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# ALTERNATIVE SPECIFICATIONS OF ADVERTISING IN THE ROTTERDAM MODEL

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# Alternative Specifications of Advertising in the Rotterdam Model

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**Abstract**

This paper examines several approaches to introduce advertising in systems of demand equations. Advertising is included in the Rotterdam model using an unrestricted specification and three restricted specifications--advertising affects demand alternatively through (1) marginal utilities as in studies by Duffy (1987, 1989, 1990) and Selvanathan (1989), (2) scaling parameters which can be viewed as indicators of product quality, and (3) translation parameters which can be viewed as indicators of basic needs. A test to choose among the alternative specifications is provided and the methodology is applied to data on demand for fruit juice products.

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# Alternative Specifications of Advertising in the Rotterdam Model

Duffy (1987, 1989, 1990) and Selvanathan (1989a, 1989b) have recently studied the impacts of advertising using a system of demand equations; in particular, the Rotterdam model (Barten, 1964a; Theil, 1965, 1975, 1980a, 1980b). The approach is theoretically appealing, accounting for demand interrelationships between products and allowing advertising on a given product to have cross-effects on related products.[1]

An aspect of demand analysis that has received considerable attention deals with specification of the equations to be estimated. In the present paper, attention is focused on specification of advertising effects in systems of demand equations. Consumer behavior theory suggests a system of demand equations obeys several basic properties; namely, expenditures by the consumer on individual goods add up to the consumer's income; absence of money illusion--a doubling of or proportionate change in prices and income leaves demand unchanged; inverse relationship between own-price and quantity, as well as related restrictions on own- and cross-price responses, with real income held constant; and symmetry of cross-price responses with real income held constant (see Phlips; Deaton and Muellbauer; Theil, 1975, 1980; among others, for discussion on basic demand properties). Imposing the basic properties of demand on a set of demand equations substantially reduces the number of parameters to be estimated and may be attractive for empirical work. Other restrictions have also been explored that reduce the parameter space further. Separability

[1]The underlying assumption of this approach is that advertising influences consumer preferences; in this case, advertising can be introduced directly into the consumer's utility function or, similarly, the indirect utility function or cost function, and interpreted as a preference shifter. An alternative approach is to treat advertising as an informational input in the household production model (Stigler and Becker); preferences are assumed to be stable and advertising affects the household's technology involved in the production of nonmarket goods or commodities from market goods, time, human and nonhuman capital and other inputs.

restrictions dealing with grouping of goods and allocation of income in stages (e.g., first, to broad commodity groups and then to specific goods in each broadly defined group) have been useful in reducing the estimation problem, as well as providing a rationale to examine commodity subgroups separately.

Most of the foregoing demand restrictions deal with price and income responses. When advertising is added to a demand system, the only basic restriction directly involved is the adding-up condition--advertising that results in demand increases on some products must be offset by demand decreases on other products to satisfy the budget constraint. However, even with the latter restriction on advertising, there may still be too many advertising effects to estimate reliably, and specific hypotheses on how advertising affects demand have been made that further reduce the estimation problem.

One hypothesis advanced by Theil (1980b) and used by Duffy (1987, 1989, 1990) and Selvanathan (1989), among others, is that advertising on a good affects demand through the consumer's marginal utility for the good; there are no cross-advertising effects on marginal utilities of other goods. The Rotterdam model under the latter hypothesis has advertising effects that work similarly to price effects but in opposite directions--minus a given percentage change in advertising times the elasticity of marginal utility with respect to advertising (an estimated parameter) times the price parameter or Slutsky coefficient (also an estimated parameter). Requiring advertising to affect demand in this manner, however, may not always be appropriate as the hypothesis may be too restrictive, statistically, and other hypotheses might be considered. Two other hypotheses considered in this paper deal with introducing advertising through scaling and translation parameters. (For general discussion on scaling and translating, see Gorman; Barten (1964b); Pollak and Wales (1980, 1981); and

Deaton and Muellbauer. For specific discussion related to advertising, see Brown and Lee). In the case of scaling, advertising can be viewed as affecting consumer perceptions of quality, and generates both direct quality effects and indirect price-related effects (more quality means price is effectively reduced). In the case of translating, advertising can be viewed as affecting perceptions of basic needs and generates income-related effects (more expenditures committed to basic needs mean less discretionary income--income in excess of expenditures on basic needs--to spend). The appeal of the hypothesis may be related to design of the advertising program; e.g., milk advertising may stress need aspects of health, while advertising for a particular brand of product may stress quality; in some cases, both need and quality aspects may be important and the two hypotheses might be combined.

The present paper provides a case study of the foregoing hypotheses of advertising and demand. Four Rotterdam model specifications with advertising are examined--(1) an unrestricted model, (2) Theil's specification with advertising affecting the marginal utilities in the specific manner mentioned, (3) scaling, and (4) translating. As a maintained hypothesis, the basic restrictions of demand are imposed.

