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DEMAND FOR ORANGE-JUICE PRODUCTS: AN APPLICATION OF THE UNIFORM SUBSTITUTE MODEL

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Abstract

The differential demand model for uniform substitutes is used to analyze demand for orange-juice products. Consumption trends are examined by allowing the marginal propensities to consume to vary with time. The results indicate ready-to-serve (frozen concentrated) orange juice has become more (less) sensitive to expenditure and prices.

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The orange-juice (OJ) market is comprised of a number of alternative brands and product types, as is the case for many commodity markets. The products in such markets are essentially the same except for slight differentiation and are described by Theil (1980) as uniform substitutes. For goods that are uniform substitutes, the effect of an additional dollar spent on one good on the marginal utility of a dollar spent on another good is independent of the two goods considered.

In this study, a differential demand model for uniform substitutes is used to analyze demands for different types of OJ. The analysis focuses on consumption trends over time by allowing the model's marginal propensities to consume (MPC's) to vary with time. The varying MPC's, in turn, result in varying price responses through the general and specific substitution effects, as well as the usual Slutsky income effect. The varying parameter specification analyzed here provides an example of how the usual assumption of constancy of the basic coefficients of the differential demand model might be relaxed--in the differential model, the price and income coefficients are usually assumed to be constant, but, as recently discussed by Theil et al., need not be, and may depend on income and price levels, as well as other factors.

The paper proceeds as follows. In the next section, the differential demand model for uniform substitutes and the time-varying parameter specification are discussed. The model is then applied to data on sales of four different types of OJ--nationally advertised branded product versus other product, with a breakdown in product type between frozen concentrated orange juice and chilled orange juice. In the last section, concluding comments are given.

Model

The OJ products analyzed were placed into four categories--nationally advertised (NA) frozen concentrated orange juice (FCOJ), other FCOJ, NA ready-to-serve chilled orange juice (COJ), and other COJ. The four types of OJ were analyzed as a group, while other goods were treated as a composite commodity (the composite commodity can be considered other foods with all foods treated as separable from nonfood goods). The analysis focuses on the conditional demand equations for the OJ group.

The differential demand model or Rotterdam model, developed by Theil (1965) and Barten (1966), provides a first-order approximation of true demand. Recent analyses by Barnett, Byron and Mountain show the approximation is comparable to other popular flexible function forms. The Rotterdam model in relative prices can be written as (Theil, 1971, 1976, 1980)

$$(1) \qquad w_i d\log q_i = \theta_i d\log Q + \phi \sum_{j=1}^{n} \theta_{ij}\left(d\log p_j - d\log P\right), \qquad i = 1, \ldots, n,$$

where subscript i indicates a particular good, $i = 1, 2, 3$ and 4 for NA FCOJ, other FCOJ, NA COJ, and other COJ, respectively, and $i = 5$ for the composite commodity; q and p are quantity and price, respectively; $w_i = \frac{p_i q_i}{x}$, the budget share for the good, x being total expenditure or income; $\theta_i = \frac{p_i \partial q_i}{\partial x}$, the marginal propensity to consume or MPC out of income; $d \log Q = \sum w_i d \log q_i$, the Divisia volume index in differential form; $\phi = \left(\frac{\partial \lambda}{\partial x} \frac{x}{\lambda} \right)^{-1}$, the income flexibility with $\lambda = \frac{\partial u}{\partial x}$, the marginal utility of income, u being utility; $\theta_{ij} = \frac{\lambda p_i p_j u^{ij}}{\phi x}$, the specific price coefficient with u^{ij} being the ij^{th} element of the matrix $\left[\frac{\partial^2 u}{\partial q_i \partial q_j} \right]^{-1}$, the inverse of the Hessian matrix for the utility maximization problem; and $d \log P = \sum \theta_i d \log p_i$, the Frisch price index. The term involving prices can be equivalently written as $\sum \pi_{ij} d \log P_j$ where $\pi_{ij} = \phi(\theta_{ij} - \theta_i \theta_j)$ with $\phi \theta_{ij}$ and $- \theta_i \theta_j$ being the Rotterdam model counterparts to the specific and general substitution effects, respectively.

