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## **AIDS AND SEPARABILITY: A NON SEQUITUR**

BY

Jonq-Ying Lee, Sr. Research Economist - FDOC

Mark G. Brown, Sr. Research Economist - FDOC

James L. Seale, Jr., Associate Professor - UF

FLORIDA DEPARTMENT OF CITRUS

Economic and Market Research Department

P.O. Box 110249

Gainesville, Florida 32611-2049 USA

Phone: 352-392-1874

Fax: 352-392-8634

Email: [mgbrown@ufl.edu](mailto:mgbrown@ufl.edu)

**[www.floridajuce.com](http://www.floridajuce.com)**

## **AIDS and Separability: a Non Sequitur**

Jonq-Ying Lee, Mark G. Brown, and James L. Seale, Jr.\*

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\*Jonq-Ying Lee and Mark G. Brown are Research Economists, Florida Department of Citrus; Adjunct Professor and Adjunct Associate Professor, respectively, Food and Resource Economics Department, University of Florida. James L. Seale, Jr., is an Associate Professor, Food and Resource Economics Department, University of Florida.

## **AIDS and Separability: a Non Sequitur**

### **Abstract**

The almost ideal demand system (AIDS) is commonly used to study demand for agricultural commodities and groups of commodities. Increasingly, separability has been utilized in conjunction with the AIDS to estimate conditional demand systems as well as to improve upon precision of parameter estimates. The paper shows that separability conditions are inconsistent with use of the AIDS and that tests of separability with the AIDS are problematic.

**Key Words:** separability, almost ideal demand system.

## **AIDS and Separability: a Non Sequitur**

The almost ideal demand system (Deaton and Muellbauer 1980b; abbreviated as the AIDS) has been a popular specification in studying demand for agricultural commodities and different commodity groupings (e.g., food (Blanciforti et al.; Capps et al.; Moschini and Green), meat (Chalfant and Alston; Eales and Unnevehr; Hayes et al.; Moschini and Meilke; Nayga and Capps), cigarettes (Haden), dairy products (Heien and Wessells), and recently, fats and oils (Gould et al.)). In the more recent studies, the AIDS has been used to estimate demand parameters and to test for separability. These studies have generally assumed the existence of weak separability among broadly defined commodity groups, with attention focused on sub-demand systems and estimation of conditional demand parameters.<sup>1</sup>

In general, the major purpose of using the separability hypothesis in demand studies is to reduce the number of demand parameters to be estimated and obtain more precise estimates for the remaining parameters. Imposing separability restrictions generally increases the degrees of freedom for statistical inference and allows estimation of demand systems given limited data availability. The separability hypothesis provides an avenue for researchers to estimate subgroup demand parameters which can be consistently linked to other demand parameters in the full demand system. Since many agricultural economists are only interested in a small part of the full demand system, the separability hypothesis becomes important in their demand studies. In order to obtain demand parameters which are consistent with demand theory, it is imperative for a researcher to ascertain that the model chosen to study the conditional demands for goods within a group of goods can be consistently linked to the full demand system (e.g., see Theil for the

Rotterdam model and Powell for the linear expenditure system). The purpose of this paper is to investigate the separability nature of the AIDS model.

### **Is the AIDS Separable?**

In the AIDS model (Deaton and Muellbauer 1980b), the budget share,  $w_i$ , for commodity  $i$ , can be written as

$$(1) \quad w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (m/P),$$

where  $m$  is total expenditure,  $p_j$  is the price of commodity  $j$ , and  $P$  is a price index defined as

$$(2) \quad \log P = \alpha_0 + \sum_k \alpha_k \log p_k + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \log p_j \log p_k.$$

The AIDS given by (1) and (2) is derived from the cost function specified by

$$(3) \quad \log c(u, p) = a(p) + ub(p)$$

where  $u$  is utility and  $a(p) = \log P$  as shown by (2) and  $b(p) = \beta_0 \prod_k p_k^{\beta_k}$ . The indirect utility function ( $u = \psi(p, m)$ ) of (3) is

$$(4) \quad \psi = (\log c(u, p) - a(p))/b(p) = (\log m - a(p))/b(p),$$

assuming a utility maximizing consumer so that  $m = c(u, p)$ . Explicit forms for the direct utility and transformation functions have not been derived for the AIDS.

