RESEARCH PAPER: 2003-3

FLORIDA CITRUS PLANTING DECISIONS: IMPACTS OF EXPECTED RETURNS AND PRICE RISKS ON VARIETAL CHOICE

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Abstract

A model is developed to explain Florida citrus planting levels by variety. The varietal choice is based on the expected prices and price variances/covariances of the varieties under consideration. Overall planting returns are maximized for a given level of price risk. The model's price coefficients are similar to those of the Theil and Barten Rotterdam demand model. As in the Rotterdam model, both absolute and relative price coefficient specifications are considered, allowing an examination of restrictions related to price risk. The empirical analysis considers two restricted specifications—a varietal independence model, based on the assumption that only the price variances are important for predicting planting levels, and a group independence model that additionally assumes the price covariance for two varieties from the same (different) group(s) is nonzero (zero). Both models substantially reduce the parameter space and may be of interest in situations where lack of degrees of freedom is an issue. A varying parameter model was also estimated, indicating that with growth in the world orange-juice market, orange prices have been perceived to be less unstable, and orange planting responses to changes in total citrus acreage planted and orange prices have been stronger.

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Introduction

This study examines how Florida citrus growers allocate acreage to different citrus varieties. A model is developed in which acreage is allocated based on the expected prices and price variances/covariances of the varieties. Allocating citrus acreage is viewed like allocating wealth across various assets in a portfolio (e.g., Bewley, 1986). The modeling approach taken in the present study is similar to that suggested by Barten and Vanloot (1996) to determine how farmers allocate land to various crops. The Barten and Vanloot model is specified in differences and its price coefficients are specified following the approach taken in the absolute price version of the Rotterdam demand model (Theil, 1971, 1975, 1976, 1980). In contrast, the model developed here to determine varietal choice is specified in levels and its price coefficients are specified following the approach taken in the relative price version of the Rotterdam model. For some analyses, modeling in terms of levels, as opposed to differences, may be called for; and the relative price specification may be useful to examine restrictions on the allocation decision. In the case of demand, the relative price version allows restrictions for various types of separability of the utility function. For the present varietal choice problem, the relative price version allows restrictions on the risk part of the planting objective function. Finally, this study examines a varying parameter specification, allowing changes in grower perceptions of orange price variances.

Acreage Allocation Model

The problem of determining the number of acres to be planted by citrus variety is viewed like the investment problem of choosing different assets in a portfolio (e.g., Bewley). In the investment problem, overall portfolio returns are maximized for a given level of risk. Similarly, overall expected returns to planting various citrus varieties are maximized for a given level of price variability. In this problem, the allocation of acreage by variety is specified as being dependent on the individual varietal returns and their covariance matrix. First, a model focusing on the variances is developed, followed by a generalization which considers both the variance and covariance terms, and a specification that allows straightforward imposition of restrictions on these model components.

Varietal Independence Model

Consider the grower objective function

(1) $\sum_i a_i p^e_i - (k/2)\sum_i a^2_i \sigma^2_i$

where a_i is the acreage allocated to variety i; p^e_i is the expected return per acre for the variety; k is a positive constant; and σ^2_i is the variance in the return for the variety.

The variance term σ^2_i is $E(p_i - p^e_i)^2$, where p_i is the actual return for variety i, and $(p_i - p^e_i)$ is the error in the return, with an expectation $E(p_i - p^e_i) = 0$. The term $a^2_i \sigma^2_i$ is the variance in the return to the acres allocated to variety i $(a_i p_i)$, and $\sum_i a^2_i \sigma^2_i$ is the *perceived* variance in overall returns. The word perceived is used because there may be unperceived variance/covariance components.

The covariance terms defined by $E(p_i - p^e_i)(p_j - p^e_j)$, $i \neq j$ will be subsequently considered but initially these terms are assumed to be zero. Even if covariances are non-zero, in some cases, growers may not be aware of their values or be able to recall exactly how the returns for any two varieties move together.[1] Overall, we allow for the possibility that grower perceptions of the variances/covariances may differ from their actual values, and in the subsequent model, the coefficients depend on perceived values, not necessarily actual ones.

Equation (1) is maximized subject to the amount of land available for planting . Formally, the constraint is $A = \sum_i a_i$, where A is the total acreage available to be planted,[2] and the Lagrangian function for choosing a_i is

(2) $L = \sum_i a_i p^e_i - (k/2)\sum_i a^2_i \sigma^2_i + \lambda(A - \sum_i a_i)$

where λ is the Lagrange multiplier.

The first order conditions for problem (2) can be written as

(3) $p^e_i - \sigma^*_i a_i = \lambda,$

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[1] The returns for some of the citrus varieties tend to move independently as the markets by variety are often driven by different factors. For example, the Florida orange market has been strongly impacted by factors in the world OJ market like Brazil's supply as well as factors in the domestic market like preference changes related to the Atkins diet; on the other hand, the grapefruit market has been impacted by preference changes related to drug interactions and exchanges rates (much of Florida's grapefruit is exported in either fresh or processed form, while a much smaller percentage of oranges is exported and little specialty citrus is exported); the specialty citrus market demand is mostly a domestic one with important factors being the availability of substitutes like Clementines and other fresh fruit.

