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THE ALMOST IDEAL DEMAND SYSTEM APPROXIMATION AND SEPARABILITY

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Abstract

The paper focuses on separability in the almost ideal demand system (AIDS) in time series analysis. The AIDS approximation based on Stone's price index is specified in terms of the Rotterdam model and block additivity is imposed for illustrative purposes. A generalization to other types of separability is then considered. The model can be viewed as an extension to Theil et al.'s Working-PI time series model. In both the Theil et al. model and the AIDS model examined here, the marginal propensity to spend on a good is allowed to depend on the good's budget share. In the model in this paper, the Slutsky coefficients are further allowed to depend on the budget shares. The model provides a consistent way to analyze subgroup demand relationships.

Keywords: Almost ideal demand system, separability, Rotterdam model.

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The Almost Ideal Demand System Approximation and Separability

The almost ideal demand system (AIDS) proposed by Deaton and Muellbauer (1980a, b) has been a popular model, frequently used in empirical demand studies. The AIDS has several features which have contributed to its popularity. It is a flexible functional form, providing a local first order approximation to any true demand system; it is based on an explicit cost function, an attractive feature for welfare analysis; and it has a linear approximation which is easy to estimate. The latter feature has often been relied upon as the full model may be difficult to estimate. Despite the popularity of the AIDS, questions remain on how best to apply it. One particular question is how to provide for separability in the AIDS. As the consumer choice problem for the AIDS is specified in terms of a cost function, the model does not straightforwardly conform to the usual definitions of separability based on the direct utility function.[1] In the present paper, an approach to allow for separability in the AIDS is suggested for time series analysis. Block additivity is assumed for illustrative purposes; a generalization of the approach is also discussed.

Other studies have considered separability and two-stage budgeting in the AIDS. For example, Eales and Unnevehr, and Winters test for separability in the AIDS, while Heien and Wessells, and Haden specify two-stage budgeting processes with a first-stage share equation which allocates total expenditure to the group under consideration, and a second-stage AIDS which allocates group expenditure to goods in the group. In the Heien and Wessells, and Haden specifications, the group share equation is a function of price indices for goods in the group and goods outside the group, total expenditure, and other study specific variables. However, while the second-stage AIDS obeys the general properties of demand--adding-up, homogeneity and symmetry--the first-stage share equation is not homogeneous. The use of group price indices in the first stage may also be a concern--as shown by Gorman, for weak separability group price indices can only be consistently defined if the utility function is strongly separable into generalized Gorman polar forms, homothetically separable, or a combination. The latter preferences, however, may be unduly restrictive (Deaton and Muellbauer; Winters). Although Heien and Wessells, and Haden do not discuss underlying preferences, Gorman's results suggest care should be taken in using group price indices in two-stage budgeting specifications.

The present study demonstrates how the two-stage budgeting process can be modeled based on the AIDS specification. The approach taken is based on recent work by Theil et al., involving application of Working's model to extend the Rotterdam model. Theil et al.'s extension (the Working-PI times series model) makes the marginal propensity to spend on a good equal to the budget share for the good plus a constant.[2] The approach used here treats the AIDS, which itself can be viewed as an extension of the Working model, similarly in context of the Rotterdam model. The approximation developed makes the Slutsky coefficients as well as the marginal propensities to spend depend on the budget shares.

The paper is organized as follows. In the next section, the AIDS model is briefly reviewed in context of the Rotterdam framework. Then a separability model for the AIDS is suggested and discussed. An extension of the approach taken in this paper is suggested in the next section. In the final section, concluding comments are given.

AIDS-Rotterdam Model

The AIDS model can be written as

$$\omega_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \log p_j + \beta_i \log \frac{x}{p}, \quad i=1, \ldots, n, \tag{1}$$

where $\omega_i = p_i q_i / x$, the budget share for good i; p_i and q_i are the price and quantity of good i, respectively; $x = \sum_j p_j q_j$, total expenditure, or, loosely, income; and p is a price index defined by

$$\log p = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j .$$

Adding-up requires $\sum_j \alpha_j = 1$, $\sum_i \gamma_{ij} = 0$, and $\sum_i \beta_i = 0$; homogeneity requires $\sum_j \gamma_{ij} = 0$; and symmetry requires $\gamma_{ij} = \gamma_{ji}$. If prices tend to be collinear, p is usually approximated by Stone's price index p^* where $\log p* = \sum_j \omega_j \log p_j$.[3]

For analysis of separability, it is useful to consider the Rotterdam version of the AIDS. Given the definition of ω_i, one has

