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A FAMILY OF INVERSE DEMAND SYSTEMS AND CHOICE OF FUNCTIONAL FORM

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In a recent study, Barten (1992) compared the Rotterdam model and almost ideal demand system (AIDS), along with two mixed models--one with Rotterdam-type price effects and AIDS-type income effects and the other with AIDS-type price effects and Rotterdam-type income effects. A synthetic model that combines the features of the latter four models and allows non-nested hypothesis testing among models has also been proposed by Barten (1992). The foregoing models are all quantity dependent and treat prices as exogenous. However, for some goods, supply may be quite inelastic and an inverse demand system with prices dependent on quantities may be required. The Rotterdam inverse demand system (RIDS) and the almost ideal inverse demand system (AIIDS) are two flexible specifications that have been proposed (Barten and Bettendorf). Laitinen and Theil have also proposed an inverse demand system comparable to the latter. These three inverse demand systems, a fourth variant and a general synthetic model are examined in this paper.

The analysis focuses on choice of functional form. Just as Barten's (1992) analysis of the Rotterdam/AIDS specifications can be viewed as allowing the Rotterdam income coefficients (marginal propensities to consume) and compensated price coefficients (Slutsky coefficients) to be variational parameters dependent on budget shares, the analysis of the different inverse demand systems here can be viewed as allowing the RIDS scale and compensated quantity or Antonelli coefficients to be variational parameters dependent on budget shares.

The models examined are applied to data on demand for different types of fresh oranges--California navel oranges, Florida navel oranges and other Florida early and midseason oranges.

The supply of oranges in a given season is largely determined by weather conditions and past tree-planting and grove-care decisions by growers and is very inelastic.

The paper proceeds as follows. In the next section, the different models analyzed are developed. The application is then discussed and concluding comments are given in the final section.

MODELS

In this section, we focus on consumer demand in order to directly compare our model specifications with previous research. However, except for interpretation, our consumer demand specifications are essentially the same as those for a producer for inputs (Theil, 1980).

We first review some of the basic results of consumer demand theory relevant for subsequent model development. The utility maximization problem for the consumer can be written as

(1) maximize $u(q_1, \ldots, q_n)$

subject to $\sum p_i q_i = x$,

where u is utility, p_i and q_i are price and quantity for good i, respectively, and x is total expenditure or income.

The first-order conditions for (1) are the budget constraint $\sum p_i q_i = x$ and

(2) $$\frac{\partial u}{\partial q_i} = \lambda p_i, \qquad i = 1, 2, \ldots, n,$$

where λ is the Lagrange multiplier and is equal to the marginal utility of income, $\frac{\partial u}{\partial x}$. A system of uncompensated inverse demand equations can be obtained from the first-order conditions using Wold's identity, i.e.,

$$(3)\quad \pi_i = \frac{\frac{\partial u}{\partial q_i}}{\sum_j \left[\frac{\partial u}{\partial q_j}\right] q_j}, \qquad i = 1, 2, ..., n,$$

where $\pi_i = \frac{p_i}{x}$. (To obtain (3), multiply (2) by q_i, sum the result over i and solve for λ, and substitute the solution for λ in (2)).

A compensated inverse demand relationship can be found by working with the distance function which is dual to problem (1). The distance function can be written as $d(u, q)$ and indicates the amount by which vector $q = (q_1, \ldots, q_n)$ must be divided to reach utility level u (see, e.g., Anderson or Deaton and Muellbauer, 1980b). The distance function is increasing in q, decreasing in u, homogeneous of degree one in q, and concave in q. Differentiation of the distance function with respect to quantity yields a system of compensated inverse demand relationships, i.e.,

$$(4)\quad \pi_i = \frac{\partial d(u, q)}{\partial q_i} = \pi_i(u, q), \qquad i = 1, 2, ..., n.$$

The Rotterdam inverse demand system can be found by totally differentiating (4), i.e.,

$$(5)\quad d\pi_i = \frac{\partial \pi_i}{\partial u} du + \sum_j \frac{\partial \pi_i}{\partial q_j} dq_j, \qquad i = 1, 2, ..., n.$$

