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A STUDY OF THE ORANGE JUICE COLOR SCORES MEASURED BY COLORFLEX CITRUS COLORIMETER

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A Study of the Orange Juice Color Scores Measured by ColorFlex Citrus Colorimeter

Section 20-65.002 of the Florida Department of Citrus (FDOC) Citrus Rules specifies six colorimeter or spectrophotometer devices to be used for measuring color scores of orange juice. The colorimeter or spectrophotometer used should be set, installed, and maintained in a manner acceptable to and approved by the Processed Products Branch, Fruit and Vegetable Division, Agricultural Marketing Service, United States Department of Agriculture (PPB/FVD/AMS/USDA), which should also be responsible to maintaining proper settings.

Any new colorimeter not specified by Section 20-65.002 of the FDOC Citrus Rules needs to be qualified by the USDA. Hunter Associates Laboratory, Inc. (HunterLab) developed the ColorFlex citrus colorimeter and with the intention to use this colorimeter in Florida orange juice processing plants to measure orange juice color scores. The ColorFlex citrus colorimeter is a standard HunterLab ColoFlex 45/0 LAV model fitted with sample clamp/port forward stand to orient ColorFlex to port forward position. The purpose of this study is to analyze the information of orange juice color scores collected in three Florida locations and determine whether the color scores measured by the ColorFlex citrus colorimeter and the color scores measured by the approved colorimeters are equal.

Data

Data were collected to study the relationship between the color scores measured by ColorFlex and Hunter D45 or Macbeth colorimeters. Three ColorFlex citrus colorimeters were placed at three locations: the USDA office in Winter Haven, Florida; Citrus World Inc. Florida's Natural Growers plant; and the Cargill Frostproof plant.

The ColorFlex citrus colorimeter (s/n 347) at the USDA office in Winter Haven was used side-by-side with the USDA Hunter D45 citrus colorimeter and a Macbeth 3100. The ColorFlex citrus colorimeter (s/n 492) at Citrus World Inc. Florida's Natural Growers plant was used side-by-side with a Macbeth 3100 colorimeter. The ColorFlex (s/n 493) at the Cargill Frostproof Plant was used side-by-side with the Hunter D45 citrus colorimeter. The information collected by the ColorFlex citrus colorimeters includes measurements of CR, CY, CN, X, Y, and Z. The information collected by the Hunter D45 colorimeters in Winter Haven and in Frostproof includes measurements of CR, CY, and CN. The information collected by Macbeth 3100 colorimeters in Winter Haven and in Citrus World Inc. Florida's Natural Growers' plants has only the measurement of CN.

Data from each location were manually entered for each location and coded into a spreadsheet. There are 331, 151, and 1,400 usable observations in Winter Haven USDA office, Cargill Frostproof plant, and Citrus World Inc. plant's records, respectively. The USDA protocol requires a minimum of 200 samples (with 25, 150, and 25 samples with color numbers less than 33.0, between 33.0 and 37.9, and greater than 37.9, respectively), or 400 samples in whatever range is available, whenever the minimum numbers of samples does not meet the ranges for instrument approval. It is evident that the number

of sample measured in the Cargill plant did not meet this minimum sample size requirement and the number of samples collected at the USDA Winter Haven office barely met this minimum requirement (see Table 1); therefore, we combined the samples from the Cargill plant and those from the USDA Winter Haven office. The samples in these two plants were mostly OJFM (orange juice for manufacturing) or FCOJ (frozen concentrated orange juice) with little additives such as calcium, pulp, and vitamin C.

The color numbers measured by the three different citrus colorimeters were converted to color scores using the table provided in the FDOC citrus codes Chapter 20-65. Note that the conversions from color number to color score are different for different OJ products. Color scores below 33 for most OJ products and 35 for COJ (canned orange juice) and COJFM (canned orange juice for manufacturing) were derived using the linear extrapolation method (see Table 2).

