DEMAND RELATIONSHIPS AMONG FRESH FRUIT AND JUICES IN CANADA

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Among developed countries, Canada is one of the world's major markets for fresh fruit and juice consumption. During the past two decades, Canadian per capita fresh fruit consumption has exhibited a modest upward trend. In the 1980's, fresh fruit consumption averaged 61.4 kilograms per capita, an increase of 3.7 percent from the comparable average of the 1970's. Growth in Canadian fresh fruit consumption has been supported by increased noncitrus (such as apples and bananas) consumption. Per capita fresh citrus consumption declined slightly over the period. During the 1980's, fresh citrus consumption averaged 14.8 kilograms per capita, a decrease of 1.3 percent from the average of the 1970's. Fresh apple consumption increased 0.5 percent between the 1970's and 1980's, increasing from 11.8 kilograms per capita in the 1970's to 11.9 kilograms per capita in the 1980's. Banana consumption increased 11.7 percent, from 9.7 kilograms per capita in the 1970's to 10.8 kilograms per capita in the 1980's. Fresh noncitrus consumption increased 5.4 percent between the 1970's and 1980's, increasing from 44.2 kilograms per capita in the 1970's to 46.6 kilograms per capita in the 1980's.

During the last two decades, Canadian per capita juice consumption has also exhibited an upward trend. In the 1980's, juice consumption averaged 42.8 kilograms (fresh equivalent) per capita, an increase of 41.9 percent from the average of 30.2 kilograms per capita during the 1970's. Apple juice consumption increased 97.9 percent between the 1970's and 1980's, increasing from 4.7 kilograms per capita in the 1970's to 9.3 kilograms per capita in the 1980's. During the same period, orange juice consumption increased from 13.4 kilograms per capita to 21.5 kilograms per capita, an increase of 59.3 percent.
During the past decade, the Canadian apple industry has been facing increasing imports of both fresh apples and apple juice concentrate. The industry is currently examining the possibility of forming a national supply management marketing agency. With this agency, the Canadian apple industry would have the power to restrict imports of apples for fresh or processing purposes as well as the power to restrict production. In order to manage effectively apple supply situations in Canada, it is imperative to understand the demand interrelationships among fresh fruit and juices. The objective of this study is to estimate the demand relationships among selected fresh fruit and juices.

In order to study the demand relationships among fresh fruit and juices, the consumer’s allocation decisions are assumed to be hierarchal in structure, allowing attention to be focused on a conditional demand system for fresh fruit and juices. This strategy is useful as it restricts the number of estimated parameters and provides more precise parameter estimates when the restrictions can be accepted. The paper is organized as follows. First, the differential approach to the consumers’ allocation problem is briefly discussed. Both the weak separability and strong separability concepts are used to derive consumer demand restrictions to test for separability among fresh fruit and juices. Finally, Canadian data are used to carry out the test, and implications are discussed.

The Differential Approach

The Rotterdam model, due to Barten (1964) and Theil (1965) takes the form

\[ w_i \ d \log q_i = \theta_i \ d \log Q + \sum_j \pi_{ij} \ d \log p_j \quad i = 1, 2, \ldots, n. \]  

Where \( w_i = (w_{it} + w_{it+1})/2 \) represents the average value share for commodity \( i \) with subscript \( t \) standing for time; \( d \log q_i = \log(q_{it}/q_{it-1}) \) is the log change in the consumption level for
commodity i; and dlog \( p_i = \log(p_{it}/p_{it-1}) \) is the log change in the price for commodity i. The term dlog \( Q \) is an index number (Divisia volume index) for the change in real income and can be written as

\[
dlog Q = \sum w_i dlog q_i.
\]

The time subscript implied by the equations are omitted for convenience. The demand parameters \( \theta_i \) and \( \pi_{ij} \) are given by

\[
\begin{align*}
\theta_i &= p_i (\partial q_i / \partial m); \\
\pi_{ij} &= (p_i p_j / m) s_{ij}; \\
s_{ij} &= \partial q_i / \partial p_j + q_j \partial q_i / \partial m;
\end{align*}
\]

where \( m \) is total outlay or the budget and \( s_{ij} \) is the \((i,j)\)th element of the Slutsky substitution matrix. The parameter \( \theta_i \) is thus the marginal budget share for commodity i and \( \pi_{ij} \) is a compensated price effect. Note that there is no strong a priori reason that the \( \theta_i \)'s and \( \pi_{ij} \)'s should be held constant. The constraints of demand theory can be directly applied to the parameters of the Rotterdam model. In particular one has

\[
\begin{align*}
(4) & \quad \text{Adding-up} \quad \sum \theta_i = 1, \sum \pi_{ij} = 0; \\
(5) & \quad \text{Homogeneity} \quad \sum \pi_{ij} = 0; \\
(6) & \quad \text{Slutsky Symmetry} \quad \pi_{ij} = \pi_{ji}.
\end{align*}
\]