The term $\frac{\partial \pi_i}{\partial q_j}$ is known as the Antonelli substitution effect. The first term on the right side of (5) involves the scale effect. Consider reference bundle q^* and scalar k where $q = kq^*$. The term $\frac{\partial \pi_i}{\partial u} du$ can then be written as $\pi_i \left[\frac{\partial \log \pi_i}{\partial \log k}\right] \left[\frac{d \log u}{\frac{\partial \log u}{\partial \log k}}\right]$, where $\frac{\partial \log \pi_i}{\partial \log k}$ is known as the scale elasticity for good i. We can also write $d \log u = \sum \left[\frac{\partial \log u}{\partial \log q_j}\right] d \log q_j$ and $\frac{\partial \log u}{\partial \log k} = \sum \frac{\partial \log u}{\partial \log q_j}$, and, noting that (3) can be written as $w_i = \frac{p_i q_i}{x} = \frac{\frac{\partial \log u}{\partial \log q_i}}{\sum_j \frac{\partial \log u}{\partial \log q_j}}$, we see that $\frac{\partial \pi_i}{\partial u} du =$ $\pi_i \frac{\partial \log \pi_i}{\partial \log k} \sum_j w_j d \log q_j$. The variable w_i is the expenditure or budget share for the consumer.

Hence, by multiplying (5) by q_i and noting $dz = z\, d \log z$ for variable z in general, we have the Rotterdam inverse demand system

(6) $w_i \, d\log \pi_i = h_i \, d\log Q + \sum h_{ij} \, d\log q_j, \qquad i = 1, 2, ..., n,$

where $d\log Q = \sum w_j \, d\log q_j$ (the Divisia volume index), $h_i = w_i \dfrac{\partial \log \pi_i}{\partial \log k}$ and $h_{ij} = w_i \dfrac{\partial \log \pi_i}{\partial \log q_j}$

. The parameters h_i and h_{ij} are thus the scale elasticity and compensated quantity elasticity (flexibility), respectively, multiplied by the budget share w_i. The following restrictions on h_i and h_{ij} hold: $\sum_i h_i = -1$ and $\sum_i h_{ij} = 0$, for adding-up; $\sum_j h_{ij} = 0$, for homogeneity (since $d(u, q)$ is homogeneous of degree one in q, π_i is homogeneous of degree zero in q); $h_{ij} = h_{ji}$, for Antonelli symmetry.

In estimating the RIDS, parameters h_i and h_{ij} are treated as constants. However, other parameterizations are possible. For example, Laitinen and Theil's inverse demand model can be derived by adding $w_i \, d \log Q$ to both sides of (6), i.e.,

(7) $w_i \left(d\log \pi_i + d\log Q \right) = \left(w_i + h_i \right) d\log Q + \sum h_{ij} \, d\log q_j$

or

$$w_i \, d\log \frac{p_i}{P} = b_i \, d\log Q + \sum_j h_{ij} \, d\log q_j$$

where $b_i = w_i + h_i$ and $d \log \pi_i + d \log Q = d \log p_i - d \log x + d \log Q = d \log p_i - d \log P$, where $d\log P = \sum w_j \, d\log p_j$ (the Divisia price index). The last result follows from the relationship $d \log x = d \log P + d \log Q$ which is obtained by totally differentiating the budget constraint. In (7), the adding-up condition requires $\sum b_i = 0$. The Laitinen-Theil model can be viewed as a variational parameter specification of the RIDS with $h_i = b_i + w_i$.

