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RESTRICTIONS ON AUTOREGRESSIVE ERROR PROCESSES IN SYSTEMS OF DEMAND EQUATIONS

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Abstract

Alternative theoretically based restrictions on autoregressive error processes in systems of demand equations are examined. Scaling, translation and a utility-based approach suggested by Theil are used to generate restrictions. A study of juice demands suggests that the restrictions examined may be useful for empirical analysis.

Restrictions on Autoregressive Error Processes in Systems of Demand Equations

Introduction

In systems of demand equations with autoregressive error processes, the adding-up property for expenditure implies certain restrictions on the error processes as shown by Berndt and Savin. For the situation where each equation's error follows a simple first-order autoregressive process with no cross-equation autocorrelation, adding up requires a common autocorrelation coefficient ρ to describe the error process in each equation. However, this particular specification, although frequently used in empirical studies, may be overly restrictive (Deaton, 1986). Berndt and Savin's general vector autoregressive process[1] provides an alternative but, to make the problem more manageable, various restrictions on the autoregressive error structure, less severe than those for the simple first-order autoregressive case, might be considered. What restrictions to consider, though, is not clear with economic theory having been used little or not at all in the specification process (Berndt and Savin; Anderson and Blundell).

In this paper, some possible linkages between consumer demand theory and autoregressive error processes in demand systems are examined. Alternative autoregressive error restrictions, motivated by particular consumer demand specifications for introducing nonprice, nonincome effects in demand systems, are proposed, and an empirical application is discussed in which the

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[1]The general vector autoregressive error process in a system of demand equations itself can be considered a particular restrictive case of a more general dynamic specification (Anderson and Blundell). Consumers may not adjust to equilibrium in every time period as suggested by the static model, and dynamic specifications recognizing such possibilities as habit persistence and inventory effects, adjustment costs, expectation errors and misinterpreted real price changes, might be more appropriate. The general dynamic specification proposed by Anderson and Blundell includes, as explanatory variables, lagged endogenous and exogenous variables, which under appropriate coefficient restrictions result in an autoregressive error process.

restrictions are examined. The analysis is limited to first-order autoregressive processes but could be extended to higher order processes.

Autoregressive Error Processes and Demand Restrictions

Following the general approach taken by Hanemann, the analysis in this paper is based on the assumption that a linkage exists between autoregressive disturbances attached to demand equations for estimation and the underlying utility function. Instead of attempting to develop a theoretical model for the linkage, general consumer demand theory results on demand effects of nonprice, nonincome variables are applied to the general autoregressive error process.

The consumer's direct utility function depends on the quantities of goods under consideration and, in general, many other factors such as physical and psychological stocks, consumer characteristics and attributes of goods, among other possibilities. In turn, the associated demand functions depend on prices and total expenditure, as well as these utility-conditioning factors. To the researcher, many conditioning nonprice, nonincome factors are unobservable and hence contribute to the random nature of the demand relationships studied (to the consumer, these factors are assumed to be known, allowing maximization of utility to occur in a deterministic fashion).

Various restrictions on the effects of nonprice, nonincome variables on demand have been suggested, and, given the general link between the equation errors in demand systems and unobservable nonprice, nonincome variables, the strategy of this study is to consider nonprice, nonincome demand restrictions in specifying autoregressive error structures.

Specification of the demand restrictions considered in this study first requires some preliminary notation. Let p_i and q_i be the price and quantity of good i; $x = \sum_{i=1}^{n} p_i q_i$ be total expenditure or income; z be some nonprice, nonincome variable; and $u = u(q,\, z)$ be utility where q is a vector containing the q_i's. The focus of attention is on the effect of z on q.

Barten (1977) has shown that, in general, the effect of a nonprice, nonincome variable z on demand can be written as

$$(1)\ \frac{\partial q_i}{\partial z} = -\frac{1}{\lambda} \sum_k s_{ik} u_{kj},$$

where $\lambda = \dfrac{\partial u}{\partial x}$, the marginal utility of income; $s_{ik} = \dfrac{\partial q_i}{\partial p_k} + q_k \dfrac{\partial q_i}{\partial x}$, the substitution effect; and

$u_{kz} = \dfrac{\partial^2 u}{\partial q_k \partial z}$, the effect of z on the marginal utility of good k.

