RESEARCH PAPER: 1993-5

THE SILENCE OF THE LAMBDAS: A TEST OF THE ALMOST IDEAL AND ROTTERDAM MODELS: COMMENT

BY

Mark G. Brown
Jonq-Ying Lee
Senior Research Economists

FLORIDA DEPARTMENT OF CITRUS
Economic and Market Research Department
P.O. Box 110249
Gainesville, Florida 32611-2049 USA
Phone: 352-392-1874
Fax: 352-392-8634
Email: mgbrown@ufl.edu

www.floridajuice.com

6/9/93
SP 93-14

The Silence of the Lambdas: A Test of the Almost Ideal and Rotterdam Models: Comment

Mark G. Brown and Jonq-Ying Lee*

*Mark G. Brown and Jonq-Ying Lee are Research Economists, Florida Department of Citrus, and Courtesy Associate Professor and Courtesy Professor, respectively, Department of Food and Resource Economics, University of Florida, Gainesville, Florida 32611.

The Silence of the Lambdas:
A Test of the Almost Ideal and Rotterdam Models:
Comment

In a study of the Almost Ideal Demand System (linear approximate) and the Rotterdam model, Alston and Chalfant (AC) propose a test to choose between the two alternative models. (See Deaton and Muellbauer for the Almost Ideal Demand system and Barten (1964) and Theil (1965) for the Rotterdam model.) Unfortunately, their test, compound models (10) and (11) as well as Rotterdam Model II in the AC paper, suffers from violation of the adding-up condition of demand equations. The adding-up problem is related to the real income variables chosen in specifying the compound models. As a result of this problem, tests (10) and (11) are not invariant to the good omitted from the system for estimation, or if all goods are included in the system for estimation, the parameter estimates will not generally satisfy adding up.

The adding-up condition requires that the demands for the goods under consideration satisfy the budget constraint. When data for a system of demand equations satisfy adding up, the disturbance covariance matrix for the system is singular, and the maximum likelihood estimation procedure can not be applied to the full system of equations (Theil, 1971). Barten (1969), however, has shown that consistent demand system estimates can be obtained by arbitrarily deleting an equation from the system and applying the maximum likelihood procedure to the system of remaining equations. The parameters for the omitted equation can be determined from the estimates of the other equations and the adding-up conditions. The maximum likelihood estimation procedure is invariant to the equation deleted. Below, we examine compound model test equation (10) in the AC paper with regard to adding up; compound model (11) can be similarly examined, with results for the two compound models being essentially the same.

Following the notation in Alston and Chalfant, let p_i and q_i be the price and quantity of good i, respectively; $x = \sum p_i q_i$ or total expenditure; $s_i = \frac{p_i q_i}{x}$, the budget share for good i, and $\bar{s}_i = \frac{s_{it} + s_{it-1}}{2}$ with subscript t indicating time. AC compound model (10) can then be written as

$$(1)\quad \left(1 - \lambda_1\right)\bar{s}_i \Delta \ln q_i + \lambda_1 \Delta s_i = \tau_i + \sum_k \theta_{ik} D_k + \sum_j \gamma_{ij} \Delta \ln p_j + \beta_i \Delta \ln \left[\frac{x}{p^*}\right],$$

where Δ is the first difference operator; $\tau_i + \sum_k \theta_{ik} D_k$ are time trend and seasonality effects; and $\Delta \ln p^* = \sum \bar{s}_j \Delta \ln p_j$.

Our examination focuses on compound model (10) real income variable which can be written as

$$(2)\quad \Delta \ln \frac{x}{p^*} = \Delta \ln x - \sum \bar{s}_j \Delta \ln p_j .$$

Normally, real income (2) is not used in the Rotterdam model but is replaced by the Divisia volume index $\sum \bar{s}_j \Delta \ln q_j$ in order to satisfy adding up. It is noteworthy that (2) and the Divisia volume index are only approximately equal; namely,

$$(3)\quad \ln s_i = \ln p_i + \ln q_i - \ln x$$

$$\text{or } \Delta \ln s_i = \Delta \ln p_i + \Delta \ln q_i - \Delta \ln x \quad \text{or}$$

$$\bar{s}_i \Delta \ln s_i = \bar{s}_i \Delta \ln p_i + \bar{s}_i \Delta \ln q_i - \bar{s}_i \Delta \ln x$$

or, after summing over *i* and rearranging,

$$\Delta \ln x - \sum \bar{s}_i \Delta \ln p_i = \sum \bar{s}_i \Delta \ln q_i - \sum \bar{s}_i \Delta \ln s_i .$$

As suggested by Theil (1971), the term $\sum \bar{s}_i \Delta \ln s_i$ tends to be close to zero, and hence use of the Divisia volume index in place of real income (2) can be expected to be a reasonable approximation.