The almost ideal inverse demand system can be obtained by further adding a specific term to each side of the Laitinen-Theil model as will be shown. However, first we show how the AIIDS can be obtained from the distance function

(8) $\log d(u, q) = a(q) - u b(q),$

$$a(q) = \alpha_0 + \sum \alpha_j \log q_j + \frac{1}{2} \sum_k \sum_l \gamma_{kl} \log q_k \log q_l,$$

$$b(q) = b_0 \pi q_k^{b_k}.$$

Differentiation of (8) with respect to log q_j yields

(9) $w_i = \alpha_i + \sum_j \gamma_{ij} \log q_j - b_i u b(q),$

or

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log q_j + b_i a(q),$$

where for utility maximization $d(u, q) = 1$ and from (8) $u\, b(q) = a(q)$. As an approximation, similar to that used to linearize the AIDS, replace $a(q)$ with $\sum w_i \log q_i$ and totally differentiate (9), further approximating the term $d\left(\sum w_i \log q_i\right)$ by $d \log Q$ following the same procedure suggested by Deaton and Muellbauer (1980a), as well as Barten and Bettendorf, in deriving the differential form for the AIDS. The result is the differential version of the AIIDS

(10a) $d w_i = b_i \, d \log Q + \sum_j \gamma_{ij} \, d \log q_j .$

As shown by Barten and Bettendorf, (10a) can also be obtained by adding $w_i(d \log q_i - d \log Q)$ to both sides of Laitinen and Theil's model, equation (7), i.e.,

$$(10b)\quad w_i\left(d\log p_i + d\log q_i - d\log P - d\log Q\right) = d w_i$$

$$= b_i d\log Q +$$

$$\sum_j \left(h_{ij} + w_i \delta_{ij} - w_i w_j\right) d\log q_j ,$$

where $\delta_{ij} = 1$ if $i = j$, else $\delta_{ij} = 0$; and $\gamma_{ij} = h_{ij} + w_i\, \delta_{ij} - w_i\, w_j$. Note that γ_{ij} are subject to the adding-up, homogeneity and symmetry conditions. As for the Laitinen-Theil model, the AIIDS can be viewed as a variational parameter specification of the RIDS with $h_i = w_i + b_i$ and $h_{ij} = \gamma_{ij} - w_i\, \delta_{ij} + w_i\, w_j$.

Another variant can be found by subtracting $w_i\, d \log Q$ from both sides of (10), i.e.,

$$(11)\quad d w_i - w_i d\log Q = \left(b_i - w_i\right) d\log Q + \sum \gamma_{ij}\, d\log q_j$$

$$= h_i d\log Q + \sum \gamma_{ij}\, d\log q_j .$$

Model (11) has RIDS scale effects and AIIDS quantity effects, and is referred to subsequently as RAIIDS. On the other hand, model (7) proposed by Laitinen and Theil has AIIDS scale effects and RIDS quantity effects.

Following Barten's (1992) approach in developing a synthetic model involving the AIDS and Rotterdam models and variants, we can also develop a synthetic model for the inverse demand models considered here. Note that, although the RIDS, AIIDS and associated variants have different left-side variables, they all have common right-side variables, and can be written as $y_j = x\, B_j$, where x is a vector of the common right-side variables (d log Q and the d log q_i's) and B_j is the associated parameter vector, j = 1, 2, 3, and 4 for equations (6), (7), (10) and (11), respectively. A synthetic model can be obtained by taking a scalar weighted average of the latter model with the weights a_i satisfying $\sum a_i = 1$, i.e.,

$$y_1 = \sum_{j=2}^{4} a_j(y_1 - y_j) + x\left[B_1 + \sum_{j=2}^{4} a_j(B_j - B_1)\right],$$

where $y_1 - y_2 = -w_i\, d \log Q$, $y_1 - y_3 = -w_i\, d \log q_i$ and $y_1 - y_4 = -w_i\, (d \log q_i - d \log Q)$. Since $(y_1 - y_2) - (y_1 - y_3) + (y_1 - y_4) = 0$, we can further write

(12) $$y_1 = (a_2 + a_3)(y_1 - y_2) + (a_4 + a_3)(y_1 - y_4) + x\left[B_1 + \sum_{j=2}^{4} a_j(B_j - B_1)\right]$$

or

$$w_i\, d \log \pi_i = (e_i - d_1 w_i)\, d \log Q + \sum_j \left(e_{ij} - d_2 w_i(\delta_{ij} - w_j)\right) d \log q_j .$$

Adding-up requires $\sum_i e_i = -1 + d_1$ and $\sum_i e_{ij} = 0$, homogeneity requires $\sum_j e_{ij} = 0$, and symmetry requires $e_{ij} = e_{ji}$.