In this study, the OJ goods are treated as uniform substitutes as defined by Theil (1980). In the following discussion, uniform substitutes are described and restrictions for the case under consideration are developed. First, consider the ij^{th} element of the matrix $[\theta^{ij}] = [\theta_{ij}]^{-1}$, i.e.,

$$(2) \quad \theta^{ij} = \frac{\phi x}{\lambda} \frac{\partial^2 u}{\partial(p_i q_i) \partial(p_j q_j)} .$$

The second-order derivative on the right side of (2) describes the effect of an additional dollar spent on good j on the marginal utility of a dollar spent on good i.

The goods in the OJ group are defined as uniform substitutes when θ^{ij}, $i \neq j$, equals a positive constant for all goods in the group (the second-order derivative in (2) is negative, and $\phi < 0$, under the assumption that the Hessian matrix is negative definite). For uniform substitutes, an extra dollar spent on one good decreases the marginal utility of a dollar spent on another good by an amount which is independent of the two goods considered. The model for uniform substitutes in the present paper also assumes that an additional dollar spent on a good in the OJ group, regardless of the good, has the same effect on the marginal utility of a dollar spent on the composite commodity, and vice versa.

Appropriateness of the uniform substitute assumption is related to the degree of product similarity. For example, whether an additional dollar is spent on beans or bread probably has an important bearing on the marginal utility of a dollar spent on meat. As the products involved become more similar, which good the additional dollar is spent on may become less important. Whether the additional dollar is spent on apple juice or grape juice is still probably important for the marginal utility of a dollar spent on OJ. However, the effect on the marginal utility of a dollar spent on one type of OJ due to another dollar spent on a second type of OJ may be about the same as the effect due to another dollar spent on a third similar type of OJ, at least as an approximation.

Formally, the foregoing uniform substitute assumptions can be written as

$$(3) \quad \left[\theta^{ij}\right] = D^{-1} + k k'$$

where D is a 5x5 diagonal matrix whose elements d_i are all positive; and $k = (a, a, a, a, b)'$ is a 5x1 vector with the first four elements equal to a and the last element equal to b, with both a and b being positive constants. The inverse of (3) is

$$(4) \quad \left[\theta_{ij}\right] = D - \frac{1}{1 + k' D k} D k k' D,$$

or

$$\theta_{ij} = d_i \Delta_{ij} - \frac{a^2\Delta + (1-\Delta)\left(ab + b^2\Delta_{ij}\right)}{1 + a^2S + b^2d_n} d_i d_j; \qquad i, j = 1, \dots, 5;$$

where $\Delta_{ij} = 1$ if $i = j$, else $\Delta_{ij} = 0$; $\Delta = 1$ if $i,j = 1, \dots, 4$, else $\Delta = 0$, and $S = \sum_{i=1}^{4} d_i$.

Based on homogeneity and adding-up properties, $\sum_{j=1}^{n} \theta_{ij} = \theta_i$, so that, based on (4)

$$\text{(5)} \quad \sum_{j=1}^{n} \theta_{ij} = \theta_i = d_i - \frac{d_i\left(a^2S + abd_n\right)}{1 + a^2S + b^2d_n},$$

$$\theta_i = d_i B, \qquad i = 1, \dots, 4,$$

where $B = \dfrac{1 + bd_n(b-a)}{1 + a^2S + b^2d_n}$. Note that B is the same for all $i = 1, \dots, 4$--the result allows specification of conditional demand equations, similar to the Rotterdam model specification for preference in dependence (Theil, 1980).

Expression (5) indicates $d_i = \dfrac{\theta_i}{B}$. Substituting this result for d_i in (4), one finds

$$\text{(6)} \qquad \theta_{ij} = \frac{\theta_i \Delta_{ij} - \left(k_1\Delta + k_2(1-\Delta)\right)\theta_i\theta_j}{B}, \qquad i = 1, \dots, 4,$$

where $k_1 = \dfrac{a^2}{1 + bd_n(b-a)}$ and $k_2 = \dfrac{ab}{1 + bd_n(b-a)}$.