The purpose of the following analysis is to compare the color score measured by the ColorFlex citrus colorimeter and those reported by either the Hunter D45 or the Macbeth 3100 citrus colorimeters. The scatter diagrams for the color numbers are presented in Figures 1 and 2 and the scatter diagrams for color scores are presented in Figures 3 and 4, respectively for Citrus World and Winter Haven/Cargill.

Analytical Method

In order to explain the method used in this study to analyze the association of color score readings from ColorFlex and those measured by either the Hunter D45 or the Macbeth citrus colorimeters, we denote y_{1t} and y_{2t} as the color score reading of the *t*th OJ sample from ColorFlex and Hunter D45/Macbeth citrus colorimeters, respectively. Since the color score readings from these meters are not exactly the same, we assume that when they are equal, in a statistical sense, we can write

(1)
$$y_{1t} = y_{2t} + \varepsilon_t,$$

where ε_t is a disturbance term or a random error of the ColorFlex readings of the *t*th OJ sample. We assume that ε_t has a normal distribution with a zero mean and a constant variance. Under the assumptions of $\alpha = 0$ and $\beta = 1$, equation (1) can also be written as

(2)
$$y_{1t} = \alpha + \beta y_{2t} + \varepsilon_t$$
,

where α is the intercept and β is the slope of the linear relationship between y_{1t} and y_{2t} , i.e., when $\alpha = 0$ and $\beta = 1$, (2) represents a 45° line that goes through the origin (see line *a* in Figure 5). When the two color score readings are statistically different, we would have $\alpha \neq 0$ and/or $\beta \neq 1$. Several different possibilities are shown in Figure 5. Lines *b* and *c* show the cases when ColorFlex color score readings are consistently higher or lower than those readings by Hunter D45/Macbeth, respectively. Lines *d* and *e* show that when ColorFlex color score readings are not only consistently higher and lower than

those reported by Hunter D45/Macbeth, but also have greater and less than 1 slope, respectively. Other possibility includes a non-linear slope β .

There are several statistical methods available to study the association of color numbers measured by ColorFlex and Hunter D45/Macbeth citrus colorimeters. These methods include the simple correlation coefficient, paired-difference test, several non-parametric tests, and the ordinary least squares regression analysis of (2). Note that the color numbers measured by the ColorFlex and Hunter D45/Macbeth citrus colorimeters are not independent because the pairs of observations are linked, i.e., the color numbers are high or low depending upon the OJ sample used in the reading. Therefore, the simple *t*-test for the equality of two population means should not be used.

Simple correlation is an indicator of the strength of the linear relationship between two variables, in this case, the color numbers measured by the ColorFlex and Hunter D45/Macbeth citrus colorimeters. Simple correlation is independent of the respective scales of measurement of the two variables involved. If all values of two variables, say y_{1t} and y_{2t} , satisfy an equation exactly, the two variables are perfectly correlated, or there is perfect correlation between these two variables and the linear correlation coefficient equals either 1 (positively correlated) or -1 (negatively correlated). This is the case when ε_t in (2) has a zero mean and a zero variance. The range of linear correlation coefficient is between -1 and 1. Any correlation coefficient that is between -1 and 1 (does not include the end points) indicates imperfect correlation. For example, if all color numbers from the two citrus colorimeters are located exactly on any one of the lines in Figure 5, then there is perfect correlation of between the two sets of color numbers. Note that simple correlation coefficient does not provide any information about the magnitudes of α and β ; therefore, it is not an ideal measure for the equivalence of the pairs of color numbers reported by two different citrus colorimeters.

As shown in the scatter diagrams, Figures 1 through 4, none of the pairs of color numbers has perfect correlation, because the pairs of color score readings do not form an exact line. The sample linear correlation coefficients for each location are presented in Table 1.