The scale and compensated quantity elasticities for (12) are

(13) $$\frac{\partial \log \pi_i}{\partial \log Q} = \frac{e_i - d_1 w_i}{w_i} = \frac{e_i}{w_i} - d_1 \quad \text{(scale)},$$

$$\frac{\partial \log \pi_i}{\partial \log q_j} = \frac{e_{ij} - d_2 w_i(\delta_{ij} - w_j)}{w_i} = \frac{e_{ij}}{w_i} - d_2(d_{ij} - w_j) \quad \text{(compensated quantity)}.$$

The uncompensated quantity elasticity is (Anderson)

(14) $$\frac{\partial \log \pi_i}{\partial \log q_j} + w_j \frac{\partial \log \pi_i}{\partial \log Q}.$$

We can also obtain the other models and their elasticities by restricting d_1 and d_2 appropriately in (12), (13) and (14); i.e.,

$$
\begin{aligned}
(15)\quad & d_1 = 0, \quad d_2 = 0 \quad \text{for the RIDS model (6);} \\
& d_1 = 1, \quad d_2 = 0 \quad \text{for the Laitinen-Theil model (7);} \\
& d_1 = 1, \quad d_2 = 1 \quad \text{for the AIIDS model (10);} \\
& d_1 = 0, \quad d_2 = 1 \quad \text{for the RAIIDS model (11).}
\end{aligned}
$$

The restrictions above underlie the testing procedure to compare models.

APPLICATION

The preceding models were applied to data on different varieties of early and midseason oranges: (1) California navel oranges, (2) Florida navel oranges, and (3) other Florida early and midseason oranges. Although these types of oranges are close substitutes, they differ in quality and appearance (e.g., California navels have few blemishes and a bright orange color; Florida oranges often have blemishes and may not be bright in color; Florida oranges are generally juicier than California oranges; and navel oranges have few seeds compared to other early and midseason oranges). The period from the first week in November through the third week in January for the 1984-85 through 1993-94 seasons was analyzed. The November-January time period is the peak harvesting and marketing period for Florida navel and other early and midseason oranges; a large volume of California navel oranges are also harvested and marketed during this period. Twelve weekly observations per season were available, providing 120 observations for the analysis.

Data were obtained from the Citrus Administrative Committee's (CAC) *Annual Statistical Report* for the different seasons studied. Quantities used in the analysis were FOB wholesale orange shipments. FOB wholesale prices were also used. The shipments are treated as predetermined since fresh oranges are marketed as they mature and are harvested. The orange shipments are divided by population to induce homoscedasticity (Theil and Clements).[1]

Using wholesale data requires a slightly different motivation for our models. The models in the preceding section were developed for consumer demand, but the data analyzed here reflect input demand. However, from the producer's cost minimization problem, we can obtain input demand equations with the same form and restrictions as those for the consumer as shown by Theil, 1980 (e.g, see Theil and Clements, Clements and Theil, and Huang for empirical studies of input demand); the problem for the firm is essentially the same as that for the consumer. The retail grocery store level of the domestic food industry is considered as a cost-minimizing producer which buys food inputs. We assume an underlying technology with different varieties of oranges being weakly separable from other foods and apply the theory of rational random behavior (Theil, 1976, 1980) to estimate conditional demand equations for oranges.[2]

The data on orange shipments exhibit a seasonal pattern. Shipments peak twice over the twelve-week period studied--once before Thanksgiving and once before Christmas. To account for seasonality in (12), the basic log change variables (log changes in quantities and normalized prices) were calculated as twelfth differences and the expenditure share variables were calculated as averages of the present and twelfth-period-lagged expenditure share values (Duffy).

Over the period studied, weekly expenditure shares for California navels, Florida navels, and other early and midseason oranges were .72 (.10), .13 (.09) and .15 (.05), respectively, with standard deviations in parentheses. As the data add up by construction--total expenditure in the models is the sum of expenditures on the three types of oranges--the error covariance matrix is singular and the equation for other early and midseason oranges was excluded from the system for estimation; the full information maximum likelihood procedure (TSP) was used to estimate the models with the errors across equations assumed to be contemporaneously correlated. As shown by Barten (1969), the maximum likelihood estimates are invariant with respect to the equation excluded from the system.