Data on demand for juice products in the U.S. are analyzed. Three types of juices are examined--(1) orange juice (OJ), (2) grapefruit juice (GJ) and (3) all remaining juice (RJ). The data were supplied by A. C. Nielsen Research and are based on a survey of retail grocery-store scanner check-out records. Stores with 2 million dollar or greater annual business were surveyed. The data are weekly, and the sample runs from week ending January 9, 1988, through week ending December 6, 1991, yielding 160 observations. Information was provided on quantity and dollar sales from which average prices were calculated. Data on A/B ads (printed material in newspapers) and displays in stores accompanied by an ad were also obtained. The advertising variables measure coverage, indicating percentages of grocery-store sales on all commodities where ads/displays were present. U.S. Department of Commerce data on the U.S. population were used to put demand on a per capita basis.

The analysis treats the juice category as separable and the demand estimates obtained are conditional. That is, the income or total expenditure variable for the demand system is total juice expenditure. A fully complete model would require data on other goods to determine how total juice expenditures are influenced by changes in income and prices (e.g., Duffy, 1987, 1989, 1990). Consistent weekly data on other goods were not available to estimate the full model. The conditional demand estimates, however, can still be useful in understanding consumer behavior, and, in the present case, how different types of advertising work across competing products.

In the present study, lagged advertising effects were found to be insignificant and omitted from the models. This was an unexpected result, especially given weekly data. Perhaps the result is related to the nature of the advertisements (newspaper printed material and in-store displays) and/or product types; the advertising variables analyzed may be providing information on deals with little effect on preferences. In contrast to lagged advertising, advertising in a particular week did have significant effects in the models. Exactly how advertising affects the demands for juice is the concern of this paper.

The remainder of the paper is as follows. In the next section, the alternative model specifications are given and the statistical tests used to compare the specifications are described. The results are then discussed and concluding comments are given.

## Model Specifications

In this section, the alternative model specifications examined in this study are described and the test statistics used to compare the specifications are developed.

The consumer choice problem (maximization of utility subject to the consumer's budget constraint) for the alternative specifications can be written as

$$(1)\quad \text{maximize} \quad u = \left(q_1, \ldots, q_n, a_1, \ldots, a_n\right)$$
$$\text{subject to} \quad \sum p_i q_i = x\,,$$

where $u$ is utility; $q_i$, $p_i$ and $a_i$ are quantity, price and the advertising level for good $i$, respectively; and $x$ is total expenditure or income. Problem (1) is the usual consumer choice problem with the addition of advertising, which is allowed to affect utility and, in turn, the bundle of goods chosen. In describing the models here, $a_i$ is treated as a single measure of advertising for good $i$, but, in general, could itself be a vector of advertising measures for different types of advertising, including present and lagged values.

The demand equations satisfying (1) have the general form

$$(2)\quad q_i = q_i\left(p_1, \ldots, p_n, a_1, \ldots, a_n, x\right)$$

indicating demand for a particular good by a utility-maximizing consumer depends on the prices for all goods, the advertising levels for all goods, and total expenditure.

A basic property of demand systems with factors such as advertising is that any demand increase(s) for product(s) as a result of a change in the factor must be offset by demand decreases for other products as total consumer expenditures are constant. In the present case, this property can be written as the differentiation of the budget constraint with respect to $a_i$, i.e.,

$$(3)\quad \sum_j p_j \frac{\partial q_j}{\partial a_i} = 0$$

or

$$\sum_j w_j \frac{\partial \log q_j}{\partial \log a_i} = 0,$$

where $w_j = \frac{p_j q_j}{x}$, the budget share for good $j$; and $\frac{\partial \log q_j}{\partial \log a_i}$ is the elasticity of demand for good $j$ with respect to advertising on good $i$. The elasticity version of (3) shows that the weighted sum of advertising elasticities (with respect to advertising for a specific product) is zero where the weights are the product budget shares.

The effects of advertising can also be related to the substitution effects of price changes (Barten, 1977; Phlips), i.e.,

$$(4)\quad \frac{\partial q_i}{\partial a_j} = -\frac{1}{\lambda} \sum_j s_{ik} v_{kj},$$

where $\lambda = \frac{\partial u}{\partial x}$, the marginal utility of income; $s_{ik} = \left.\frac{\partial q_i}{\partial p_k}\right|_{\bar{u}}$, the Slutsky substitution effect

or demand price slope with utility held constant; and $v_{kj} = \frac{\partial\left(\frac{\partial u}{\partial q_k}\right)}{\partial a_j}$, the effect of advertising on marginal utility.