In the empirical example of this study, the generalized addilog demand system or GADS (Bewley; Bewley and Young) is used, and an alternative version of the above result applicable to the GADS can be obtained by multiplying (1) by $\dfrac{p_i}{x}$; i.e.,

$$(2)\quad \frac{p_i q_i}{x}\frac{1}{q_i}\frac{\partial q_i}{\partial z} = -\sum_k \frac{p_i p_k}{x} s_{ik} \frac{\partial\left[\dfrac{\partial u}{\partial q_k}\right]}{\partial z} \frac{1}{\dfrac{\partial u}{\partial q_k}}$$

or

$$w_i \frac{\partial \log q_i}{\partial z} = -\sum_k \pi_{ik} r_{kj},$$

where $w_i = \frac{p_i q_i}{x}$, the budget share for good i; $\pi_{ik} = \frac{p_i p_k}{x} s_{ik}$, the Slutsky coefficient (see the GADS or Rotterdam model (Barten, 1964a; Theil, 1965); $r_{kj} = \frac{\partial \log\left[\frac{\partial u}{\partial q_k}\right]}{\partial z}$; and, based on the first-order conditions for utility maximization, λ has been replaced by $\frac{p_k}{\frac{\partial u}{\partial q_k}}$.

Result (2) can be useful as a source of restrictions in the GADS or Rotterdam models. Suppose that, instead of a single variable z, product-specific nonprice, nonincome variables, denoted by z_i, affect demand, i.e., result (2) applies straightforwardly with a product-specific subscript added to z. In subsequent analysis, the lagged error in an equation will be treated like a product-specific z_i. One possible demand restriction in such a case is that the effect of z_i is limited to its effect on the marginal utility of good i. Theil (1980) has proposed this particular assumption for the effects of advertising on demand, and Duffy's (1987, 1990) research results indicate the assumption is reasonable for describing demand for wines and spirits. For this restriction, result (2) becomes

$$(2) \quad w_i \frac{\partial \log q_i}{\partial z_j} = -\pi_{ij} r_{jj} .$$

Treating π_{ij} and r_{jj} as parameters to be estimated, the estimation problem has become much more manageable since only the diagonal terms of $[r_{ij}]$ need to be estimated. The general strategy of obtaining demand restrictions in this fashion, i.e., through the effects on marginal utilities, is like that used to generate various separability restrictions (Theil, 1975, 1976).

Other restrictions on the effects of the z_j's can be obtained using scaling and translation methods (Gorman; Barten, 1964b; Pollak and Wales; Deaton and Muellbauer, 1980a). Consider the cost function $c(u, p) = \sum p_i^* \gamma_i + c^*(u, p^*)$ where $p_i^* = \frac{p_i}{\phi_i}$, and γ_i and ϕ_i are translation and scaling parameters, respectively, with $p = (p_1, ..., p_n)$ and $p^* = (p_1^*, ..., p_n^*)$. The translation term $\sum p_i^* \gamma_i$ can be viewed as fixed cost with γ_i indicating basic needs, while p_i^* can be viewed as an adjusted price, dependent on the scaling term ϕ_i, which can be viewed as a measure of perceived quality or basic needs. In general, however, the parameters γ_i and ϕ_i can simply be viewed as indicators of preferences.

The z_j variables can be included in the cost function through the scaling and translation parameters. In particular, the assumption $\gamma_i = \gamma_i(z_i)$ and $\phi_i = \phi_i(z_i)$ is considered (e.g., Brown and Lee).

Using Shepherd's lemma, the cost function can be differentiated with respect to price p_i to obtain the Hicksian demand equation for good i, i.e.,

$$(4)\quad q_i = \frac{1}{\phi_i}\left(\gamma_i + q_i^*(p^*;\, u)\right),$$

where $q_i^* = \dfrac{\partial c^*}{\partial p_i^*}$.

Letting $x = c(p,\, u)$, the cost function can be inverted to find the indirect utility function $u = \psi\left(p^*;\, x - \sum p_i^* \gamma_i\right)$, which can be substituted into (4) to obtain the Marshallian demand equation

$$(5)\quad q_i = \frac{1}{\phi_i}\left(\gamma_i + q_i^*(p^*;\, x^*)\right),$$

where $x^* = x - \sum p_i^* \gamma_i$.