The pair-difference test (Mendenhall et al., pp. 282-9) uses the statistics derived from the difference, d_t , of y_{1t} and y_{2t} is

(3)
$$d_t = y_{1t} - y_{2t},$$

and a *t*-statistic is used to test the hypothesis that d_t has a zero mean. This approach is equivalent to test whether $(\alpha + \varepsilon_t)$ has a zero mean under that assumption that $\beta = 1$, or we can rewrite (2) as

(4)
$$d_t = y_{1t} - \beta y_{2t} = y_{1t} - y_{2t} = \alpha + \varepsilon_t.$$

Generally speaking, the assumption of $\beta = 1$ is probably a strong assumption, therefore, the pair-difference test is not a proper statistical method for this study. In addition, when

the true $\beta \neq 1$, the pair-difference results in a large standard error for d_i , hence, results in not rejecting the hypothesis that $d_i = 0$ (see Appendix A for the derivation of variance of d_i). Other non-parametric methods for testing pair-differences, such as the sign test (Mendenhall, et al., pp. 290-5) or the rank test (Mendenhall, pp. 369-75), suffer similar assumption-related problems, and therefore were not used in this study.

Due to the above reasons, we will use the ordinary least squares regression method and a *F*-statistic to test the hypotheses of $\alpha = 0$ and $\beta = 1$ (Johnston, p. 185) in this study. Two sets of regression results will be estimated respectively for color numbers (CN) and color scores. Sample means and simple correlation coefficients are presented in Table 3; regression results, *F*-statistics, and *F*-test table values at $\alpha = .05$ level are presented in Table 4.

Results

As shown in Table 3, the simple correlations between the color numbers measured by ColorFlex and Hunter D45 and the resulting color scores are about 0.99 and 0.95 for Winter Haven/Cargill samples and about 0.91 and 0.81 for the color numbers measured by ColorFlex and Macbeth citrus colorimeters at the Citrus World plant, respectively. These simple correlation coefficients are relatively high, an indication that the color numbers measured by all citrus colorimeters and the resulting color scores are highly correlated. If we conduct the pair-difference test using the information provided in Table 3, we found that we cannot reject the hypothesis that there is no difference between the color numbers (and color scores) measured by ColorFlex and Hunter D45/Macbeth citrus colorimeters.

However, regression results indicate the opposite. Two sets of regression lines (2) using color numbers and color scores, respectively, were estimated. As shown in Table 4, all coefficient estimates for the intercept α are statistically different from zero, the intercept estimates for the Citrus World sample were statistically greater than zero (i.e., the color numbers and color scores obtained from ColorFlex citrus colorimeter were consistently higher than those obtained from Macbeth citrus colorimeter) while the intercept estimates for the Winter Haven/Cargill sample were statistically less than zero (i.e., the color numbers and color scores obtained from ColorFlex citrus colorimeter) while the intercept estimates for the Winter Haven/Cargill sample were statistically less than zero (i.e., the color numbers and color scores obtained from ColorFlex citrus colorimeter were consistently lower than those obtained from Hunter D45 citrus colorimeter). The slope coefficient estimates are either less than one (for the Citrus World sample) or greater than one (for the Winter Haven/Cargill sample). The joint *F*-test statistics reject the hypothesis that $\alpha = 0$ and $\beta = 1$ for the model represented by (2) for both cases. In other words, the color numbers measured by ColorFlex citrus colorimeter and the resulting color scores are statistically different from those measured by either the Hunter D45 or the Macbeth citrus colorimeter.

The OJ samples are translucent and are measured in a curved tube. The addition and homogeneity of various degrees of pulp, calcium, and vitamins, as well as the different juice types (POJ, OJFC, FCOJ, COJ, OM, etc.) will affect the translucency of the OJ sample, and contribute to the variation in the measurement. In order to examine the influence of these additives and juice types on the association of color numbers and color scores found with ColorFlex and Hunter D45/Macbeth, we added several dummy variables to equation (2). These dummy variables include CA (CA = 1 if the sample is calcium added), High Pulp (= 1 if the sample has additional pulp), No Pulp (=1 if the sample has no pulp), POJ (= 1 if the sample is pasteurized OJ), OJFC (= 1 if the sample of OJ from concentrate), CCOJ (= 1 if the sample is canned OJ), and OJM (= 1 if the sample is OJ for manufacturing). We hope that the inclusion of these dummy variables in the analysis would help identify whether the intercept equals zero and the color number (score) slope is unity. Regression results are presented in Table 4. Again, the *F*-statistics show that we reject the hypothesis that $\alpha = 0$ and $\beta = 1$.