Given information about the structure of $[v_{kj}]$, (4) can be used to reduce the number of parameters to be estimated in a demand system. Theil (1980b) and Duffy (1987, 1989, 1990) assumed $v_{kj} = 0$ for $k \neq j$. In this case, (4) can be written as

$$(5) \qquad \frac{\partial q_i}{\partial a_j} = -\frac{1}{\lambda} s_{ij} v_{jj}$$

or

$$\frac{p_i q_i}{x} \frac{\partial q_i}{\partial a_j} \frac{a_j}{q_i} = -\frac{p_i p_j}{x} s_{ij} \frac{\partial\left(\frac{\partial u}{\partial q_j}\right)}{\partial a_j} \frac{a_j}{\lambda p_j}$$

or

$$w_i \frac{\partial \log q_i}{\partial \log a_j} = -\Pi_{ij} \alpha_j ,$$

where $\Pi_{ij} = \frac{p_i p_j}{x} s_{ij}$, the Slutsky coefficient of the Rotterdam model, and

$$\alpha_j = \frac{\partial \left( \log \left( \frac{\partial u}{\partial q_j} \right) \right)}{\partial \log a_j},$$ the elasticity of the marginal utility of good $j$ with respect to its own-advertising (note $\frac{\partial u}{\partial q_j} = \lambda p_j$ for utility maximization). Specification (5) is the first restriction tested in this paper.

The unrestricted differential demand model or Rotterdam model associated with (2) can be written as Selvanathan (1989b)

$$(6) \quad w_i d \log q_i = \theta_i d \log Q + \sum_j \Pi_{ij} d \log p_j + \sum_j \beta_{ij} d \log a_j$$

where $\theta_i = w_i \frac{\partial \log q_i}{\partial \log x} = p_i \frac{\partial q_i}{\partial x}$, the marginal propensity to consume; $d \log Q$

$= d \log x - \sum_j w_j d \log p_j = \sum w_j d \log q_j$, the Divisia volume index; and $\beta_{ij} = w_i \frac{\partial \log q_i}{\partial \log a_j}$.

As an approximation, the $\theta_i$'s, $\Pi_{ij}$'s and $\beta_{ij}$'s are treated as constants to be estimated. To obtain the specification used by Theil (1980b) and Duffy (1987, 1989, 1990), impose (5) on (6), setting $\beta_{ij} = -\Pi_{ij}\, \alpha_j$.

Next, the scaling and translating model specifications are considered. For scaling and translating, utility maximization problem (1) can be rewritten as

$$(7)\quad \text{maximize} \qquad u = u\left(q_1^*, \ldots, q_n^*\right)$$

$$\text{subject to} \qquad \sum p_i^* q_i^* = x^*$$

where $q_i^*$, $p_i^*$ and $x^*$ are quantity, price and income variables, respectively. For scaling,

$q_i^* = \phi_i q_i$, $p_i^* = \dfrac{q_i}{\phi_i}$, $x^* = x$, and $\phi_i = \phi_i(a_i)$; for translating, $q_i^* = q_i - \gamma_i, p_i^* = p_i$,

$x^* = x - \sum p_j \gamma_j$, and $\gamma_j = \gamma_j(a_i)$. Advertising is introduced through parameters $\phi_i$ and $\gamma_i$.

For scaling, $\phi_i$ can be viewed as a parameter indicating the consumer's perception of product quality. In this case, $q_i^*$ and $p_i^*$ can be viewed as the perceived quantity and price of good $i$, with advertising influencing perceptions.

For translating, $\gamma_i$ can be viewed as a needs or subsistence parameter as in the linear expenditure system (Stone; Phlips). In this case, $x^*$ is viewed as supernumerary income or excess income after needs are met. Advertising is assumed to influence perceptions of needs and hence the amount of supernumerary income to spend after perceived needs are met.

The general form of the demand equations satisfying (7) is

$$(8)\quad q_i^* = q_i^*\left(p_1^*, \ldots, p_n^*, x^*\right).$$

For scaling, (8) becomes

$$(9)\quad q_i^* = q_i^*\left(p_1^*, \ldots, p_n^*, x\right)$$

or

$$q_i = \frac{1}{\phi_i(a_i)} q_i^*\left(\frac{p_1}{\phi_1(a)}, \ldots, \frac{p_n}{\phi_n(a_n)}, x\right).$$

For translating, (8) becomes

$$(10)\quad q_i^* = q_i^*\left(p_1, \ldots, p_n, x*\right)$$

or

$$q_i = \gamma_i(a_i) + q_i^*\left(p_1, \ldots, p_n, x - \sum \gamma_j(a_j)\right).$$

