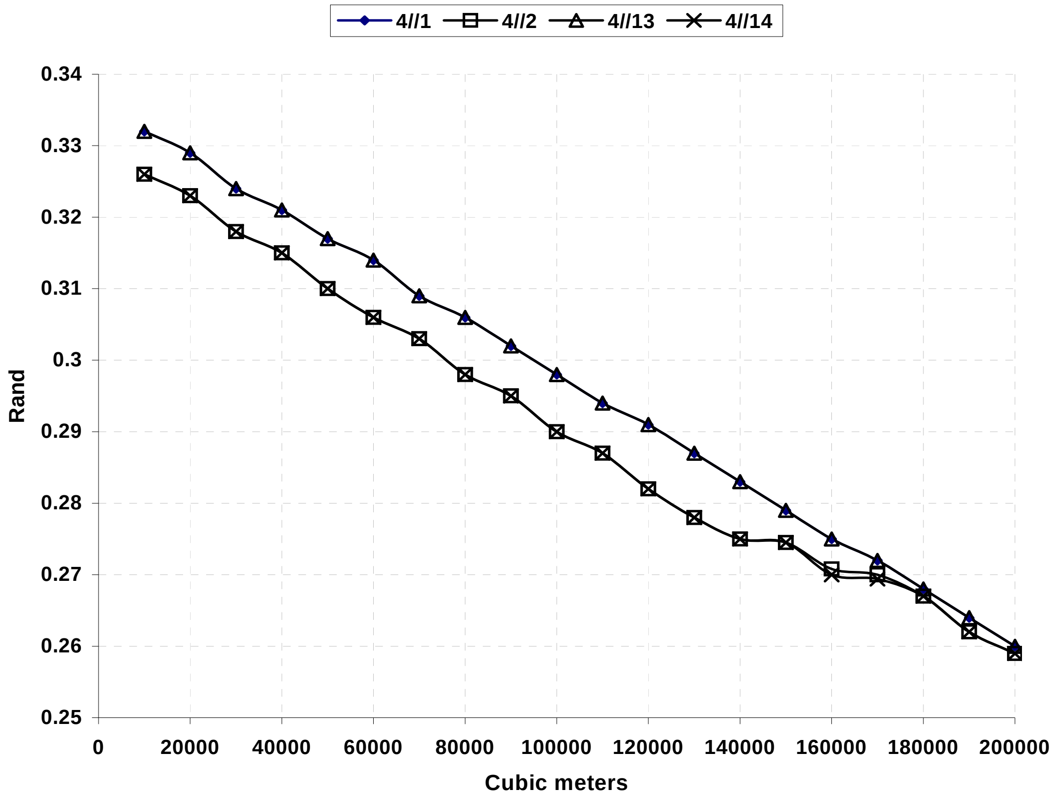

Figure 6: MVP in Rand per cubic meter of CS contents for CS and IS 25% of BC

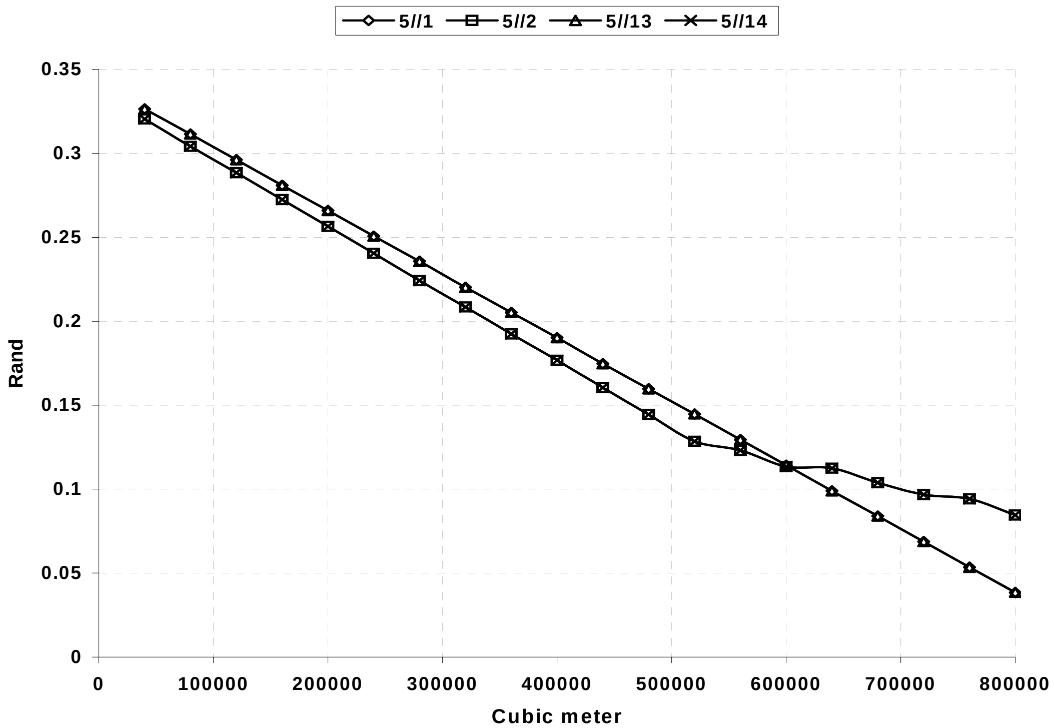

Figure 7: MVP in Rand per cubic meter of CS contents when IS doubles

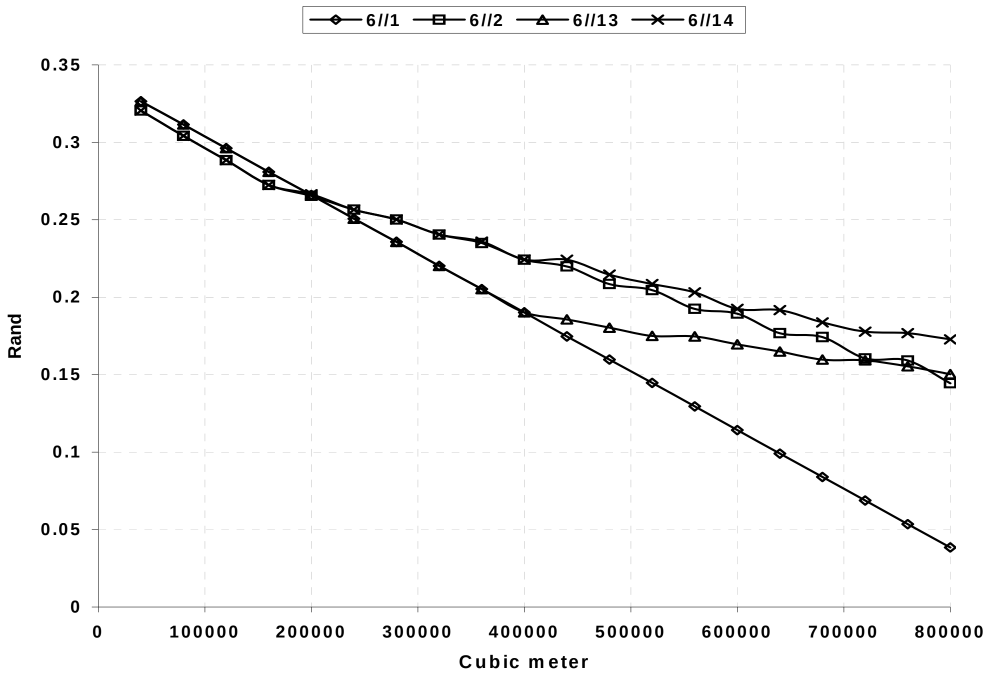

Figure 8: MVP in Rand per cubic meter of CS contents when IS halves

- Summer water saving begins at lower CS contents and is more valuable than in the base case.

- Winter water saving begins at lower CS contents, with the number of seasons remaining making a marked difference from just over 400,000m^3

Scenario 7 (IS same as BC but CS halves) - Figure 9

- The cut-off along the horizontal axis is at 400,000m^3.

- No water saving occurs, leaving both winter and summer MVPs identical to those from the LP, due to the high inflows relative to CS storage capacity.

Scenario 8 (CS and IS double BC) - Figure 10

- With one season left in the planning horizon (8//1) no summer saving occurs, resulting in the MVP becoming zero beyond 800,000m^3 which is the maximum quantity that the farm can profitably use.

- With many seasons to go, contents in excess of current season requirements have low positive MVP's.