[2] One might consider how A is determined, but for this study we take it as given based on grower investment decisions in land and the acreage available for replanting due to tree losses.

where $\sigma^*_i = k\sigma^2_i$, and, the constraint,

(4) $A = \sum_i a_i$.

The Hessian matrix is a diagonal matrix with negative diagonal elements, $-k\sigma^2_i$, given the variances (σ^2_i) are positive. Hence, the Hessian matrix is negative definite and the second order conditions are meet.

To obtain our model from these results, we first solve for the Lagrange multiplier, by dividing equations (3) by σ^*_i and summing over i to find

(5) $\sum_i p^e_i / \sigma^*_i - \sum_i a_i = \sum_i \lambda/\sigma^*_i$,

or, rearranging, and using constraint (4),

(6) $\lambda = (\sum_i p^e_i / \sigma^*_i - A) / \sum_i (1/\sigma^*_i)$.

Substituting result (6) into equation (3), and rearranging, we obtain our acreage allocation model

(7) $a_i = \beta_i A + \phi \beta_i (p^e_i - \sum_j \beta_j p^e_j)$,

where

(8) $\beta_i = (1/\sigma^*_i)/(\sum_j 1/\sigma^*_j)$
$= (1/\sigma^2_i)/(\sum_j 1/\sigma^2_j)$,

since $\sigma^*_i = k\sigma^2_i$ (the term k in the numerator and denominator cancels out), and

(9) $\phi = \sum_j (1/\sigma^*_j)$
$= (1/k)\sum_j \sigma^2_j$.

The term ($1/\sigma^2_i$) can be viewed as a measure of the certainty of p_i; if the variance of p_i is large we have less certainty in the price. The term $\sum_j (1/\sigma^2_j)$ can be viewed as measure of the overall price certainty across varieties. Thus, β_i indicates the relative price certainty for variety i, while the term ϕ measures the general impact of price uncertainty in the model (the more important price risk is in problem (1) or the higher k, the lower ϕ, and the lower the impact of prices in the planting decision).[3]

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[3] Although we will initially treat the coefficients β_i and ϕ as constant in estimating model (7), note that an increase in the variance σ^2_i, or a decrease in the level of price certainty of variety i, results in decreases in β_i and π_{ii}; i.e., $\partial\beta_i/\partial\sigma^2_i = (\beta_i-1)\beta_i/\sigma^2_i < 0$ and $\partial\pi_{ii}/\partial\sigma^2_i = (\beta_i-1)\pi_{ii}/\sigma^2_i < 0$. Given constraint (8), the increase in σ^2_i further results in increases in the other β_j's, i.e., $\partial\beta_j/\partial\sigma^2_i = \beta_i\beta_j/\sigma^2_i > 0$. Likewise, given constraints (8), (9), and (10), the increase in

Given variances are positive, the parameters (8) and (9) obey the inequalities $0 < \beta_i < 1$ and $\phi > 0$.

In equation (7), own- and cross-price effects can be defined by $\pi_{ij} = \phi\beta_i(\Delta_{ij} - \beta_j)$, where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if $i=j$; $\Delta_{ij} = 0$ if $i \neq j$. The price effects of the above model are similar to those in Theil's preference independent demand model or those in his uniform substitute demand model (Theil, 1980) . Thus, model (7) will be subsequently referred to as a varietal independence model, given the underlying price covariances are zero. In estimating this model, the parameters ϕ and the β_i's will initially be treated as constants. Alternatively, if the price coefficients were specified with the π_{ij}'s as constants, we would have a model similar to the absolute price version of the Rotterdam model.

When $dA = 1$ or A increases by one acre, the change in acreage allocated to variety i is β_i. Hence, the term β_i can be viewed as a scale coefficient.

If only the expected price of variety i changes (dp^e_i), the change in acreage allocated to variety i is determined by $\phi\beta_i(1-\beta_i)dp^e_i$. Hence, $\pi_{ii} = \phi\beta_i(1-\beta_i)$ can be viewed as an own-price coefficient. This parameter is positive, given ϕ is positive and β_i is between zero and one as noted above.

Likewise, when only the expected price of variety j changes (dp^e_j), the change in acreage allocated to variety i is determined by $-\phi\beta_i\beta_j\, dp^e_j$, and $\pi_{ij} = -\phi\beta_i\beta_j$ is viewed as a cross-price coefficient. This parameter is negative.

These scale and price coefficients obey the following properties.

Adding-up ($\sum_i a_i = A$):

(10) $\sum_i \beta_i = 1, \qquad \sum_i \pi_{ij} = 0.$

Homogeneity (changing all expected prices by the same amount, with total acreage constant, leaves the allocations unchanged):

(11) $\sum_j \pi_{ij} = 0.$

Symmetry of price effects:

(12) $\pi_{ij} = \pi_{ij}.$

σ^2_i results in increases in π_{ji} and π_{ij} $j \neq i$. It also follows from the foregoing changes and constraint (8), (9) and (10) that the increase in σ^2_i results in decreases in the other own-and cross- price coefficients.