Next, we want to specify versions of the Rotterdam model that approximate (9) and (10). First, the basic Rotterdam demand equation for scaling specification (9) is

$$(11)\quad w_i d\log q_i^* = \theta_i d\log Q^* + \sum \Pi_{ij} d\log p_j^*$$

where $d\log Q^* = \sum w_j d\log q_j^*$ (note $w_i^* = \dfrac{p_i^* q_i^*}{x} = \dfrac{p_i q_i}{x} = w_i$). Given the scaling definitions of $q_i^*$ and $p_j^*$, $d \log q_i^* = d \log q_i + d \log \phi_i$, $d \log p_j^* = d \log p_j - d \log \phi_i$, and $d\log Q^* = \sum w_j\left(d\log q_j + d\log\phi_j\right)$. Also, we can write $d\log\phi_i = \left(\dfrac{\partial\log\phi_i}{\partial\log a_i}\right) d\log a_i$. Hence, (11) can be written as

$$(12)\quad w_i\, d\log q_i = -w_i n_i\, d\log a_i + \theta_i\left(d\log Q + \sum_j w_j n_j\, d\log a_j\right) + \sum_j \Pi_{ij}\left(d\log p_j - n_j\, d\log a_j\right),$$

where $n_j = \dfrac{\partial \log \phi_i}{\partial \log a_i}$. Note that the last summation on the right-hand side (RHS) of (12) is the same as in the Theil Rotterdam model, except that $n_j$ and $\alpha_j$ (see equation (5)) are defined differently. However, the first term on the RHS of (12), $-w_i\, n_i\, d \log a_i$, and the associated adjustment in the Divisia volume index, $\sum w_j n_j\, d\log a_j$, are absent from the Theil specification. In (12), advertising can be viewed as working directly through the first of these two terms and indirectly through the second, as well as the term $-\sum_j \Pi_{ij} n_j\, d\log a_j$. The latter term involving the Slutsky coefficients shows that a change in advertising in the scaling model generates effects similar to price effects. For testing (12) against unrestricted model (6), we see that $\beta_{ij} = \left(w_j\left(\theta_i - \Delta_{ij}\right) - \Pi_{ij}\right)n_j$ where $\Delta_{ij} = 1$ if $i = j$, else $\Delta_{ij} = 0$.

For translating specification (10), the Rotterdam model can be written as

$$(13)\quad w_i^{*}\, d\log q_i^{*} = \theta_i\, d\log Q^{*} + \sum \Pi_{ij}^{*}\, d\log p_j$$

where $w_i^* = \frac{p_i q_i^*}{x^*}$; $d \log q_i^* = \frac{d q_i - d \gamma_i}{q_i^*}$; $d \log Q^* = \sum_j w_j^* d \log q_j^*$; $\theta_i = p_i \frac{\partial q_i^*}{\partial x^*} =$

$p_i \frac{\partial q_i}{\partial x}$; and $\Pi_{ij}^* = \frac{p_i p_j}{x^*}$. Multiplying through by $\frac{x^*}{x}$ and using the latter definitions for

the starred terms, equation (13) can be written as

$$(14) \quad w_i d \log q_i = m_i d \log a_i + \theta_i \left( d \log Q - \sum_j m_j d \log a_j \right) + \sum_j \Pi_{ij} d \log p_j,$$

where use has been made of the relationship $\frac{p_i \gamma_i}{x} d \log \gamma_i = \frac{p_i \gamma_i}{x} \frac{\partial \log \gamma_i}{\partial \log a_i} d \log a_i = m_i d \log a_i$,

$m_i = \frac{p_i \gamma_i}{x} \frac{\partial \log \gamma_i}{\partial \log a_i}$. The first term on the RHS of (14) is the direct effect of advertising

while the adjustment involving advertisement in the second term is an indirect effect similar to an income effect. For testing (14) against unrestricted model (6), we see that $\beta_{ij} = (\Delta_{ij} - \theta_i) m_j$.

We are interested in testing the alternative advertising specifications against the unrestricted model. An asymptotic chi-square test--Wald test--was used. Unrestricted model (6) was estimated by the full information maximum likelihood method, providing coefficient estimates $\hat{\theta}_i$, $\hat{\Pi}_{ij}$, and $\hat{\beta}_{ij}$, as well as the coefficient covariance matrix (the econometric software package TSP was used). The test for the alternative advertising specifications is similar to the proportionality test for demographic separability proposed by Deaton et al.

For each specification, the restrictions on the coefficients of (6), required for the specification to be accepted, can be written as

$$(15)\quad \beta_{ij} = f_{ij}\beta_j$$

or

$$\beta_j = g_{ij}$$

$$g_{ij} = \frac{\beta_{ij}}{f_{ij}}$$

where $f_{ij} = -\Pi_{ij}$, $\beta_j = \alpha_j$ for the Theil model;

$f_{ij} = w_j(\theta_i - \Delta_{ij}) - \Pi_{ij}$, $\beta_j = n_j$ for the scaling model;

$f_{ij} = (\Delta_{ij} - \theta_i)$, $\beta_j = m_j$ for the translation model.

Like parameters $\theta_i$, $\Pi_{ij}$ and $\beta_{ij}$, $\beta_j$ is treated as a constant proportionality factor, and the test is made at the mean budget share values for scaling. Note that $g_{ij}$ is a function of coefficients that can be estimated from the unrestricted model.