Note that regression results show that the addition of calcium to the OJ and the juice types had significant influence on the differences between the color numbers (scores) measured by ColorFlex and Hunter D-45/Cargill citrus colorimeters.

Multiple linear regression equations were calculated using the ColorFlex color numbers (CN) as the dependent variable and the CIE X, Y, and Z values as independent variables (e.g., Buslig and Wagner 1985, 1988; Buslig et al.). Results shown in Table 5 indicate that CIE X, Y, and Z values explained about 87% and 93% of the variations in color numbers in the Citrus World and Winter Haven/Cargill datasets, respectively. The difference in the explanatory power of CIE X, Y, and Z values may be attributed to that more different types of OJ were measured in the Citrus World plant than those measured in the Winter Haven/Cargill plants. Perhaps, an indication that citrus color measurements are sensitive to additives, such as calcium in OJ, and the types of OJ measured.

Additional Analysis of Winter Haven Data

Even though the USDA Winter Haven dataset has less than 400 observations, but the richness of the measurements of CR, CY, and CN in this dataset provides an opportunity for us to analyze the relationships between the color numbers measured by Hunter D45 and Macbeth citrus colorimeters and the relationships of CR and CY measurements found with Hunter D45 and ColorFlex citrus colorimeters.

Regression results presented in Table 6 indicate (1) the color numbers measured by Macbeth and Hunter D45 citrus colorimeters are not equal; (2) the CY measured by ColorFlex and Hunter D45 are not equal; and (3) the CR measured by ColorFlex and Hunte D45 citrus colorimeters are equal. Results (2) and (3) may explain the differences between the color numbers measured by ColorFlex and Hunter D45 citrus colorimeters.

Conclusion

In this study, we analyzed the relationship between the color numbers and color scores measured by ColorFlex and Hunter D45/Macbeth citrus colorimeters. Based on the regression analyses and *F*-statistics, we found that these two sets of color numbers or color scores are not equal.

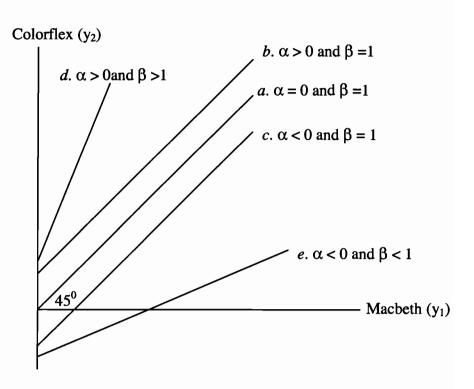


Figure 5. Regression line $y_{1t} = \alpha + \beta y_{2t} + \varepsilon_t$

	Measured by	<33.0	33.0 - 37.9	> 37.9
Minimum		25	150	25
Citrus World	Macbeth	5	1,393	2
	ColorFlex		1,384	16
Cargill	Hunter D45	1	137	13
-	ColorFlex	18	125	8
Winter Haven	Macbeth	27	275	29
	Hunter D45	24	279	28
	ColorFlex	30	275	26
Cargill+WH	Hunter D45	28	412	42
_	ColorFlex	48	400	34

Table 1.	Minimum san	nple size a	nd actual	sample sizes

Color Number	Color Score for			
Color Number	POJ and OJFC ^a	FCOJ and CCOJ	COJ and COJFM ^b	
_				
30.5-31.4 ^c	31	31	33	
31.5-32.4 ^c	32	32	34	
32.5-33.4	33	33	35	
33.5-34.4	34	34	36	
34.5-35.4		35	36	
34.54.9	35			
35.0-36.4	36			
35.5-36.4		36	37	
36.5-37.4	37	37	38	
37.5-38.4	38	38	38	
38.5-39.4	39	39	39	
39.5-40.4	40	40	40	

 Table 2.
 Color number and color score conversions

^a Includes OJFM, aseptic, and ATFI.
 ^b Includes dehydrated OJ, OM, tanker farm, and OMT.