The Rotterdam model is a particular parameterization of system of differential demand equations, where the demand parameters \( \theta_i \)'s and \( \pi_{ij} \)'s are assumed to be constant. An alternative parameterization is based on Working's Engel model, i.e.,

\[
(7) \quad w_i = a_i + \beta_i \log m, \quad i=1,2,...,n.
\]

As the sum of the budget shares is unity, it follows from (7) that \( \sum a_i = 1 \) and \( \sum \beta_i = 0 \). To derive the marginal shares implied by Working's model, one multiplies (7) by \( m \) and then differentiates with respect to \( m \), which results in
(8) \[ \frac{\partial (q_{ij})}{\partial m} = \alpha_i + \beta_i (1 + \log m) \]
\[ = w_i + \beta_i. \]

Hence, under Working’s model the \( i \)th marginal share differs from the corresponding budget share by \( \beta_i \); as the budget share is not constant with respect to income, neither is the associated marginal share. The income elasticity corresponding to (8) is

(9) \[ \eta_i = 1 + \beta_i/w_i. \]

Expression (9) shows that a good with positive (negative) \( \beta_i \) is a luxury (necessity). As the budget share of a luxury increases with income, prices remaining constant, it follows from (9) that increasing income causes the \( \eta_i \) for such a good to fall toward 1. The income elasticity of a necessity also declines with increasing income under (9). Thus as the consumer becomes more affluent, all goods become less luxurious under Working’s model, which seems to be a plausible outcome.

Replacing \( \theta_i \) in (1) with (8) and rearranging terms slightly, one obtains

(10) \[ w_i (d\log q_i - d\log Q) = \beta_i d\log Q + \sum j \pi_{ij} d\log p_j, \]

where \( \beta_i \) and \( \pi_{ij} \) are constant coefficients (Keller and van Driel; Theil and Clements).

Equation (10) will be referred to as the CBS model following Keller and van Driel.

In general, for models (1) and (10), the income elasticity for each commodity group \( (\eta_i) \) can be derived by dividing the marginal budget share parameter by the corresponding budget share, and the compensated price elasticity estimates \( (\eta_{ij}) \) can be derived by dividing the Slutsky parameter by the budget share, i.e.,

(11) \textbf{Income Elasticity:} \quad \eta_i = \beta_i/w_i \quad \text{or} \quad \eta_i = \beta_i/w_i + 1; \]

\textbf{Compensated Price Elasticity:} \quad \eta_{ij} = \pi_{ij}/w_i.
In (11) both income and price elasticities are functions of expenditure shares. Note that even though (1) and (10) are not nested, they are special cases of a more general model (Barten, 1990)

\[ w_i \Delta q_i = d_i \Delta q_i + \sum_{j} \pi_{ij} \Delta p_j + \delta w_i \Delta q_s + \epsilon_i. \]

Model (12) collapses to the Rotterdam model when \( \delta = 0 \), and to the CBS model when \( \delta = 1 \). Therefore, the Rotterdam and the CBS models are nested in (12) and the restrictions leading to each model can be tested. The demand restrictions on the parameters for (12) are

- Adding-up \( \sum_i d_i = 1 - \delta \) and \( \sum_i \pi_{ij} = 0 \);
- Homogeneity \( \pi_{ij} = 0 \); and,
- Symmetry \( \pi_{ij} = \pi_{ji} \).

The income and compensated price elasticities for (12) are

\[
\begin{align*}
\text{Income Elasticity} & \quad \eta_i = \frac{d_i}{w_i} + \delta_i; \\
\text{Price Elasticity} & \quad \eta_{ij} = \frac{\pi_{ij}}{w_i}.
\end{align*}
\]

A comparison of (13) with (11) shows the additional flexibility offered by the general model. For example, with negative \( d_i \) and positive \( \delta_i \), a good can be a luxury for high values of \( w_i \) and an inferior good for sufficiently low values of \( w_i \). The reverse is true for positive \( d_i \) and negative \( \delta_i \).