Next, substitute (6) into demand specification (1) for i = 1, . . . , 4 to obtain

$$w_i \, d\log q_i = \theta_i \, d\log Q + \frac{\phi}{B} \sum_{j=1}^{4} \left(\theta_i \Delta_{ij} - k_1 \theta_i \theta_j\right)\left(d\log p_j - d\log P\right) - \tag{7}$$

$$\frac{\theta}{B} k_2 \theta_i \theta_5 \left(d\log p_5 - d\log P\right), \qquad i = 1, \ldots, 4.$$

To obtain the conditional demand equations, first sum (7) over $i = 1, \ldots, 4$ to find the group demand

$$\sum_{i=1}^{4} w_i \, d\log q_i = \theta_A \, d\log Q + \frac{\phi}{B} \sum_{j=1}^{4} \left(\theta_j - k_1 \theta_A \theta_j\right)\left(d\log p_j - d\log P\right) - \tag{8}$$

$$\frac{\phi}{B} k_2 \theta_A \theta_5 \left(d\log p_5 - d\log P\right),$$

where $\theta_A = \sum_{j=1}^{4} \theta_i$, the group MPC.

Finally, multiply (8) by $\frac{\theta_i}{\theta_A}$ and subtract the result from (7) to find the conditional demand equations, which, after further dividing by $W_A = \sum_{i=1}^{4} w_i$, can be written as

$$\frac{w_i}{W_A} d\log q_i = \frac{\theta_i}{\theta_A} d\log Q_A + \frac{\phi \theta_A}{B W_A} \frac{\theta_i}{\theta_A} \left(d\log p_i - d\log P_A\right), \qquad i = 1, \ldots, 4, \tag{9}$$

where $Q_A = \sum_{i=1}^{4} \frac{w_i}{W_A} d\log q_i$, the Divisia volume index for the group; and $P_A = \sum_{i=1}^{4} \frac{\theta_i}{\theta_A} d\log p_i$, the Frisch price index for the group. The terms $\frac{\theta_i}{\theta_A}$ and $\frac{w_i}{W_A}$ are referred to as the conditional MPC and budget share, respectively. Result (9) is like that for preference independence except for the term B.

To allow for trends in consumption and changes in tastes, constant terms are sometimes included in the Rotterdam model (e.g., Theil, 1976; Barten, 1969; Deaton; Deaton and Muellbauer). In the present case, a constant term T_i might be added to (9), to account for trends due, for example, to build-up of habits, advertising effects and lifestyle changes. The latter, of course, is a rough approximation and may not always capture the effects of time-related dynamic forces. Statistically insignificant estimates of the constant terms may not indicate absence of trends. Dynamic effects may be more complicated and involve other parameters. The general time-varying parameter model suggests alternative approximations (e.g., Singh et al., Judge et al.). In this study, the possibility that the conditional MPC is dependent on time is considered. Specifically, the hypothesis is

$$(10) \quad \frac{\theta_i}{\theta_A} = a_i + b_i t ,$$

where a_i and b_i are parameters, and t is time.

Using t also as a subscript to indicate an observation, let $\frac{w_{it}}{W_{At}}$ be approximated by

$\frac{w'_{it} + w'_{it-1}}{2}$ where $w'_{it} = \frac{p_{it} q_{it}}{\sum_{j=1}^{4} p_{jt} q_{jt}}$. Likewise, $d \log q_{it}$ and $d \log p_{it}$ can be approximated by $\log\left(\frac{q_{it}}{q_{it-1}}\right)$ and $\log\left(\frac{p_{it}}{p_{it-1}}\right)$, respectively. The parameters to be estimated are the conditional MPC's, which involve coefficients a_i and b_i in the case of specification (10), and the term $\frac{\phi\,\theta_A}{B\,W_A}$.

For the conditional demand system given by (9), adding-up requires $\sum_{i=1}^{4}\frac{\theta_i}{\theta_A} = 1$, and $\sum_{i=1}^{4} a_i = 1$ and $\sum_{i=1}^{4} b_i = 0$ when $\frac{\theta_i}{\theta_A}$ is specified by (10); symmetry holds with the conditional Slutsky coefficient defined as $\pi_{ij}^A = \frac{\phi\,\theta_A}{B\,W_A}\left(\frac{\theta_i}{\theta_A}\Delta_{ij} - \frac{\theta_i}{\theta_A}\frac{\theta_j}{\theta_A}\right)$; and homogeneity holds provided adding-up holds, and vice versa. The conditional uncompensated income and price elasticities for (10) are $\dfrac{\frac{\theta_i}{\theta_A}}{\frac{w_i}{W_A}}$ and $\dfrac{\pi_{ij}^A - \frac{w_j}{W_A}\frac{\theta_i}{\theta_A}}{\frac{w_i}{W_A}}$, respectively.