For specification (5), the translation and scaling elasticities can be written as

$$(6)\quad \frac{\partial \log q_i}{\partial \log \gamma_j} = \Delta_{ij}\frac{\gamma_j}{\phi_i q_i} - \frac{\partial \log q_i}{\partial \log x} w_j^*$$

and

$$(7)\quad \frac{\partial \log q_i}{\partial \log \phi_j} = -\Delta_{ij} - \frac{\partial \log q_i}{\partial \log p_j}$$

where Δ_{ij} is the Kronecker delta ($\Delta_{ij} = 1$ if $i = j$, $= 0$ if $i \neq j$) and $w_j^* = \dfrac{p_j^* \gamma_j}{x}$.

Given (6) and (7), $w_i\left[\dfrac{\partial \log q_i}{\partial \log \gamma_j}\right] = \Delta_{ij} w_j^* - \theta_i w_j^*$ and $w_i\left[\dfrac{\partial \log q_i}{\partial \log \phi_j}\right] = -\Delta_{ij} w_j - \pi_{ij}$ $+ w_j \theta_i$ where $\theta_i = w_i \dfrac{\partial \log q_i}{\partial \log x}$ is the marginal propensity to consume for good i; and, letting h_j $= w_j^* \dfrac{\partial \log \gamma_j}{\partial z_j}$ and $l_j = \dfrac{\partial \log \phi_j}{\partial z_j}$, the effect of z_j on the demand for good i can be written as

(8) $$w_i \frac{\partial \log q_i}{\partial z_j} = \Delta_{ij}\left(h_j - w_j l_j\right) + \theta_i\left(w_j l_j - h_j\right) - \pi_{ij} l_j$$

or

$$w_i \frac{\partial \log q_i}{\partial z_j} = \left(\Delta_{ij} - \theta_i\right) m_j - \pi_{ij} l_j,$$

where $m_j = h_j - w_j l_j$.

The foregoing results indicate that the Theil, scaling and translation approaches involve the following demand restrictions for the effect of z_j:

(9) $w_i \dfrac{\partial \log q_i}{\partial z_j} = -\pi_{ij} r_{jj}$ for the Theil hypothesis.

(10) $w_i \dfrac{\partial \log q_i}{\partial z_j} = \left(\Delta_{ij} - \theta_i\right) h_j$ for translation hypothesis with scaling parameters fixed; i.e., $l_j = 0$.

(11) $w_i \dfrac{\partial \log q_i}{\partial z_j} = \left[\left(\theta_i - \Delta_{ij}\right) w_j - \pi_{ij}\right] l_j$ for scaling hypothesis with translation parameters fixed, i.e., $h_j = 0$.

(12) $w_i \frac{\partial \log q_i}{\partial z_j} = (\Delta_{ij} - \theta_i) m_j - \pi_{ij} l_j$ for combined scaling and translation hypotheses.

Subsequently, restrictions (9) through (12) will be used to examine the autoregressive error process in context of the GADS model.

Model

The GADS model can be written as

(13) $$\bar{w}_i \log \left[\frac{q_i}{\bar{w}} \right] = \alpha_i + \bar{\theta}_i \log \left[\frac{x}{\bar{p}} \right] + \sum_j \bar{\pi}_{ij} \log p_j ,$$

where $\log \bar{w} = \sum_j \bar{w}_j \log w_j$; $\log \bar{p} = \sum_j \bar{w}_j \log p_j$; and the bar over a variable or coefficient indicates a value at a specific point, the mean in the present study. The GADS is a flexible functional form and can be considered a levels version of the usual Rotterdam model since the income and price parameters (the $\bar{\theta}_i$'s and $\bar{\pi}_{ij}$'s) of the GADS are the same as those in the Rotterdam model, except the GADS parameters are defined at specific budget share values (see Barten, 1989, for an alternative levels version of the Rotterdam model).

The term $\log \bar{w}$ on the left-hand side of (13) stems from the underlying logit specification

(14) $$w_i = \frac{e^{f_i}}{\sum_j e^{f_j}} ,$$

where f_i is a function of prices and total expenditure; i.e., $f_i = a_i + b_{i0} \log x + \sum_j b_{ij} \log p_j$.