^c Extrapolation.

Abbreviations: POJ: pasteurized orange juice; OJFC: orange juice from concentrate; FCOJ: frozen concentrated orange juice; CCOJ: canned concentrated orange juice; COJ: canned orange juice and dehydrated orange juice; OM or COJFM: concentrated orange juice for manufacturing, OJFM: orange juice for manufacturing; ATFI:

Variable	Mean	Std Dev	Minimum	Maximum	Simple Correlation
	·	Citru	s World (N =	1,400)	•
Color Number					
ColorFlex	36.36	0.83	33.08	39.75	
Macbeth	36.17	0.84	32.60	38.40	0.9076 ^a
Difference ^a	0.1862	0.3582	-1.97	4.64	
Color Score					
ColorFlex	36.46	0.75	33	40	
Macbeth	36.42	0.73	33	38	0.8122^{a}
Difference ^a	0.0414	0.4550	-2	4	
		Winter I	Haven/Cargill ((N = 482)	
Color Number					
ColorFlex	35.68	1.81	30.97	39.86	
Hunter D45	35.89	1.71	32.20	40.20	0.9870 ^b
Difference ^b	-0.2157	0.2999	-1.43	1.50	
Color Score					
ColorFlex	36.43	1.54	32	40	
Hunter D45	36.65	1.40	33	40	0.9480 ^b
Difference ^b	-0.2241	0.4949	-2	2	

Table 3. Sample color number statistics

^a Correlation or difference between ColorFlex and Macbeth color numbers. ^b Correlation or difference between ColorFlex and Hunter D45 color numbers.

Table 4. Regress								
	ColorFlex Color Number			ColorFlex Color Score				
Variable	Parameter	Standard	Parameter	Standard	Parameter	Standard	Parameter	Standard
	Estimate	Error	Estimate	Error	Estimate	Error	Estimate	Error
				Citrus	World			
Intercept	3.8562*	0.4022	4.2572*	0.4032	6.0338*	0.5846	6.9384*	0.6059
Macbeth Number	0.8985*	0.0111	0.8920*	0.0113				
Macbeth Score					0.8354*	0.0161	0.8157*	0.0169
Calcium			-0.2777*	0.0224			-0.1965*	0.0289
High Pulp			-0.0061	0.0324			-0.0020	0.0421
No Pulp			-0.0209	0.0739			0.0102	0.0959
POJ			-0.1131*	0.0333			-0.1386*	0.0431
OJFC			-0.1285*	0.0347			-0.2402*	0.0451
CCOJ			0.0105	0.0735			0.2990*	0.0981
R-sq	0.8237		0.8465		0.6596		0.6855	
Adj R-sq	0.8236		0.8457		0.6594		0.6840	
F-Statistic ^a	241.8423		149.5289		58.7937		98.0293	
			v	Vinter Hav	ven/Cargill			
Intercept	-1.7885*	0.2790	-1.7227*	0.2807	-1.9286*	0.5885	-2.0465*	0.6047
Hunter Number	1.0438*	0.0078	1.0428*	0.0078				
Hunter Score					1.0465*	0.0160	1.0504*	0.0167
ОЈМ			-0.0489*	0.0272			-0.0406	0.0477
R-square	0.9741		0.9743		0.8986		0.8988	
Adj R-sq	0.9741		0.9742		0.8984		0.8984	
F-Statistic ^b	148.6122		54.3663		51.1428		24.0002	

Table 4.	Regression	result.	F-statistics,	and F	'table values
14010 1.	regression	roualt,	I DIMIDUOU,		

*Statistically different from zero at $\alpha = 0.05$ level. ^a Table value for $F_{(2, 1398), \alpha=0.05} = 4.60$. ^b Table value for $F_{(2, 480), \alpha=0.05} = 4.65$.