The Data

The data used in this study were obtained from two sources. Per capita consumption, population, and per capita disposable income data were obtained from Agriculture Canada; and price information was obtained from Statistics Canada. Annual data from 1960 through
1987 were used. Table 1 shows the budget or expenditure shares for the different fruit product groups for the years studied. The average conditional expenditure shares for the period were 0.17, 0.04, 0.22, 0.12, 0.27, 0.09, and 0.09, for oranges, grapefruit, apples, bananas, orange juice, apple juice, and tomato juice, respectively. In general, over time the group budget shares have decreased for all four fresh fruits and tomato juice and increased for orange juice and apple juice. For example, the expenditure share for orange juice increased from 22.3% in 1960 to 35.5% in 1987 and the expenditure shares for fresh oranges decreased from 22.1% to 12.4% during the same period.

The foregoing products can be viewed as belonging to three groups — fresh fruits (oranges, grapefruit, apples, and bananas), fruit juices (orange juice and apple juice), and vegetable juice (tomato juice). Since these three groups of fruit and juices differ in form and are prepared and consumed in different ways, one suspects that the marginal utility between groups may be independent. In order to verify the independence of marginal utility among these three groups of fruit and juices, separability hypotheses are tested. If the independence hypothesis can not be rejected, then it will be imposed on the demand model.

The test for weak separability proposed in this study is based on Goldman and Uzawa's result. The necessary and sufficient condition for weak separability is that the off-diagonal term in the Slutsky substitution matrix is proportional to the income derivatives of the two separable goods. That is, if fresh fruit \( k \) and juice \( i \) are separable, then

\[
(14) \quad s_{ik} \propto \phi^{gi}(\partial q_i/\partial m)(\partial q_k/\partial m),
\]

for all \( i \in \text{group } g \) and \( k \in \text{group } h \);

where \( s_{ik} \) is the appropriate element in the Slutsky substitution matrix and \( \phi^{gi} \) is a factor of proportionality between group \( g \) and group \( h \). If one multiplies both sides of (14) by \( p_ip_j/\ell m \),
one has

(15) \[ \pi_{ik} = (p_i p_k / m) s_{ik} = \varphi^g \Theta_i \Theta_k. \]

The necessary and sufficient condition for strong separability is that the off-diagonal term in the Slutsky substitution matrix is proportional to the income derivatives of the two separable goods with the factor of proportionality independent of the groups. That is, if fresh fruit \( k \) and juice \( i \) are separable, then

(16) \[ s_{ik} = \varphi (\partial q_i / \partial m)(\partial q_k / \partial m), \quad \text{for all } i \in \text{group } g \text{ and } k \in \text{group } h; \]

where \( s_{ik} \) is the appropriate element in the Slutsky substitution matrix and \( \varphi \) is a factor of proportionality independent of groups \( g \) and \( h \). If one multiplies both sides of (16) by \( p_i p_k / m \), one has

(17) \[ \pi_{ik} = (p_i p_k / m) s_{ik} = \varphi^* \Theta_i \Theta_k, \]

where \( \varphi^* = \varphi / m \).

Note that nested models with either the symmetry or homogeneity conditions, or restriction (15) or (17) can be derived from (1), (10), and (12). Therefore, the likelihood ratio test (LRT) for the hypotheses that the set of equations represented by (15) or (17) for all \( i, j \in \text{group } g \) and \( k \in \text{group } h \) is (Amemiya, pp. 141-6)

(18) \[ \text{LRT} = 2[\log L(\theta^*) - \log L(\theta)], \]

where \( \theta \) is the vector of parameter estimates with either restrictions (15) or (17), \( \theta \) is the vector of parameter estimates without the restrictions, and \( \log L(.) \) is the log value of the likelihood function. Under the null hypothesis (15) or (17), the test statistic LRT has an asymptotic distribution of \( \chi^2(q) \) where \( q \) is the degrees of freedom (the difference between the number of parameters without restrictions and with restrictions).
The three groups of fresh fruit and juices — fresh fruit (oranges, grapefruit, apples, and bananas), fruit juices (orange juice and apple juice), and tomato juice — are analyzed using models (1), (10), and (12) and the following restrictions are tested:

Homogeneity: $\Sigma_j \pi_{ij} = 0$.

Homogeneity and Symmetry: $\Sigma_j \pi_{ij} = 0$ and $\pi_{ij} = \pi_{ji}$.

Weak Separability: $\Sigma_j \pi_{ij} = 0$, $\pi_{ij} = \pi_{ji}$, and $\pi_{ij} = \phi \theta_i \theta_j$ for all $i \neq g$, $j \neq h$, and $g \neq h$.

Strong Separability: $\Sigma_j \pi_{ij} = 0$, and $\pi_{ij} = \pi_{ji} = \phi \theta_i \theta_j$ for all $i \neq g$, $j \neq h$, and $g \neq h$.

The adding-up conditions $\Sigma_i \theta_i = 1$ and $\Sigma_i \pi_{ij} = 0$, can not be tested since the data add up by construction.