To test (15), we hypothesize

$$(16)\quad H_0 : d_{ij} = g_{ij} - \sum_{i=1}^{n} \frac{g_{ij}}{n} = 0 \qquad i = 1, \ldots, n;\quad j = 1, \ldots, n.$$

For each good $i$, there is a value $g_{ij}$ which is a measure of the proportionality factor $\beta_j$, and the average proportionality factor $\beta_j$ over the $n$ goods is given by the last term on the RHS of (16). The test statistic for (16) can be described in more detail as follows. Stack $g_{ij}$, letting $g_j = \left(g_{ij}, \ldots, g_{nj}\right)'$ and $g = \left(g_1, \ldots, g_n\right)'$. Define $A_1 = I - \left(\frac{i i'}{n}\right)$, where $I$ is an $nxn$ identity matrix and $i$ is an $n$x1 unit vector; and $A = \begin{bmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_1 \end{bmatrix}$, an ($nxn$)x($nxn$) block diagonal matrix. In matrix notation, the null hypothesis can be written as $Ag = 0$. Let $b$ be the vector of estimated coefficients--the $\hat{\theta}_i$'s, $\hat{\Pi}_{ij}$'s and $\hat{\beta}_{ij}$'s--and $V(b)$ be the estimated covariance matrix for $b$. Given each $g_{ij}$ is a function of $b$, the covariance matrix for $Ag$ is estimated by $V(Ag) = A\left(\frac{\partial g}{\partial b}\right)' V(b)\left(\frac{\partial g}{\partial b}\right)A'$ where $\frac{\partial g}{\partial b} = \left(\frac{\partial g}{\partial \theta}, \frac{\partial g}{\partial \Pi_{11}}, \ldots\right)'$. The test statistic is then $A'g'\,V(Ag)^{-1}Ag$ and is distributed asymptotically as a chi-square variable with $n(n$-1) degrees of freedom under the null hypothesis ($g$ involves $n^2$ constraints but only $n(n$-1) independent ones as each $d_{ij}$ expresses $g_{ij}$ as a deviation from its mean over $i$).

## Application

The alternative model specifications were applied to the Nielsen grocery-store scanner data to estimate a system of demand equations for OJ, GJ and RJ. As the data add up by construction--income in the model is total per capita consumer expenditure on the three types of juice--the error covariance matrix is singular and the equation for RJ was excluded (Barten, 1969). The errors across equations were assumed to be contemporaneously correlated, and the full information maximum likelihood procedure (TSP) was used to estimate the alternative models with homogeneity and symmetry holding.

Estimates of the unrestricted model were obtained first, and Wald tests for the alternative advertising specifications were made. Each of the alternative specifications was tested against the unrestricted model. Six advertising variables--(1) A/B ads and (2) displays with ads, by type of juice--were included in each demand equation.[2] The unrestricted model had eighteen advertising coefficients, with twelve (for OJ and GJ) estimated directly and six (for RJ) estimated from the adding-up restriction. Each Wald statistic had twelve degrees of freedom--six degrees of freedom were lost in estimating the average proportionality factors ($\gamma_j$, $n_j$, and $m_j$ for the Theil, scaling and translation models, respectively) as previously discussed. The Wald test results are shown in Table 1 and indicate that, for this application, the scaling specification can probably be accepted, while the other two specifications should probably be rejected. The Theil specification had the largest Wald statistic,

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[2]As mentioned earlier, lagged advertising effects were not found to be significant in the present analysis. Both present and lagged advertising variables were included in the unrestricted model and a Wald test was made for the joint hypothesis that the coefficients on the lagged advertising variables were zero. The asymptotic chi-square statistic was 13.14 with twelve degrees of freedom (the model had 18 lagged advertising coefficients but only twelve were independent due to the adding-up constraint), indicating the lagged advertising variables were not significantly affecting juice demands.

indicating rejection of the specification at most reasonable levels of significance; the translation specification had a smaller Wald statistic, indicating the specification might be accepted if the test criterion is relatively weak (a Type I error larger than .10). The Wald statistic for the scaling model was relatively small, indicating the specification should be accepted at most reasonable levels of significance.

Further comparison of the alternative model specifications was made by estimating each specification separately, using the full information maximum likelihood procedure, and comparing the expenditure elasticities, uncompensated price elasticities and advertising elasticities for the different specifications. The elasticity estimates discussed are conditional, applying to the juice subsystem; for convenience, the word conditional is usually not repeated in subsequent discussion. Table 2a shows the alternative elasticity estimates while Table 2b shows the corresponding asymptotic t values. The expenditure elasticities are $e_i = \dfrac{\partial \log q_i}{\partial \log x}$; the uncompensated price elasticities are $e_{ij} = \dfrac{\partial \log q_i}{\partial \log p_j} = e_{ij}^{*} - w_j e_i$, where