In general, separability can be defined over the direct utility function, indirect utility function, cost function, and transformation function. The most commonly used preference structure is weak separability of the direct utility function. However, since the direct utility function and transformation function of the AIDS do not have explicit forms, the emphasis of the following discussion will be placed on the cost function and indirect utility function of the AIDS.

According to Blackorby et al. (p. 67 and p. 70), the Leontief-Sono separability conditions for the indirect utility function and cost function can be written as

$$(5) \quad \partial(\psi_i/\psi_j)/\partial(p_k/m) = 0, \text{ and}$$

$$(6) \quad \partial(c_i/c_j)/\partial p_k = 0,$$

respectively, where  $\psi_i = \partial\psi/\partial(p_i/m)$ , and  $\psi_i/\psi_j$  is the ratio of Marshallian demands for the  $i^{\text{th}}$  and  $j^{\text{th}}$  commodities;  $c_i = \partial c/\partial p_i$  is the Hicksian demand for commodity  $i$ ; and commodities  $i$  and  $j$  are in group  $G$  while commodity  $k$  is not in group  $G$ . In other words, if an indirect utility function satisfies the separability condition, the ratio of Marshallian demands for the  $i^{\text{th}}$  and  $j^{\text{th}}$  commodities in group  $G$  is independent of normalized prices that are not in the  $G^{\text{th}}$  group; likewise, if a cost function satisfies the separability condition, the ratio of Hicksian demands for any two commodities in group  $G$  is independent of prices of commodities not in group  $G$ . Note that  $\psi_i/\psi_j$  depends on  $m$  while  $c_i/c_j$  depends upon  $u$ . Due to the non-linear nature of  $a(p)$  and  $b(p)$  in (3), it is evident that neither the cost function nor the indirect utility function of the AIDS, in general, satisfies the Leontief-Sono separability conditions. Essentially, under the AIDS specification,  $\psi_i/\psi_j = (p_j/p_i)(w_i/w_j)$ , where  $w_i$  is given by (1), and  $c_i/c_j = (p_j/p_i)(w_i/w_j)$  where  $w_i$  is given by (1) with  $\log(m/P)$  replaced by  $ub(p)$ .

Winters (p. 253) has noted, only under extremely restrictive conditions the AIDS can be made separable

"...A homothetic AIDS can, with constant parameters, be made separable only at one point of the data space at a time..."

Checking conditions (5) and (6) for the AIDS, it can be seen that this statement is true if one is willing to sacrifice the flexibility of the AIDS by assuming all  $\beta_k$ s equal zero and  $\gamma_{ij}=0$  for  $i \in G$ ,

$j \in H$ , and  $G \neq H$ . Note that for this restriction the Marshallian demands equal the Hicksian demands. These conditions are generally overly restrictive and can easily be tested empirically. Essentially, separability imposed at one data point (which one?) is a trivial artifact of the AIDS.

When the assumption of weak separability is applied to the utility function (e.g., Eales and Unnevehr; Nayga and Capps), the lower-stage expenditure (or cost function) of the AIDS model is

$$(7) \quad \log m_G = a(p_G) + u_G b(p_G).$$

Deaton and Muellbauer (1980a, p. 133) show that, if the cost function is implicitly separable, the subgroup cost functions should be functions of total utility instead of subgroup utilities as indicated by (7). Blackorby et al. (p. 101 and p. 114) further indicate that the usefulness of the above practice is limited because such subgroup functions cannot generally be aggregated consistently into macro functions:

"...as conjugate implicit separability does not imply explicit separability, these sectoral utility functions cannot generally be aggregated into macro utility functions. Hence their usefulness is limited (p. 101) ....(Strict) separability of the utility functions does not imply, nor is it implied by, (strict) separability of the cost or transformation functions (p.114)..."