The likelihood ratio test was used to test restrictions (15). Under the null hypothesis of the restricted model, twice the difference between the maximum logarithmic likelihood value for the unrestricted and that value for the restricted model is asymptotically distributed as a chi-square statistic. The number of degrees of freedom for the statistic is equal to the number of restrictions imposed (the difference between the number of free parameters in the unrestricted and restricted models). For testing the different models indicated by (15), there are two degrees of freedom.

Synthetic model (12) with homogeneity and symmetry imposed was the unrestricted model for testing (15). The homogeneity and symmetry restrictions were not rejected at the 10% level of significance, based on the likelihood ratio test. The maximum likelihood values for (12) without homogeneity and symmetry imposed, with homogeneity imposed, and with homogeneity and symmetry imposed were 535.091, 534.514 and 533.013, respectively. The chi-square test statistics with degrees of freedom in parentheses were 1.154 (2) for homogeneity and 4.156 (3) for homogeneity and symmetry.

Table 1 shows the logarithmic likelihood values for the synthetic model and restricted versions. For any reasonable level of significance, the RIDS, Laitinen-Theil model, AIIDS and RAIIDS are rejected against the synthetic model. The hypotheses that d_1 is zero with d_2 being free and d_2 is zero with d_1 being free are also rejected.

The synthetic model estimates for d_1 and d_2 were 1.14 (.122) and .26 (.090), respectively, with asymptotic standard errors in parentheses, indicating both estimates are different from zero at the 10 % level of significance. The coefficients of determination for the synthetic model were .981, .816 and .768 for California navels, Florida navels and other early and midseason oranges, respectively. Although the results in Table 1 indicate that the synthetic model be chosen for further analysis, we first compare the (conditional) scale elasticities and price flexibilities for the different models to see if the basic demand responses for the synthetic model differ from those for the restricted versions (comparison of the elasticities/flexibilities is more straightforward than comparison of the parameter estimates which have less clear interpretations). Table 2 shows the scale and price flexibility estimates for the synthetic model, RIDS, Laitinen-Theil model, AIIDS and RAIIDS. The estimates were made using the sample mean expenditure shares. Comparison across models indicates substantial differences in the estimated demand responses. For example, the own-price flexibility for Florida navels varies from -.20 for the RIDS to-.57 for the AIIDS, with a value of -.33 for the synthetic model. A number of other substantial differences in demand responses across models can also be seen. The differences in the basic demand responses show that model choice is important, and, given the results in Table 1, the synthetic model is chosen to further describe the demand for these varieties of oranges.

All the demand response estimates for the synthetic model in Table 2 are twice or larger than their corresponding asymptotic standard error estimates. The scale elasticity estimates indicate that a one-percent increase in aggregate quantity of oranges would result in decreases in the normalized prices of 1.13 %, 1.06 % and.96 % for Florida navels, other Florida early and midseason oranges and California navels, respectively. With a scale elasticity value of -1 indicating the expenditure share is constant as the scale changes, these results indicate that the (conditional) expenditure share for either Florida variety (California navels) can be expected to decrease (increase) slightly with an increase in scale.

The (conditional) uncompensated own-quantity elasticities or flexibilities for the synthetic model in Table 2 are consistent with theory with negative signs. The cross flexibilities are also negative, indicating substitute relationships. Overall, the magnitudes of the flexibilities indicate that prices for the Florida varieties are relatively more sensitive to California navel shipments than own shipments. California navel oranges have the highest own-flexibility at -.77, with Florida early and midseason oranges, and navel oranges having own-flexibilities of -.39 and -.33, respectively. The cross-flexibility estimates indicating the impacts of California navel shipments on the prices of Florida early and midseason oranges, and navels are -.53 and -.63, respectively.

