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ALLOCATION OF SHELF SPACE: SOME THEORETICAL CONSIDERATIONS

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Allocation of Shelf Space: Some Theoretical Considerations*

The amount of shelf space allocated to each product in a grocery store ,or any store in general, has an opportunity cost, knowledge of which allows determination of the most profitable allocation of space between alternative products competing for the store's limited shelf space. Factors to consider in determining the optimal allocation include the per unit profit of each product and the demand levels for the products which, in turn, can expected to be dependent on such factors as prices, consumer income and advertising. Shelf space, itself, can also be expected to influence demands (Kepner). For example, a product's shelf space may act as a message to consumers, indicating, perhaps, the product's popularity, or simply putting the product on the tops of consumers' minds. Regardless, a product's sales can be expected to be dependent on the amount of shelf space allocated to the product, as well as the location of the space. In this case,the optimal allocation of shelf space would depend, in part, on the impacts of shelf space on the demands of products. Other factors such as the cost of stocking product on shelves may also be relevant, in which case they should be included in the allocation problem. In this paper, we assume that a store can handle additional stocking, which may result from a reallocation of shelf space, with existing labor, without additional cost, i.e., labor is not so tight that it can not be juggled to accommodate such additional stocking. Hence, we focus on demand and its relationship with shelf space. The paper proceeds by examining more closely the question of optimal shelf space using a simple mathematical model. The results indicate where store

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management might look, and perhaps experiment with, in allocating shelf space to maximize profits.

Model

A profit maximization model is used to reveal the basic aspects of the shelf space allocation problem. The problem is to allocate a limited amount of shelf space between products such that store or store department profits are maximized. Letting S, s(i), p(i), w(i), q(i), x be the total fixed shelf space; shelf space allocated to product i; retail price of product i; wholesale price of product i; quantity demanded of product i; and a vector containing retail prices, consumer income, advertising and other demand explanatory variables, respectively, the maximization problem can be written

(1) maximize $V=m'q$, subject to $S=l's$,

where $m'=(p(1)-w(1),...)$, $q=(q(1),...)$, l is a unit vector, and $s'=(s(1),...)$. The term m(i) measures the profit per unit of product i, with retail and wholesale prices assumed to be fixed by competitive forces. The demands for products can further be written as functions of x and s, i.e., $q=f(x,s)$.

The Lagrangian for problem (1) can be written as $L=m'q+r(S-l's)$, where r is the Lagrangian multiplier, and the first order conditions are

(2a) $dL/ds(i) = m(i)*dq(i)/ds(i) - r = 0, \quad i=1,...,n.$

(2b) $dL/dr \quad = S - l's = 0,$

where, in general, dy/dz is the partial derivative of y with respect z.

Approximating demand by the double log function $q(i)=A(i)*s(i)**e(i)$, where $e(i)$ is the elasticity of demand for product i with respect to shelf space, and $A(i)$ is a function of x with cross-product-shelf effects ignored for simplicity, (2a) can be written as

$$(3a) \quad m(i)*A(i)*e(i)*s(i)**e(i)=r*s(i),$$

or

$$(3b) \quad B(i)=r*s(i),$$

where $B(i)=m(i)*q(i)*e(i)$ and (2a) has been multiplied through by $s(i)$.

Summing (3) over i, the solution for the Lagrangian multiplier can be written as

$$(4a) \quad l'B=r*l's,$$

or

$$(4b) \quad r=l'B/S,$$

where $B=(B(1),...)$ and we have used the constraint $S=l's$. One can show that r is marginal profit obtained by relaxing the shelf space constraint by a unit.

Substituting (4b) into first-order conditions (3a), the solution for $s(i)$ can be written as

$$(5) \quad s(i)/S = B(i)/l'B.$$

Solution (5) indicates that the share of shelf space allocated to product i is product i's share contribution to the profit term $l'B$. The term $B(i)$ can be interpreted as product i's profit resulting from changing its allocation of shelf space from zero to $s(i)$, i.e., $B(i)=m(i)*q(i)*e(i)$ or $B(i)= m(i)*(dq(i)/ds(i))*s(i)$, and $l'B$ is the overall profit of shelf space S.