$(\log w_i = f_i - \log \sum e^{f_j}$; $\sum \bar{w}_i \log w_i = \sum \bar{w}_i f_i - \log \sum e^{f_j}$; $\log w_i - \log \bar{w} = f_i - \sum \bar{w}_j f_j$;

$\bar{w}_i\left(\log q_i - \log \bar{w}\right) = \bar{w}_i f_i - \sum_j \bar{w}_i \bar{w}_j f_j + \bar{w}_i \log x - \bar{w}_i \log p_i$; given the specification for f_i, equation (13) follows.) The logit specification has the appealing feature that predicted budget shares will always be in the zero-one interval (this is not necessarily true for other demand models such as popular budget share specifications, the almost ideal demand system (Deaton and Muellbauer, 1980b) and the translog model (Christensen, et al.).

An error term e_{it} with a time subscript t can be added to (13), and the model in terms of matrices can be written as

(15) $y_t = B x_t + e_t$,

where

$$y_t = \left[\bar{w}_{1t} \log \frac{q_{1t}}{\bar{w}_t}, \ \ldots, \ \bar{w}_{nt} \log \frac{q_{nt}}{\bar{w}_t} \right]',$$

$$B = \begin{bmatrix} \alpha_1 & \bar{\theta}_1 & \bar{\pi}_{11} & \cdots & \bar{\pi}_{1n} \\ \vdots & \vdots & \vdots & & \vdots \\ \alpha_n & \bar{\theta}_n & \bar{\pi}_{n1} & \cdots & \bar{\pi}_{nn} \end{bmatrix},$$

$$x_t = \left[1, \ \log \frac{x_t}{\bar{p}_t}, \ \log p_{1t}, \ \ldots, \ \log p_{nt} \right]', \text{ and}$$

$$e_t = \left(e_{1t}, \ \ldots, \ e_{nt}\right)'.$$

The basic restrictions of demand—adding up, homogeneity and symmetry—require

$$\sum \alpha_i = 0,$$

$$\sum_i \bar{\theta}_i = 1 \quad \text{and} \quad \sum_i \bar{\pi}_{ij} = 0 \qquad \text{(adding up)},$$

$$\sum_j \bar{\pi}_{ij} = 0 \qquad \text{(homogeneity),}$$

and

$$\bar{\pi}_{ij} = \bar{\pi}_{ji} \qquad \text{(symmetry).}$$

Premultiplying (15) by the unit vector results in $\sum_i \bar{w}_{it} \log q_{it} - \sum_i \bar{w}_{it} \log w_{it} = \log x_t - \sum_i \bar{w}_i \log p_{it} + \sum_i e_{it}$ which implies $\sum_i e_{it} = 0$ (note that $\log w_i = \log p_i + \log q_i - \log x$; hence $\sum_i \bar{w}_i \log w_i = \sum_i \bar{w}_i \log p_i + \sum_i \bar{w}_i \log q_i - \log x$). This result shows that the covariance matrix for (15) is singular, which for estimation purposes can be handled by arbitrarily deleting an equation (Barten, 1969).

Following Berndt and Savin, the error terms in model (15) may follow some autoregressive error process such as

(16) $e_t = R\, e_{t-1} + v_t$

where v_t are independent, identically distributed, normal, random vectors with mean zero and covariance matrix Ω; and $R = \left[R_{ij} \right]$ is an nxn matrix of parameters.

Berndt and Savin show that model (15) with e_t defined by (16) can be consistently estimated by arbitrarily deleting an equation, say the n^{th} one, and using the constraint $\sum_i e_{i,t-1} = 0$ to reduce (16) to $\bar{R}^n e_{t-1}^n$ where $\bar{R}^n = \left[R_{ij} - R_{in} \right]$, $i, j = 1, \ldots, n\text{-}1$, and $e_t^n = (e_{1t}, \ldots, e_{n-1,t})$, the error vector excluding the n^{th} error. Letting y_t^n, B^n and v_t^n indicate y_t, B and v_t, respectively, with the n^{th} row deleted, the model can be written as

$$(17) \quad y_t^n = B^n x_t + \bar{R}^n e_{t-1}^n + v_t^n$$

or

$$y_t^n = B^n x_t + \bar{R}^n \left(y_{t-1}^n - B^n x_{t-1} \right) + v_t^n .$$

For estimation, B^n and $\bar{R}^n$ are treated as constants.