When researchers use the AIDS cost function for lower-stage and upper-stage expenditures (e.g., Hayes et al.), they implicitly assume

$$(8) \quad \log m_G = a(p_G) + u b(p_G) \text{ and}$$

$$(9) \quad \log m = a(p_1, \dots, p_N) + u b(p_1, \dots, p_N),$$

respectively; equation (8) assumes the expenditure on commodities in group  $G$  depends only on group prices and utility, while equation (9) assumes total expenditure on all commodities depends on all group price vectors and total utility. Solving (8) and (9) for utility, (8) indicates



indirect utility is a function of group expenditure and prices of commodities in the subgroup while (9) indicates that it is a function of all prices and total expenditures. Accordingly, under the AIDS, these two conditions can not be satisfied simultaneously and (8) and (9) in combination are not plausible. Additionally, since total expenditure equals the sum of group expenditures (i.e.,  $m = \sum m_G$ ), (8) implies

$$\sum m_G = \sum \exp[a(p_G) + ub(p_G)]$$

which is obviously not equal to  $\exp[a(p_1, \dots, p_N) + ub(p_1, \dots, p_N)]$  as indicated by (9).

It can be further shown that subgroup expenditures commonly used in the AIDS expressed either by (7) or by (8) are not consistent with the separability concept applied to the indirect utility function (i.e., equation (5)). Separability applied to the indirect utility function requires that the indirect utility function has the following functional form,

$$(10) \quad \psi(m, p) = f(\psi^1(m, p_1), \dots, \psi^N(m, p_N)).$$

Note that each subgroup indirect utility function is a function of total expenditure, not group expenditure. Furthermore, the AIDS indirect utility function, (4), is non-linear in prices and does not have the same structure as specified by (10).

If one were only interested in the conditional demand relationships among commodities in a subgroup, one might argue for ignoring the upper stage of the consumer allocation problem and how these stages are related. By assuming that the direct utility function satisfies the separability conditions, one can use subgroup direct utility, indirect utility, cost, and transformation functions to rationalize a conditional demand system. However, as shown above, the AIDS does not conform to this type of rationalization. Further, conditional demand equations in agricultural economics are typically for narrowly defined commodity groups (e.g., meats) and are

generally dependent on demand factors outside the group, in which case, ignoring the upper stage(s) is inappropriate<sup>2</sup>.

### **Testing Separability with the AIDS**

Three distinctive approaches have been used with AIDS to test for the existence of separability among broadly defined commodity groups. In the first approach, additional price variables are added to the AIDS and, based on the t-ratios of these additional variables, conclusions about separability are made (e.g., Alston et al.; Winters). In the second approach, restrictions based on the direct utility function are used to test for separability (e.g., Eales and Unnevehr; Nayga and Capps). In the third approach, the cost function is used to test for the existence of separability (e.g., Hayes et al.).

Each of the above approaches fails to provide a consistent test for separability, especially when accounting for transition from the lower stage budget allocation to the upper stage budget allocation. Consider the first approach. According to Goldman and Uzawa and to Blackorby et al. (p. 52), the utility function  $u(q)$  is weakly separable with respect to a partition, if the marginal rate of substitution between two commodities  $i$  and  $j$  from group  $G$  is independent of the quantities of commodities not in that group. In other words, the ratio of the marginal utilities is independent of the quantity of any commodity outside the group such that

$$(11) \quad \partial(u_i/u_j)/\partial q_k = 0 \quad \text{for all } i, j \in G \text{ and } k \notin G,$$

where  $u_i = \partial u / \partial q_i$ .

Barten (1964) shows that the substitution effect of a price change ( $k_{ij} = \partial q_i / \partial p_j + q_j \partial q_i / \partial m$ ) can be expressed as the sum of the specific substitution effect and the general substitution effect:

specific substitution effect:	$\lambda u^{ij}$
general substitution effect:	$-\lambda(\partial\lambda/\partial m)(\partial q_i/\partial m)(\partial q_j/\partial m)$ .