Application

Model (9) was applied to Nielsen scanner data for grocery stores with annual sales of 4 million dollars or greater (the stores surveyed account for roughly 80% of all retail OJ sales). The data are weekly, and the sample runs from November 14, 1987, through November 3, 1990, providing 156 observations. The basic data include single-strength-equivalent gallon and dollars sales for the four different types of OJ discussed in the previous section. Prices were calculated by dividing dollar sales by gallon sales. U.S.

Department of Commerce data on the U.S. population were used to put demand on a per capita basis.

As the data add-up by construction--income in the model is total consumer expenditure on the four types of OJ--the error covariance matrix is singular and the equation for other COJ was excluded (Barten, 1969). The errors across equations were assumed to be contemporaneously correlated, and the full information maximum likelihood procedure was used to estimate the model with homogeneity and symmetry holding.

Maximum likelihood estimates for conditional demand system (9) with alternative trend specifications are shown in Table 1. Results for three specifications are given--model A: specification (9) with intercept term T_i; Model B: specification (9) with intercept term T_i and the conditional MPC specified by (10); and Model C: specification (9) with just the MPC specified by (10). The results for Model A suggest absence of consumer trends with all intercept estimates being insignificant. Model B further shows that the intercept terms are insignificant; however, the estimates for the MPC trend coefficients, the b_i's, are all more than twice their corresponding standard errors; the estimates for the other MPC coefficients, the a_i's, and the estimate for the proportionality term $\frac{\phi\,\theta_A}{B\,W_A}$ are also more than twice their corresponding standard error estimates. The results for Model C, which restricts the intercept terms to zero, are about the same as for Model B. Overall, the results in Table 1 indicate that, contrary to the initial suggestion of absence of consumer trends, sales of OJ do seem to be subject to trends. The following discussion focuses on Model C.

The R^2's for NA FCOJ, other FCOJ, NA COJ and other COJ were .87, .76, .89 and .66, respectively, indicating reasonably good fits for specifications in first differences;

however, the R^2 values are interpreted cautiously, as the equations do not have intercepts and have been estimated jointly as a system (Barten and Bettendorf). Autocorrelation did not seem to be a problem--estimation of the model assuming first-order autocorrelation yielded similar results.[1]

The results for Model C indicate the conditional MPC's for NA FCOJ and other FCOJ have been decreasing over time, while the conditional MPC's for NA COJ and other COJ have been increasing over time. The estimate for the proportionality term is negative indicating negative own-price and positive cross-price compensated demand responses for the subsystem, for all sample values of t, as well as all of values of t reasonably close to the sample values.

Uncompensated conditional expenditure and price elasticities for the beginning and end of the time period analyzed are reported in Table 2. The elasticities further show how the demand responses for orange juice may have been changing over time. Without going into the details, the pattern in Table 2 is clear--the demands for the two types of FCOJ have been becoming less responsive to group expenditure and prices, while the demands for the types of COJ have been becoming more responsive. The model does not provide an explanation for the pattern, but one factor underlying the results may be advertising.

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[1]The Durbin-Watson (DW) statistics for NA FCOJ, other FCOJ and NA COJ were 2.62, 2.61 and 2.89, respectively. For demand systems obeying the adding-up property, the DW statistic is not entirely appropriate as a measure of autocorrelation (Bewley)--for instance, if each equation is subject to first-order autocorrelation, with no assumed autocorrelation across equations, the autocorrelation parameter ρ should be the same in each equation (Berndt and Savin), and the DW test should reach the same conclusion across equations. In this case, the DW statistics only provides a guideline. Estimation of Model C assuming first-order autocorrelation (each equation has the same ρ) yielded an estimate of -.39 for ρ with an asymptotic standard error of .05. The estimates for the other coefficients were very similar to the estimates for Model C when autocorrelation was not assumed (ρ = 0).

Advertising for OJ, as well as other products, commonly involves price discounts; in this case, the price response may be stronger than otherwise. The latter conjecture is consistent with the results for COJ but not for FCOJ. One explanation may be that consumer trends toward convenience exist and work synergistically with advertising to produce a larger response for COJ (COJ is a ready-to-serve product relative to FCOJ, which needs to be prepared). The rate of change in the MPC over time for NA COJ is also about twice as fast as that for other COJ, possibly reflecting the effectiveness of national advertising.