Generalization

Considering both variances and covariances in the citrus returns, the grower objective function can be written as

(13) $a'p^e - (k/2)a'(M + g\iota\iota')a$

where $a = (a_1, ..., a_n)$ and $p^e = (p^e{}_1, ..., p^e{}_n)$ are nx1 vectors with a_i and $p^e{}_i$ defined as above; k is again a positive constant; and $M + g\iota\iota'$ is the covariance matrix for expected returns with M being an nxn matrix of specific covariance terms, ι being an nx1 unit vector, and g being a positive scalar representing a common tendency for returns to move together. That is, for the risk part of the objective function, the error in returns is $(p - p^e)$; its expectation is $E(p_i - p^e{}_i) = 0$; and its covariance matrix is $E(p - p^e)(p - p^e)' = M + g\iota\iota'$, where p is the vector of actual returns. Hence, $a'(p - p^e)$ is the error in overall expected returns across the allocated acres, and $a'(M + g\iota\iota')a$ is its covariance matrix.

The specific covariance matrix M reflects risks originating say from market conditions, while the general covariance matrix $g\iota\iota'$ reflects common risks, originating say from weather and Statewide growing conditions. As shown below the common variation does not play a role in the allocation. On the other hand, the structure of the specific matrix M underlies the subsequent model restrictions analyzed.

Like equation (1), equation (13) is maximized subject to the constraint $A = \iota'a$ or the amount of land available for planting. Formally, the Lagrangian function for choosing a_i is

(14) $L = a'p^e - (k/2)a'(M + g'\iota\iota')a + \lambda(A - \iota'a)$.

The first order conditions for problem (14) can be written as

(15) $p^e - k(M + g\iota\iota')a = \lambda\iota$,

and, the constraint,

(16) $A = \iota'a$.

The Hessian matrix is $-k(M + g\iota\iota')$ which is negative definite, given k is positive and $M + g\iota\iota'$ is a positive definite covariance matrix. Thus, the second order conditions are meet.

Based on constraint (16), equation (15) can be expressed as $p^e - kMa - kgA\iota = \lambda\iota$. Pre-multiplying both sides of this expression by $(1/k)\,M^{-1}$, and solving for vector a gives

(17) $a = (1/k)M^{-1}p^e - gAM^{-1}\iota - \lambda(1/k)M^{-1}\iota$.

Equation (17) shows the acreage allocation, conditional on the Lagrange multiplier λ.

The solution for λ is[4]

(18) $\lambda = (\iota' M^{-1}\iota)^{-1}\iota' M^{-1}p^e - k((\iota' M^{-1}\iota)^{-1} + g)A.$

Substituting result (18) into equation (17) yields

(19) $a = (1/k)M^{-1}p^e - gAM^{-1}\iota - [(\iota' M^{-1}\iota)^{-1}\iota' M^{-1}p^e - k((\iota' M^{-1}\iota)^{-1} + g)A](1/k)M^{-1}\iota,$

$= (\iota' M^{-1}\iota)^{-1}M^{-1}\iota A + (1/k)\iota' M^{-1}\iota\, [\, (\iota' M^{-1}\iota)^{-1}\, M^{-1} - (\iota' M^{-1}\iota)^{-1}\, (\iota' M^{-1}\iota)^{-1}\, M^{-1}\iota\, \iota' M^{-1}]p^e,$

$= \beta A + (\phi_0/k)\, [B - \beta\, \beta']p^e,$

where

(20) $\beta = [\beta_i] = \phi_0^{-1}M^{-1}\iota,$

and

(21) $B = \phi_0^{-1}\, M^{-1},$

with $\phi_0 = \iota' M^{-1}\iota$. Given M is positive definite, its inverse is positive definite and ϕ_0 is positive.

Equation (19) is our generalized acreage allocation model. Note that in this model the common variation involving the term g cancels out and does not play a role in determining the varietal planting levels.

In terms of individual elements equation (19) can be written as

(22) $a_i = \beta_i A + (\phi_0/k)\sum_j(\beta_{ij} - \beta_i\, \beta_j)p^e_j,$

$= \beta_i A + (\phi_0/k)[\sum_j\beta_{ij}\, p^e_j - \beta_i \sum_k\beta_k p^e_k],$ (distributive law)

$= \beta_i A + (\phi_0/k)[\sum_j\beta_{ij}\, p^e_j - \sum_j\beta_{ij} \sum_k\beta_k p^e_k],$ ($\sum_j \beta_{ij} = \beta_i$)

$= \beta_i A + (\phi_0/k)\sum_j\beta_{ij}\, (p^e_j - \sum_k\beta_k p^e_k),$ (distributive law)

where $[\beta_{ij}] = B$; and $\sum_j \beta_{ij} = \beta_i$ and $\sum_i \beta_{ij} = \beta_j$, given $[\beta_i] = \phi_0^{-1}M^{-1}\iota$. Given M is symmetric, $\beta_{ij} = \beta_{ji}$. Further, note that adding-up, homogeneity and symmetry—restrictions (10), (11) and (12), respectively—also hold for model (22) with $\pi_{ij} = (\phi_0/k)(\beta_{ij} - \beta_i\, \beta_j)$. As in the case for the varietal

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[4] To find result (18) multiply both sides of result (17) by ι' and use constraint (16) to find $A = (1/k)\iota' M^{-1}p^e - gA\iota' M^{-1}\iota - \lambda(1/k)\iota' M^{-1}\iota$. Multiplying both sides of this result by k $(\iota' M^{-1}\iota)^{-1}$ and rearranging terms find $\lambda = k\, (\iota' M^{-1}\iota)^{-1}\, [(1/k)\iota' M^{-1}p^e - (1 + g\iota' M^{-1}\iota)A]$ or equation (18).

independent model, the own-price effects (π_{ii}) are positive;[5] the scale coefficients (β_i), however, no longer need be positive.