Variable	Parameter	Standard
variable	Estimate	Error
	Citrus	World
Intercept	38.9059	0.0517
X	0.9754	0.0480
Y	-0.4320	0.0512
Z	-1.4133	0.0211
R-square	0.8723	
	Winter Hav	ven/Cargill
Intercept	40.99429	0.084237
X	3.259465	0.125082
Y	-3.12625	0.121249
Z	-0.57336	0.017174
R-square	0.9309	

Table 5. Regression analysis for color number and CIE X, Y, and Z values^a

^a ColorFlex citrus colorimeter measurements.

Variable	Parameter	Standard	F Statistics
v allable	Estimate	Error	r Statistics
	CN (Hu	nter D45 dependent	variable)
α (Intercept)	1.5865	0.1091	107.1964
β (Macbeth CN)	1.0440	0.0030	$F_{(2,329)} = 4.64^{a}$
R-square	0.9973		
		CY (ColorFlex)	
α (Intercept)	-10.6406	0.6817	338.0971
β (Hunter D45 CY)	1.1247	0.0089	$F_{(2,329)} = 4.64^{a}$
R-square	0.9797		
		CR (ColorFlex)	
α (Intercept)	0.5179	0.4908	3.7488
β (Hunter D45 CR)	0.9920	0.0152	$F_{(2, 329)} = 4.64^{a}$
R-square	0.9282		<u> </u>

Table 6. Regression analysis of the Winter Haven dataset

^a Table value for the F test at $\alpha = .05$ level. If F-statistics is greater than the table F-value, then we can reject the hypothesis that $\alpha = 0$ and $\beta = 1$.

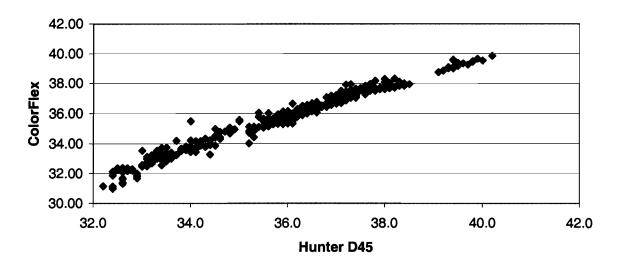
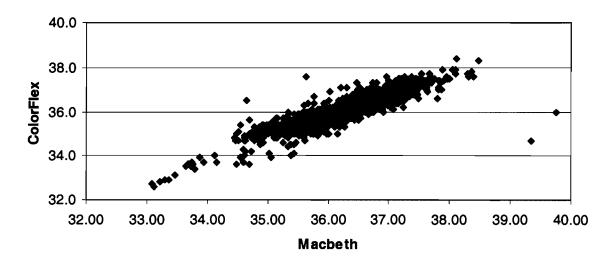


Figure 1. Color Number (Citrus World)

Figure 2. Color Number (Winter Haven/Cargill)



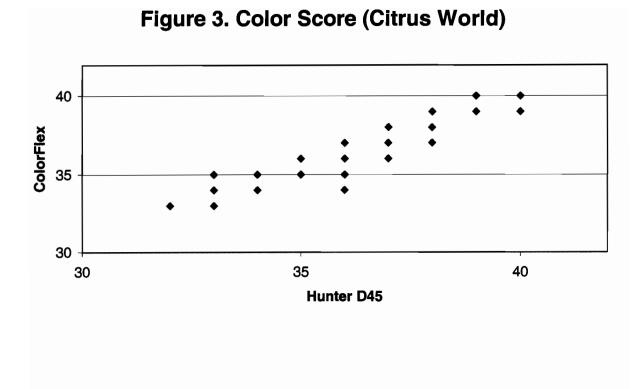
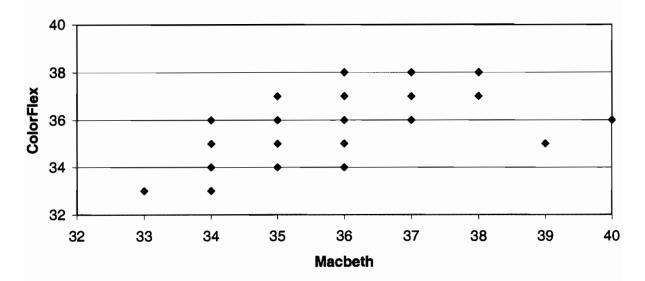