(2) $$\omega_i \, d \log q_i = \omega_i \, d \log \omega_i + \omega_i \, d \log x - \omega_i \, d \log p_i \, .$$

From (1), one obtains

(3) $$\omega_i \, d \log \omega_i = \sum_j \gamma_{ij} \, d \log p_j + \beta_i (d \log x - d \log p).$$

As suggested by Deaton and Muellbauer (1980a, b), one approximation for $d \log p$ is $\sum_j \omega_j \, d \log p_j$. Using this approximation, substitute (3) into (2) and add and subtract $\omega_i \sum_j d \log p_j$ to obtain the Rotterdam version of the AIDS[4]

(4) $$\omega_i \, d \log q_i = u_i \, d \log Q + \sum_j \Pi_{ij} \, d \log p_j \, ,$$

where

$$d \log Q = \sum_j \omega_j \, d \log q_i = d \log x - \sum \omega_j \, d \log p_j$$

$$u_i = \omega_i + \beta_i$$

$$\Pi_{ij} = -\omega_i \Delta_{ij} + \gamma_{ij} + \omega_i \omega_j \, , \qquad \begin{array}{l} \Delta_{ij} = 1 \;\; if \; j = i \\ \Delta_{ij} = 0 \;\; if \; j \neq i \, . \end{array}$$

In equation (4), u_i is the marginal propensity to spend on good i and Π_{ij} is the Slutsky coefficient, as can be verified by obtaining the income elasticities (e_i's) and price elasticities (e_{ij}'s) from (1) with p^* used in place of p and ω_i treated as a constant for approximation purposes, and using the relationships that $u_i = \omega_i e_i$ and $\Pi_{ij} = \omega_i e_{ij} + \omega_i \omega_j e_i$. Equation (4) shows that, in the AIDS approximation, the marginal propensity to spend on good i is equal to the budget share plus a constant, while the Slutsky coefficient is equal to the product of budget shares for the two goods in question plus a constant minus the budget share for the own price case.

Separability

A separability model for the AIDS approximation can be specified following a similar approach as used to define separability in the Rotterdam model (Theil). Block additivity, an assumption frequently made in applied analysis, is assumed.

In general, the Slutsky coefficient can be written as (Barten)

(5)
$$\Pi_{ij} = v_{ij} - \phi\, u_i\, u_j$$
$$v_{ij} = \left(\frac{p_i\, p_j}{x}\right) \lambda u^{ij}$$
$$\phi = 1/((\partial\lambda/\partial x)(x/\lambda))$$

where λ is the marginal utility of income and u^{ij} is the $i\,j^{\text{th}}$ element of the inverse of the Hessian matrix for the utility maximization problem.

The Working-PI time series model can be obtained by substituting (5) into (4) and treating v_{ij} as a constant while maintaining $u_i = \omega_i + \beta_i$.

Assuming adding-up and homogeneity and noting that Π_{ij} equals $(p_i p_j/x)s_{ij}$ where s_{ij} is the substitution effect, it is easy to show

(6)
$$\sum_i v_{ij} = \phi u_j$$
$$\sum_j v_{ij} = \phi u_i\ .$$

Under the separability assumption of block additivity, $u^{ij} = 0$ for i and j belonging to different groups, and one can derive a subgroup demand system for group R (Theil); i.e.,

$$\omega_i \, d \log q_i = u_i' \omega_R \, d \log Q_R + \sum_{j \in R} \Pi_{ij}^R \, d \log p_j \tag{7}$$

where

$$d \log Q_R = \sum_{j \in R} \frac{\omega_j}{\omega_R} \, d \log q_j$$

$$\omega_R = \sum_{j \in R} \omega_j$$

$$u_i' = \frac{u_i}{M_R}$$

$$M_R = \sum_{j \in R} u_j$$

$$\Pi_{ij}^R = v_{ij} - \frac{\phi u_i u_j}{M_R}$$

$$= \Pi_{ij} + \phi u_i u_j \left[1 - \frac{1}{M_R} \right].$$

The last expression for Π_{ij}^R is obtained by noting $v_{ij} = \Pi_{ij} + \phi \, u_i \, u_j$ based on (5).

Substituting u_i and Π_{ij} from (4) into (7) results in a subgroup demand system for the AIDS; i.e.,

$$\omega_i \, d \log q_i = \frac{\omega_i + \beta_i}{\sum_{j \in R} (\omega_j + \beta_j)} \omega_R \, d \log Q_R \tag{8}$$

$$+ \sum_{j \in R} \left[-\omega_i \, \Delta_{ij} + \gamma_{ij} + \omega_i \, \omega_j + \phi(\omega_i + \beta_i)(\omega_j + \beta_j) \left(1 - \frac{1}{\sum_{j \in R} (\omega_j + \beta_j)} \right) \right] d \log p_j \, .$$

In equation (8), homogeneity requires $\sum_{j \in R} \pi_{ij}^R = 0$ and symmetry requires $\gamma_{ij} = \gamma_{ji}$ as before. Adding-up is, of course, satisfied automatically.