Based on the above results, $\beta_{ii} = \beta_i - \sum_{j \neq i} \beta_{ij}$, and equation (22) can be written as

$$(23) \quad a_i = \beta_i A + (\phi_0/k)(\beta_i - \sum_{j \neq i} \beta_{ij})(p^e_i - \sum_j \beta_j p^e_j) + (\phi_0/k)\sum_{j \neq i} \beta_{ij} (p^e_j - \sum_j \beta_j p^e_j),$$
$$= \beta_i A + (\phi_0/k)\beta_i(p^e_i - \sum_j \beta_j p^e_j) + (\phi_0/k)\sum_{j \neq i} \beta_{ij} (p^e_j - p^e_i).$$

Equation (22) is a convenient form of the model as cross restrictions on the covariance matrix M imply restrictions on the coefficients β_{ij}.

In context of the literature on differential demand, model (22) or (23) is a relative-price specification with the coefficients β_i, β_{ij} and $\phi = \phi_0/k$ treated as constants. In contrast, in the absolute-price specification, the price coefficients would be defined as $\pi_{ij} = (\phi_0/k)(\beta_{ij} - \beta_i \beta_j)$; and the π_{ij}'s, along with the scale coefficients, the β_i's, would be treated as constants. That is, formally, the absolute-price specification is

$$(24) \quad a_i = \beta_i A + \sum_j \pi_{ij} p^e_j.$$

As shown by Theil, the relative price version cannot be estimated unless at least one restriction is placed on the model's coefficients. For example, if $[\beta_{ij}]$ were diagonal ($\beta_{ij} = 0$, $i \neq j$), the last term on the right-hand side of model (23)—second line— vanishes leaving us with the varietal independence model, equation (7). But less severe restrictions might also be considered. In this study, we have three groups of citrus, each comprised of similar varieties; and we examine the possibility that only the β_{ij} for i and j belonging to different groups are zero (β_{ij} for i and j for two varieties within a group can be nonzero).

Given $\beta_i = \partial a_i/\partial A$ and $\pi_{ij} = \partial a_i/\partial p^e_j$, the elasticities for the acreage allocation model are

$$(25) \quad \epsilon_i = \beta_i (a_i / A) \qquad \text{(elasticity of variety i acres with respect to total acreage planted)}$$

and

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[5] The price coefficients in terms of matrices are

$$\text{(i)} \quad \pi = (\phi_0/k)(\phi_0^{-1}M^{-1} - \phi_0^{-1} \phi_0^{-1} M^{-1}\iota\iota' M^{-1})$$
$$= (1/k)(M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1}).$$

Pre- and post multiplying the covariance matrix M by π gives

$$\text{(ii)} \quad \pi M \pi = (1/k)(M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1})M (M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1})$$
$$= (1/k)(I - \phi_0^{-1} M^{-1}\iota\iota')(M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1})$$
$$= (1/k)(M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1}) - \phi_0^{-1} M^{-1}\iota\iota' M^{-1} + \phi_0^{-1} M^{-1}\iota\iota' M^{-1}$$
$$= (1/k)(M^{-1} - \phi_0^{-1} M^{-1}\iota\iota' M^{-1})$$
$$= \pi.$$

Let α be a vector whose elements are zero except for element i which is one. Then, $\alpha' \pi M \pi \alpha = \alpha' \pi \alpha = \pi_{ii}$. Given M is positive definite $\pi_{ii} > 0$.

(26) $\epsilon_{ij} = \pi_{ij} (a_i / p^e_j)$ (elasticity of variety i acres with respect to the expected price of variety j).

Application

The empirical study examines Florida acreage planting levels for five citrus varieties—1) early and midseason oranges, 2) late season, Valencia oranges, 3) white seedless grapefruit[6], 4) red seedless grapefruit, and 5) specialty citrus (Temple oranges, tangelos, and tangerines). Data on acres planted were obtained from various issues of the"Commercial Citrus Inventory," published by the Florida Agricultural Statistics Service (FASS). Citrus returns were measured by on-tree prices per acre based on on-tree revenue reported by the Florida Agricultural Statistics Service (FASS) in various issues of "Citrus Summary." On-tree prices per acre were calculated as total on-tree revenues divided by the associated number of bearing acres. Annual planting levels from 1964 through 2003 were studied, using on-tree prices from 1956 to 2003 to construct the expected price variables.[7] The on-tree prices were deflated by the consumer price index (U.S. Department of Labor, Bureau of Labor Statistics). Descriptive statistics for the acres planted and prices are shown in Table 1.