The term  $\lambda$  equals  $\partial u/\partial m$ , the marginal utility of money, while  $u^{ij}$  is the (i,j) element of the inverse of the Hessian matrix,  $[\partial^2 u/\partial q_i \partial q_j]^{-1}$ . Separability restrictions on the substitution effects are obtained by focusing on the  $u^{ij}$ s in the specific substitution effects. For strong separability  $u^{ij} = 0$  for  $i \in G$  and  $j \in H$  ( $G \neq H$ ), and  $k_{ij} = \phi(\partial q_i/\partial m)(\partial q_j/\partial m)$  where the factor of proportionality,  $\phi$ , is the same for all cross-substitution effects. For weak separability,  $u^{ij}$  does not vanish but the substitution effects,  $k_{ij}$ s, for goods in different groups are

$$(12) \quad k_{ij} = \phi_{GH}(\partial q_i/\partial m)(\partial q_j/\partial m)$$

for all  $i \in G$ ,  $j \in H$ , and  $G \neq H$  where the factor of proportionality,  $\phi_{GH}$ , is the same for all cross-substitution effects between pairs of commodities with one member belonging to subset G and the other member belonging to subset H.

As shown by Deaton and Muellbauer (1980a, p. 76), the relationship between  $k_{ij}$  and the price parameter,  $\gamma_{ij}$ , of the AIDS is

$$(13) \quad \gamma_{ij} = \pi_{ij} - \beta_i \beta_j \log(m/P) + w_i \delta_{ij} - w_i w_j$$

where  $\pi_{ij} = k_{ij}(p_i p_j/m)$  is referred to as the Slutsky coefficient. Since  $\gamma_{ij}$  can not be readily decomposed into specific and general substitution effects, the separability test carried out by Winters and later by Alston et al. (i.e., test for specific  $\gamma_{ij}=0$ ) is, in general, inconclusive. One can obtain further insight by realizing that the price parameter,  $\gamma_{ij}$ , in the AIDS is a constant which embodies not only the compensated price effect  $\partial c_i/\partial p_j$  (which is restricted under separability) but also the budget shares  $w_i$  and  $w_j$  as well as the real income term,  $\log(m/P)$  (Deaton and Muellbauer, 1980a). Thus, a zero parameter estimate for an added price variable

does not imply a restriction on the compensated price effect. The second approach used by Eales and Unnevehr,<sup>4</sup> and Nayga and Capps) to test for weak separability with the first-difference form of the AIDS and is also based on (12); however, based on (13), weak separability can only be imposed at one data point using the AIDS, limiting the usefulness of their test. Further, this approach ignores the fact that an explicit direct utility function has not been derived for the AIDS.

The third approach ignores the fact that the AIDS cost function and indirect utility function do not satisfy separability conditions (5) and (6). Even if the AIDS cost function satisfied separability condition (6), price and quantity indices have to be developed over the macro function (i.e., the cost function for the top stage of two-stage budgeting (Deaton and Muellbauer, p. 134)).

Even if one ignores in the third approach the problem of how to consistently link upper and lower stage budget allocations, one still faces other problems in testing for separability in the AIDS. Note that the AIDS is derived from a specific cost function, equation (3). Deaton and Muellbauer (p. 133) indicate that

"...Preferences are .... implicitly separable if and only if the cost function can be written in the form

$$(14) \quad c(u, p) = c[u, c_1(u, p_1), \dots, c_G(u, p_G), \dots, c_N(u, p_N)]$$

where, ... the goods are partitioned into N groups with price subvectors  $p_1, \dots, p_N$ . The function  $c_G(u, p_G)$  is increasing in  $u$  and  $p_G$  .... Note carefully that it is *total* utility that appears in each of the function  $c_G(u, p_G)$ ; in sharp contrast to weak separability there are no group subutilities..."<sup>3</sup>

Similar results are discussed by Blackorby et al. (p. 70). Differentiating (14) with respect to  $p_{i \in G}$ , where good  $i$  belongs to group  $G$ , one obtains the Hicksian demand for good  $i$

$$(15) \quad q_{i \in G} = (\partial c / \partial c_G)(\partial c_G / \partial p_{i \in G}).$$