In the next section, model (17) is used to examine alternative restrictions on the autoregressive process in a demand study of fruit juices. The elements of $\bar{R}^n$ are restricted to be functions of the elements of B^n as indicated by (9) through (12), treating $e_{j,t-1}$ like z_j in the latter equations. The parameters r_{jj}, h_j and m_j in (9) through (12) are treated as constants and since (11) and (12) depend on the budget shares, mean budget share values are used to impose these two restrictions.

Application

The alternative autoregressive error structures proposed were examined in a demand study of fruit juices. A conditional demand system for four types of juices—orange juice (OJ), grapefruit juice (GJ), apple juice (AJ) and remaining juice (RJ)—was studied.

Data on retail sales for fruit juices were obtained from A.C. Nielsen Marketing Research. Weekly data for the period from week ending November 14, 1987, through December 26, 1992, (268 observations) were studied. The Nielsen data reflect total retail sales in outlets with annual sales of $4 million or greater. (For the period from December 1990 through November 1991, an estimated 82% of total U.S. retail sales of OJ and 74% of total U.S. retail sales of GJ occurred in outlets with annual sales of $4 million or greater; insufficient information precluded an estimate for AJ and RJ.) Average conditional budget shares for OJ, GJ, AJ and RJ were .597, .048, .146 and .209, respectively. In creating the dependent variable for the GADS, Nielsen quantity sales were transformed to per capita sales using U.S. Department of Commerce population data.

In addition to the basic explanatory variables—prices and total expenditure—seasonality was also found to be important over the 52 weeks comprising a year, and, following Duffy (1990), the data were 52^{nd} differenced.

The basic restrictions of demand—adding up, homogeneity and symmetry—were imposed as part of the maintained hypothesis. Given the data add up by construction, the adding-up restrictions are automatically satisfied. The alternative models were estimated by the full information maximum likelihood method (TSP) with the equation for RJ omitted. All model estimates are invariant to the equation deleted, which was verified.

The likelihood ratio test was used to compare alternative restricted autoregressive specifications (9) through (12), as well as the simple autoregressive process involving a common autocorrelation coefficient across equations, against the unrestricted autoregressive process (see, e.g., Berndt and Savin; Barten, 1969; or Deaton, 1974; for application of the likelihood ratio test in examining demand-system restrictions). Table 1 shows logarithmic likelihood values and associated model fit information. Twice the difference between the logarithmic likelihood value for the unrestricted model and that value for the alternative is asymptotically chi-squared with degrees of freedom equal to the difference in the number of free parameters in the two models compared. The results in Table 1 indicate that the combined translation-scaling and scaling restrictions are acceptable at any reasonable level of significance, and, surprisingly, the simple autoregressive process is acceptable at the 10% level of significance.

Coefficient estimates for the alternative models are provided in Table 2. For each model, all Slutsky coefficients and marginal propensities to consume were significant, to the extent their coefficient estimates were twice or greater than their corresponding asymptotic standard error estimates. All own-Slutsky coefficients were negative, as predicted by theory, and all cross-Slutsky coefficients were positive, indicating substitute relationships as might be expected given the competitive nature of the juice market. All marginal propensities to consume were positive, indicating normal goods. Overall, the price and income coefficient estimates differ very little between models and seem to provide a reasonable description of juice demands.

The autoregressive coefficient estimates for the translation (h_j), Theil (r_{jj}) and scaling (l_j) specifications were all positive and twice or greater than their corresponding asymptotic standard errors, except the scaling estimate for OJ which was nearly twice (1.9) the value of its asymptotic standard error. The results for the Theil model indicate that, for each good, the error in the previous period positively affects the marginal utility in the present period; the results for the scaling and translation models similarly indicate that the error in the previous period positively affects the consumer's present perception of needs, quality or whatever interpretation is given to the scaling and translation parameters. The autoregressive estimates for combined translation-scaling indicate a dominance of the parameters related to income (m_j), all being positive and significant, while the parameters related to price (l_j) are all negative with relatively large asymptotic standard errors.