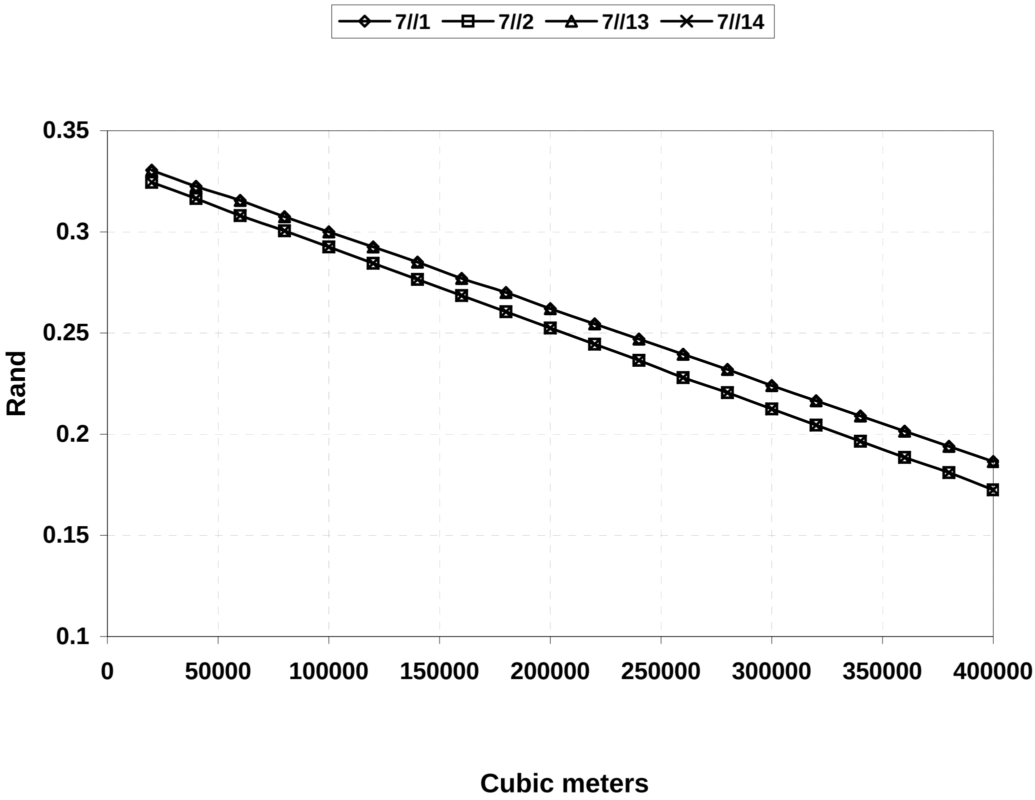

Figure 9: MVP in Rand per cubic meter of CS contents when CS halves

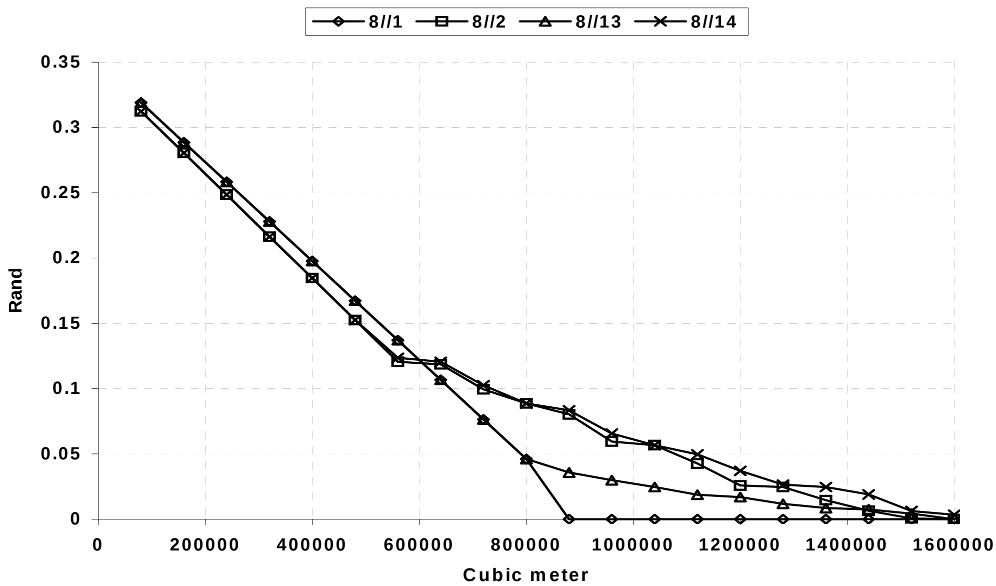

Figure 10: MVP in Rand per cubic meter of CS contents when CS and IS double

- Winter MVP's are rather similar regardless of the number of seasons left.