Therefore, the expenditure on good  $i$ ,  $m_{i \in G}$ , equals

$$(16) \quad m_{i \in G} = p_{i \in G} q_{i \in G} = (\partial c / \partial c_G)(\partial c_G / \partial p_{i \in G}) p_{i \in G}.$$

Hence, total expenditure on all goods in group  $G$  is

$$(17) \quad m_G = (\partial c / \partial c_G) c_G,$$

where  $c_G = \sum_{i \in G} (\partial c / \partial p_{i \in G}) p_{i \in G}$ . Since  $c_G$  is linearly homogeneous in the  $p_{i \in G}$ s,  $c_G$  is considered the price index for group  $G$ , and the functions  $c_G$  are not group expenditures. To apply (14), one needs to specify price indexes for commodity groups so that the macro-cost function can thus be defined (Deaton and Muellbauer 1980a, p. 133-4). Therefore, treating the cost function of the AIDS as implicitly separable without defining the price indexes ( $c_G$ ) is also inconsistent with the separability concept and utility theory (e.g., Hayes et al.).

As a last comment, note that in (13), the  $\beta_i$ s and  $\pi_{ij}$  are constants in the AIDS model. Equation (13) involves variables  $w_i$ s, which are the dependent variables according to (1) and are typically not constants. Hence, using relationships (12) and (13) to test the weak separability hypothesis (e.g., Eales and Unnevehr, Nayga and Capps, and Moschini and Green) implicitly *reparameterizes* the AIDS, i.e.,  $\gamma_{ij}$  becomes a variable instead of a constant as specified by (1) (More discussion on this issue is presented in the Appendix.). In practice, the hypothesis is tested at sample means in the latter studies. When discussing the constraints on the elasticities for the double-log demand model, Barten (1989, p. 444) indicates that

"...If one is only interested in saving degrees of freedom, one could work with constant elasticities, using a single set of  $w_i$  in the constraints. That means, *inter alia*, that [the budget constraint] is not respected for the explained  $q_j$ , except for the sample point for which the selected  $w_i$  are valid. It is clearly more desirable to

work with a parameterization that allows the use of constraints without impairing the simulation properties of the demand equations..."

Even if separability is found to exist at the sample means, it does not follow that separability exists at every data point; to the extent separability does not hold elsewhere it may be inappropriate to use this approach to improve the precision of the estimates.

### **Concluding Remarks**

The AIDS cost and indirect utility functions are not separable, and explicit forms for the direct utility and transformation functions do not exist for the AIDS. Since the AIDS model is not consistent with separability, maintained hypotheses such as weak separability become untenable. Using the AIDS model to test for separability may not provide accurate information about the structure of demand.

Of course one can always treat the AIDS as a general functional form and test for separability at sample means as is sometimes done, for example, using the double logarithmic demand specification (Pudney; Barten 1989).<sup>5</sup> However, if separability is found to exist at the sample means, should not the AIDS parameters be restricted to be consistent with separability at the mean values only? One may find restricting the model in such a manner an unsatisfactory solution. As Deaton and Muellbauer (1980b, p. 315) state

"There is a large number of parameters ... and on most data sets these are unlikely to be all well determined. It is thus important that there should exist some straightforward procedure for eliminating unnecessary parameters without untoward consequences for the properties of the model. In the AIDS, this can be done by placing whatever restrictions on  $\gamma_{ij}$  parameters are thought to be empirically or theoretically plausible [there is no mention of separability]..."

As the AIDS functional form is not consistent with the usual separability concepts, the suggested approach by Deaton and Muellbauer, to restrict the model, seems to be the best one can do without degenerating the model.

### Footnotes

<sup>1</sup>Recently, an entire selected papers session at the 1991 American Agricultural Economics Association Annual Meetings was devoted to studies using the AIDS model.

<sup>2</sup>Another problem is that group expenditure may be endogenous, see, for example, Theil, Attfield, and LaFrance for a discussion of this issue.