$e_{ij}^{*} = \left( \dfrac{\partial \log q_i}{\partial \log p_j} \right) \Bigg|_{\bar{u}}$, i.e., the price elasticity with utility held constant; and the advertising

elasticities are $\dfrac{\partial \log q_i}{\partial \log a_j}$ where $a_j$ could be either the level of A/B advertising or display

activity with advertising. For the Rotterdam model, $\theta_i = w_i e_i$ and $\Pi_{ij} = w_i e_{ij}^{*}$; hence, $e_i$ is estimated by $\frac{\hat{\theta}_i}{w_i}$, while $e_{ij}$ is estimated by $\frac{\hat{\Pi}_{ij}}{w_i} - \frac{w_j \hat{\theta}_i}{w_i}$. The advertising elasticities were calculated as $\frac{\hat{\beta}_{ij}}{w_i}$ for the unrestricted model, $\frac{-\hat{\Pi}_{ij} \hat{\gamma}_j}{w_i}$ for the Theil model, $\frac{\left(\left(\hat{\theta}_i - \Delta_{ij}\right) w_j - \hat{\Pi}_{ij}\right) \hat{n}_j}{w_i}$ for the scaling model, and $\frac{\left(\Delta_{ij} - \hat{\theta}_{ij}\right) \hat{m}_j}{w_i}$ for the translation model. All elasticities were calculated at mean budget share values (.61, .05, and .34 for OJ, GJ, and RJ, respectively).

The expenditure elasticity estimates for all juices and all model specifications were positive and significant; i.e., significant to the extent the elasticity estimate is twice its asymptotic standard error estimate or its asymptotic t-statistic is greater than two. The values of the expenditure elasticities were relatively consistent for the different specifications--1.04 to 1.05 for OJ, .86 to .94 for GJ, and .92 to .93 for RJ.

The results indicate each type of juice is a normal good with respect to total juice expenditure (an increase in total juice expenditure results in an increase in demand for each type of juice). In addition, OJ is a superior good with respect to group expenditure (an increase in total juice expenditure results in an increase in the OJ budget share since the OJ expenditure elasticity exceeds unity).

For each model specification, the own-price elasticity estimates are negative and significant. The estimates are relatively consistent across specification, ranging from -1.15 to -1.20 for OJ; -1.20 to -1.45 for GJ; and -1.17 to -1.20 for RJ. The results indicate the conditional demand for each type of juice is elastic (a lower own price, all else constant, increases dollar sales and vice versa).

All cross-price elasticity estimates are either positive and significant, or not significantly different than zero (a positive value indicates a substitute relationship). The sizes of the cross-price elasticities are relatively small, ranging from 0 to .4. In general, the results are fairly consistent across model specifications; however, a larger number of elasticity estimates are significant in each of the three restricted models than in the unrestricted model--two out of six cross-price elasticity estimates were significant in the unrestricted model while four out of the six cross-price elasticity estimates were significant in each of the other specifications. Using a restricted model may give one undue confidence in the cross-price elasticities, although the Wald test suggests the scaling model may be acceptable.

A larger number of the advertising elasticity estimates were also significant for the restricted models compared to the unrestricted model--out of the 18 advertising elasticity estimates, eleven, eight, twelve and five were significant in the Theil, scaling, translation and unrestricted specifications. Based on these results, one would tend to have more confidence in the advertising estimates if the Theil or translation specification were used; the scaling specification is somewhat closer to the unrestricted model in terms of number of significant advertising estimates. Overall, the elasticities for advertising are quite small, ranging from -.02 to .03, the former being the cross elasticity for RJ displays with ads in the OJ equation and the latter being the own elasticity for displays with ads in the RJ equation.

The elasticity estimates for both types of OJ advertising (A/B ads and displays with ads) are insignificant in each of the three juice demand equations in each model; however, except for the GJ equation in the unrestricted model, the elasticity estimates have the expected sign--positive in the OJ equations and negative in the GJ and RJ equations (a negative sign for a cross elasticity suggests competition between types of juice). The insignificance of the OJ advertising elasticities may reflect competition among different brands of OJ--brand advertising, in this case, may be reallocating sales among brands, and not expanding the overall OJ category.

Both types of GJ advertising have significant, positive impacts on GJ demand, except in the unrestricted model where the A/B ad elasticity for the GJ equation, although positive, is insignificant. Both types of GJ advertising also impact the other juice demands negatively or insignificantly.

The strongest advertising results are for the two types of advertising on RJ. Each model specification shows RJ advertising having a significant, positive impact on RJ demand and a significant, negative impact on OJ demand; the impacts on GJ demand are either insignificant or negative and significant.

## Concluding Comments

Advertising can be introduced in systems of demand equations using alternative specifications, each appealing to some degree. In this paper, three alternative specifications were considered--Theil's specification where advertising affects demand through the consumer's marginal utility for each good, a scaling model where advertising affects demand through quality-type parameters, and a translation model where advertising affects demand through a needs-type parameter. In a study of a juice demand subsystem, the three specifications were tested against an unrestricted model using a Wald test. For this particular application, the Theil and translation specifications were rejected while the scaling specification was accepted. In the scaling model, advertising generates direct quality-type effects and indirect price effects; in contrast, in the Theil specification, advertising works entirely through the Slutsky price term; while, in the translation model, advertising works through the income term.