An adaptive expectations specification was used to model the expected prices in the model as in the Barten and Vanloot study:

(27) $p^e_{i,t} = \gamma p_{i,t-1} + (1-\gamma)p^e_{i,t-1},$

where γ is a scalar between zero and one. Through repeated substitution, equation (27) can also be written as

(28) $p^e_{i,t} = \sum_{k=1 \text{ to } t} \gamma(1-\gamma)^{k-1} p_{i,t-k} + (1-\gamma)^t p^e_{i,0}.$

Based on result (28), the model's price term, $\sum_j \pi_{ij}\, p^e_j$, can be written as

(29) $\sum_j \pi_{ij} (\sum_{k=1 \text{ to } t} \gamma(1-\gamma)^{k-1} p_{j,t-k} + (1-\gamma)^t p^e_{j,0}),$

or,

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[6] Includes seedy grapefruit.

[7] The first commercial citrus inventory, from which the planting levels were obtained, was made in 1965, while complete data on all prices were available back to 1956.

(30) $\sum_j \pi_{ij} p^*_j + \alpha_i (1-\gamma)^{t+1}$,

where $p^*_j = \sum_{k=1 \text{ to } t} \gamma(1-\gamma)^{k-1} p_{j,t-k}$ and $\alpha_i = \sum_j \pi_{ij} p^e_{j,0}$. The term p^*_j is that part of the expected price based on the observed prices, while the term $(1-\gamma)^t p^e_{j,0}$ is that part based on the pre-sample expected price. In the empirical analysis, $(1-\gamma)^{t+1}$ is treated as a variable and α_i is treated as a coefficient to be estimated.

The equation error terms were assumed to be contemporaneously correlated, and the system of planting equations were jointly estimated by the maximum likelihood procedure (TSP), obtained by iterating the seemingly unrelated regression method. The homogeneity and symmetry restrictions were imposed as part of the maintained hypothesis. As the data add up—the sum of the dependent variables, acres planted (a_i), equals one of the independent variables, total acreage (A)— the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten,1969). The parameters for the excluded equation can be obtained using conditions (10), (11) and (12) or by re-estimating the model omitting a different equation. The expected price parameter γ was set to maximize the likelihood value of the system of equations, based on a grid search (Maddala). The value of γ chosen was .72 .

Three specifications of the acreage allocation model were estimated—unrestricted model (24), varietal independence model (7), and a group independence model, based on a restriction imposed on model (23). For defining the group model as well as for reporting the various model estimates, the variety subscript is defined as follows:

i=1 for early and midseason oranges,
i=2 for Valencia oranges,
i=3 for white seedless grapefruit,
i=4 for red seedless grapefruit,
i=5 for specialty citrus.

For the group independence model, there are three groups—a) oranges (i=1 and 2), b) grapefruit (i= 3 and 4) and c) specialty (i=5). For this model the coefficient $\beta_{ij} = 0$ or the covariance between prices i and j is zero when i and j are from different groups. Thus, in equation (23), the only nonzero β_{ij}'s are β_{12} (=β_{21}) and β_{34} (=β_{43}).

The logarithmic likelihood values for these three models are shown in Table 2, along with the likelihood test results. Under the null hypothesis, twice the difference between the logarithmic likelihood value for the unrestricted model and that for the restricted model is asymptotically distributed as a chi-square statistic with the number of degrees of freedom equal to the number of restrictions (difference in the number of free parameters in the two models). The likelihood test results do not provide strong support for either of the restricted models, with the probabilities of exceeding the test values being .11 and .20 for the price independent and group independent models, respectively. However, since these probabilities are not extremely small, particularly for the group independent model, we will consider these restricted models further.

The coefficient estimates for the three models are shown in Table 3. For the unrestricted model, 12 out of 18 of the free parameters were significantly different than zero at the $\alpha = .10$ level; for the price independent and group independent models, 8 out of 9 and 9 out of 11 of the free parameters were significant, respectively. All marginal propensities (β_i) across the models were positive and significant, indicating an increase in total acres results in increased plantings across varieties. In the unrestricted model, the own-price coefficient for each variety was positive, indicating an increase in the variety's price results in an increase in that variety's planting level; all own-price coefficients were significant except that for red seedless grapefruit; all cross price coefficients were negative (indicating substitution) or not significantly different than zero. The pre-sample price impact (α_i) was significant across models, except in the equation for early and midseason oranges. In the price independent model, the factor of proportionality was positive and significant, which along with the estimates of the β_i's in the zero-one interval imply positive and negative own- and cross-price impacts, respectively. In the group independent model, the factor of proportionality and β_i's were also positive, but the coefficient β_{34} between the white and red seedless grapefruit varieties was large enough to result in a cross- price effect between these two varieties (π_{34}) that was positive but not significantly different than zero.

The model elasticity estimates, calculated at the sample means, are shown in Table 3. The planting responses to own-price changes tend to be inelastic, with the own-price elasticity estimates (ϵ_{ii}) being less than unity, except for specialty citrus in the unrestricted model. The cross-price elasticity estimates (ϵ_{ij}) tend to be smaller (in absolute value) than the own-price elasticities. The total acre elasticities (ϵ_i) across model were relatively close to unity for early and midseason oranges; exceeded unity for Valencia oranges and red seedless grapefruit, except for red seedless grapefruit in the price independent model; and were less than unity for white seedless and specialty citrus. That is, the marginal propensity β_i tends to equal (be greater than; be less than) the corresponding average propensity, a_i /A, for Early and midseason oranges (Valencia and red seedless grapefruit; white seedless grapefruit and specialty citrus).