<sup>3</sup>Hayes et al. (p. 561) state

"...quasi-separability if the function can also be written as  
 (10)  $m = G[g_1(p_1, u), \dots, g_r(p_r, u)]$ ,  
 where  $G(\cdot)$  and the functions  $g_r(\cdot)$  also have the general properties of cost functions, goods are portioned into  $r$  groups with price subvectors  $p_1, \dots, p_r$ , and the function  $g_r(p_r, u)$  are increasing in  $u$  and  $G$ ..."

<sup>4</sup>Eales and Unnevehr (p. 523) defined  $s_{ij} = \phi_{ij}(\partial q_i / \partial m)$ , where  $s_{ij} = k_{ij}$  as defined by (12).

<sup>5</sup>For the double logarithmic model,

$$\log q_i = \alpha_i + \sum_j \eta^*_{ij} \log p_j + \eta_i (\log m - \sum_k w_k \log p_k).$$

The term  $\eta^*_{ij} = \eta_{ij} + \eta_i w_i$  (the compensated price elasticity) can be used to derive the separability restrictions on parameters.



## Appendix

The relationships between the AIDS price parameters ( $\gamma_{ij}$ 's) and Slutsky coefficients ( $\pi_{ij}$ 's), the latter which can be directly related to the underlying utility function, is (Deaton and Muellbauer 1980)

$$(1A) \quad \gamma_{ij} = \pi_{ij} - \beta_i \beta_j \log \frac{m}{P} + w_i \delta_{ij} - w_i w_j,$$

where  $P$  is a price index,  $m$  is total expenditure or income;  $w_i = \frac{p_i q_i}{m}$ , the budget share for good

$i$ , where  $p_i$  and  $q_i$  are the price and quantity for good  $i$ , respectively;  $\delta_{ij} = 1$  if  $i = j$  and  $= 0$  if  $i \neq j$ ;

$\beta_i$  is the AIDS income parameter for good  $i$ ; and  $\pi_{ij} = \frac{p_i p_j}{m} \left( \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial m} \right)$ . The parameters

$\gamma_{ij}$  and  $\beta_i$  are assumed to be constant for the AIDS parameterization.

As shown by Barten (1964), the Slutsky coefficient can further be decomposed into specific and general terms, i.e.,

$$(2A) \quad \pi_{ij} = \varphi (\theta_{ij} - \theta_i \theta_j),$$

where  $\varphi = \left( \frac{\partial \log \lambda}{\partial \log x} \right)^{-1}$ ,  $\lambda = \frac{\partial u}{\partial m}$ , the marginal utility of money, where  $u$  is utility;

$$\theta_{ij} = \frac{p_i p_j u^{ij}}{\varphi m}, \text{ where } u^{ij} \text{ is the } ij^{\text{th}} \text{ element of the inverse of the Hessian matrix } \left[ \frac{\partial u}{\partial q_i \partial q_j} \right]$$

(i.e.,  $[u^{ij}] = \left[ \frac{\partial u}{\partial q_i \partial q_j} \right]^{-1}$ ); and  $\theta_i = \frac{\partial(p_i q_i)}{\partial x}$ , the marginal propensity to consume for good  $i$ .

The term  $\phi\theta_{ij}$  is the specific substitution term while the term  $-\phi\theta_i\theta_j$  is the general substitution term.

For additive (strong) separability ( $u = \sum f_i(q_i)$ ), the Hessian matrix and its inverse are diagonal ( $u^{ij} = 0$  for  $i \neq j$ ) and the Slutsky coefficient becomes

$$(3A) \quad \pi_{ij} = \theta_i(\delta_{ij} - \theta_j)$$

( $\theta_{ij} = 0$  for  $i \neq j$  and homogeneity implies  $\sum_j \pi_{ij} = \phi \sum_j (\theta_{ij} - \theta_i \theta_j) = 0$  or  $\theta_{ii} = \theta_i$ ,

since  $\sum_j \theta_j = 1$  by Engel aggregation.)