Figure 4. Color Score (Winter Haven/Cargill)



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Appendix A The Variance of Pair-Differences

Regression model (2)

(A1)
$$y_{1t} = \alpha + \beta y_{2t} + \varepsilon_t, \quad t = 1, 2, ..., T;$$

where y_{1t} and y_{2t} are the color number (or score) readings of the *t*th OJ sample from ColorFlex and Hunter D45/Macbeth citrus colorimeters, respectively; and $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \sigma^2 I$. The true pair-difference model is

(A2)
$$y_{1t} - y_{2t} = \alpha + (\beta - 1)y_{2t} + \varepsilon_t, \text{ or} d_t = \alpha + (\beta - 1)y_{2t} + \varepsilon_t \qquad t = 1, 2, \dots, T$$

In matrix notation, (A2) can be written as

(A3)
$$D = i\alpha + Y_2(\beta - 1) + \epsilon$$

where D is a column vector of d_{ts} , *i* is a column vector of 1s, Y₂ is a column vector of y_{2ts} , and ε is a column vector of ε_{ts} . The mis-specified model is

$$(A4) D = i\alpha + u,$$

where $u = Y_2(\beta - 1) + \epsilon$. Under the assumption that E(u) = 0 and $E(uu') = \sigma^{*2}I$, we have

(A5)
$$\hat{\alpha} = (i'i)^{-1}i'D$$
 and $E(\hat{\alpha}) = \alpha + (\beta - 1)\overline{y}_2$, and
var $(\hat{\alpha}) = \sigma^{*2}(i'i)^{-1}$.

Where $\overline{y}_2 = \sum_t y_{2t}/T$. The variance σ^{*2} becomes

(A6)

$$\sigma^{*^{2}} = e^{\prime}e^{\prime}(T-1)$$

$$= D^{\prime}MD/(T-1)$$

$$= (1/(T-1))(i^{\prime}\alpha + Y_{2}'(\beta - 1) + \epsilon^{\prime})M(i\alpha + Y_{2}(\beta - 1) + \epsilon)$$

$$= (1/(T-1))(\epsilon^{\prime}M\epsilon + (\beta - 1)Y_{2}'MY_{2}(\beta - 1)), \text{ and}$$

$$\tilde{Exp}(\sigma^{*^{2}}) = \sigma^{2} + (\beta - 1)^{2}Y_{2}'MMY_{2}/(T-1)$$

$$= \sigma^{2} + (\beta - 1)^{2}\Sigma_{t} (y_{2t} - \bar{y}_{2})^{2}/(T-1);$$

where $e = D - i\alpha$ and $M = I - i(i'i)^{-1}i'$ is an idempotent matrix (i.e., M' = M and MM = M). Therefore, when $\beta = 1$, $\sigma^{*2} = \sigma^2$. However, when $\beta \neq 1$, the estimate of σ^{*2} would be greater than the true variance σ^2 , since $(\beta - 1)^2 \sum_t (y_{2t} - \overline{y}_2)^2 / (T - 1)$ is a positive quantity.

Hunter D45

$$CR = 200(1.277 X - 0.213 Z)Y - 1$$

$$CY = 100(1 - 0.8477/Y)$$

$$CN = 22.51 + 0.165 CR + 0.111 CY, or$$

$$CN = 0.61 + 42.14 X/Y - 16.43 Z/Y$$

ColorFlex

CR = -469.800 + 542.475 X/Y - 54.707 Z/Y + 75.447/Y
CY = 134.900 – 28.329 X/Y – 117.722 Z/Y – 19.163/Y
CN = -40.600 + 86.597 X/Y - 22.073 Z/Y + 18.309/Y