CONCLUDING COMMENTS

For modelling quantity-dependent demand, Barten (1992) has shown that the Rotterdam and AIDS models may be too rigid and a mixed model with features of both models may perform better. In this paper, we show that, for modelling price-dependent demand, a similar mixed model may be more appropriate than the Rotterdam inverse demand system or almost ideal inverse demand system by themselves. A synthetic model which combines the features of the RIDS and AIIDS can be used to test alternative model specifications and can serve by itself as a demand model.

FOOTNOTES

[1]Presence of heteroscedasticity was indicated by Goldfeld-Quandt tests. The sample was divided into two partitions---one where the population variable was at relatively high levels, and the other where this variable was relatively low. For each partition, non-per-capita inverse demand equations for the different types of oranges were estimated. The Goldfeld-Quandt F statistic for each type of orange was the sum of squares for the high population partition divided by the sum of squares for the low population partition, with 50 degrees of freedom in both the denominator and numerator. The F values were 2.41, 5.20 and .94 for California navels, Florida navels and other Florida early and midseason oranges, respectively, indicating that the California navel and Florida navel equations have heteroscedastic errors, at most reasonable levels of significance.

[2] Insufficient data on other goods precluded testing for weak separability.

Table 1. Maximum likelihood test statistics.

System	$\hat{d}_1$	$\hat{d}_2$	MLV[a]	LRT[b]
RIDS	0	0	475.333	115.360 (2)
Laitinen-Theil	1	0	524.229	17.568 (2)
AIIDS	1	1	461.831	142.368 (2)
RAIIDS	0	1	402.354	261.318 (2)
Free d_1, Zero d_2	0.969	0	524.304	17.418 (1)
Free d_2, Zero d_1	0	-.066	475.720	114.570 (1)
Synthetic	1.144	.257	533.013	--

[a]Maximum likelihood value.

[b]2 x (MLV - MLV for synthetic); degrees of freedom in parentheses.

Table 2. Scale elasticity and flexibility estimates for the five alternative models

	Scale Elasticity Estimate	Flexibility Estimate		
		Cal. Navel	Fla Navel	Fla E&M
		Synthetic Model		
Cal. Navel	-0.963 (0.032)[a]	-0.775 (0.015)	-0.095 (0.009	-0.093 (0.013)
Fla Navel	-1.131 (0.064)	-0.629 (0.037)	-0.330 (0.032	-0.172 (0.041)
Fla E&M	-1.063 (0.117)	-0.526 (0.054)	-0.148 (0.041	-0.389 (0.054)
		RIDS		
Cal. Navel	-0.995 (0.047)	-0.755 (0.019)	-0.122 (0.015	-0.119 (0.017)
Fla Navel	-0.867 (0.117)	-0.557 (0.047)	-0.199 (0.041	-0.111 (0.049)
Fla E&M	-1.146 (0.145)	-0.690 (0.060)	-0.139 (0.046	-0.317 (0.067)
		Laitinen-Theil Model		
Cal. Navel	-0.974 (0.030)	-0.775 (0.013)	-0.103 (0.009	-0.096 (0.013)
Fla Navel	-1.117 (0.066)	-0.654 (0.035)	-0.240 (0.033	-0.222 (0.036)
Fla E&M	-1.022 (0.113)	-0.502 (0.045)	-0.191 (0.037	-0.329 (0.055)
		AIIDS		
Cal. Navel	-0.933 (0.038)	-0.760 (0.014)	-0.075 (0.011	-0.097 (0.015)
Fla Navel	-1.091 (0.104)	-0.515 (0.046)	-0.566 (0.037	-0.010 (0.042)
Fla E&M	-1.245 (0.115)	-0.700 (0.047)	-0.029 (0.039	-0.515 (0.053)
		RAIIDS		
Cal. Navel	-0.949 (0.063)	-0.739 (0.024)	-0.092 (0.020	-0.118 (0.022)
Fla Navel	-0.846 (0.189)	-0.419 (0.078)	-0.518 (0.070	0.091 (0.058)
Fla E&M	-1.390 (0.155)	-0.894 (0.058)	0.010 (0.049)	-0.506 (0.070)

[a]Asymptotic standard errors in parentheses.

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