Similarly, for weak separability ( $u = u(f_A(q_A), f_B(q_B), \dots)$  where  $q_R$  is the vector of goods in group  $R$ ), the Slutsky coefficient is

$$(4A) \quad \pi_{ij} = \phi_{RS} \theta_i \theta_j,$$

for goods  $i$  in group  $R$  and goods  $j$  in group  $S$  (Theil; Deaton and Muellbauer; among others). If goods  $i$  and  $j$  are from the same group, there is no restriction on the Slutsky coefficient.

The issue is whether the AIDS parameterization is consistent with the foregoing separability conditions. We examine the weak separability case (the same argument applies to additive separability or additive separability between groups of goods). Combining (1A) and (2A) and imposing the condition for weak separability (4A), the AIDS price coefficients are

$$(5A) \quad \gamma_{ij} = \pi_{ij} \delta_{RS} + \phi_{RS} \theta_i \theta_j (1 - \delta_{RS}) - \beta_i \beta_j \log \frac{m}{p} + w_i \delta_{ij} - w_i w_j,$$

where  $\delta_{RS} = 1$  if  $R = S$  and  $= 0$  if  $R \neq S$ . For the AIDS, the marginal propensity to consume is  $\theta_i = w_i + \beta_i$ , so that (5A) can be written as

$$(6A) \quad \gamma_{ij} = \pi_{ijt} \delta_{RS} + \phi_{RS,t} (w_{it} + \beta_i) (w_{jt} + \beta_j) (1 - \delta_{RS}) - \beta_i \beta_j \log \frac{m_t}{p_t} + w_{it} \delta_{ij} - w_{it} w_{jt},$$

where a  $t$  subscript, indicating an observation, has been attached to each term, other than the Kronecker delta terms  $\delta_{RS}$  and  $\delta_{ij}$  and the constant parameters  $\gamma_{ij}$  and  $\beta_i$ .

If equation (6A) holds for some observations, there is no guarantee it will hold for other observations. The AIDS demand equations are

$$(7A) \quad w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \frac{m}{p}, \quad i = 1, 2, \dots, n.$$

Suppose prices and income change, leaving  $\frac{m}{p}$  unchanged but changing the budget shares.

Based on (6A), the implication of such price and income changes is that the AIDS price coefficients may change in contradiction to the parameterization assumption. For example, the implied between-group AIDS price coefficient change is

$$(8A) \quad \Delta \gamma_{ij,t} = \Delta (\phi_{RS,t}) \theta_{it} \theta_{jt} + \phi_{RS,t-1} \Delta (\theta_{it} \theta_{jt}) - \Delta (w_{it} w_{jt}),$$

where  $\Delta x = x_t - x_{t-1}$  for variable  $x$  in general. Now suppose  $w_{it}$  and  $w_{jt}$  do not change ( $i \in R$  and  $j \in S$ ) so that the last two terms on the right side of (8A) are zero, implying  $\Delta \phi_{RS,t} = 0$  to obtain the result  $\Delta \gamma_{ij,t} = 0$  and preserve the constancy of  $\gamma_{ij}$ . At the same time, suppose the budget shares for two other goods ( $k \in R$  and  $l \in S$ ) change so that both  $\Delta(w_{kt} w_{lt})$  and  $\Delta(\theta_{kt} \theta_{lt})$  are nonzero, implying  $\Delta \phi_{RS,t} \neq 0$  in order for  $\Delta \gamma_{kl,t} = 0$ . In turn,  $\Delta \phi_{RS,t} \neq 0$  implies  $\Delta \gamma_{ij,t} \neq 0$ . Hence, we have

a contradiction and conclude that the AIDS price coefficients can not be constant assuming weak separability. Most tests for weak separability treat  $\phi_{RS}$  as a constant across observations, in which case  $\gamma_{ij}$  varies directly through the marginal propensities to consumer ( $\theta_i$ 's) and the budget shares ( $w_i$ 's). If one allows  $\phi_{RS}$  to take a different value for each observation the number of parameters to be estimated will increase, in contradiction to the objective of using separability to reduce the parameter space for estimation.

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