**Analysis and Results**

The models analyzed in this study were estimated by the maximum likelihood method. The likelihood values and their corresponding test statistics for each of the models are presented in Table 2. The first-order autocorrelation coefficient estimates for these models were not statistically different from zero indicating the autocorrelation was not a problem. The numbers in the first column are the likelihood values; the numbers in the second column are the values calculated from equation (18); the number in parentheses in the second column is the number of restrictions (the difference in the number of parameters in the free or unrestricted model and the restricted model) for each of the respective models; and the numbers in the last column are the corresponding table values of the $\chi^2$ statistic for the $\alpha = .10$ level of significance. As shown in Table 2, the hypotheses of homogeneity, symmetry and strong separability can not be rejected by all three systems. Note that the values of the likelihood function for the Rotterdam model and the general
model (12) are similar, i.e., an indication that the Rotterdam type income effect works better than those of the CBS type for this data set. Selected parameter estimates for the three systems with strong separability restrictions are presented in Table 3; to avoid overburdening the reader with results, only the income and own-price parameter estimates are presented and subsequent discussion centers on the results for the general model (12).

In general, all income parameter estimates are statistically different from zero at \( \alpha = .05 \) level of significance for the Rotterdam and General model; those for the CBS model are statistically different zero except for orange, apples, and orange juice. The results for the CBS model indicate that among the seven commodities studied apple juice is a luxury and grapefruit, bananas, and tomato juice are necessities; the estimates indicate orange, apple, and orange juice have unitary income elasticities. The own-price parameter estimates \((\pi_i)'s\) are negative and similar across the three models studied; most of the own-price parameter estimates are statistically different from zero at \( \alpha = .05 \) level except for some of the estimates for the juices.

Income and compensated own-price elasticity estimates for the alternative models (see equations (11) and (13)) calculated at sample expenditure share means and at expenditure shares for 1960 and 1987 are presented in Table 4. Generally speaking, the income elasticities for oranges, grapefruit, bananas, and tomato juice\(^1\) have increased and those for apples and fruit juices have decreased. Conditional income elasticities are useful when the analyst is interested in the effect of a change in the consumption volume of the commodity group on a conditional budget share. The​il showed that an increase in the demand for group g raises the associated conditional budget shares for those commodities
that have conditional income elasticities larger than unity. The conditional income elasticity estimates for 1987 indicate that given a one-percent increase in expenditures on fruit and juices by Canadians, expenditures on oranges and apples would increase by more than one percent; while expenditures on other fruits and juices would increase by less than one percent.

As shown in Table 4, compensated own-price elasticity estimates are all negative and all are less than .5 in absolute value except those for fresh grapefruit which averaged -0.5 during the study period. The compensated own-price elasticities presented in Table 4 indicate that during the study period, the demands for all fresh fruits except apples and tomato juice have become more price elastic and the demands for apples and juices have become less elastic. To evaluate the demand relationships among fruits and juices, demand parameter estimates for the general model (12) are presented in Table 5.

Results presented in Table 5 indicate that cross-price effects between fresh fruits and juices and those between fruit juices and tomato juice are insignificant. However, most cross-price effects between pairs of fresh fruits are statistically different from zero at \( \alpha = .05 \) level. Cross-price parameter estimates for fresh fruits indicate that oranges and grapefruit are complements; apples and oranges, apples and grapefruit, bananas and orange, and bananas and grapefruit are substitutes; and apples and bananas are neutral. Cross-price elasticity estimates calculated at sample expenditure share means and at expenditure shares for 1960 and 1987 for the General model are presented in Table 6. As shown in Table 6, all cross-price elasticities have increased over time except those between apple and oranges and between apples and grapefruit.
Concluding Remarks

The results of this study indicate that if Canadian consumers were to allocate larger portions of their budgets to the consumption of fresh fruit and juices, expenditure shares on oranges and apples would increase, with oranges benefitting the most. Furthermore, the results indicate that the own-price elasticities for apples and apple juice are smaller than unity; hence, provided the conditional elasticity estimates are similar in magnitude to the corresponding unconditional elasticities, an increase in price through supply management (either by restricting production or imports or both) would increase revenue to the Canadian apple industry. The study results also indicate that oranges and grapefruit are substitutes for apples; therefore, an increase in the price of fresh apples would increase the consumption of citrus and citrus imports would increase.
Footnotes

1Results presented in Table 2 indicate that the Rotterdam and General models are better than the CBS model; therefore, the declining income elasticities derived from the CBS parameter estimates for fresh grapefruit and tomato juice were discounted in this study.
Table 1  Budget shares, 1960 through 1987