Using (9) through (12), alternative estimates of $\bar{R}^n$ were obtained to compare further the different autoregressive assumptions. The results are shown in Table 3 and, as expected, indicate the estimates of $\bar{R}^n$ differ little among the unrestricted, combined and scaling models; the estimates for the other models are also generally similar to the unrestricted estimates. Overall, the estimates in Table 3 confirm the likelihood ratio test results in Table 1. It is also noted that the estimates of the alternative autoregressive processes are stationary. By successive substitution of $\bar{R}^n e_{t-1}^n + v_t^n$ for e_t^n, model (17) can be written as $y_t^n = B^n x_t + \left(\bar{R}^n\right)^s e_{t-s}^n + \sum_{k=1}^{s-1} \left(\bar{R}^n\right)^k v_{t-k}^n$ $+ v_t^n$ where $\left(\bar{R}^n\right)^s$ is $\bar{R}^n$ multiplied by itself s times. Stationarity requires that $\left(\bar{R}^n\right)^s$ converges to zero as $s \to \infty$ as was verified for each model. Convergence of $\left(\bar{R}^n\right)^s$ to zero implies the latent roots of $\bar{R}^n$ are all less than one in absolute value.

The findings of this study are, of course, data specific and the various restrictions examined here may perform quite differently in other applications. The present results, however, do suggest that at least in some applications a relatively restrictive autoregressive process, based perhaps on scaling, translation, or Theil's approach, may be appropriate, and the simple first-order autoregressive process with no cross-equation autocorrelation may even be acceptable.

Conclusions

Demand equation errors can be viewed as aggregate effects of unobservable nonprice, nonincome factors, and restrictions that have been suggested for such factors may be useful in specifying autoregressive error process in systems of demand equations.

In this study, restrictions associated with scaling, translation, combined translation and scaling, and an approach suggested by Theil to measure nonprice, nonincome demand effects were examined. An empirical study of demand for different fruit juices indicated that the scaling and combined translation-scaling restrictions provide reasonable descriptions of the autoregressive process. Although such findings are data specific, the results suggest that, at least for some applications, the restrictions examined in this study may be useful.

Finally, although this study suggests general linkages between underlying utility functions and demand error processes, detailed relationships that might exist were not explored. Further research in this direction may provide additional insight on the autoregressive error process in demand systems.

Table 1. Logarithmic likelihood values and coefficients of determination for alternative autoregressive specifications.

Model	Logarithmic Likelihood Value	Free Parameters	$2(L(\hat{\beta}) - L(\tilde{\beta}))$[a]	ρ Value[b]	Coefficient of Determination			
					OJ	GJ	AJ	RJ
Unrestricted	2,912.03	18	--	--	0.99	0.89	0.96	0.90
Theil	2,904.75	13	14.56	0.01	0.99	0.88	0.96	0.90
Translation	2,906.93	13	10.20	0.07	0.99	0.89	0.96	0.90
Scaling	2,908.77	13	6.52	0.26	0.99	0.88	0.96	0.90
Translation-Scaling	2,912.02	17	0.02	0.89	0.99	0.89	0.96	0.90
Simple	2,905.53	10	13.00	0.11	0.99	0.88	0.96	0.90

[a] Twice the difference between the log likelihood value for the unrestricted model, $L(\hat{\beta})$, and the log likelihood value for the restricted model, $\tilde{\beta}$.

[b] Chi-square probability greater than $2(L(\hat{\beta}) - L(\tilde{\beta}))$ with degrees of freedom equal to the difference in the number of free parameters between the unrestricted and restricted models.

Table 2. Full information maximum likelihood estimates of the GADS for fruit juices under alternative autoregressive error restrictions.