Concluding Comments

For closely related products that can be considered as uniform substitutes, the Rotterdam model simplifies to a form like that for preference independence. In the present study, different types of OJ were treated as uniform substitutes, and an OJ demand subsystem was estimated. A simple trend intercept was included in the model, but proved to be insignificant. Further analysis of trends in consumption, however, indicated that the MPC's were changing over time, making COJ more responsive to expenditures and prices, and FCOJ less responsive. The model does not provide an explanation for the MPC trends, and more detailed analysis including information on the underlying dynamic factors, will be required for a better understanding. Nevertheless, the results show that allowing the basic coefficients of the Rotterdam model to be dependent on time or, in general, other factors, provides for additional flexibility and may be useful for obtaining better estimates of demand.

Table 1. Maximum likelihood estimates for the uniform substitute differential model for orange-juice products with alternative trend specifications.

Parameter		Model A: Intercept Only		Model B: Intercept and MPC		Model C: MPC Only	
Intercept	T_1	$-2\text{x}10^{-4}$	$(5\text{x}10^{-4})$[a]	$-2\text{x}10^{-4}$	$(5\text{x}10^{-4})$		
	T_2	$-2\text{x}10^{-4}$	$(5\text{x}10^{-4})$	$-1\text{x}10^{-4}$	$(5\text{x}10^{-4})$		
	T_3	$5\text{x}10^{-4}$	$(9\text{x}10^{-4})$	$4\text{x}10^{-4}$	$(9\text{x}10^{-4})$		
MPC	a_1	.235	(.009)	.295	(.022)	.295	(.022)
	b_1			$-7\text{x}10^{-4}$	$(3\text{x}10^{-4})$	$-7\text{x}10^{-4}$	$(3\text{x}10^{-4})$
	a_2	.204	(.010)	.278	(.024)	.278	(.024)
	b_2			$-8\text{x}10^{-4}$	$(2\text{x}10^{-4})$	$-8\text{x}10^{-4}$	$(2\text{x}10^{-4})$
	a_3	.403	(.014)	.322	(.027)	.321	(.027)
	b_3			$10\text{x}10^{-4}$	$(3\text{x}10^{-4})$	$10\text{x}10^{-4}$	$(3\text{x}10^{-4})$
Proportionality Factor	$\frac{\phi\,\theta_A}{B\,W_A}$	-2.178	(.078)	-2.162	(.080)	-2.162	(.079)

[a]Asymptotic standard errors in parentheses.

Table 2. Conditional price and expenditure elasticity estimates for the orange-juice group.

Product	Point in Sample	Expenditure	Price: NA FCOJ	Price: Other FCOJ	Price: NA COJ	Price: Other COJ
NA FCOJ	Beg.[a]	1.280 (.095)[c]	-2.247 (.138)	.524 (.076)	.418 (.074)	.025 (.049)
	End[b]	1.055 (.124)	-2.052 (.191)	.152 (.046)	.621 (.074)	.224 (.050)
Other FCOJ	Beg.	1.460 (.127)	.593 (.086)	-2.558 (.176)	.477 (.079)	.028 (.056)
	End	.833 (.102)	.182 (.042)	-1.681 (.190)	.490 (.072)	.177 (.044)
NA COJ	Beg.	.871 (.073)	.354 (.046)	.357 (.045)	-1.599 (.089)	.017 (.033)
	End	1.095 (.061)	.239 (.045)	.158 (.045)	-1.724 (.057)	.232 (.052)
Other COJ	Beg.	.505 (.086)	.205 (.040)	.207 (.041)	.165 (.032)	-1.082 (.163)
	End	.908 (.099)	.198 (.040)	.131 (.039)	.534 (.070)	-1.771 (.164)

[a]Week ending November 14, 1987; conditional budget shares for NA FCOJ, Other FCOJ, NA COJ and Other COJ were .23, .19, .37 and .21, respectively.

[b]Week ending November 3, 1990; conditional budget shares for NA FCOJ, Other FCOJ, NA COJ and Other COJ were .17, .18, .43 and .22, respectively.

[c]Asymptotic standard errors in parentheses.

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