The system can be completed by relating $\omega_R \, d \log Q_R$ to $d \log Q$ and prices inside and outside the group. Assume for simplicity that all goods outside group R comprise a composite commodity, good n; i.e., goods in group R are denoted by $i = 1, \ldots, n\text{-}1$ and the composite commodity is denoted by n. Summing equation (4) over all goods in group R and using (5), (6), $v_{in} = v_{ni} = 0$, and $u_n = 1 - M_R$, yields[5]

$$\omega_R \, d \log Q_R = M_R \, d \log Q + \phi \, M_R(1 - M_R)\left[\sum_{j=1}^{n-1} u_j' \, d \log p_j - d \log p_n\right]. \tag{9}$$

The complete AIDS under block additivity is then comprised of equations (8) and (9) and can be estimated treating $\omega_R \, d \log Q_R$ as endogenous.[6,7] To estimate the model, let $d \log z$ be measured by $\log(z_t/z_{t-1})$ where t indicates time and z stands for q_i, p_i, etc. The budget share variables ω_{it} can be measured by $\bar{\omega}_{i,t} = \frac{\omega_{i,t} + \omega_{i,t-1}}{2}$ and, as suggested by Theil, might be treated as exogenous, allowing the budget shares to appear on both the right- and left-hand sides of equations (8) and (9). However, with budget shares on both sides of the equations, endogeneity may be a problem. One approach to this problem is to use lagged budget shares on the left-hand side of the equations. Such an approach would be similar to the use of lagged budget shares to calculate Stone's price index for use in estimating the AIDS approximation.

Extensions

In this section, attention is focused on extending the Rotterdam model (4) to allow for other types of separability. In an empirical study of demand, different types of separability might be considered. As discussed by Pudney, three basic types of separability are: (1) direct separability based on the direct utility function; (2) indirect separability based on the indirect utility function; and (3) quasi separability based on the cost function. Block additivity, of course, is a special case of direct separability. The more general case of direct separability (weak separability or blockwise dependence) has been developed by Theil. The other two types of separability are considered here.

Letting Ri indicate good i in group R and Sj indicate good j in group S, the compensated price elasticities $e^*_{Ri,Sj}$'s for indirect and quasi separability can be written as

(10) Indirect: $e^*_{Ri,Sj} = \phi_{RS} \, \omega_{Sj} + e_{Sj} \, \omega_{Sj} + e_{Ri} \, \omega_{Sj} + \beta_{Ri,Sj}$

(11) Quasi: $e^*_{Ri,Sj} = \phi_{RS} \, \omega_{Sj} + \beta_{Ri,Sj}$

where ϕ_{RS} is a factor of proportionality defined for each type of separability. Except for $\beta_{Ri,Sj}$, details on deriving expressions (10) and (11) are provided by Pudney. For indirect separability,

$\beta_{Ri,Sj} = -\frac{1}{\lambda}\frac{\partial H}{\partial h_R}\frac{\partial^2 h_R}{\partial p_{Ri}\,\partial p_{Sj}} \cdot \frac{p_{Sj}}{q_{Ri}}$ if $R = S$, else $\beta_{Ri,Sj} = 0$; where $u = H\left[h_R\left[\frac{P_R}{x}\right], h_S\left[\frac{P_S}{x}\right], \ldots\right]$ and P_R and P_S are price vectors for groups R and S, respectively. For quasi separability, $\beta_{Ri,Sj} = \frac{\partial G}{\partial g_R}\frac{\partial^2 g_R}{\partial p_{Ri}\,\partial p_{Sj}} \cdot \frac{p_{Sj}}{q_{Ri}}$ if $R = S$, else $\beta_{Ri,Sj} = 0$; where $x = G\left(g_R(P_R, u), g_S(P_S, u), \ldots, u\right)$.

Given $\Pi_{Ri,Sj} = \omega_{Ri}\, e^{*}_{Ri,Sj}$, the Slutsky coefficients for indirect and quasi separability are

(12) Indirect: $\Pi_{Ri,Sj} = \phi_{RS}\,\omega_{Ri}\,\omega_{Sj} + \omega_{Ri}\,u_{Sj} + \omega_{Sj}\,u_{Ri} + \beta^{*}_{Ri,Sj}$

(13) Quasi: $\Pi_{Ri,Sj} = \phi_{RS}\,\omega_{Ri}\,\omega_{Sj} + \beta^{*}_{Ri,Sj}$

where $\beta^{*}_{Ri,Sj} = \omega_{Ri}\,\beta_{Ri,Sj}$.