In each of the above models, the scale and price coefficients have been assumed to be constants over the time period studied. Constancy of these coefficients further implies constancy of the price variance/covariance terms (M), given these coefficients are functions of these terms—equations (20) and (21). Below, we consider relaxing these constancy assumptions.

A major development over the period studied has been the emergence of Brazil as the dominant orange and orange-juice (OJ) producer in the world. In the 1960s, the United States was the largest orange and OJ producer in the world. Florida was the dominant producer in the United States. In 1962-63, Florida's citrus production was sharply reduced as a result of a freeze. The shortfall in Florida orange and OJ production resulted in large orange price increases which in turn stimulated Brazil's orange and OJ production and its OJ exports to the Untied States. Freezes in Florida in subsequent years, particularly those in the 1980s, further resulted OJ price increases and expansion in the Brazil industry. Additionally, the world market for OJ grew, particularly in Europe. As a result of these supply and demand changes, OJ prices have become relatively more stable—major OJ supply shifts in Florida, for example, no longer tend to impact prices as sharply as in earlier years when they were largely determined by Florida supply and U.S. demand. For example, the standard deviation in the early and midseason (Valencia) on-tree orange price was \$868/acre (849/acre) from 1994 through 1993 verus \$241/acre (\$253/acre) from 1994 trough 2003. Thus, in our model, the perceived price variance/ covariance terms for oranges may have changed over time.

To allow for the above possibility, consider the varietal independence model. The coefficient β_i in this model is $(1/\sigma^2_i)/(\sum_j 1/\sigma^2_j)$ and ϕ is $(1/k)\sum_j (1/\sigma^2_j)$. The aim is to specify these coefficients as varying parameters. The variance terms for the two orange varieties (σ^2_i, i=1,2) will be allowed to change over time.

Consider estimating the σ^2_is and k. Note that σ^2_i in β_i and ϕ can not be identified— a proportional change in all variances leaves β_i unchanged; while in the definition of ϕ the parameter k can be changed to offset any proportional change in the variances. This problem can be solved by normalizing the variances in some fashion. Here, we multiply and divide β_i by the variance for the last variety (i=5 or specialty citrus) which results in

$$(31)\quad \beta_i = (\sigma^2_5/\sigma^2_i)/(1+\textstyle\sum_{j=1\ \mathrm{to}\ 4} \sigma^2_5/\sigma^2_j) = \rho_i/(1+\textstyle\sum_{j=1\ \mathrm{to}\ 4} \rho_j),$$

where $\rho_i = \sigma^2_5/\sigma^2_i$.

Similarly, we specify

$$(32)\quad \phi = (1/(k\sigma^2_5))(1+\textstyle\sum_j (\sigma^2_5/\sigma^2_j) = k^*(1+\textstyle\sum_{j\ j=1\ \mathrm{to}\ 4} \rho_j),$$

where $k^* = 1/(k\sigma^2_5)$.

The term ρ_i (i=1 and 2) are then specified as functions of the log of time, i.e.,

$$(33)\quad \rho_i = \delta_{1i} + \delta_2 \log(t).$$

In general, the coefficient δ_2 in equation (33) could also have a variety subscript i but due to collinearity between the two orange prices separate values of δ_2 could not be estimated. Moreover, it was found that when the coefficient restriction $\delta_{11} = \delta_{12}$ was imposed the log likelihood value changed little and the restriction could not be rejected at any reasonable level of significance. That is, the two orange varieties have the same price variance which may not be too surprising since both are similar ingredients (close substitutes) in OJ products. The parameters ρ_i (i=2,3 and 4) and k^* were treated as constants. Assuming that the coefficients β_i and ϕ only varied over the sample period when the structure of the OJ industry changed, the coefficients α_i were continued to be treated as constants.

A varying parameter specification for the group independence model was also estimated where the β_i and ϕ were allowed to vary as in the varietal independence with the cross-price parameter between the two orange varieties, β_{12}, further specified as a linear function of the log of time. The likelihood value for this model declined relatively little compared to that for the above time varietal independence model with varying parameters, and, based on the likelihood ratio test, we will focus on the varietal independence model (twice the difference in the log likelihood values for the varietal and group models was 2.74; the difference in the number of free parameters in the two models (degrees of freedom) was 3; and area of the chi-square distribution with 3 degrees of freedom exceeding 2.74 was .43).

The estimates of the varietal independence model with time varying parameters are shown in Table 5. Eight out of 10 of the model coefficients, shown in left hand side of the table, were statistically significant (the two insignificant estimates were for two of the pre-sample price effects). The positive value of the parameter δ_2 indicates that the perceived orange-price variance has been decreasing over time as expected. In the right hand side of the table, the values of the scale (β_i), proportionality (ϕ) and price(π_{ij}) coefficients are shown for three select years—the beginning of the sample, 1964; the middle of the sample, 1984; and the end of the sample, 2003. The scale parameter for oranges has been increasing while those for the other varieties have been decreasing. With the decreased, variance over time, the factor of proportionality for the price effects has been increasing. These results indicate the own-price effects have been increasing for the orange varieties, but have been relatively stable for grapefruit and specialty citrus. Thus, these estimates are consistent with the hypothesis that with the growth in the Brazil OJ industry and the world OJ market, growers have viewed orange price as being less unstable which has resulted in stronger orange planting responses to total citrus acre and orange-price changes.