Substitution of the Divisia volume index in the Rotterdam assures that the data for the model satisfy the adding-up condition. Without the substitution, the Rotterdam model (AC model II) or AC compound model (10) with $\lambda_1 = 0$ can be estimated without deletion of an equation, and, in this case, the demand estimates will not, in general, satisfy adding up. Alternatively, AC compound model (10) results with $\lambda_1 = 0$, which are shown in Table 3 in the Alston and Chalfant paper, are specific to the good deleted, fish. If fish were included in the model and, say, pork deleted, the results would change, although the magnitude of the change would of course be data specific.

In general, the adding-up problem with compound models (10) and (11) in the AC paper can be seen by summing the equations over *i*, imposing the usual Almost Ideal Demand and Rotterdam parameter restrictions (the actual parameter estimates of (10), however, will not generally satisfy these restrictions), and noticing contradictions with respect to the left- and right-hand sides of the summations. For example, for (10) the summation is

$$(4)\quad \sum_i \left(1 - \lambda_i\right)\bar{s}_i \Delta \ln q_i + \lambda_1 \sum_i \Delta s_i = \sum_i \tau_i + \sum_k \sum_i \theta_{ik} D_k + \sum_j \sum_i \gamma_{ij} \Delta \ln p_j + \sum_i \beta_i \Delta \ln \frac{x}{p^*}$$

or $$\left(1 - \lambda_1\right)\sum_i \bar{s}_i \Delta \ln q_i = \sum_i \beta_i \Delta \ln \frac{x}{p^*} = \left(\sum_i \beta_i\right)\left(\Delta \ln x - \sum_j \bar{s}_j \Delta \ln p_j\right)$$

or $$\sum_i \bar{s}_i \Delta \ln q_i = \Delta \ln x - \sum_j \bar{s}_j \Delta \ln p_j$$

where, in summing over *i*, the term $\lambda_1 \sum \Delta s_i = 0$, and use has been made of Almost Ideal Demand and Rotterdam restrictions which require that the time, seasonality and price parameters sum to zero; and consistent with the implied linear combination of the Almost Ideal Demand and Rotterdam models, we note that $\beta_i = \left(1 - \lambda_1\right)\beta_i' + \lambda_1 \beta_i''$ where β_i' and β_i'' are the income parameters for the Rotterdam and Almost Ideal Demand models, respectively, so that $\sum_i \beta_i = 1 - \lambda_1$, since $\sum \beta_i' = 1$ for the Rotterdam model and $\sum \beta_i'' = 0$ for the Almost Ideal Demand model. As shown previously by (3), the left-hand side of equation (4), which is the Divisia volume index, is not equal to the real income variable on the right-hand side of (4), and hence (10) in the AC paper involves a contradiction. A similar contradiction can be shown for compound model (11).

Summarizing, the adding-up problem with compound test model (10) in the AC paper, and similarly model (11), prevents these models from consistently choosing the Rotterdam model

($\lambda_1 = 0$ in (10) or $\lambda_2 = 1$ in (11)), as the summation of the dependent variables over goods for the Rotterdam model is the Divisia volume index which is not equal to the real income variable in either (10) or (11). Hence estimation of the compound models using all demand equations will fail to satisfy adding up, or if an equation is deleted, the results will not be invariant to the deleted equation.

One solution to the foregoing problem is to use the Divisia volume index as the real income variable in both AC models (10) and (11). This solution follows from the summation over *i* of both sides of (10) and (11). We illustrate the summation for (10) using OLS estimates; the result extends to the full information maximum likelihood estimates (Barten, 1969; Bewley). Let $Y_i = \left[(1 - \lambda_1)\bar{s}_i \Delta \ln q_i + \lambda_1 \Delta s_i\right]$ be a *T*x1 vector for the observations for good *i*; and *X* be a *T*x*K* explanatory matrix for (10) with the K^{th} or last column being the vector of real income variables $\left[\Delta \ln x - \sum \bar{s}_j \Delta \ln p_j\right]$. Hence, equation (10) can be written as $Y_i = X\beta_i$ where β_i are the parameters to be estimated. For any given value of λ_1, the OLS estimates are $\hat{\beta}_i = (X'X)^{-1}X'Y_i$ and $\sum \hat{\beta}_i = (X'X)^{-1}X'\sum Y_i$ which does not satisfy adding up in general since $\sum Y_i$ does not equal or is proportional to or is a linear combination of any of the columns of *X* (Bewley). However, suppose the last column of *X* is $\left[\sum \bar{s}_i \Delta \ln q_i\right]$, the vector of Divisia volume index values. Given that $\sum Y_i = (1 - \lambda_1)\left[\sum \bar{s}_i \Delta \ln q_i\right] = (1 - \lambda_1)X\alpha$, where α

$= (0, \ldots, 0, 1)'$ is a Kx1 vector with the first K-1 elements being zero and the K^{th} element being one, the summation of parameter estimates $\sum \beta_i = (X'X)^{-1} X'X(1 - \lambda_1)\alpha = (1 - \lambda_1)\alpha$ which indicates adding up is satisfied. This solution, of course, is the same as used in specifying the usual Rotterdam model (Theil, 1971).