Neither solution (4b) or (5) are in reduced form as the s(i)'s are on the right sides of the solutions. To obtain an explicit closed-form solution, further structure needs to be given to the problem. For example, suppose all e(i) are the same, i.e., $e=e(i)$. This is the constant elasticity of substitution (C.E.S.) assumption examined in demand and production models (e.g.,Varian). In this case, the shelf space solution can be written as $s(i)/S=C(i)/1'C$, where $C(i)=(m(i)*A(i))**k$, $k=1/(1-e)$, and $C=(C(1),...)$.[1] We assume that the shelf space elasticities are in the zero-one interval ($0<e<1$), assuring that the second order conditions are meet (this seems to be a reasonable assumption given stores have not grossly under or over allocated shelf space).

Although not in reduced form, (5) can be used to find a solution for s(i), using some iterative procedure (of course, this assumes that the second order conditions are fulfilled so that a solution exists). For example, initial values for the s(i)'s can be substituted into the right-hand-side of (5) to obtain updated s(i)'s which can then be substituted into (5) to obtain further updates. This procedure can be repeated until convergence is achieved, i.e., the values of s(i) are the same on both sides of (5).

With some further simplifying assumptions, solution (5) can also be used to examine summary data on shelf space and sales, excluding detailed demand estimates. For example, suppose shelf space for a department in a store is fixed and similarity of products allows the C.E.S. approximation to be used. In this case, (5) becomes $s(i)/S=m(i)*q(i)/m'q$, and if all retail prices are approximately marked up by the same percentage---$p(i)=(1+g)*w(i), g>0$---then $s(i)/S=p(i)*q(i)/p'q$, where $p'=(p(i),...)$. That is, a product's share of department retail dollar sales equals the share of department shelf space that should be allocated to the product, for profit maximization. From this result, we can also see that each product's average dollar sales per unit of shelf space is the same for an optimal allocation, i.e., p(i)*q(i)/p'q=s(i)/S or, after rearranging, p(i)*q(i)/s(i)=p'q/S for all i. Similarly, if the C.E.S. assumption is maintained and all products have the same absolute markup---m(i)=m---then s(i)/S= q(i)/l'q, or the share of department quantity sales accounted for by a product equals its share of shelf space. Of course, one would prefer to have knowledge of per unit profits, demand levels and demand impacts of shelf space, so that (5) could be applied more precisely.

Application

Data on shelf space, and dollar and gallon sales for juice and juice-drink products in refrigerated and frozen departments of grocery stores were examined for consistency with the above theoretical results. The data were provided by Nielsen Marketing Research and are for U.S. grocery stores doing $4 millon or greater business, for the week ending March 12,1994. Refrigerated and frozen departments are normally found in different store locations and the data for these two departments were examined separately. (A complete analysis would need to link the amounts of shelf space in these two departments to each other and total store shelf space.) Shelf space was measured by number of visible product facings and corresponding number of linear feet.

Table 1 shows the basic data. In each of the departments, the largest difference in a product's shelf space share (shelf space shares were only available for facings; footage was estimated as proportional to facings) and its dollar or gallon share was for orange juice---

chilled ready-to-serve orange juice (COJ) in the refrigerated department and frozen concentrate orange juice (FCOJ) in the frozen department. For both these products, dollar and gallon shares were substantially greater than the corresponding shelf space shares---COJ (FCOJ) had 51% (36%) of the department facings versus 70% (47%) of the department dollar sales and 67% (50%) of the gallon sales. Does this suggest that shelf space is under allocated for COJ and FCOJ? Based on result (5), it is possible that profit per unit of product, demand level and impact of shelf space on demand could be such that an outcome as in Table 1 is consistent with profit maximization. However, with similarity of juice-and-juice-drink products, the C.E.S. specification and across-product equality of mark-ups (either in percentage or absolute terms) may be reasonable approximations. In this case, both COJ and FCOJ appear to have less shelf space than would be optimal, based on discussion in the Model section.

To show that COJ and FCOJ have less than optimal shelf space for the C.E.S. model, first note that, for equal percentage mark ups across products, profits V are

$$V=k1*((p(1)*q(1)+p(2)*q(2)+...),$$

where k1 is some percentage. Given the C.E.S. parameter assumptions, the change in profits for a reallocation of shelf space is

$$dV=k1*e*(avg(1)*ds(1)+avg(2)*ds(2)+...),$$

where avg(i)=p(i)*q(i)/s(i) or average retail dollar sales per foot of shelf space for product i; and ds(i) is the change in product i's shelf space. Given the shelf space constraint, note ds(1)+ds(2)+...+ds(n)=0 or ds(1)=-ds(2)-ds(3)-...-ds(n). Substituting the latter result into dV, we find

$$dV = k1*e*((avg(2)-avg(1))*ds(2) + (avg(3)-avg(1))*ds(3) + ...).$$

That is, if product 1 were COJ (FCOJ) profits could be increased by increasing COJ's (FCOJ's) shelf space and decreasing the shelf space for the remaining products, since, as shown in Table 1, COJ (FCOJ) has the greatest average retail dollar sales per foot of shelf space, i.e., $avg(i)-avg(1) < 0$ for $i = 2,3,...,n$, implying ds(2) through ds(n) should be negative for an increase in profit.