The main purpose of the analysis of citrus variety planting levels has been to provide examples for the various model specifications, but it should be noted that the estimated citrus varietal planting equations can be important in themselves for analyzing the interaction between citrus supply and demand. Barten and Vanloot show how planting equations for various agriculture crops in Europe can be integrated into a supply and demand model to examine price dynamics. Similarly, citrus planting equations can be included in a larger model. Planting levels are a major determinant of future production, along with tree loses and yields (e.g., Spreen, Brewster and Brown). To analyze, for example, the impact of changes in the U.S. citrus tariff structure on U.S. citrus production as in the Spreen et al study or to simply estimate future production for industry planning purposes requires some assumption or estimates on planting levels. In short, modeling planting levels by variety can be an important input for the larger citrus supply-demand situation that one may be interested in.

Concluding Comments

In this study, a model was developed to explain Florida citrus planting levels by variety. Planting different citrus varieties is viewed like choosing alternative assets in a portfolio. The choice of variety to plant is based on the expected prices and price variances/covariances of the varieties under consideration. Overall planting returns are maximized for a given level of price risk. The model's price coefficients are similar to those of the Theil and Barten Rotterdam demand model. As in the Rotterdam model, both absolute and relative price coefficient specifications were considered, allowing in the present study an examination of restrictions related to price risk. The model here is also specified in terms of levels as opposed to differences in the Rotterdam model and in a planting model suggested by Barten and Vanloot. A varying parameter model model allowing the price variances to change over time was also considered in the present study.

In the empirical application, two restricted models were considered, each showing some promise in explaining the citrus planting data analyzed. The first model was a price independent specification which is based on the assumption that only the price variances are important for predicting varietal planting levels. This model greatly reduces the parameter space and may be of interest in situations where lack of degrees of freedom is an issue. The second restricted specification was a group independent model allowing more flexibility in the cross- price responses between varieties in a group. The unrestricted model, against which these two models were tested, allows all price covariances to be non-zero.

Over the period studied, both world supply and demand for OJ has grown, and the variance in orange prices has tended to decrease. The varying parameter model indicates that over time growers have perceived orange prices as being relatively less unstable. As a result, the estimated grower planting responses to total citrus acre and price changes have become larger.

Table 1. Descriptive Statistics of Florida Citrus Acres Planted and Prices.

Variable	Mean	Std. Dev.
Annual Acres Planted[a]		
Early and Midseason Oranges	12,737	9,149
Valencia Oranges	12,224	8,487
White Seedless Grapefruit	1,996	1,933
Red Seedless Grapefruit	2,398	1,917
Specialty Citrus	1,968	2,670
Total	31,324	20,790
Share of Total Acres Planted[a]		
Early and Midseason Oranges	39.4%	9.1%
Valencia Oranges	38.9%	9.5%
White Seedless Grapefruit	7.6%	7.2%
Red Seedless Grapefruit	8.6%	6.3%
Specialty Citrus	5.5%	4.4%
Annual Dollar On-Tree Return Per Acre[b,c]		
Early and Midseason Oranges	2,434	970
Valencia Oranges	2,390	867
White Seedless Grapefruit	2,257	1,011
Red Seedless Grapefruit	2,669	1,248
Specialty Citrus	2,478	992

Source: Florida Agricultural Statistics Service---various issues of the Commercial Citrus Inventory and Citrus Summary.

[a] Based annual data from 1964 through 2003.

[b] Based annual data from 1956 through 2003.

[c] Deflated by the consumer price index (in 2004 dollars).

Table 2. Logarithmic Likelihood Values for Alternative Rotterdam Model Specifications.

Item	Model		
	Unrestricted	Price Independent	Group Independent
Logarithmic Likelihood Value	-1355.86	-1363.08	-1360.76
Likelihood Ratio Test Value[a]	---	14.44	9.8
Free Parameters	18	9	11
Degrees of Freedom[b]	---	9	7
P-Value[c]	---	0.108	0.200

[a] Twice the difference between the logarithmic likelihood value for the unrestricted model and that value for the restricted model.

[b] Number of free parameters in the unrestricted model minus the number of free parameters in the restricted model.

[c] Probability of obtaining likelihood ratio values exceeding the test value shown in the table.

Table 3. Maximum Likelihood Estimates of the Citrus Acreage Allocation Model.