- Winter water saving begins at about 75% of maximum seasonal farm requirements.

MVP from SDP-with-simulation for long-term decisions

By using the optimal water decisions from the SDP model in a simulation model it is possible to determine marginal value products for different CS and IS sizes. This is important information for long-term decisions when transferring part of a CS or IS to other users or sectors. The MVP's in figure 11 shows what a farmer will loose in annual gross margins if he has to give up 1000 cubic meters of water, for different CS and IS sizes. These gross margins should however, be replaced by net margins if the fixed costs of the farm will also change as a result of changes in CS and/or IS.

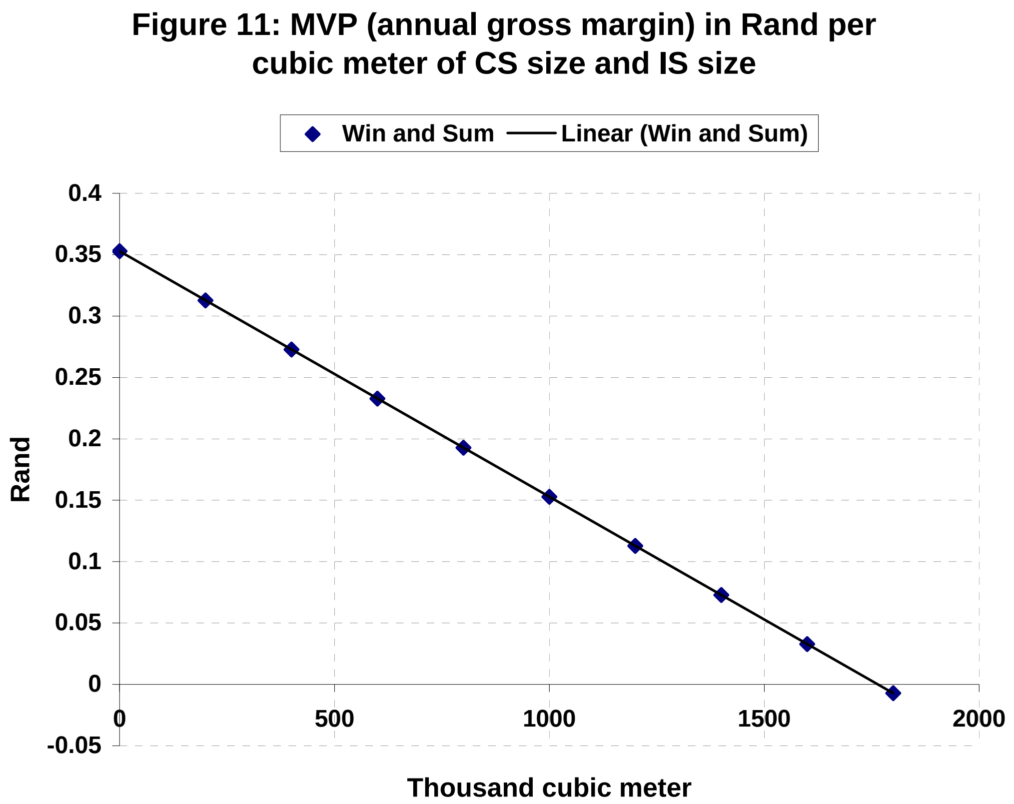

Figure 11: MVP (annual gross margin) in Rand per cubic meter of CS size and IS size

CONCLUSION

The MVP's from LP simulation, SDP and SDP–with-simulation provides useful information for water management decision making. Each of the three types of MVP's has its specific application. The MVP's from LP simulation are for immediate inter season decisions about water purchases (sales) from (to) a source other than the CS reservoir, e.g. groundwater [or surface water downstream of the dam]. The MVP's from SDP are for inter year decision-making, helping the farmer in optimal decision-making (in use, saving and water trading) over a number of years within the capacity share or inflow share constraints. The MVP's from SDP-with-simulation provide information to trade/transfer part of the inflow share size or capacity share size to other users or sectors. As time passes, and the relative scarcity of water increases, water trading/transfer will become more important, increasing the relevance of information on MVP's.

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