Conclusions

The results of this study show how shelf space can be allocated between products in a profit maximizing framework. The basic problem and equations might also be useful for allocation of other fixed factors among alternative uses. For example, a fixed amount of advertising expenditure might be allocated between different products or different markets to maximize profits or sales---the question of an optimal allocation can be examined through the advertising impacts on demands and profits using the same approach outlined here.

To fully apply the results of this study, one needs estimates of product demands, including the impacts of shelf space or any other similar factor being examined, along with product mark ups or per unit profits. Demand interactions between shelf space and factors such as advertising and promotion might also be considered in an application. Given the foregoing information, shelf space allocation can be determined straightforwardly by (5) or some variant with an alternative demand specification, i.e., the double log demand specification in (5) need not be maintained. Using this approach, fine-tune adjustments of shelf space for demand changes resulting from changes in the explanatory variables under consideration might be considered.

TABLE 1. DOLLARS AND GALLONS PER FACING AND PER SHELF-SPACE FOOTAGE, TOTAL US, BY PRODUCT CATEGORY, FOR WEEK ENDING MARCH 12,1994.

PRODUCT	NO. FAC. (000)	SHARE OF DEPT. FAC.	FEET (000)	DOL. (000)	SHARE OF DEPT. DOL.	GAL. (000)	SHARE OF DEPT. GAL.	DOL. PER FOOT	GAL. PER FOOT
REFRIGERATED DEPT. a									
JUICE/JUICE DRKS	3375	1.00	1230	49564	1.00	13899	1.00	40.28	11.30
ORANGE JUICE	1718	0.51	626	34476	0.70	9266	0.67	55.05	14.80
GRAPEFRUIT JUICE	170	0.05	62	1297	0.03	283	0.02	20.96	4.57
OTHER JUICE	512	0.15	186	3688	0.07	818	0.06	19.77	4.39
JUICE DRINKS	976	0.29	356	10104	0.20	3531	0.25	28.40	9.92
FROZEN DEPT. b									
JUICE/JUICE DRKS	4652	1.00	676	28941	1.00	10142	1.00	42.82	15.01
ORANGE JUICE	1654	0.36	240	13577	0.47	5082	0.50	56.51	21.15
GRAPEFRUIT JUICE	125	0.03	18	316	0.01	97	0.01	17.45	5.35
OTHER JUICE	1382	0.30	201	6706	0.23	2204	0.22	33.39	10.98
JUICE DRINKS	1491	0.32	217	8342	0.29	2759	0.27	38.50	12.74

a:Chilled ready-to-serve juice and juice drink products.
b:Frozen concentrate juice and juice drink products.

Footnotes

1 To obtain the C.E.S. specification note

(a) s(i) = (e*m(i)*A(i)*s(i)**e / (e*m'q)) * S,

or

(b) s(i)**(1-e) = (m(i)*A(i) / m'q) * S;

or

(c) s(i) = ((m(i)*A(i) / m'q) * S)**(1/(1-e))),

or

(d) m(i)*A(i)*s(i)**e = m(i)*A(i)* ((m(i)*A(i) / m'q) * S)**(e/(1-e))),

or

(e) m(i)*q(i) = (m(i)*A(i))**(1/(1-e)) * (m'q)**(-e/(1-e))*S**(e/(1-e)),

or, summing over i

(f) m'q = l'C * (m'q)**(-e/(1-e)) * S**(e/(1-e)),

where as defined previously C(i)=m(i)*A(i)**(1/(1-e)) and C=(C(1),...); hence

(g) (m'q)**(1/(1-e)) = l'C * S**(e/(1-e)),

or

(h) m'q = (l'C)**(1-e) *S**e.

Substituting (h) into (c) results in

(i) s(i) = (C(i) / l'C) * S.