Coefficient	Unrestricted		Variety Independent		Group Independent	
	Coeff. Est.	Std. Error	Coeff. Est.	Std. Error	Coeff. Est.	Std. Error
β1	0.403*	0.012	0.410*	0.011	0.406*	0.011
β2	0.428*	0.012	0.420*	0.010	0.423*	0.010
β3	0.043*	0.007	0.048*	0.006	0.043*	0.007
β4	0.083*	0.006	0.076*	0.005	0.082*	0.006
β5	0.043*	0.006	0.046*	0.005	0.046*	0.005
ϕ			11.003*	2.105	10.450*	2.254
β12					-0.171	0.135
β34					0.039*	0.023
α1	44353	37492	28953	36248	38740	35936
α2	-155584*	20333	-154660*	31650	-163522*	31676
α3	74694*	34814	84791*	18719	83986*	18801
α4	-92525*	17522	-104034*	16828	-104242*	17158
α5	129061*	18586	144950*	17346	145037*	17341
π11	4.539*	1.204				
π12	-3.159*	1.156				
π13	-1.002*	0.482				
π14	0.351	0.393				
π15	-0.729	0.483				
π22	4.312*	1.398				
π23	0.086	0.491				
π24	-0.485	0.410				
π25	-0.755	0.536				
π33	0.360	0.360				
π34	0.119	0.265				
π35	0.437	0.290				
π44	0.473*	0.245				
π45	-0.459*	0.228				
π55	1.506*	0.379				

* indicates the estimate is significantly different than zero at the $\alpha = .10$ level.

Table 4. Elasticity Estimates of the Citrus Acreage Allocation Model.[a]

Equation	Variable	Unrestricted Coeff. Est.	Unrestricted Std. Error	Variety Independent Coeff. Est.	Variety Independent Std. Error	Group Independent Coeff. Est.	Group Independent Std. Error
EM	Total Acreage	0.99*	0.03	1.01*	0.03	1.00*	0.03
	EM Price	0.80*	0.21	0.47*	0.09	0.76*	0.20
	Val Price	-0.56*	0.20	-0.34*	0.07	-0.63*	0.20
	W. Price	-0.18*	0.09	-0.04*	0.01	-0.03*	0.01
	R. Price	0.08	0.09	-0.08*	0.01	-0.08*	0.02
	Spec. Price	-0.14	0.09	-0.04*	0.01	-0.04*	0.01
Val	Total Acreage	1.10*	0.03	1.08*	0.03	1.08*	0.03
	EM Price	-0.58*	0.21	-0.35*	0.07	-0.66*	0.21
	Val Price	0.80*	0.26	0.49*	0.10	0.80*	0.21
	W. Price	0.02	0.09	-0.04*	0.01	-0.04*	0.01
	R. Price	-0.11	0.10	-0.08*	0.01	-0.08*	0.02
	Spec. Price	-0.15	0.11	-0.04*	0.01	-0.04*	0.01
White Gft.	Total Acreage	0.67*	0.11	0.76*	0.09	0.68*	0.11
	EM Price	-1.13*	0.54	-0.24*	0.05	-0.21*	0.05
	Val Price	0.10	0.55	-0.25*	0.06	-0.22*	0.05
	W. Price	0.41	0.41	0.58*	0.13	0.02	0.32
	R. Price	0.17	0.38	-0.06*	0.01	0.53	0.35
	Spec. Price	0.54	0.36	-0.03*	0.01	-0.03*	0.01
Red Gft.	Total Acreage	1.09*	0.08	0.99*	0.07	1.07*	0.08
	EM Price	0.33	0.37	-0.32*	0.06	-0.33*	0.07
	Val Price	-0.46	0.39	-0.33*	0.06	-0.34*	0.07
	W. Price	0.11	0.25	-0.04*	0.01	0.36	0.24
	R. Price	0.56*	0.29	0.92*	0.16	0.44	0.28
	Spec. Price	-0.47*	0.23	-0.04*	0.01	-0.04*	0.01
Specialty	Total Acreage	0.68*	0.10	0.72*	0.08	0.73*	0.08
	EM Price	-0.83	0.55	-0.23*	0.05	-0.22*	0.05
	Val Price	-0.86	0.61	-0.24*	0.05	-0.23*	0.05
	W. Price	0.51	0.34	-0.03*	0.01	-0.02*	0.01
	R. Price	-0.66*	0.33	-0.05*	0.01	-0.06*	0.01
	Spec. Price	1.88*	0.47	0.60*	0.12	0.57*	0.13

[a] At sample mean values.

* indicates the estimate is significantly different than zero at the $\alpha = .10$ level.

Table 5. Maximum Likelihood Estimates of the Citrus Varietal Independence Acreage Allocation Model with Time Varying Parameters.

Coefficient	Coeff. Est.	Std. Error	Parameter	Value by Year 1964	1984	2003
δ1[a]	-6.832*	3.127	β1	0.32	0.41	0.43
δ2[a]	4.519*	1.021	β2	0.32	0.41	0.43
ρ3	1.018*	0.108	β3	0.10	0.05	0.04
ρ4	1.545*	0.190	β4	0.16	0.08	0.06
k*	0.595*	0.119	β5	0.10	0.05	0.04
α1	140550*	56568	ϕ	5.81	12.11	14.82
α2	-22782	53280	π11	1.26	2.93	3.63
α3	15371	33806	π22	1.26	2.93	3.63
α4	-206030*	45238	π33	0.54	0.58	0.58
α5	72892*	30796	π44	0.77	0.85	0.86
			π55	0.53	0.57	0.57

[a] The restriction ρ1=ρ2 was imposed.

* indicates the estimate is significantly different than zero at the α = .10 level.

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