Comparison of demand elasticity estimates for the different specifications indicates a degree of robustness, as well as differences in signs and significance of some estimates. Generally, the income, own-price and own-advertising elasticities were fairly consistent across specifications. Most of the differences were related to the cross-price and cross-advertising elasticities. The restricted models suggested many of the cross-elasticities could be estimated with a moderate degree of precision; the unrestricted model, as well as the scaling model to a lesser degree, indicated many of the cross-elasticities could not be determined very precisely.

The results of this study are specific for the U.S. juice market, and other applications might find one of the other advertising specifications to be more appropriate. The tests suggested here are easy to apply and may be helpful in choosing a model.

Table 1. Wald tests for alternative advertising specifications against the unrestricted model.[a]

| Model | Chi-Square Statistic[b] | P-Value[c] |
|---|---|---|
| Theil | 22.304 | .034 |
| Scaling | 14.427 | .274 |
| Translating | 18.376 | .105 |

[a]Model (6).

[b]Twelve degrees of freedom for each model.

[c]Probability for a value greater than the estimated chi-square statistic.

Table 2a. Demand elasticities for alternative model specifications.[a]

| Product | Model[b] | Elasticity | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expen-diture | Price | | | A/B Ad | | | Display/Ad | | |
| | | | OJ | GJ | RJ | OJ | GJ | RJ | OJ | GJ | RJ |
| OJ | UN | 1.055* | -1.150* | 0.019 | 0.076 | 0.010 | -0.001 | -0.016* | 0.006 | -0.003 | -0.019* |
| | TH | 1.041* | -1.200* | 0.030* | 0.129* | 0.006 | -0.002* | -0.011* | 0.003 | -0.001* | -0.015* |
| | SC | 1.049* | -1.186* | 0.023* | 0.114* | 0.010 | -0.002* | -0.014* | 0.004 | -0.002* | -0.018* |
| | TR | 1.040* | -1.188* | 0.028* | 0.120* | 0.004 | -0.001* | -0.010* | 0.002 | -0.001* | -0.013* |
| GJ | UN | 0.860* | 0.344* | -1.450* | 0.246 | -0.024 | 0.023 | 0.012 | 0.005 | 0.019* | 0.009 |
| | TH | 0.941* | 0.421* | -1.292* | -0.070 | -0.011 | 0.025* | -0.006 | -0.005 | 0.021* | -0.008* |
| | SC | 0.918* | 0.358* | -1.261* | -0.015 | -0.018 | 0.025* | 0.002 | -0.008 | 0.021* | 0.002 |
| | TR | 0.943* | 0.397* | -1.353* | 0.013 | -0.006 | 0.025* | -0.009* | -0.004 | 0.021* | -0.012* |
| RJ | UN | 0.923* | 0.217* | 0.033 | -1.173* | -0.015 | -0.002 | 0.027* | -0.012 | 0.003 | 0.033* |
| | TH | 0.935* | 0.296* | -0.010 | -1.221* | -0.009 | -0.001* | 0.021* | -0.005 | -0.001* | 0.028* |
| | SC | 0.925* | 0.280* | -0.003 | -1.202* | -0.014 | 0.000 | 0.024* | -0.007 | 0.000 | 0.031* |
| | TR | 0.937* | 0.277* | 0.002 | -1.216* | -0.006 | -0.001* | 0.019* | -0.004 | -0.001* | 0.026* |

[a]At mean budget shares--.61 for OJ, .05 for GJ and .34 for RJ.

[b]UN, TH, SC and TR stand for the unrestricted model, Theil model, scaling model and translating model, respectively.

Table 2b. Asymptotic t-statistics for demand elasticities for alternative model specifications.[a]

| Product | Model[b] | Asymptotic t-Statistics | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Expen-diture | Price | | | A/B Ad | | | Display/Ad | | |
| | | | OJ | GJ | RJ | OJ | GJ | RJ | OJ | GJ | RJ |
| OJ | UN | 52.708 | -23.530 | 1.231 | 1.741 | 0.965 | -0.279 | -2.276 | 1.440 | -1.320 | -4.344 |
| | TH | 52.537 | -31.654 | 2.557 | 3.559 | 0.692 | -2.859 | -2.176 | 0.719 | -3.075 | -4.425 |
| | SC | 53.470 | -30.770 | 2.280 | 3.096 | 1.081 | -2.313 | -2.290 | 1.011 | -2.737 | -4.777 |
| | TR | 52.049 | -27.670 | 2.057 | 3.021 | 0.431 | -3.084 | -2.107 | 0.557 | -3.173 | -4.151 |
| GJ | UN | 15.128 | 2.084 | -9.378 | 1.452 | -1.184 | 1.883 | 0.647 | 0.439 | 2.589 | 0.640 |
| | TH | 20.211 | 3.124 | -8.939 | -0.745 | -0.695 | 2.943 | -1.745 | -0.707 | 3.301 | -2.212 |
| | SC | 19.068 | 3.114 | -9.613 | -0.413 | -1.089 | 2.662 | 0.404 | -0.942 | 3.327 | 0.415 |
| | TR | 20.546 | 2.545 | -8.920 | 0.096 | -0.432 | 3.087 | -2.103 | -0.557 | 3.174 | -4.017 |
| RJ | UN | 26.046 | 2.970 | 1.369 | -17.163 | -0.845 | -0.274 | 2.368 | -1.763 | 0.700 | 4.610 |
| | TH | 25.914 | 4.895 | -0.672 | -19.115 | -0.691 | -2.004 | 2.184 | -0.720 | -2.253 | 4.428 |
| | SC | 25.839 | 4.552 | -0.349 | -18.422 | -1.068 | 0.319 | 2.302 | -1.016 | 0.321 | 4.782 |
| | TR | 26.021 | 4.231 | 0.108 | -18.545 | -0.431 | -3.062 | 2.107 | -0.557 | -3.141 | 4.148 |