Alternative versions of the Rotterdam model under the assumptions of indirect separability and quasi separability can be obtained by substituting (12) and (13) into (4), respectively; if one is prepared to treat the β^{*}'s (or β's), u's and ϕ_{RS}'s as constants and the budget shares as exogenous, or use lagged budget shares, as suggested before, the different versions can be estimated straightforwardly.

Concluding Comments

The AIDS is often applied to a subgroup of commodities based on the assumption of separability. The demand estimates in such cases are thus conditional. To obtain unconditional estimates, separability assumptions need to be explicitly modeled. In this paper, an approximation to the AIDS is modeled under the assumption of block additivity. The modeling approach taken can be used to analyze other types of separability.

The AIDS specification suggested here is based on the Rotterdam model and can be thought of as an extension of the Working-PI time series model. The latter model makes the marginal propensity to spend on a good equal to the budget share for the good plus a constant. The AIDS specification in this paper maintains the latter definition for the marginal propensities while further making the Slutsky coefficients depend on the budget shares.

The AIDS-Rotterdam model offers a straightforward way to tie subgroup expenditure to total expenditure and prices. The latter may be important in analyzing demand responses for a subgroup, particularly where subgroup expenditure is sensitive to changes in the system.

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Footnotes

1. The cost function for the AIDS has the form log $c = a(P) + u\, b(P)$ where a and b are functions of prices P and u is utility. The indirect utility function for the AIDS is then $u = (\log x - a(P))/b(P)$ where c has been set to total expenditure x. For the AIDS specification, the indirect utility function can not be expressed as a weakly separable function $u = u(u_1(Q_1), \ldots, u_n(Q_n)) = u(\Psi_1(x_1, P_1), \ldots, \Psi_n(x_n, P_n))$ where u_i and Ψ_i are group direct and indirect utility functions, respectively; Q_i and P_i are price and quantity vectors, respectively, for group i; and x_i is expenditure on group i.

2. The Working-PI model, which is the primary focus of attention in the Theil et al. study, involves an approach to introduce price variation across countries.

3. Use of Stone's price index may involve an endogeneity problem as the ω_j's are specified as endogenous. One approach to this problem in time series analysis is to use lagged budget shares in the index; e.g., Eales and Unnevehr. Regardless of the latter, one may object to using Stone's price index based on the observation that the index has the undesirable feature that its value can change when prices are constant if the budget shares change.

4. An inverse Rotterdam version for the AIDS approximation can be obtained in a similar fashion as shown by Barten and Bettendorf.

5. See Duffy for an application of the Rotterdam model based on specification (9).

6. For comparison purposes, a consistent two-stage AIDS budgeting process following Heien and Wessells', and Haden's approach can be defined as follows. Let $\gamma_{ij} = 0$ for j outside group R and sum (1) over all i in group R to find the group budget share

$$\omega_R = \alpha_R + \beta_R \log \frac{x}{p} ,$$

where $\alpha_R = \sum_{i \in R} \alpha_i$ and $\beta_R = \sum_{i \in R} \beta_i$. The term $\sum_{i \in R} \sum_{j \in R} \gamma_{ij} \log p_j = 0$, assuming adding-up; i.e., $\sum_{i \in R} \gamma_{ij} = 0$.

In context of the AIDS approximation equation (4), setting $\gamma_{ij} = 0$ would be approximately equivalent to setting $\Pi_{ij} = 0$, provided ω_i and ω_j are small.

Solving the group budget share for log x/p and substituting the result into (1) yields

$$\omega_i = \left[\alpha_i - \frac{\beta_i}{\beta_R} \alpha_R \right] + \sum_{j \in R} \gamma_{ij} \log p_j + \frac{\beta_i}{\beta_R} \omega_R .$$

Equations for ω_R and ω_i define consistent group and subgroup demand equations. Note the restrictive nature of setting $\gamma_{ij} = 0$; namely, changes in prices only indirectly affect the overall budget share ω_R through real income x/p. The extreme case where all $\gamma_{ij} = 0$ and $\omega_i = \alpha_i + \beta_i \log x/p$ is very likely to be too restrictive as found by Deaton and Muellbauer (1980a).

7. In Heien and Wessells', and Haden's applications, group expenditure in the AIDS equations is treated as exogenous. The specification of ω_R as endogenous, however, suggests that group expenditure should also be endogenous.