Coefficient[a]	Unrestricted		Combined Translation-Scaling		Translation		Theil		Scaling		Simple	
$\overline{\theta}_1$	0.62	(57.23)[b]	0.62	(57.99)	0.61	(58.75)	0.62	(61.57)	0.62	(64.86)	0.61	(58.44)
$\overline{\theta}_2$	0.05	(22.51)	0.05	(22.92)	0.05	(24.79)	0.05	(23.33)	0.05	(23.24)	0.05	(24.96)
$\overline{\theta}_3$	0.13	(17.16)	0.13	(17.26)	0.13	(17.73)	0.14	(18.42)	0.13	(19.27)	0.13	(17.38)
$\overline{\pi}_{11}$	-0.28	(-13.11)	-0.28	(-13.27)	-0.29	(-13.07)	-0.29	(-13.35)	-0.27	(-13.40)	-0.29	(-13.04)
$\overline{\pi}_{12}$	0.04	(7.48)	0.04	(7.54)	0.04	(7.73)	0.03	(7.38)	0.03	(10.82)	0.04	(7.66)
$\overline{\pi}_{13}$	0.10	(7.54)	0.10	(7.56)	0.10	(7.66)	0.11	(9.77)	0.10	(8.99)	0.10	(7.80)
$\overline{\pi}_{22}$	-0.08	(-12.91)	-0.08	(-12.92)	-0.07	(-13.05)	-0.08	(-13.44)	-0.08	(-13.88)	-0.07	(-13.36)
$\overline{\pi}_{23}$	0.02	(4.72)	0.03	(4.88)	0.02	(4.62)	0.02	(7.29)	0.02	(6.09)	0.02	(4.64)
$\overline{\pi}_{33}$	-0.20	(-17.27)	-0.20	(-17.47)	-0.21	(-17.78)	-0.20	(-19.96)	-0.20	(-18.73)	-0.20	(-18.29)
$\overline{R}^4_{11}$, m_1, h_1, ρ	0.92	(19.41)	2.73	(0.85)	0.93	(14.05)					0.96	(74.08)
$\overline{R}^4_{12}$, m_2, h_2	0.00	(0.04)	1.39	(2.76)	0.93	(38.80)						
$\overline{R}^4_{13}$, m_3, h_3	-0.04	(-0.56)	1.39	(3.04)	0.98	(45.28)						
$\overline{R}^4_{21}$, m_4, h_4	0.03	(2.07)	1.35	(3.10)	0.97	(30.37)						
$\overline{R}^4_{22}$, l_1, r_{11}	0.94	(35.68)	-2.74	(-0.65)			1.93	(6.86)	19.87	(1.89)		
$\overline{R}^4_{23}$, l_2, r_{22}	0.03	(1.49)	-5.80	(-0.94)			10.21	(10.82)	26.91	(4.83)		
$\overline{R}^4_{31}$, l_3, r_{33}	-0.02	(-0.69)	-1.71	(-1.02)			3.76	(16.41)	6.16	(3.97)		
$\overline{R}^4_{32}$, l_4, r_{44}	0.04	(0.66)	-1.20	(-0.93)			2.75	(10.39)	8.98	(3.46)		
$\overline{R}^4_{33}$	0.94	(22.47)										

[a] $\overline{R}^4_{ij}$'s are for the unrestricted model; m_i's and l_i's are for the combined translation-scaling model; h_i's are for the translation model; r_{ii}'s are for the Theil model; l_i's by themselves are for the scaling model; and the ρ is for the simple model.

[b] Asymptotic t statistics in parentheses.

Table 3. Alternative estimates of the autoregressive error process for fruit juices.

Coefficient	Unrestricted		Combined Translation-Scaling		Translation		Theil		Scaling		Simple	
$\overline{R}^4_{11}$	0.924	(19.414)[a]	0.923	(19.592)	0.951	(38.781)	0.943	(39.005)	0.931	(27.428)	0.956	(74.078)
$\overline{R}^4_{12}$	0.005	(0.039)	0.014	(0.165)	0.022	(0.857)	0.047	(2.153)	0.008	(0.199)	0.000	(--)
$\overline{R}^4_{13}$	-0.040	(-0.560)	-0.043	(-0.612)	-0.006	(-0.278)	-0.020	(-1.159)	-0.024	(-0.603)	0.000	(--)
$\overline{R}^4_{21}$	0.025	(2.066)	0.025	(2.086)	0.002	(0.451)	0.018	(1.931)	0.028	(2.812)	0.000	(--)
$\overline{R}^4_{22}$	0.936	(35.679)	0.935	(37.567)	0.932	(40.905)	0.906	(33.691)	0.918	(39.666)	0.956	(74.078)
$\overline{R}^4_{23}$	0.026	(1.486)	0.026	(1.487)	4×10^{-4}	(-0.278)	0.021	(1.561)	0.028	(2.175)	0.000	(--)
$\overline{R}^4_{31}$	-0.018	(-0.693)	-0.018	(-0.699)	0.005	(0.450)	-0.011	(-0.739)	-0.023	(-1.159)	0.000	(--)
$\overline{R}^4_{32}$	0.043	(0.662)	0.041	(0.653)	0.005	(0.856)	0.030	(1.235)	0.057	(1.638)	0.000	(--)
$\overline{R}^4_{33}$	0.940	(22.470)	0.941	(22.652)	0.974	(48.452)	0.948	(41.282)	0.933	(32.142)	0.956	(74.078)

[a] Asymptotic t statistics in parentheses.

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