[a]Asymptotic t-statistic for corresponding elasticities in Table 2a.

[b]UN, TH, SC and TR stand for the unrestricted model, Theil model, scaling model and translating model, respectively.

## References


Barten, A.P. "The Systems of Consumer Demand Functions Approach: A Review." *Econometrica* 45(1977):23-51.

Barten, A.P. "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review* 1(1969):7-73.

Barten, A.P. "Consumer Demand Functions Under Conditions of Almost Additive Preferences." *Econometrica* 32(1964a):1-38.

Barten, A.P. "Family Composition, Prices and Expenditure Patterns," in P.E. Hart, G. Mills, and J.K. Whittaker (eds.), *Econometric Analysis for National Economic Planning*. London: Butterworth (1964b):277-292.

Brown, M. and J. Lee. "Theoretical Overview of Demand Systems Incorporating Advertising Effects." *Proceedings* from Northeast Conference on Commodity advertising and Promotion, Orlando, Florida, February 22-23, 1989, in press.

Deaton, A.S., J. Ruiz-Castillo, and D. Thomas. "The Influence of Household Composition on Household Expenditure Patterns: Theory and Spanish Evidence." *Journal of Political Economy* 97(1989):179-200.

Deaton and Muellbauer. *Economics and Consumer Behavior*. Cambridge, MA: Cambridge University Press. 1980.

Duffy, M. "Advertising and Alcoholic Drink Demand in the UK: Some Further Rotterdam Model Estimates." *International Journal of Advertising* 9(1990):247-257.

Duffy, M. "Measuring the Contribution of Advertising to Growth in Demand: An Econometric-Accounting Framework." *International Journal of Advertising* 8(1989):95-110.

Duffy, M. "Advertising and the Inter-product Distribution of Demand--A Rotterdam Model Approach." *European Economic Review* 31(1987):1051-1070.

Gorman, W.M. "Tricks with Utility Functions," in M.J. Artis and A.R. Nobay (eds.), *Essays in Economic Analysis*. Cambridge: Cambridge University Press (1976):211-243.

Nielsen Market Research. Scanner data prepared for the Florida Department of Citrus, Atlanta, Georgia.

Phlips, L. *Applied Consumer Analysis*. Amsterdam: North Holland Publishing Co. 1974.

Pollak, R.A. and T.J. Wales. "Demographic Variables in Demand Analysis." *Econometrica* 49(1981):1533-1551.

Pollak, R.A. and T.J. Wales. "Comparison of the Quadratic Expenditure System and Translog Demand Systems with Alternative Specifications of Demographic Effects." *Econometrica* 48(1980):595-611.

Selvanathan, E.A. "Advertising and Alcohol Demand in the UK: Further Results." *International Journal of Advertising* 8(1989a):181-188.

Selvanathan, E.A. "Advertising and Consumer Demand: A Differential Approach." *Economic Letters* 31(1989b):215-219.

Stigler, G.J. and G.S. Becker. "De Gustibus Non Est Disputandum." *American Economic Review* 67(1977):76-90.

Stone, R. "Linear Expenditure System and Demand Analysis: An Application to the Pattern of British Demand." *Economic Journal* 64(1954):511-527.

Theil, H. *The System-Wide Approach to Microeconomics*. University of Chicago Press. 1980a.

Theil, H. *System-Wide Explorations in International Economics, Input-Output Analysis, and Marketing Research*. Amsterdam: North Holland Publishing Co. 1980b.

Theil, H. *Theory and Measurement of Consumer Demand*, Vol. I, Vol. II. Amsterdam: North Holland Publishing Co. 1975, 1976.

Theil, H. "The Information Approach to Demand Analysis." *Econometrica* 33(1965):67-87.

U.S. Department of Commerce, Bureau of Census. "Population Estimates and Projections." Series P-25, various issues.