Lastly, we note that if the Divisia volume index were used in specifying the test model in the AC paper, the resulting model would essentially be a special case of Barten's synthetic model (Barten, 1990) which combines the features of the Almost Ideal Demand and Rotterdam models. At risk of confusion with notation in the AC paper, which we have tried to follow to this point, the synthetic model is shown below using the common income and price parameter notation for the Almost Ideal Demand and Rotterdam models, omitting time trend and seasonality effects for convenience. The usefulness of showing this specification lies with the interpretation of the underlying model parameters.

The linear combination of the two models (equation (10) in the AC paper) can be written as

$$(5) \quad (1 - \lambda_1)\bar{s}_i \Delta \ln q_i + \lambda_1 \Delta s_i = (1 - \lambda_1)\left[\sum_j \pi_{ij} \Delta \ln p_j + \theta_i \Delta \ln\left[\frac{x}{p^*}\right]\right]$$
$$+ \lambda_1 \left[\sum_j \gamma_{ij} \Delta \ln p_j + \beta_i \Delta \ln\left[\frac{x}{p^*}\right]\right],$$

where π_{ij} and θ_i are the Slutksy coefficient and marginal propensity to consume, respectively, for the Rotterdam model; and γ_{ij} and β_i are the price and income parameters, respectively, specifically for the Almost Ideal Demand System, in contrast to their general interpretation in the AC paper.

The total differential of s_i is $ds_i = s_i(d\ln p_i + d\ln q_i - d\ln x)$, so that Δs_i is approximately

(6) $\Delta s_i \simeq \bar{s}_i(\Delta \ln p_i + \Delta \ln q_i - \Delta \ln x)$.

Substituting (6) for Δs_i in (5), adding and subtracting $\lambda \bar{s}_i \Delta \ln p^*$ on the right-hand side of the equation and rearranging terms, the compound model can be written as

(7) $\bar{s}_i \Delta \ln q_i = (\alpha_i + \lambda_1 \bar{s}_i) \Delta \ln \frac{x}{p^*} + \sum_j \left(c_{ij} - \lambda_1 \bar{s}_i (\delta_{ij} - \bar{s}_j)\right) \Delta \ln p_j$

where $\delta_{ij} = 1$, if $i = j$, otherwise $\delta_{ij} = 0$; $\alpha_i = \theta_i + \lambda_1(\beta_i - \theta_i)$ and $c_{ij} = \pi_{ij} + \lambda_1(\gamma_{ij} - \pi_{ij})$.

Replacing the real income variable $\Delta \ln \frac{x}{p^*}$ in (7) by the Divisia volume index, to satisfy adding up, results in a special case of Barten's (Barten, 1990) synthetic model (model (7) has one additional parameter λ_1, as opposed to two additional parameters in Barten's more general synthetic model). Specifying the compound model in the synthetic format as in (7) may be useful for seeing the inherent variational parameter interpretation that can be given to the model marginal propensities $(\alpha_i + \lambda_1 \bar{s}_i)$ and Slutsky coefficients $\left(c_{ij} - \lambda \bar{s}_i(\delta_{ij} - \bar{s}_j)\right)$; i.e., these coefficients now depend on the budget shares, increasing the flexibility of demand responses.

References

Alston, J.M. and J.A. Chalfant. "The Silence of the Lambdas: A Test of the Almost Ideal and Rotterdam Models." *American Journal of Agricultural Economics* 75 (May, 1993): 304-13.

Barten, A.P. "Consumer Demand Functions under Conditions of Almost Additive Preferences." *Econometrica* 32 (January-April, 1964):1-38.

Barten, A.P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review* 1 (1969):7-73.

Barten, A.P. "Consumer Allocation Models: Choice of Functional Form." Catholic University of Leuven and CORE, Discussion Paper, December, 1990.

Bewley, R. *Allocation Models: Specification, Estimation and Applications*. Cambridge, Massachusetts: Ballinger Pub. Co., 1986.

Deaton, A.S. and J. Muellbauer. "An Almost Ideal Demand System." *American Economic Review* 70 (June, 1980):312-26.

Theil, H. "The Information Approach to Demand Analysis." *Econometrica* 33 (January, 1965): 67-87.

Theil, H. *Principles of Econometrics*. New York: John Wiley and Sons, Inc., 1971.