<table>
<thead>
<tr>
<th>Year</th>
<th>Orange</th>
<th>Grapefruit</th>
<th>Apples</th>
<th>Bananas</th>
<th>Orange Juice</th>
<th>Apple Juice</th>
<th>Tomato Juice</th>
</tr>
</thead>
<tbody>
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<td>1960</td>
<td>0.2212</td>
<td>0.0427</td>
<td>0.1627</td>
<td>0.1916</td>
<td>0.2228</td>
<td>0.0555</td>
<td>0.1035</td>
</tr>
<tr>
<td>1961</td>
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<td>0.0496</td>
<td>0.1879</td>
<td>0.1785</td>
<td>0.2444</td>
<td>0.0465</td>
<td>0.1105</td>
</tr>
<tr>
<td>1962</td>
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<td>0.0414</td>
<td>0.2308</td>
<td>0.1486</td>
<td>0.2091</td>
<td>0.0566</td>
<td>0.1069</td>
</tr>
<tr>
<td>1963</td>
<td>0.1807</td>
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<td>0.2572</td>
<td>0.1408</td>
<td>0.2047</td>
<td>0.0725</td>
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</tr>
<tr>
<td>1964</td>
<td>0.1918</td>
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<td>0.1407</td>
<td>0.2203</td>
<td>0.0763</td>
<td>0.0980</td>
</tr>
<tr>
<td>1965</td>
<td>0.1976</td>
<td>0.0464</td>
<td>0.2439</td>
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<td>0.1819</td>
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<td>0.1080</td>
</tr>
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<td>1966</td>
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<td>0.1785</td>
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<td>1967</td>
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<td>0.2081</td>
<td>0.1530</td>
<td>0.1982</td>
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<td>0.1976</td>
<td>0.0652</td>
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<tr>
<td>1969</td>
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<td>0.2696</td>
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<td>0.0713</td>
</tr>
<tr>
<td>1970</td>
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<td>1971</td>
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<tr>
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<tr>
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<td>1986</td>
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<td>0.3553</td>
<td>0.1336</td>
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Average 0.1681 0.0422 0.2228 0.1210 0.2705 0.0885 0.0869
<table>
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<th>Model</th>
<th>Log of Likelihood Function</th>
<th>LRT (Eq. (18))</th>
<th>$\chi^2_{20}$</th>
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<tr>
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*Numbers in parentheses are degrees of freedom for the $\chi^2$ test.
### Table 3  Demand parameter estimates – with strong separability restrictions

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<th>CBS Model (10)</th>
<th>General Model (12)*</th>
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<td>$\pi_j$</td>
<td>$\beta_i$</td>
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<td>(0.0551)</td>
<td>(0.0216)</td>
<td>(0.0537)</td>
</tr>
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<td>(0.0155)</td>
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<td>Apples</td>
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<td>(0.0740)</td>
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<td>(0.0203)</td>
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<td>Orange Juice</td>
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<td>0.0426</td>
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<td>(0.0622)</td>
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<td>(0.0417)</td>
<td>(0.0179)</td>
<td>(0.0393)</td>
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* $\delta = -0.4357$ with a standard error of 0.3250.
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<td>Average</td>
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<td>Average</td>
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<td>1.3667</td>
<td>1.7971</td>
<td>-0.1975</td>
<td>-0.2675</td>
<td>-0.3517</td>
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<td>0.0297</td>
<td>0.0365</td>
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<td>-0.6167</td>
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<td>-0.2768</td>
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<td>-0.2129</td>
</tr>
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<td>0.6238</td>
<td>-0.0550</td>
<td>-0.0688</td>
<td>-0.1068</td>
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<td>1.1454</td>
<td>1.1636</td>
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<td>Apple J.</td>
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Table 4 Income and compensated own-price elasticity estimates

Rotterdam Model (1)

CBS Model (10)

General Model (12)
Table 5  Demand parameter estimates for the general model (12)

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<td></td>
<td>Orange</td>
<td>Grapefruit</td>
<td>Apples</td>
<td>Bananas</td>
<td>Orange Juice</td>
<td>Apple Juice</td>
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<td>(0.0250)</td>
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<td>(0.0282)</td>
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<td>(0.0028)</td>
<td>(0.0011)</td>
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*Numbers in parentheses are standard errors of estimates.
Table 6  Compensated cross-price elasticity estimates for fresh fruits

<table>
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<th>Grapefruit</th>
<th>Apples</th>
<th>Bananas</th>
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</tr>
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</table>

*Elasticity estimate is not statistically different from zero.
References


Statistic Canada, "Consumer Prices and Price Indexes," selected issues.

